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Design of Robust Near-Field Multi-Beam Forming Based on Improved LCMV Algorithm

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Abstract. *Most existing approaches for multi-beam forming use algorithm in sub-array respectively and switch the direction of beam to realize multi-area coverage. In this paper, we develop a method for multi-beam based on the traditional algorithm LCMV (Linear constrained minimum variance) which can produce multi-beam simultaneously out of sub-array. Although LCMV is the most common existing method to overcome the problem of uncertainty in the signal look direction, its performance in near-field multi-beam can not realize what we desired. So, we improve this algorithm to provide robustness against steering vector error and interference. This improved LCMV algorithm upon an uncertain set for near-field multi-beam is presented in this paper. This algorithm can accurately distinguish and effectively receive desired signals of multiple directions in complex environment without changing the structure of array. And because the beampattern null of adaptive beamfroming is very narrow in interference position, the performance degradation of interference suppression occurs as a result of array nonstationarities, weight lags or interference disturbance. For that, direct constraint direction vector is proposed in this paper to broaden nulls to a certain range of Direction of Arrival (DOA) of interference keeping the required width of nulls. Through simulation results, we can see that the improved algorithm can direct its main beams toward actual signal directions while putting broadened nulls to the interference directions in beampattern, the proposed algorithm can be expected to provide sufficient robustness improvements.*
Keywords: LCMV; Multi-beam forming; Near-field; null broadening; robustness.

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1. **Introduction.** As array signal processing is widely used in radar, communication, radio astronomy and medical imaging and other areas. Beamforming as an essential part of array signal processing is becoming more and more important in modern society. Meanwhile, with increasing development of electronic information technology, the electromagnetic environment became more and more complicated than ever before, so how the multiple target signals are effectively distinguished, identified and monitored in complex environment is being a severe test of modern sonar, radar, and all kinds of reconnaissance system. Now, the main technology which simultaneously obtains wide airspace coverage and high sensitivity is multi-beam forming technique [1]. Thus multi-beam forming technology is being a hot research point, and the studies are becoming necessary. Most existing methods for multi-beam forming ,by getting beam forming of the sub-array respectively or switch the direction of beam according to a certain frequency to realize multi-area coverage[2]. In array signal processing, near-field application is more difficulty than traditional far-field methods [3]. However in on-board system, telephone

conference or communication system of a small confined space, if we still take the far-field assumption, the output performance of beamforming would drop dramatically.

[4]-[6] research multi-beam forming technology based on far-field, including pointing optimization of multi-beam, the method of multi-beam based on NLMS which makes array antenna produce multi-beam simultaneously out of sub-array, and application of digital multi-beam phased array antenna are all in the condition of far-field studied in those papers. [7] discusses the test of multi-beam formation technologies in far-field and near-field, it proposes the near-field is different from far-field in phase and energy.[8]-[10] present a closed approximate solution of diagonal loading factor in near-field, it greatly reduces the amount of calculation, while the performance of multi-beam using this method remains to be discussed. LCMV (Linearly Constrained Minimum Variance) which is based on MVDR adds constraints to restrain signals from different directions and solve the minimum of output power, and it improves robustness by adding constrains. While LCMV is very sensitive to the error of expect direction signals [11]. In [12], it puts forward a method of pretreating target signal sources, and it codes the signal sources before processing. To some extent, by this method the robustness can be improved, but the main shortcoming is bit error and it makes the process of signal processing more complicated. In [13], a given normalized frequency bands of signal source is used to resolve constraint matrix and response vector. It can increases robustness, but in practical application the estimation of signal location may has error, so that system output can not meet requirements. While article [14] proposed near-field LCMV algorithm by region constraint, weight vector is decomposed into two orthogonal components. However this method requires the signals must be in a certain range of frequency band.

LCMV is known to be able to provide robustness against uncertainty in the signal look direction, while in near-field multi-beam beamforming the performance of this algorithm is not ideal[17] [18]. In [17] [18], the LCMV method has wider main-beam and beam pointing error, and the detailed information will be introduced in section 3-5. In this paper, we will focus on the robustness against steering vector and nulls, we restrains the constraints of directional constraints LCMV by an uncertain set, and this improved LCMV can direct its main beams toward actual signal direction. Adaptive beam former is well known for narrow nulls, so when weight has lags or interference is disturbed, the performance of interference suppression degrades significantly. So in this paper we change the steering vector near interference position to achieve null broadening. Simulation results show that this algorithm provide sufficient robustness improvements against nulls and direction vector error.

The paper is organized as follows. In section 2, we give the near-field signal model which is different from far-field signal model. In section 3 we discuss traditional LCMV method restrained by directional constraints. In section 4, we expound multi-beam LCMV, there are two parts in this section, on the one hand, a modified LCMV based on uncertain set is proposed, on the other hand, a new method of null broadening is proposed. Section 5 is simulation. In section 6, we describes conclusion of this paper.

2. **Near-field Signal Model.**

Signal waveform in near-field is spherical while in far-field is plane, so traditional signal expression can not be used in this case. By spherical wave equation, signal expression (1) in near-field is obtained [15][16].

$$x(r,t) = \frac{Q}{r} exp[j(\omega t - kr)] \tag{1}$$

Where Q is constant, ω is angular frequency of the signal, k is wave number, k=$2\pi/\lambda$. Expression (1) shows that not only does the distance between signal source and array influence phase but also it is in inverse proportion to signal amplitude.

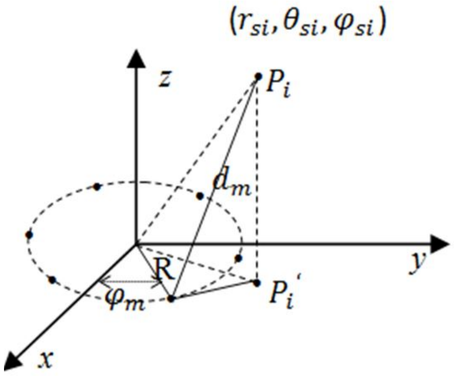

FIGURE 1. Near-field array

We assume that the array is a uniform circular array composed of M same omni-directional sensors, and the radius of array is R, as show in Figure.1. Signal P_i is located at an arbitrary position assumed as (r_{si},θ_{si},φ_{si}), in this paper we consider the array and signal source are in the same plane, i.e. θ_{si}=90°, then the distance between P_i and mth sensor is:

$$d_m = \sqrt{R^2 + r_{si}^2 - 2Rr_{si}cos(\varphi_{si} - \varphi_m)} \tag{2}$$

φ_m is the angle between mth element and coordinate axis x, then the mth element received signal is:

$$x_m(d_m, t) = \frac{Q}{d_m}exp[j(\omega t - kd_m)] = \frac{Q}{d_m}exp[-jkd_m] \cdot exp(j\omega t) = a_m s(t) \tag{3}$$

In (3), signal is divided into two parts, the part of a_m=$\frac{Q}{d_m}$exp[-jkd_m] is impacted by d_m, it can be seen that a_m influences the phase and amplitude of signal, while s(t)=exp(jωt) is irrelevant to distance and it has no effect on performance of algorithm.

So beampattern can be described as:

$$F(r_{si}, \theta_{si}, \varphi_{si}) = \mathbf{w}^H(r_{si}, \theta_{si}, \varphi_{si}) \cdot a_m \tag{4}$$

3. **Linearly Constrained Minimum Variance (LCMV).** LCMV is well known to increase robustness of beam former by adding constraints. To steering vector deviation, multiple directional constraints are used to broaden beam pointing near signal direction to ensure the beam former can receive target signal. So several distortionless constraints are added near the direction of assumed desired signal [17], and two constraints are added on φ_{si} like this:

$$\begin{cases} \mathbf{w}^H \boldsymbol{a}(\varphi_{si} + \Delta\varphi) = 1 \\ \mathbf{w}^H \boldsymbol{a}(\varphi_{si} - \Delta\varphi) = 1 \end{cases} \tag{5}$$

Where $\Delta\varphi$ is assumed error of directional angle. For single beam forming:

$$\begin{cases} min_{\mathbf{w}}\mathbf{w}^H\mathbf{R}\mathbf{w} \\ s.t.\mathbf{w}^H\mathbf{A} = \boldsymbol{f} \end{cases} \tag{6}$$

For single beamforming coming from φ_{si}, constraint matrix $\boldsymbol{A}$ is $\boldsymbol{A}$=[$\boldsymbol{a}(\varphi_{si})$,$\boldsymbol{a}(\varphi_{si}$-$\Delta\varphi)$,$\boldsymbol{a}(\varphi_{si}$+$\Delta\varphi)$], $\boldsymbol{f}$ is the response vector and $\boldsymbol{f}$=[1,1,1], then multiple target signals can have the same gain through beam former. This kind of amplitude constraint can also ensure the response of beam former in a range of angle [18]. In this way it improves the robustness against mismatch between presumed and actual direction of signal.

4. **Multi-beam LCMV.** In multi-beam forming, we divide spatial domain into several different subspaces based on single beam forming model, using the method of single beam forming within each subspace, then superimposing the constraints of every different spatial domain, this is valid on receiving multiple target signals. This approach which is called sub-space is simpler comparing with common sub-array method. Suppose that there are N target signals which come from $\varphi_{s1}\varphi_{s2},\ldots,\varphi_{sN}$, so the steering vector is that $\boldsymbol{a}$=[$\boldsymbol{a}_1$,$\boldsymbol{a}_2$,$\boldsymbol{a}_3$,$\ldots$,$\boldsymbol{a}_N$], where $\boldsymbol{a}_1$=$\boldsymbol{a}(\varphi_{s1})$,$\boldsymbol{a}_2$=$\boldsymbol{a}(\varphi_{s2})$,$\ldots$,$\boldsymbol{a}_N$=$\boldsymbol{a}(\varphi_{sN})$, multi-beam LCMV expression based on direction constraint is:

$$\begin{cases} min_{\mathbf{w}}\mathbf{w}^H\mathbf{R}\mathbf{w} \\ s.t.\mathbf{w}^H\mathbf{C} = \boldsymbol{g} \end{cases} \tag{7}$$

Where $\mathbf{C}$=[$\boldsymbol{a}$,$\boldsymbol{a}'$,$\boldsymbol{a}''$], $\boldsymbol{a}'$=[$\boldsymbol{a}(\varphi_{s1}$-$\Delta\varphi)$,$\boldsymbol{a}(\varphi_{s2}$-$\Delta\varphi)$,$\ldots$,$\boldsymbol{a}(\varphi_{sN}$-$\Delta\varphi)$], $\boldsymbol{a}''$=[$\boldsymbol{a}(\varphi_{s1}$+$\Delta\varphi)$,$\boldsymbol{a}(\varphi_{s2}$+$\Delta\varphi)$,$\ldots$,$\boldsymbol{a}(\varphi_{sN}$+$\Delta\varphi)$], $\boldsymbol{g}$ is 1×3N array response vector. However from the simulation results (Figure.2 and Figure.3), it can be seen that the beam-pointing extension is wider than 2*$\Delta\varphi$ which is we desired, so signals we received are composed of target signals and any others which are not desired signals in the region larger than 2*$\Delta\varphi$. This affects the robust performance of beam former seriously. For solving this problem, we apply traditional LCMV restrained by directional constraints to the whole steering vector, that is, restrain the error bound of the whole direction vector instead of the constraints of angle. While these multiple distortionless response conditions reduce the degree of freedom of adaptive sensor array, so it significantly depresses the ability to eliminate noise and interference signal, it causes a very high sidelobe value. Besides, the beampattern has a deviation and the results are still not ideal (as Figure.2 and Figure.3 show).

4.1. **Modified LCMV based an uncertain Set.** Considering the problems proposed in last section, we present that the direction extension vector is bounded by ellipsoidal restriction set in this section. Suppose that $\tilde{\boldsymbol{a}}$ is a bounded uncertain set which is $\{\tilde{\boldsymbol{a}} | \|\tilde{\boldsymbol{a}}\| \leq \varepsilon\}$ instead of a certain value, where ε is a nonnegative constant. Then for each subspace when the entire spatial domain is divided into N different subspaces there is:

$$\begin{cases} \mathbf{w}^H\boldsymbol{a}(\varphi_{si}) = 1 \\ \mathbf{w}^H[\boldsymbol{a}(\varphi_{si}) - \tilde{\boldsymbol{a}}] = 1 \\ \mathbf{w}^H[\boldsymbol{a}(\varphi_{si}) + \tilde{\boldsymbol{a}}] = 1 \end{cases} \tag{8}$$

Then we can get

$$\begin{cases} \mathbf{w}^H[\boldsymbol{a}(\varphi_{si}) + \tilde{\boldsymbol{a}}] = 1 \\ \|\tilde{\boldsymbol{a}}\| \leq \varepsilon \end{cases} \tag{9}$$

Then beamforming expression is:

$$\begin{cases} min_{\mathbf{w}}\mathbf{w}^H\mathbf{R}\mathbf{w} \\ s.t.\mathbf{w}^H[\boldsymbol{a}(\varphi_{si}) + \tilde{\boldsymbol{a}}], \|\tilde{\boldsymbol{a}}\| \leq \varepsilon \end{cases} \tag{10}$$

We assume $\mathbf{E}(\varepsilon)$=$\{\boldsymbol{s}|\boldsymbol{s}$=$\boldsymbol{a}(\varphi_{si})$+$\boldsymbol{e}$,$\|\boldsymbol{e}\|$≤$\varepsilon\}$, where $\boldsymbol{a}$=$\boldsymbol{a}(\varphi_{si})$+$\boldsymbol{e}$, when $\boldsymbol{e}$ is equal to $\tilde{\boldsymbol{a}}$, the steering vector of actual coming signal must be any member of the spheroid constraint set. In order to improve robustness, a constraint is imposed on the all steering vectors belong to $\mathbf{E}(\varepsilon)$.

$$|\mathbf{w}^H \boldsymbol{s}| \geq 1, \forall \boldsymbol{s} \in \mathbf{E}(\varepsilon) \tag{11}$$

(11) shows that any value of spheroid constraint set $\mathbf{E}(\varepsilon)$ can satisfy (11), when the minimal response value of $|\mathbf{w}^H \boldsymbol{s}|$ is not less than 1. Now above question can be interpreted in terms of worst case performance optimization [19]. Expression can be rewritten as (12)

$$\begin{cases} min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ s.t. min_{\boldsymbol{s}} |\mathbf{w}^H \boldsymbol{s}| \geq 1, \forall \boldsymbol{s} \in \mathbf{E}(\varepsilon) \end{cases} \tag{12}$$

Hence, beam former can ensure that the response is undistorted output. According to definition of $\mathbf{E}(\varepsilon)$, triangle inequality, Cauchy-Schwarz, and $\|\boldsymbol{e}\| \leq \varepsilon$, the (12) can be rewritten as (13).

$$\begin{cases} min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ s.t. |\mathbf{w}^H \boldsymbol{a}(\varphi_{st})| - \varepsilon \|\mathbf{w}\| \geq 1 \end{cases} \tag{13}$$

While from (13), we can see it isn not a convex problem, so solution must be very complex. Observing objective function will never change even if weight vector rotates any phase. Then (13) can be changed as (14).

$$\begin{cases} min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ s.t. |\mathbf{w}^H \boldsymbol{a}(\varphi_{st})| - \varepsilon \|\mathbf{w}\| \geq 1 \\ Im\{\mathbf{w}^H \boldsymbol{a}(\varphi_{st})\} = 0 \end{cases} \tag{14}$$

Expression (14) is convex, then second-order cone (SOC) programming and well-established interior point method can be used to solve this expression. Changing second order objective function into a simple liner function will simplify the solving process. In practical application received interference and noise generally mix with desired signal. So in this paper we uses sample autocorrelation matrix $\mathbf{R}$ to substitute interference signal and noisy cross-variance matrix $\mathbf{R}_{i+n}$. In order to simplify the solution process, we factorize the ample autocorrelation matrix $\mathbf{R}$ by Cholesky.

$$\mathbf{R} = \mathbf{L}^H \mathbf{L} \tag{15}$$

Second order function can be represented as:

$$\mathbf{w}^H \mathbf{R} \mathbf{w} = \mathbf{w}^H \mathbf{L}^H \mathbf{L} \mathbf{w} = (\mathbf{L}\mathbf{w})^H (\mathbf{L}\mathbf{w}) = \|\mathbf{L}\mathbf{w}\|^2 \tag{16}$$

Constraint of $\|\mathbf{L}\mathbf{w}\|$ is equal to $\mathbf{w}^H \mathbf{R} \mathbf{w}$, so the initial optimization problem is converted into convex optimization, and the essence of the optimization is to get a optimize value to meet the minimal output power of beam former. So we introduce a nonnegative constant τ, and $\|\mathbf{L}\mathbf{w}\| \leq \tau$ to solve the minimal value of function and to get a suitable value τ to obtain maximal output SINR. Then the expression of beam former converts into the following problem:

$$\begin{cases} min_{\mathbf{w},\tau} \|\mathbf{L}\mathbf{w}\| \leq \tau \\ s.t. |\mathbf{w}^H \boldsymbol{a}(\varphi_{st})| - \varepsilon \|\mathbf{w}\| \geq 1 \\ Im\{\mathbf{w}^H \boldsymbol{a}(\varphi_{st})\} = 0 \end{cases} \tag{17}$$

(17) is the subspace which look direction is φ_{si}. Multi-beam former superimposes several different subspaces and the number of constraints which are equal to the number of subspace. When there are N desired signals, spatial domain is divided into N subspaces,

and the corresponding expression of constraint is a constraint set which includes N inequality constraints and N equality constraints. This achieves the effective reception of each desired signals from N directions. In this way beam former shows better performance and robustness.

4.2. **Null Broadening.** From Figure.2 and Figure.3 we can see beampattern null is narrow and deep in strong interference position, this is one of the characteristics of adaptive beamforming algorithm. While in practical application adaptive weight may have hysteresis, there may be data mismatch when speed and real-time of beam former is restrained in batch processing, moreover the vibration of antenna platform and interference disturbance may also cause the position mismatch of interference direction and beampattern null. Mailloux [20] and Zatman [21] studied null-broadening and both of them proposed their solution. Both [22]-[24] rebuild new covariance matrix based on the mind of virtual interference signal source instead of the old one, however it needs much more computing time to get the matrix many times and influences the system processing speed greatly, in this paper we directly change the steering vector near interference signal, forms multiple null within the scope of Δ. Beampattern can be described as:

$$F = |\mathbf{w}^H \boldsymbol{a}| \tag{18}$$

The weight solved by Section 4.1 forms a narrow and deep null in the position of interference, if we want to achieve null broadening form multiple nulls within the scope of Δ, we can only change the steering vector $\boldsymbol{a}$ in Δ.

$$\boldsymbol{a}(\varphi_j + \delta) = \boldsymbol{a}(\varphi_j) \tag{19}$$

Where δ belongs to $[-\Delta/2,\Delta/2]$, φ_j is the direction of interference. From (18) we can see that when we change the steering vectors within scope of Δ near interference direction into $\boldsymbol{a}(\varphi_j)$, the beampattern will also change.

5. **Simulation.** In this section, we will test two sets of simulations to verify the robustness of the proposed modified LCMV method against steering vectors and nulls. For the simulation parameters, suppose that R=0.25,M=16 ,signal source is SNR=0dB,f=1700Hz, wave speed is c=340m/s, and $\Delta\varphi$=1.5, spatial noise is a Gaussian white noise with variance 1, P_1 and P_2 are the interference signals and there are P_1 ($-60°$,3.5m),P_2(70°,5m), INR=[30dB,20dB], frequency is fi=[1500,2000]Hz.

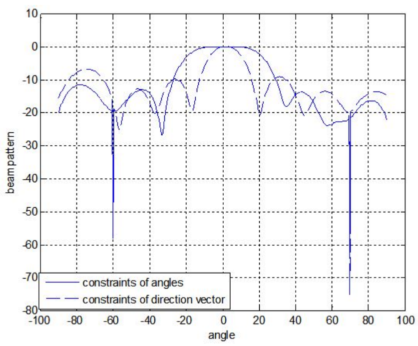

Figure 2. Single beam forming under two kinds of constraints

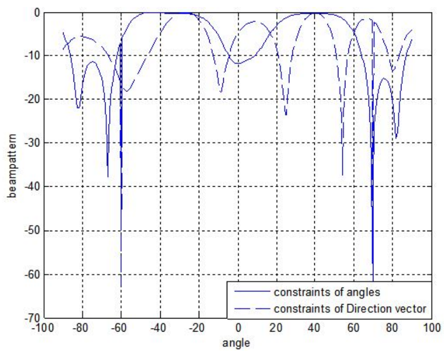

Figure 3. Multi-beam forming under two kinds of constraints

5.1. **Modified LCMV.** Figure.2 shows that signal source is (0°,4m), Figure.3 is (−30°,4m), (40°,4m). In Figure.2 the curve for constraints of angles is based on equation (5), and the curve for constraints of direction vector is based on equation (8). In Figure.3 the curve for constraints of angles is based on equation (6), and the curve for constraints of direction vector is based on equation (8). From Figure.2 and Figure.3, we can see the mainlobe of beampattern is broadened, and it is broadened about 20°, it is much more different than what we expect as described in Section 4. From Figure.3 we can see that beampattern has a deviation, so restraining the error bound of the whole direction vector is still not ideal as described in Section 4. Figure.4 shows that the modified LCMV based on an uncertain set has solved the problem, that is, the region of received signals is about 20° larger than we expect $2*\Delta\varphi=3°$ which is presented in Section 4. Comparing with the method proposed in Section 4 which just restrains by directional vector, from Figure.3 we can see it is a little distorted, while modified LCMV has a better robustness.

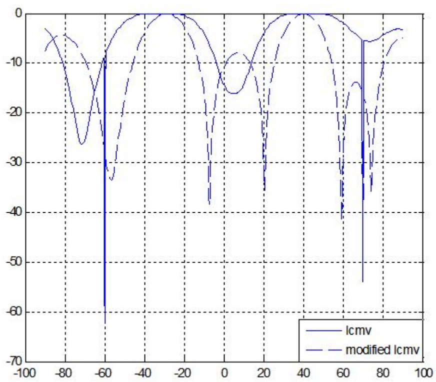

Figure 4. Modified LCMV

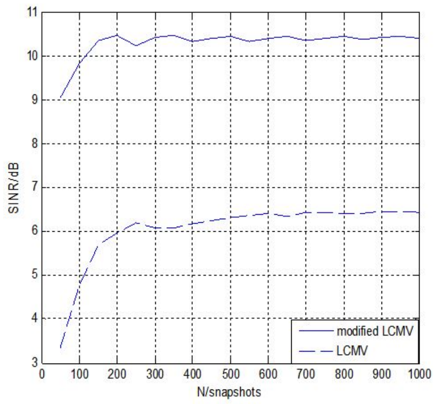

Figure 5. Output SINR

Figure.5 is the influence of different samplings on output SINR, we can see the performance of modified LCMV is better than traditional LCMV. From Figure.5 we can see that the output SINR curve is not smooth, this is because that uncertain set constrains multiple signal sources simultaneously to ensure weight vector meets two signals output without distortion. This figure shows that there are ups and downs in a small range, so it does not have too much effect on the system overall output performance. In general the algorithm has good output performance.

5.2. **Null Broadening.** We assume that null scope Δ is 8°, boundary value e is 1.5. As Figure.6 shows that when weight has hysteresis, and interference direction disturbance is within Δ, system can still restrain interferences effectively then receive target signals correctly. When there is error 2° of steering vector, that is to say actual direction of signal sources are −18° and 38°, the improved algorithm proposed in this paper can still achieve that beam former receives target signals correctly as Figure.7

From Figure.8 we can see the different boundary value e of an uncertain set affects output SINR of single beamforming in subspace. When e is smaller, output SINR raises with the increasing of e, there is a relatively stable state around 1.5, after the value 1.5 it sharply declines with the increasing of e. The reason of this condition is that the process of receiving signals may extract interference as desired signals. Though it expands the scope of deviation and improved the robustness, it declines the output performance, so in practical application we should change a suitable e to get a prefect output performance.

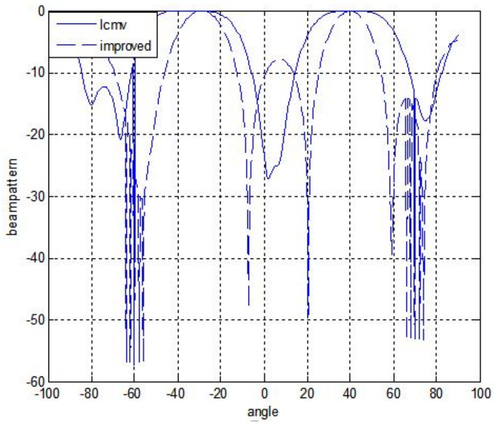

Figure 6. Improved LCMV comparing with directional constraints LCMV

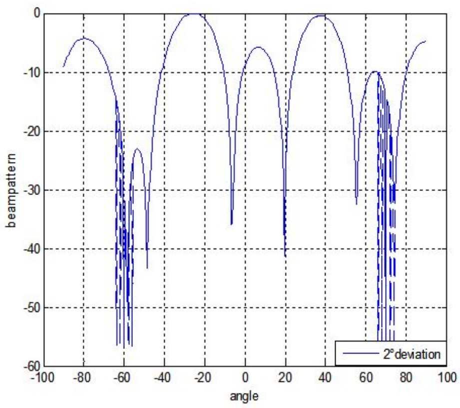

Figure 7. The actual and desired direction of signals have 2° deviation

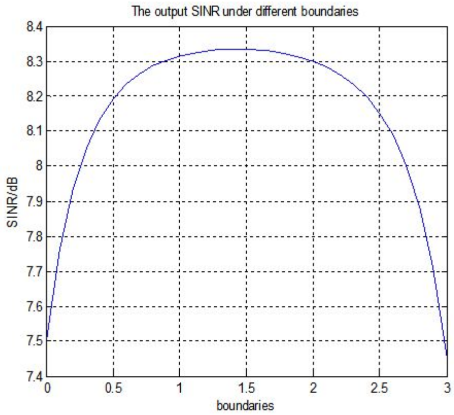

Figure 8. The output SINR under different boundaries

6. **Conclusion.** Conventional directional constraints LCMV is applied in near-field in this paper, although this method can receive desired signals when steering information is inaccurate, from Figure.2 and Figure.3 we can see the beam-pointing broadening is much wider than expected, at this point a lot of interference signals will be extracted as desired signals, and because of characteristic of adaptive beamforming algorithm, the null of beampattern is so narrow that when weight has hysteresis, ability of interference suppression may be lost. Modified algorithm proposed in this paper solves above problems effectively, and it has a good robustness. However, from the simulation we can see there still are a little high sidelobe, the beampattern is not the best. We should improve it in the future studies.

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