

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Letter of Clarification to the ISOLDE and Neutron Time-of-Flight  
Committee

(Following HIE-ISOLDE proposal CERN-INTC-2012-055/P-356)

Determination of the fission barrier height in fission of heavy  
radioactive beams induced by the (d,p)-transfer

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**Abstract:** A theoretical framework is described, allowing to determine the fission barrier height using the observed cross sections of fission induced by the (d,p)-transfer with accuracy, which is not achievable in another type of low-energy fission of neutron-deficient nuclei, the  $\beta$ -delayed fission. The proposed experiment [1] at the HIE-ISOLDE, using the ACTAR TPC, will thus provide the experimental information, which is not available at the moment and which is highly interesting for nuclear theory.



# Introduction

In October 2012, the proposal for experiment at HIE-ISOLDE CERN-INTC-2012-055/P-356 "(d,p)-transfer induced fission of heavy radioactive beams" [1] was submitted by us to INTC, following our Letter of Intent CERN-INTC-2010-026/I-095 "Transfer-induced fission of heavy radioactive beams", [2]. The decision of the INTC was negative with the statement that "it was however not clear how the fission barriers can be derived from the observed data". In this Letter of Clarification, we provide a detailed explanation of the procedure of extraction of fission barrier heights from the observed (d,pf)-transfer reactions of heavy RIBs, and also explain the advantages over extraction of fission barrier heights from our earlier data on  $\beta$ -delayed fission [3, 4].

## Probability of low energy fission

The probability of low energy fission can be determined using the expression:

$$P_{\text{LEf}} = \frac{\int_0^{E_{\text{max}}^*} W(E^*) \frac{\Gamma_f(E^*)}{\Gamma_f(E^*) + \Gamma_\gamma(E^*)} dE^*}{\int_0^{E_{\text{max}}^*} W(E^*) dE^*}, \quad (1)$$

where  $E^*$  is the excitation energy,  $W(E^*)$  is the probability of population of a given excited state,  $\Gamma_f$  is the fission width and  $\Gamma_\gamma$  is the  $\gamma$ -decay width. Typically, only the two most dominant channels, i.e. fission and  $\gamma$ -rays emission, are considered at sufficiently low excitation energies, such as e.g. in  $\beta$ -delayed fission, where the upper limit of excitation energy  $E_{\text{max}}^*$  is determined by the value of decay energy  $Q_{\text{EC}}$ . At such low excitation energies, emission of neutrons is hindered by the high neutron separation energy in the neutron-deficient nuclei. The emission of protons, which have a lower separation energy than neutrons in the neutron-deficient nuclei, is hindered because of the Coulomb barrier. The  $\beta$ -delayed fission of  $^{178,180}\text{Tl}$  was investigated recently at ISOLDE [3] and the fission barriers were determined using the experimental probabilities of  $\beta$ -delayed fission in the work [4], using the version of the formula (1) for  $\beta$ -delayed fission [5], with  $W(E^*) \propto F(Q_\beta - E^*)S_\beta(E^*)$  where  $F(Q_\beta - E^*)$  is the statistical Fermi function, and  $S_\beta(E^*)$  is the  $\beta$ -strength function. In the work [4], the measured probabilities of the  $\beta$ -delayed fission for  $^{178,180}\text{Tl}$  were used to deduce the fission-barrier heights of the daughter isotopes  $^{178,180}\text{Hg}$ , undergoing low-energy fission. Four alternative  $\beta$ -decay strength functions and four variants of the statistical model of de-excitation of the daughter nucleus were used to determine the fission-barrier height for  $^{180}\text{Hg}$ . Depending on the choice of the model, the deduced fission-barrier heights appeared to be between 10 and 40 % smaller than theoretical estimates. This observation was verified also for fission-barrier heights extracted using the probability of  $\beta$ -delayed fission of  $^{178}\text{Tl}$ . The spread in extracted fission-barrier heights resulted mainly from uncertainties in the magnitude of the pairing gap at the saddle configuration.

## Probability of fission in the (d,pf)-reaction

As described in the Letter of Intent [2] and Proposal [1], the use of the (d,pf)-reaction of the post-accelerated heavy RIB's, delivered by the HIE-ISOLDE with energies up to 5.5 AMeV, can overcome the limitations, faced in the study of  $\beta$ -delayed fission. Specifically, when using the RIBs of odd isotopes of odd elements (with even neutron number), one can observe fission of odd-odd nuclei and thus remove the uncertainties due to unknown pairing gap in the saddle configuration. Furthermore, the use of experimental setup with the active target TPC ACTAR [6] allows to measure the fission excitation function at once, and thus will allow to extract significantly more precise values of fission barrier height, since, understandably, the final value of fission barrier height will be much better constrained by multiple measured fission probabilities, obtained for different kinetic energies of the radioactive beam at different positions in the ACTAR TPC.

For determination of the fission barrier height in the (d,pf) reaction, the formula (1) needs to be made more specific by substituting the probability  $W(E^*)$  by the differential cross section of the (d,p) transfer reaction  $(d\sigma_{(d,p)}(E_{beam})/dE^*)$ , leading to heavy target-like residue with excitation energy  $E^*$ , at beam energy  $E_{beam}$ , corresponding to a given position in the ACTAR TPC. Then the formula (1) becomes

$$P_{LEf}(E_{beam}) = \frac{\int_0^{E_{max}^*} (d\sigma_{(d,p)}(E_{beam})/dE^*) \frac{\Gamma_f(E^*)}{\Gamma_f(E^*) + \Gamma_\gamma(E^*)} dE^*}{\int_0^{E_{max}^*} (d\sigma_{(d,p)}(E_{beam})/dE^*) dE^*}, \quad (2)$$

and the value of the fission barrier height can be determined after adopting proper approximations for  $\Gamma_f$  and  $\Gamma_\gamma$ . The values of  $(d\sigma_{(d,p)}(E_{beam})/dE^*)$  can be determined during the proposed experiment either directly from kinematics (as described later), if statistics will be sufficient, or otherwise, it can be sufficient to measure the inclusive cross section  $\sigma_{(d,p)}(E^*, E_{beam})$  which is equal to the expression in the denominator of the formula

(2)  $\sigma_{(d,p)}(E_{beam}) = \int_0^{E_{max}^*} (d\sigma_{(d,p)}(E_{beam})/dE^*) dE^*$ , and the differential cross section can be determined using the simple two-body kinematics of the reaction.

The fission width of the excited nucleus in the formula (2) can be expressed [7]

$$\Gamma_f(E^*) = \frac{1}{2\pi\rho_c(E^* - \Delta)} \int_0^{E^* - B_f - \Delta_{sp}} \rho_{sp}(E^* - B_f - \Delta_{sp} - E') dE', \quad (3)$$

where  $\rho_c(E^* - \Delta)$  and  $\rho_{sp}(E^* - B_f - \Delta_{sp} - E')$  are level densities of the excited intermediate nucleus after  $\beta$ -decay and of its deformed saddle configuration, respectively,  $B_f$  is the fission-barrier height,  $\Delta$  is the pairing gap causing a sharp cutoff in the level density of the daughter nucleus and  $\Delta_{sp}$  is the pairing gap of the fissioning nucleus in its saddle configuration (which is zero for odd-odd fissioning nucleus). The level density of the nucleus with excitation energy  $E^*$  can be expressed within the framework of the Fermi-gas model, with the level-density parameter  $a_n$  (the index n is introduced to distinguish it from the level density at the saddle configuration  $a_f$ ) taken according to the formula

of Ignatyuk [8], which takes into account the shell correction to the ground-state mass and its damping with excitation energy by the constant  $E_d = 18.5$  MeV, leading to the asymptotic value  $\tilde{a}_n$  at high excitation energy. For the saddle configuration, usually no shell correction is assumed and therefore the level density in the saddle configuration  $a_f$  is fixed at a constant value  $a_f = \tilde{a}_n$ . Alternatively, the level density formula of Gilbert and Cameron [9] can be used in both cases, with the shell correction set to zero in the case of saddle configuration.

The statistical model for the decay of a compound nucleus combines the expression for  $\Gamma_f$  given by Eq. (3) with Hauser-Feshbach expressions for the emission widths of neutron, proton and  $\alpha$  particles. In order to reproduce the observed evaporation-residue cross sections for neutron-deficient nuclei at excitation energies varying from 30 up to 150 MeV, a reduction of theoretical fission barriers by 20–30 % is necessary. Detailed analysis of dependence of  $\Gamma_f/\Gamma_n$  of neutron-deficient Ra nuclei as a function of excitation energy [10] established that the reduction of fission barrier using a fixed level-density parameter in the fission channel ( $a_f = \tilde{a}_n$ ) is preferred over the alternative possibility to enlarge  $a_f$  over  $\tilde{a}_n$  by 5 – 10 %. Moreover, compared to compound-nucleus fission,  $\beta$ -delayed fission occurs at low excitation energies and thus the eventual variation of  $a_f$  with respect to  $\tilde{a}_n$  practically does not influence the resulting fission probabilities.

The  $\gamma$ -decay width  $\Gamma_\gamma$  can be determined using the empirical formula of Stolovy and Harvey, see [11], which is based on experimental  $\gamma$ -width data of heavy nuclei and therefore can be considered as a reasonable estimate of the widths for the  $\gamma$ -emission at excitation energies up to 10 MeV. It was introduced as :

$$\Gamma_\gamma(E^*) = 0.00053 A^{2/3} D(E^* - \Delta)^{0.25} (E^* - \Delta)^{4.3} [\text{meV}], \quad (4)$$

where the level spacing  $D(E^* - \Delta)$  is given in units of eV, the excitation energy is given in MeV and the result is obtained in meV. The level spacing is defined as  $D(E^* - \Delta) = \rho^{-1}(E^* - \Delta)$ . Here,  $\rho(E^* - \Delta)$  can be calculated using the empirical formula of Gilbert and Cameron, see [9].

Thus, complementing formula (2) with the model framework presented above allows us to determine the fission-barrier height  $B_f$  from the experimental values of fission cross sections, measured in the (d,pf) reaction of heavy RIBs at HIE-ISOLDE.

As a complementary method, the capabilities of the ACTAR TPC can be exploited even further and the excitation energy  $E^*$  of the projectile-like residue can be determined directly from the observed reaction kinematics. For this, the events resulting from the (d,p) reaction can be used, and, using the known two-body kinematics of the reaction, the two measured angles can be used to estimate the excitation energy of the residue. Such a dependence of excitation energy on proton angle and beam energy can be applied as well to the (d,pf) reaction, since the fission occurs as a secondary decay of the target-like residue which can have no influence on the properties of proton emitted in the transfer reaction, which occurred earlier. Using the expected angular resolution of the ACTAR of  $1^\circ$  for proton tracks, and the laws of two-body kinematics, the expected uncertainty of the determined excitation energy of the projectile-like residue can be estimated as not exceeding 0.5 MeV. Such a value appears as sufficient for additional verification of the values obtained using the statistical description by this direct method.

## Conclusions

When combined, the methods, described above, provide a framework, allowing to determine the fission barrier height using the observed fission cross sections with accuracy, which is not achievable in another type of low-energy fission of neutron-deficient nuclei, the  $\beta$ -delayed fission. The proposed experiment [1] at the HIE-ISOLDE, using the ACTAR TPC, will thus provide the experimental information, which is not available at the moment and which is highly interesting for nuclear theory.

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