PARADOXES IN NUMERICAL CALCULATIONS

J. Brandts,* M. Křížek,† Z. Zhang‡

Abstract: When solving problems of mathematical physics using numerical methods we always encounter three basic types of errors: modeling error, discretization error, and round-off errors. In this survey, we present several pathological examples which may appear during numerical calculations. We will mostly concentrate on the influence of round-off errors.

Dedicated to Prof. Ivo Babuška on his 90th birthday

1. Introduction

The general computational scheme for solving problems of mathematical physics is sketched in Fig. 1. In general, three basic types of errors e_0, e_1, and e_2 are introduced.
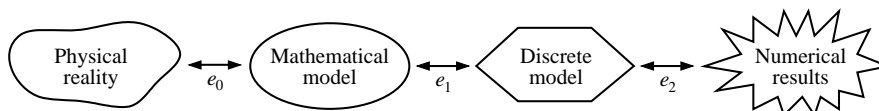

Fig. 1 *General computational scheme: The modeling error e_0 is the difference between physical reality and its mathematical model. The difference between the mathematical model and the discrete model is called the discretization error e_1. Finally, round-off errors are included in e_2.*

*Jan Brandts, Korteweg-de Vries Institute for Mathematics, University of Amsterdam, P.O. Box 92248, 1090 GE Amsterdam, The Netherlands, E-mail: janbrandts@gmail.com

†Michal Křížek – Corresponding author, Institute of Mathematics, Czech Academy of Sciences, Žitná 25, CZ-115 67 Prague 1, Czech Republic, E-mail: krizek@cesnet.cz

‡Zhimin Zhang, Beijing Computational Science Research Center, Beijing 100094, China, E-mail: zmzhang@csrc.ac.cn; Department of Mathematics, Wayne State University, 1131 Faculty/Administration Bldg., 656 West Kirby, Detroit, MI 48202, U.S.A., E-mail: zzhang@math.wayne.edu