James Stirling: Mathematician and Mine Manager

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Does your hometown have any mathematical tourist attractions such as statues, plaques, graves, the café where the famous conjecture was made, the desk where the famous initials are scratched, birthplaces, houses, or memorials? Have you encountered a mathematical sight on your travels? If so, we invite you to submit an essay to this column. Be sure to include a picture, a description of its mathematical significance, and either a map or directions so that others may follow in your tracks.

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Although James Stirling enjoyed a substantial reputation as a mathematician among his contemporaries in Britain and in some other European countries, he published remarkably little after his book *Methodus Differentialis: sive Tractatus de Summatione et Interpolatione Serierum Infinitarum* in 1730. An article with valuable information on the achievements of James Stirling during his time as mine manager in Scotland appeared in the *Glasgow Herald* of August 3, 1886, see [8]. The *Herald* article was reprinted in Mitchell's *The Old Glasgow Essays* of 1905 [7]. Detailed accounts of Stirling's mathematical achievements and correspondences with his contemporaries can be found in [12, 13], and [14].

The Stirlings and Their Estates

James Stirling was the third son of Archibald Stirling (March 21, 1651 - August 19, 1715) of the estate Garden (about 20 km west of Stirling) in the parish of Kippen, Stirlingshire, Scotland.

In 1180, during the reign of King William I of Scotland, a Stirling acquired the estate of Cawder in Lanarkshire, and it has been in the possession of the family ever since. Sir Archibald Stirling (1618 - 1668) was a conspicuous Royalist in the Civil War, and was heavily fined by Cromwell; but his loyalty was rewarded at the Second Restoration (1661), and he ascended the Scottish bench with the title of Lord Garden, a Lord of Session of some distinction in the reign of Charles II (1660 - 1685). On his death in 1668, Lord Garden left Garden to his second son Archibald, the mathematician's father, who became Laird of Garden in 1668 [14]. The family was a strong supporter of the Jacobite cause; this was to have a significant influence on the life of the mathematician James Stirling. (Jacobitism was the political movement dedicated to the restoration of the Stuart kings to the thrones of England, Scotland, and Ireland.) Figure 1 shows the memorial tablet to the Stirlings of Garden in Dunblane Cathedral, Perthshire. Sir Archibald Stirling, Lord Garden (mentioned above), and some of his descendants, were buried in this cathedral.

In January 1686 Archibald Stirling, the mathematician's father, married Anna, the eldest daughter of Sir Alexander Hamilton of Haggs, near Linlithgow. They had a family of four sons and five daughters. James, the mathematician, was the third son. He was born in the old Tower of Garden, previous to May 11, 1692, and was baptised on August 1 of that year [3].

James Stirling at Balliol

Little is known of James Stirling's early years. He reached Oxford toward the close of the year 1710. He was

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Figure 1. Memorial tablet in Dunblane Cathedral. Photograph: Hendry [3].

nominated Snell Exhibitioner on December 7, 1710, and he matriculated on January 18, 1711, at Balliol College. The Snell Exhibitions to Balliol College were established by the will of an Ayrshire man, John Snell (~1629 - 1679) [1, 12]. Their main provision was for the maintenance of Scottish students from Glasgow at Oxford. From the Register of Admissions and Degrees, Balliol College (not legible at places) [10]:

1711. Jan 4 Jacobus Sterling Filius Tertius Archibaldi Sterling in Parochiâ sancti Niniani in comitadu Sterling in Scotiâ admissus est comm̃unarius et deinde sc ill : Jan; 10 exhibitionarius snellianus in locum Jacobi carnegy.

Through the interest of the Earl of Mar, James Stirling was also nominated Warner Exhibitioner. No requirement to take Holy Orders (this being frustrated by the 1690 re-establishment of Presbyterianism in Scotland) or to return to Scotland was imposed on the Snell Exhibitioners. Anne, Queen of England, Scotland, and Ireland, died in August 1714, and the German George I of the House of Hanover acceded to the British throne. In 1715 there was another Jacobite Rebellion, which melted away after the Battle of Sheriffmuir on November 13, 1715. The Records of Balliol bear witness to Stirling's tenure of the Snell and Warner Exhibitions to September, 1716. There is, however, no indication of his expulsion from Oxford, though the last mention of him informs us that he had lost his scholarship for refusing to take 'The Oaths' of allegiance to the House of Hanover [3, 14]. Certainly Stirling could not graduate from Oxford, and he left in 1716 [13]. In the minutes of a meeting of the Royal Society of London on April 4, 1717, when Brook Taylor lectured on extracting roots of equations and on logarithms, it was recorded that "Mr Sterling of Balliol College in Oxford had leave to be present" [11, 12].

James Stirling's reputation among his contemporaries was off to an early start under the guidance of John Keill. In 1694, Keill (from Edinburgh) was admitted at Balliol as a Senior Commoner, and in 1712 he was elected Savilian Professor of Astronomy. He was a frequent correspondent

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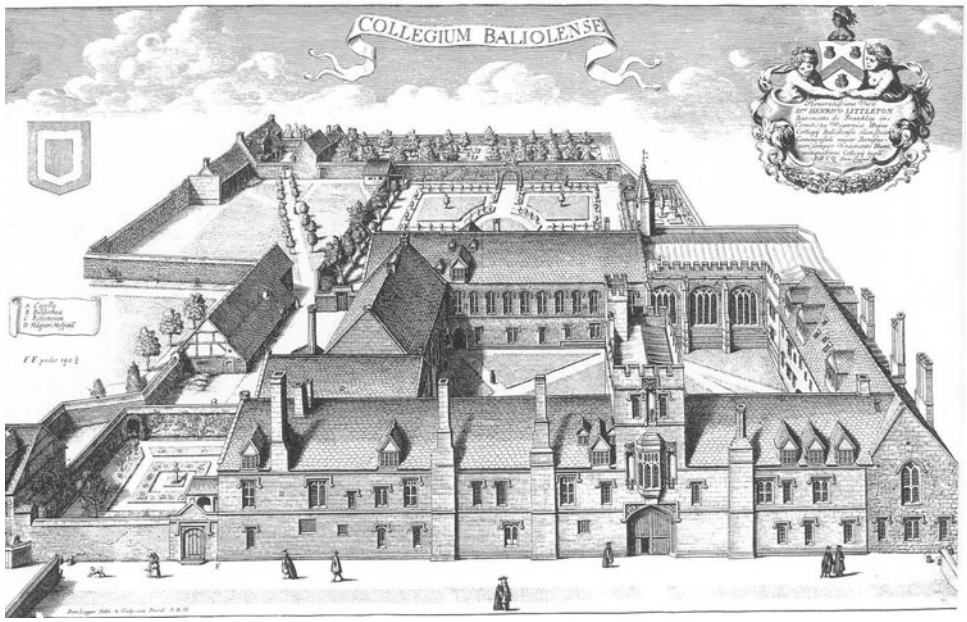

Figure 2. Balliol College. Print by David Loggan, 1675.

of Newton's, and became thereby the pivotal figure in a protracted and fierce controversy over the discovery of the differential calculus. As a diversion from the main dispute, in 1715, Leibniz sent a complex problem in the properties of a certain kind of hyperbola to mathematicians in England, by way of a challenge. On February 24, 1715, Keill was able to write to Newton to say that he had a solution, adding that "Mr Sterling, an undergraduate here, has likewise solved this problem." The precocious undergraduate was James Stirling [4].

First publication

At about the same time, Stirling became acquainted with John Arbuthnot, the well-known mathematician, physician, and satirist [13]. Stirling must already by this time have been working on his first publication *Lineæ Tertii Ordinis Neutonianae sive Illustratio Tractatus D. Newtoni De Enumeratione Linearum Tertii Ordinis. Cui subjungitur, Solutio Trium Problematum*. This work was mainly a commentary on Newton's enumeration of curves of the third degree. Isaac Newton had in his *Tractatus de Quadratura Curvarum* and *Enumeratio linearum tertii Ordinis* (1704) made great advances into the theory of higher plane curves, and brought order into the classification of cubics, but he furnished no proofs of his statements. James Stirling proved all Newton's theorems up to, and including, the enumeration of cubics. Stirling's connections with Arbuthnot enabled him to publish this 128-page book in Oxford. Afterwards, Colin Maclaurin and P. Murdoch made major contributions to the organic descriptions of curves in their respective works *Geometria Organica* (1720) and *Genesis Curvarum per Umbras* (1740). Another edition of Stirling's *Lineae Tertii Ordinis* was published in Paris in 1787 as *Isaaci Newtoni Enumeratio Linearum Tertii Ordinis. Sequitur illustratio eiusdem tractatus Iacobo Stirling*.

In Venice

The Venetian ambassador Nicholas Tron left London to return to Venice in June 1717, and it is almost certain that Stirling accompanied him. Stirling had been offered the Chair of Mathematics at the University of Padua (then in the Republic of Venice), but, for some reason that is not known, the appointment fell through [14]. Not much is known about his stay in Venice, but he certainly continued his mathematical research.

In 1718 Stirling submitted, through Newton, his first Royal Society paper *Methodus Differentialis Newtoniana Illustrata*, Phil. Trans. 30 (No.362) Sept - Oct. 1719, 1050 - 1070. (Royal Society, June 18, 1719.) The first part of this paper is an explanation of Newton's treatise *Methodus Differentialis* (1711) and is devoted to interpolation and quadrature by finite differences, Newton's forward difference formula, Stirling's interpolation formula, Bessel's interpolation formula, and the closed Newton-Cotes quadrature formulae for 3, 5, 7, 9, and 11 ordinates. In the second part of his paper, Stirling pursued the transformation of a slowly converging series into one that converges rapidly to the same sum, and he introduced a process for finding limits, now known as the "Stirling-Schellbach algorithm" [6, pp. 276 - 280].

In August 1719, Stirling wrote from Venice thanking Newton for his kindness and offering to act as intermediary with Nicolaus I Bernoulli [14]. In 1721, Stirling was in Padua where he attended the university and where Nicolaus I Bernoulli occupied the chair from 1716 until 1722. According to Hendry [3], Stirling had to leave Venice in a hurry for fear of assassination because of his discovery of the secrets of the Venetian glass manufacturers, and Tweedle wrote that Stirling had, at the request of certain London merchants, acquired information regarding the manufacture of plate glass [14].

In London

It is not clear what Stirling did between 1722 and late 1724, but it is known that, at least from 1722, he had the intention of becoming a teacher in London: In August 1722, Colin Maclaurin (1698 - 1746) visited Newton in London and Newton showed him a letter from Stirling in which the latter wrote that he intended to set himself up as a mathematics teacher in London. At that stage, Maclaurin was professor of mathematics (since August 1717 already, at the age of 19) at Marischal College, University of Aberdeen, but he moved to Edinburgh in November 1725 as professor of mathematics. The minutes of the meeting of the Royal Society on October 27, 1726, recorded that "Mr Sterling was proposed for a Fellow by Dr Arbuthnot and recommended by Sir Alexander Cumming." He was elected on November 3, 1726, and admitted on December 8, 1726 (Journal Book of the Royal Society, see [13]). Stirling enjoyed the friendship of Sir Isaac Newton and was corresponding on equal terms with most of the leading European scientists. About this time, Stirling was appointed to succeed Benjamin Worster as one of the partners of William Watt's Little Tower Street Academy, Covent Garden, London, which was, according to references [2] and [11], "... one of the most successful schools in London; ... although he had to borrow money to pay for the mathematical instruments he needed." The Academy's prospectus of 1727 listed 'A Course on Mechanical and Experimental Philosophy' given by Stirling and others. The syllabus included mechanics, hydrostatics, optics, and astronomy.

While in London, Stirling published his most important work, namely, *Methodus Differentialis, sive Tractatus de Summatione et Interpolatione Serierum Infinitarum*, G. Strahan, London, 1730.[‡] Regarded as one of the early classics of numerical analysis, this book contains the results and ideas for which Stirling is mainly remembered today: Stirling numbers, Stirling's interpolation formula, and the speeding up of the convergence of series. Stirling's formula for $\ln n!$ and the asymptotic formula for $n!$, for which he is best known, appear as Example 2 to proposition 28. After Abraham De Moivre (1667 - 1754) published his *Miscellanea Analytica* in 1730, Stirling wrote to him, pointing out some errors that he had made in a table of logarithms, and told De Moivre about Example 2 to Proposition 28. De Moivre was able to extend his earlier results using Stirling's ideas and published a Supplement to *Miscellanea Analytica* a few months later. The influence of *Metodus Differentials* has now extended for more than 280 years. A vast number of articles and books have been devoted to 'Stirling's formula' (proofs, extensions, and so on), and the modern development of combinatorial theory has ensured a central place for the Stirling numbers. Stirling showed that

$$\ln m! = \left(m+\frac{1}{2}\right)\ln\left(m+\frac{1}{2}\right)+\frac{1}{2}\ln(2\pi)-\left(m+\frac{1}{2}\right) - \frac{1}{24(m+\frac{1}{2})}+\cdots \tag{1}$$

Neglect all the terms involving reciprocal powers of $m+\frac{1}{2}$ and exponentiate:

$$m! \approx \sqrt{2\pi}\left(\frac{m+\frac{1}{2}}{e}\right)^{m+\frac{1}{2}} \tag{2}$$

Formulae (1) and (2) are Stirling's own versions of the expressions that are generally labelled 'Stirling's formula'. De Moivre obtained the expression

$$\ln(m-1)! = \left(m-\frac{1}{2}\right)\ln m+\frac{1}{2}\ln(2\pi)-m+\frac{1}{12m} - \frac{1}{360m^3}+\frac{1}{1260m^5}\cdots \tag{3}$$

Neglect the reciprocal powers of m and exponentiate. Then

$$(m-1)! \approx \sqrt{\frac{2\pi}{m}}\left(\frac{m}{e}\right)^m$$

or

$$m! \approx \sqrt{2\pi m}\left(\frac{m}{e}\right)^m \tag{4}$$

Unfortunately, (3) and (4) are also labelled 'Stirling's formula', probably because they are a little simpler than Stirling's own versions. Maclaurin placed great reliance upon Stirling's judgement, and frequently consulted Stirling while writing his *Treatise of Fluxions* [14]. In London, Stirling mixed on easy and familiar footing with Henry St. John, Lord Bolingbroke, and his circle of Tory supporters, which included Jonathan Swift, Alexander Pope, and other influential people. With such people around him, it was expected that he would soon be rewarded with a post of high academic standing. But it was not along such lines that Stirling's future was to be shaped.

Leadhills

At the age of forty-three, James Stirling was to enter upon an entirely different career that in due course would link his name with the precursors of the Scottish Industrial Revolution. In fact, Stirling is often described as one of the forgotten pioneering figures in the opening stages of the Scottish Industrial Revolution [3]. During the summers of 1734 - 1736 he was employed by the Scotch Mining Company at Leadhills, Lanarkshire, in Scotland. Leadhills, today only fifteen minutes' drive from Abington, was once a busy lead-mining village; now it is a peaceful place of some 300 inhabitants where clusters of white and grey stone cottages nestle on the hillside. This village lies at the head of the barren and heathy valley through which the Sonar Burn flows down to the river Clyde. Lead has been mined in these hills since the thirteenth century; the last shaft in this area was closed in 1959. These hills also yield gold. A ring of Leadhills gold was presented to Queen Mary (1867 - 1953, the Queen Consort of George V), and the mace for the new Scottish Parliament has a thick band of Leadhills-Wanlockhead gold around it [5].

[‡]It was re-issued in 1753 by Richard Manby, Ludgate Hill, London, and in 1764 by J. Whiston and B. White, Fleet Street, London. A translation by Francis Holliday was published in 1749 by E. Cape, St. John's Gate, London. An annoted translation by Ian Tweddle was published by Springer in 2003 [13].

At the beginning of May 1737, Stirling returned to Leadhills with a permanent resident appointment as Chief Agent at a salary of £210 per year [12]. He proved extremely successful as a practical administrator [11]. There had been little systematic method in the running of the mines and they had been approaching bankruptcy. All this was altered by new methods and rules that he introduced, and with the prosperity of the mines the welfare and amenities for the miners themselves also improved. His measures to improve output and productivity as well as working conditions and social environment made the Leadhills mines not only a profitable undertaking but a model for future social reformers. For example, he divided the workers into four classes, namely, miners, labourers, washers, and smelters, he organised them into decent shifts, encouraged them to cultivate plots of land and crofts, and they were also made to contribute toward the maintenance of the sick and aged of the village [3, 7, 8]. He did not give up mathematics; in fact, there is a discussion of unpublished mathematical work in notebooks of Stirling that were probably written between 1730 and 1745 [12]. Although he continued his mathematical correspondence with Euler, Maclaurin, and others, it is quite clear that most of his energy was spent on mining affairs.

On December 6, 1733, Stirling read a paper entitled *Twelve propositions concerning the figure of the Earth* to the Royal Society of London. This paper could be regarded as the first major contribution by a British scientist to the theoretical study of the figure of the earth and its gravitational forces since the seminal work of Newton and Huygens [12, pp. 102, 169 - 171].[§] Stirling was considered the leading British expert on the subject for the next few years by all, including Maclaurin and Robert Simson, who went on to make major contributions themselves. As Stirling's unpublished manuscripts show, he did go much further than the 1735 paper, but probably the pressure of work at the mining company gave him too little time to polish the work [12]. He explained in a letter to Maclaurin in 1738 why he had not published despite pressure to do so:

> I got a letter this last summer from Mr Machin wholly relating to the figure of the Earth and the new mensuration, he seems to think this a proper time for me to publish my proposition on that subject when everybody is making a noise about it; but I choose rather to stay till the French arrive from the south, which I hear will be very soon. And hitherto I have not been able to reconcile the measurements made in the north to the theory ... [11].

The French expedition to Ecuador, referred to by Stirling as 'the south', left in 1735 but did not return until 1744. To this period there belonged only one paper by Stirling, namely a very short article entitled *A Description of a machine to blow Fire by the Fall of Water*. This machine is known to engineers as 'Stirling's Engine', and furnishes an ingenious mechanical contrivance to create a current of air, due to falling water, sufficiently strong to blow a forge or to supply fresh air in a mine.[¶]

Figure 3. Stirling's house in Leadhills. Photograph: P. Maritz.

The Scotch Mining Company built James Stirling a house supplied with a well stocked library and wine cellar, and provided him with a carriage and a pair of horses [3]. The house (also known as Woodlands Hall) still stands in cherished preservation on slightly rising ground above the village. Harriet Martineau, who visited Leadhills many years after Stirling's time, wrote a delightful description of the house, which subsequently appeared in Dickens's *Household Words*; the house was included in a list of the most distinguished gardens of the eighteenth century. Stirling successfully managed the mines for thirty-five years, during which time he transformed them into one of the most profitable industrial enterprises in Scotland.

Allan Ramsay, born in Leadhills in 1686, had worked in the mines as a washer's boy. By 1718 he had become known as a poet, and then started business as a bookseller, adding a circulating library in 1725 in the Luckenbooths of Edinburgh, apparently the first in Britain. Ramsay contributed to the revival of vernacular Scottish poetry. It was through his influence and the vision of the enlightened mine manager of the day, James Stirling, that a 'subscribed' library was formed in 1741, allowing for the first time lead-miners and their families access to literature. It is the oldest subscription library in the British Isles. Of the 23 founder members at Leadhills, all were miners except the minister and the schoolmaster. The early books were mainly religious in character, and included many volumes of sermons. Today the library contains various relics of the past life of the village and the mines, as well as a book collection [5, 6].

In the year 1752, at the request of the magistrates of Glasgow, James Stirling carried out a survey and submitted proposals for the deepening of the upper reaches of the River Clyde. This survey paved the way for the later schemes of well-known engineers such as Goldburn, James Watt, and others [8]. In that year Stirling was presented with a silver kettle and stand.

[§]An extended version of Stirling's results appeared as the article: *Of the figure of the Earth, and the variation of gravity on the surface*, Phil. Trans 39 (No. 438), July - Sept. 1735 (1735 - 1736), 98 - 105. (Royal Society, 1735.)

[¶]This machine is not to be confused with the much better-known hot-air 'Stirling Engine' invented in 1816 by the Scottish clergyman Robert Stirling.

Figure 4. Leadhills Miners Library. Photograph: P. Maritz.

Figure 5. Name-plate of Leadhills Miners Library. Photograph: P. Maritz.

The kettle bears the inscription: "A compliment made by the Town Council of Glasgow to James Stirling, mathematician, for his services, pains, and trouble in surveying the river (Clyde) towards deepening it by locks. 1st July 1752".

Stirling's wife, Barbara (née Watson), died in February 1753. The date of his marriage does not seem to be known, but it was no earlier than 1745. By his marriage with Barbara, Stirling had a daughter, Christian, who married her cousin, Archibald Stirling of Garden, and their descendants retain possession of the estate of Garden [14]. Archibald succeeded his father-in-law as manager of the mines. In 1746, James Stirling was elected to membership of the Royal Academy of Berlin. In 1753, he resigned from the Royal Society of London as he was in debt to the Society and could no longer afford the annual subscriptions. It cost him £20 to resign.

Greyfriars Church

In 1638, the National Covenant was signed in the Greyfriars Church in Edinburgh. The covenant rejected the attempts by Charles I to reintroduce episcopacy and a new English prayer book, and affirmed the independence of the Scottish Church.

A major Jacobite rebellion took place in 1745, and Maclaurin played an active role in the defence of Edinburgh against the Jacobites. There was no trace of James Stirling being implicated, though his uncle of Cawder was imprisoned by the government [14]. In 1746, Maclaurin died, partly as a consequence of the battles of the previous year, and he was buried in Greyfriars Kirkyard.

Figure 6. Silver tea-kettle presented to James Stirling.

Figure 7. Plaque of James Stirling in Greyfriars Kirkyard. Photograph: P. Maritz.

Stirling was considered for MacLaurin's chair at Edinburgh. However, his strong support for the Jacobite cause meant that such an appointment was impossible, especially in the year after the Rising [11, 12]. Charles and his Jacobites were finally defeated on April 16, 1746, by Cumberland's forces at the Battle of Culloden, and effectively ended Jacobitism as a serious force in Britain. James Stirling died in Edinburgh on December 5, 1770, at the age of 78 years, when on a visit there to obtain medical treatment.

The plaque is wall-mounted, deeply incised and in good condition. The inscription reads: PROPE HVNC LOCVM SEPVLTVS JACET JACOBVS STIRLING COGNOMINE VENETIANVS MATHEMATICVS ILLVSTRVS QVI A. D. D. MDCXCII NATVS ARCHIBALDI STIRLING DE GARDEN FILIVS QVARTVS ANNO MDCCLXX MORTVVS EST

John Ramsay assesses Stirling thus [9]: "This gentleman may be regarded as an excellent specimen of the Scotsmen of the last age, who began their course without patrons and without money, yet being well taught, and obliged to avail themselves of time and chance, their spirit of industry and address enabled them to surmount every difficulty, raising them to eminence, and commanding the esteem of all who knew them."

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Permission to use the photograph in Figure 1 was granted by the Curator of the Scotland's Magazine Archives. The author is grateful for the opportunity granted by the Librarian of Balliol College, Oxford, to consult the documents on James Stirling. The copyright of the print by David Loggan in Figure 2 belongs to Balliol College, Oxford, and permission to use it was granted by Anna Sander, Lonsdale Curator of Archives and Manuscripts, Balliol College, Oxford. The illustration in Figure 6 is used by the courtesy of Colonel James Stirling of Garden.

REFERENCES

[1] W. Innes Addison: *The Snell Exhibitions. Founder, Foundation, Foundationers*. James MacLehose & Sons, Publishers to the University of Glasgow, 1901.

[2] *Dictionary of Scientific Biography. Volume XIII*. Editor in Chief C. C. Gillespie. Charles Scribner's Sons, New York, 1970 – 1990, 67 – 70.

[3] W. B. Hendry: James Stirling 'The Venetian', *Scotland's Magazine*, October 1965, 33 – 35.

[4] J. Jones: *Balliol College. A History*. Second Edition. Oxford University Press, 1997.

[5] Leadhills Reading Society. Available at URL http://www.lowtherhills.fsnet.co.uk (accessed June 25, 2003).

[6] Leadhills Library website. Available at URL http://www.miners-library.fsnet.co.uk (accessed March 28, 2008).

[7] John Oswald Mitchell: *The Old Glasgow Essays*. MacLehose, 1905.

[8] Muir Collection, MSB 691, 4(10), J. Stirling. The National Library of South Africa, Cape Town.

[9] John Ramsay: *Scotland and Scotsmen in the eighteenth century*. Editor Alexander Allardyce, 2 volumes. W. Blackwood and Sons, Edinburgh and London, 1888.

[10] *Register of Admissions and degrees, 1682 – 1833*. Balliol College, Oxford.

[11] James Stirling. Available at URL http://www-history.mcs.st-andrews.ac.uk/history/mathematicians/Stirling.html (accessed March 28, 2008).

[12] I. Tweddle: *James Stirling: 'This about series and such things.'* Scottish Academic Press, Edinburgh, 1988.

[13] I. Tweddle: *James Stirling's Methodus Differentiales*. An Annotated Translation of Stirling's text. Springer-Verlag. London. 2003.

[14] C. Tweedie: *James Stirling: a sketch of his life and works along with his scientific correspondence*. Clarendon Press, Oxford, 1922.