

Analytic Lifshitz black holes in higher dimensions

Eloy Ayón-Beato

*Departamento de Física, CINVESTAV-IPN,
Apdo. Postal 14-740, 07000, México D.F., México.
E-mail: ayon-beato-at-fis.cinvestav.mx*

Alan Garbarz

*Departamento de Física, Universidad de Buenos Aires FCEN - UBA,
Ciudad Universitaria, Pabellón 1, 1428, Buenos Aires, Argentina.
E-mail: alan-at-df.uba.ar*

Gaston Giribet

*Departamento de Física, Universidad de Buenos Aires FCEN - UBA and CONICET,
Ciudad Universitaria, Pabellón 1, 1428, Buenos Aires, Argentina.
E-mail: gaston-at-df.uba.ar*

Mokhtar Hassaine

*Instituto de Matemática y Física, Universidad de Talca,
Casilla 747, Talca, Chile.
E-mail: hassaine-at-inst-mat.otalca.cl*

ABSTRACT: We generalize the four-dimensional R^2 -corrected $z = 3/2$ Lifshitz black hole to a two-parameter family of black hole solutions for any dynamical exponent z and for any dimension D . For a particular relation between the parameters, we find the first example of an extremal Lifshitz black hole. An asymptotically Lifshitz black hole with a logarithmic decay is also exhibited for a specific critical exponent depending on the dimension. We extend this analysis to the more general quadratic curvature corrections for which we present three new families of higher-dimensional $D \geq 5$ analytic Lifshitz black holes for generic z . One of these higher-dimensional families contains as critical limits the $z = 3$ three-dimensional Lifshitz black hole and a new $z = 6$ four-dimensional black hole. The variety of analytic solutions presented here encourages to explore these gravity models within the context of non-relativistic holographic correspondence.

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1. Introduction

In the last years, promising attempts to extend the AdS/CFT correspondence to other areas of physics have attracted much attention. Within the context of non-relativistic physics, holographic techniques were recently considered with remarkable success. Pioneer work on this matter, where gravity duals to non-relativistic systems were proposed, has been done in Refs. [1] and [2]. Of particular importance is also the construction of Ref. [3], where the authors proposed new gravitational duals to scale invariant Lifshitz fixed points with no Galilean invariance. These gravity backgrounds have the form

$$ds^2 = -\frac{r^{2z}}{l^{2z}} dt^2 + \frac{l^2}{r^2} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (1.1)$$

where \vec{x} is a $(D - 2)$ -dimensional vector. These geometries are usually called Lifshitz spacetimes, and admit the following anisotropic scaling symmetry

$$t \mapsto \lambda^z t, \quad r \mapsto \lambda^{-1} r, \quad \vec{x} \mapsto \lambda \vec{x}, \quad (1.2)$$

as part of their isometry group. This is the geometric realization of the scale invariance exhibited by their non-relativistic dual systems, which are thought to be formulated on the $(D - 1)$ -dimensional space located at infinite r . In this sense, this picture completely mimics the prescription of the standard AdS/CFT correspondence.

The natural extension of the construction of [3] is to look for black hole configurations that asymptote the Lifshitz spacetimes (1.1). Holographically, they should describe the finite temperature behavior of the non-relativistic theories. Black hole solutions of this type are known with the name of “Lifshitz black holes”, and the quest for such solutions has received much attention recently. Analytic Lifshitz black hole solutions are scarce and they are actually hard to be found. In spite of the fact their metrics are not of a particularly abstruse form, these are reluctant to appear as exact solutions of theories of gravity with physically sensible matter sources. The main obstacle for these spacetimes to exist are the Birkhoff theorems, which happen to hold for generic models and restrict the subspace of static solutions in a strong way. Nevertheless, some few examples of black hole solutions that are asymptotically Lifshitz spaces were recently found in the literature. One of the first analytic examples was reported in Ref. [4] for a sort of higher-dimensional dilaton gravity without restricting the value of the dynamical exponent z . In Ref. [5], a topological black hole solution which happens to be asymptotically Lifshitz with $z = 2$ was found. An example with $z = 4$ and with spherical topology was given in Ref. [6]. Numerical solutions for more general values of z were explored in Refs. [7, 5, 6, 8]. More examples of analytic Lifshitz black holes were studied in Refs. [9, 10, 11], and the solution found in [10] is particularly interesting as it corresponds to a remarkably simple analytic example with $z = 2$ in $D = 4$ dimensions. The difficulty of embedding Lifshitz black holes in string theory was also investigated in Refs. [12, 13, 14]. The holographic description of asymptotically Lifshitz spacetimes was studied in [15]. More recent investigations related to Lifshitz black holes can be found in Refs. [16, 17, 18, 19, 20].

In [21] a remarkably simple solution with $z = 3$ in absence of matter fields was found for the New Massive Gravity theory [22], which consists of special square-curvature corrections to three-dimensional gravity. Previously, in Ref. [23], it was shown that square-curvature corrections to gravity generically can support the Lifshitz spacetimes (1.1). The example of Ref. [21] is the first to show that these theories also allows the existence of Lifshitz black holes. Another example with $z = 3/2$ was subsequently found for a four-dimensional theory with R^2 -corrections in Ref. [24].

Inspired in the results of [21] and [24], we will investigate in this paper how the introduction of higher-curvature corrections to the Einstein-Hilbert action leads to find a large zoo of analytic Lifshitz black hole solutions in D dimensions. We will begin our search of Lifshitz black holes by considering the simplest example of quadratic corrections. Already in the simplest case we will find interesting solutions, provided a suitable parameterization of the coupling constants, and which hold for generic z in D dimensions. Interestingly enough, we will also exhibit an extremal Lifshitz black hole and an asymptotically Lifshitz black hole with logarithmic decay at infinity. Motivated by the richness of examples we find in the simplest case, we will then consider the most general square-curvature corrections, and we will present several classes of analytic Lifshitz black hole families of solutions in $D \geq 5$ dimensions. Curiously, one of these higher-dimensional families leads, through some particular limiting procedure, to the three-dimensional $z = 3$ Lifshitz black hole of [21] as well as to a new solution in $D = 4$ with critical exponent $z = 6$.

2. R^2 -corrected Lifshitz black holes for any dimension

We first consider a gravity theory with R^2 -corrections

$$S[g_{\mu\nu}] = \int d^D x \sqrt{-g} (R - 2\lambda + \beta_1 R^2), \quad (2.1)$$

giving the following field equations

$$G_{\mu\nu} + \lambda g_{\mu\nu} + 2\beta_1 g_{\mu\nu} \square R - 2\beta_1 \nabla_\mu \nabla_\nu R + 2\beta_1 R R_{\mu\nu} - \frac{1}{2} \beta_1 R^2 g_{\mu\nu} = 0. \quad (2.2)$$

These equations allow Lifshitz spacetimes (1.1) as solutions for a generic value of the dynamical exponent z in any dimension, provided a suitable choice of the cosmological constant λ and the coupling constant β_1 that is given by

$$\lambda = -\frac{2z^2 + (D-2)(2z+D-1)}{4l^2}, \quad (2.3a)$$

$$\beta_1 = -\frac{1}{8\lambda}. \quad (2.3b)$$

It is well known that this kind of theory (2.1) can be generically mapped into scalar-tensor theories through a conformal transformation of the metric with conformal factor $\Omega^2 = 1 + 2\beta_1 R$. However, for the particular choice of the coupling constant (2.3), this trick does not work since $R = 4\lambda$ for Lifshitz spacetimes. In this sense, the model corresponds to a genuine pure gravity theory. The black hole solutions we will derive below present the same degeneracy for the conformal transformation, and thus have no scalar-tensor counterpart.

Our purpose is to explore whether there exist some black hole solutions which asymptote the Lifshitz spacetimes (1.1). This analysis is motivated by the existence of a four-dimensional Lifshitz black hole solution for these theories with a specific value of the dynamical exponent $z = 3/2$ found in Ref. [24]. In fact, we can show that for any dimension D , there exists a two-parametric family of solutions given by

$$ds^2 = -\frac{r^{2z}}{l^{2z}} \left(1 - \frac{M^- l^{\alpha_-}}{r^{\alpha_-}} + \frac{M^+ l^{\alpha_+}}{r^{\alpha_+}} \right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{M^- l^{\alpha_-}}{r^{\alpha_-}} + \frac{M^+ l^{\alpha_+}}{r^{\alpha_+}} \right)^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (2.4a)$$

$$\alpha_\pm = \frac{3z + 2(D-2) \pm \sqrt{z^2 + 4(D-2)(z-1)}}{2}, \quad (2.4b)$$

for which the coupling constants are the same as in the purely Lifshitz case (2.3). It is important to mention that this family of geometries has the same constant scalar curvature of the Lifshitz spacetimes.

First, it is clear from the expression of α_\pm , given by (2.4b), that the dynamical exponent may take the values $z \in (-\infty, z_-] \cup [z_+, \infty)$ where

$$z_\pm = 4 - 2D \pm 2\sqrt{(D-1)(D-2)}. \quad (2.5)$$

On the other hand, the solution (2.4) represents an asymptotically Lifshitz black hole for $\alpha_{\pm} > 0$, and this occurs for $z \geq z_+$. It is easy to see that the four-dimensional solution of reference [24] corresponds to the particular case $M^+ = 0$ with $z = 3/2$, that is $\lambda = -33/8l^2$ and $\alpha_- = 3$.

For a specific relation between the constants M^{\pm} given by

$$M^+ = \alpha_-(\alpha_+ - \alpha_-) \frac{\alpha_+ - \alpha_-}{\alpha_-} \left(\frac{M^-}{\alpha_+} \right)^{\frac{\alpha_+}{\alpha_-}}, \quad (2.6)$$

the solution (2.4) has zero temperature, i.e. it is an extremal black hole

$$ds^2 = -\frac{r^{2z}}{l^{2z}} \left[1 - \frac{\alpha_+}{\alpha_+ - \alpha_-} \left(\frac{r_e}{r} \right)^{\alpha_-} + \frac{\alpha_-}{\alpha_+ - \alpha_-} \left(\frac{r_e}{r} \right)^{\alpha_+} \right] dt^2 + \frac{l^2}{r^2} \left[1 - \frac{\alpha_+}{\alpha_+ - \alpha_-} \left(\frac{r_e}{r} \right)^{\alpha_-} + \frac{\alpha_-}{\alpha_+ - \alpha_-} \left(\frac{r_e}{r} \right)^{\alpha_+} \right]^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (2.7)$$

where the extremal radius r_e is expressed as

$$r_e = l \left(\frac{\alpha_+ - \alpha_-}{\alpha_+} M^- \right)^{1/\alpha_-}. \quad (2.8)$$

The interest on solution (2.4) increases once one notices that when the dynamical exponent approach the value $z = z_+$, defined in (2.5), there exists an additional solution which asymptotes the Lifshitz spacetime in a much slower way

$$ds^2 = -\frac{r^{2z_+}}{l^{2z_+}} \left\{ 1 - \frac{l^{\alpha_0}}{r^{\alpha_0}} \left[M_1 + M_2 \ln \left(\frac{r}{l} \right) \right] \right\} dt^2 + \frac{l^2}{r^2} \left\{ 1 - \frac{l^{\alpha_0}}{r^{\alpha_0}} \left[M_1 + M_2 \ln \left(\frac{r}{l} \right) \right] \right\}^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (2.9)$$

where the parameter α_0 is given by

$$\alpha_0 = 3\sqrt{(D-1)(D-2)} - 2(D-2). \quad (2.10)$$

The fact of having a weakened (logarithmic) fall-off as a next-to-leading contribution in the asymptotic behavior is well known in the standard AdS/CFT correspondence. In particular, this was one of the key points in recent discussions on three-dimensional massive gravity (see [25] and reference therein).

The extremal version of the logarithmic black hole (2.9) is found for

$$M_1 = \frac{M_2}{\alpha_0} \left[1 - \ln \left(\frac{M_2}{\alpha_0} \right) \right],$$

and then the corresponding spacetime geometry reads

$$ds^2 = -\frac{r^{2z_+}}{l^{2z_+}} \left\{ 1 - \frac{r_e^{\alpha_0}}{r^{\alpha_0}} \left[1 + \alpha_0 \ln \left(\frac{r}{r_e} \right) \right] \right\} dt^2 + \frac{l^2}{r^2} \left\{ 1 - \frac{r_e^{\alpha_0}}{r^{\alpha_0}} \left[1 + \alpha_0 \ln \left(\frac{r}{r_e} \right) \right] \right\}^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (2.11)$$

where the extremal radius is defined by

$$r_e = l \left(\frac{M_2}{\alpha_0} \right)^{\frac{1}{\alpha_0}}.$$

Let us stress that spacetimes (2.7) and (2.11) are the first examples of asymptotically Lifshitz black hole solutions with an extremal horizon. Remarkably, the solution (2.9) is additionally the first solution with a logarithmic decay.

In the list of curiosities, we can also mention that the family of solutions (2.4) contains an asymptotically conformal limit at $z = 1$; namely

$$ds^2 = - \left(\frac{r^2}{l^2} - \frac{M^- l^{D-2}}{r^{D-2}} + \frac{M^+ l^{D-3}}{r^{D-3}} \right) dt^2 + \left(\frac{r^2}{l^2} - \frac{M^- l^{D-2}}{r^{D-2}} + \frac{M^+ l^{D-3}}{r^{D-3}} \right)^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2. \quad (2.12)$$

For $M^- = 0$ ($M^+ < 0$), the resulting spacetime is nothing but the Schwarzschild-Tangherlini-AdS topological black hole with toroidal horizon ($k = 0$ in standard notation) for $\lambda = -D(D-1)/(4l^2)$. For $M^+ = 0$, the solution corresponds to a different asymptotically AdS toroidal black hole with a faster decay. As an appealing remark, for $D = 4$, the solution (2.12) is precisely the Reissner-Nordstrom-AdS topological black hole with $k = 0$.

As a final comment, we would like to point out that the Lagrangian of the gravity action (2.1) for the coupling constant given by (2.3b) can be written as a perfect square,

$$R - 2\lambda - \frac{1}{8\lambda} R^2 = -\frac{1}{8\lambda} (R - 4\lambda)^2. \quad (2.13)$$

Therefore, for this choice of the coupling constant, the gravity action is definite positive and reaches its minimal (vanishing) value for $R = 4\lambda$, which is precisely the case of the solutions (2.4). Then, this solution (or its Euclidean continuation) can be seen as a sort of gravitational instanton. This has to do with the degeneracy of the field equations (2.2) in the following sense: for the case of constant scalar curvature solutions, equations (2.2) become

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} = 0,$$

where $f(R)$ is the Lagrangian expressed in terms of the scalar curvature. Apart from considering $f'(R) \neq 0$, which yields Einstein equations with an effective cosmological constant and hence no Lifshitz configurations, the only option for constant scalar curvature solutions to exist is that the value of the scalar curvature be a double root of the Lagrangian $f(R)$. Then, it is clear that the family of black holes obtained in (2.4) will be solutions of any gravity theory with Lagrangian $f(R) = (R - 4\lambda)^2 H(R)$ where H is a function regular at $R = 4\lambda$.

3. More general quadratic corrections

Due to the new and interesting results of the R^2 -corrected theory (2.1) presented above, it is natural to extend the analysis and explore the existence of asymptotically Lifshitz black hole

configurations with the most general quadratic corrections. We will proceed in the same way as before, by first establishing the purely Lifshitz configurations and then by presenting three different classes of asymptotically Lifshitz black hole solutions which correspond to different ranges of the dynamical exponent z . As shown below, for any value of the dynamical exponent z at least one family of black hole solutions exists.

We consider now the gravity action that includes the most general quadratic-curvature corrections in D -dimensions; namely

$$S[g_{\mu\nu}] = \int d^D x \sqrt{-g} \left(R - 2\lambda + \beta_1 R^2 + \beta_2 R_{\alpha\beta} R^{\alpha\beta} + \beta_3 R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \right). \quad (3.1)$$

It follows from the Gauss-Bonnet theorem in four dimensions and the vanishing of the Gauss-Bonnet term in three dimensions that in the case $D < 5$, it is sufficient to consider only two of the three quadratic invariants in the Lagrangian. This makes necessary to split the analysis in two parts by first considering the higher-dimensional cases and then to analyze separately the lower dimensional ones, $D = 3$ and $D = 4$.

The action (3.1) gives rise to the following field equations

$$\begin{aligned} G_{\mu\nu} + \lambda g_{\mu\nu} + (\beta_2 + 4\beta_3) \square R_{\mu\nu} + \frac{1}{2} (4\beta_1 + \beta_2) g_{\mu\nu} \square R - (2\beta_1 + \beta_2 + 2\beta_3) \nabla_\mu \nabla_\nu R \\ + 2\beta_3 R_{\mu\gamma\alpha\beta} R_\nu{}^{\gamma\alpha\beta} + 2(\beta_2 + 2\beta_3) R_{\mu\alpha\nu\beta} R^{\alpha\beta} - 4\beta_3 R_{\mu\alpha} R_\nu{}^\alpha + 2\beta_1 R R_{\mu\nu} \\ - \frac{1}{2} \left(\beta_1 R^2 + \beta_2 R_{\alpha\beta} R^{\alpha\beta} + \beta_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \right) g_{\mu\nu} = 0. \end{aligned} \quad (3.2)$$

As in the purely R^2 -case, Lifshitz spacetimes (1.1) are solutions of these field equations for a generic value of the dynamical exponent z in any dimension, provided that

$$\lambda = -\frac{1}{4l^2} \left(2z^2 + (D-2)(2z+D-1) - \frac{4(D-3)(D-4)z(z+D-2)\beta_3}{l^2} \right), \quad (3.3a)$$

$$\beta_2 = \frac{l^2 - 2[2z^2 + (D-2)(2z+D-1)]\beta_1 - 4[z^2 - (D-2)z + 1]\beta_3}{2(z^2 + D - 2)}. \quad (3.3b)$$

Notice that the above parameterizations coincide with the values previously found for the New Massive Gravity in $D = 3$ [21], where $\beta_3 = 0$ and $\beta_2 = -(8/3)\beta_1 = -1/m^2$. In turn, this generalizes the previous authors' result. We shall now proceed to present three more different Lifshitz black hole families in $D \geq 5$ dimensions.

3.1 An asymptotically Lifshitz black hole family for $z > 2 - D$

The first family of solutions we present here is described by the following line element

$$ds^2 = -\frac{r^{2z}}{l^{2z}} \left(1 - \frac{Ml^{(z+D-2)/2}}{r^{(z+D-2)/2}} \right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{Ml^{(z+D-2)/2}}{r^{(z+D-2)/2}} \right)^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (3.4a)$$

and represents an asymptotically Lifshitz black hole solution of the field equations (3.2) for the dynamical exponent $z > 2 - D$. The coupling constants allowing the existence of the solution (3.4a) are parameterized in term of the dynamical exponent z by

$$\lambda = \frac{(D-2)}{4l^2} \left\{ (197D - 389)z^4 + 4(19D^2 - 200D + 325)z^3 + (D-2)[2(5D^2 - 73D + 356)z^2 + 4(D^3 - 2D^2 + 15D - 62)z + (D+2)(D-1)(D-2)^2] \right\} / P_4(z), \quad (3.4b)$$

$$\beta_1 = l^2 \left\{ 27z^6 - 18(3D-4)z^5 + 3(19D^2 - 168D + 356)z^4 - 12(11D^3 - 84D^2 + 196D - 120)z^3 - (D-2)[(19D^3 - 330D^2 + 2052D - 3640)z^2 + 2(3D^4 - 30D^3 + 124D^2 - 536D + 1024)z + (D+2)(D-2)^2(D^2 - 4D + 36)] \right\} / \left(2(D-3)(D-4)(z+D-2)^2 P_4(z) \right), \quad (3.4c)$$

$$\beta_2 = -2l^2 [3z^2 + (D+2)(D-2)] \left\{ 9z^4 - 6(3D-4)z^3 - 8(D^2 - 10)z^2 + 2(D^3 - 4D^2 + 32D - 80)z - (D-2)[D^3 + 2D^2 - 12(D-2)] \right\} / \left((D-3)(D-4)(z+D-2)^2 P_4(z) \right), \quad (3.4d)$$

$$\beta_3 = l^2 [3z^2 + (D+2)(D-2)] \left\{ 9z^3 - 3(9D-14)z^2 - (D-2)[(5D-62)z + D^2 - 4D + 36] \right\} / \left(2(D-3)(D-4)(z+D-2)P_4(z) \right), \quad (3.4e)$$

where P_4 is a polynomial of degree four in the dynamical exponent z given by

$$P_4(z) = 27z^4 - 4(27D - 45)z^3 - (D-2)[2(5D - 116)z^2 + 4(D^2 - D + 30)z + (D+2)(D-2)^2].$$

It is clear from the expressions of the β_i that the solution is defined only for higher dimensions $D \geq 5$. The analysis of the lower dimensional cases will be done in the next section. As in the purely R^2 -case, there exists a conformal limit $z = 1$ of the family (3.4) which is given by the following asymptotically AdS black hole

$$ds^2 = - \left(\frac{r^2}{l^2} - \frac{Ml^{(D-5)/2}}{r^{(D-5)/2}} \right) dt^2 + \left(\frac{r^2}{l^2} - \frac{Ml^{(D-5)/2}}{r^{(D-5)/2}} \right)^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2. \quad (3.5)$$

More precisely, this spacetime (3.5) is a solution of the Einstein-Gauss-Bonnet gravity with a fine-tuned coupling constant, since for $z = 1$ the parameterizations (3.4b)-(3.4e) become

$$\lambda = -\frac{(D-1)(D-2)}{4l^2}, \quad (3.6a)$$

$$\beta_1 = -\frac{1}{4}\beta_2 = \beta_3 = \frac{l^2}{2(D-3)(D-4)}, \quad (3.6b)$$

yielding to the Lagrangian

$$R - 2\lambda - \frac{(D-1)(D-2)}{8(D-3)(D-4)\lambda} \mathcal{L}_{\text{GB}}, \quad (3.7)$$

where $\mathcal{L}_{\text{GB}} = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ is the Gauss-Bonnet term. Note that for $D = 5$, the above black hole becomes diffeomorphic to a warped product having as base AdS_3 with a 2-plane fiber. Moreover this case precisely corresponds to the Chern-Simons gravity in $D = 5$.

3.2 An asymptotically Lifshitz black hole family for $z > 1$

The second family of asymptotically Lifshitz black holes we find is valid for $z > 1$,

$$ds^2 = -\frac{r^{2z}}{l^{2z}} \left(1 - \frac{Ml^{2(z-1)}}{r^{2(z-1)}} \right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{Ml^{2(z-1)}}{r^{2(z-1)}} \right)^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (3.8a)$$

and it exists for the following choice of coupling constants

$$\lambda = -\frac{(z-1)[z^2 - Dz - (D-1)(D-2)]}{2l^2(z-D)}, \quad (3.8b)$$

$$\beta_1 = l^2 \left[3(D-1)(D-2)z^3 - (2D^3 - 2D^2 - 11D + 20)z^2 + (3D^3 - 14D^2 + 19D + 10)z + (D+2)(D-4) \right] / \left[2(D-2)(D-3)(D-4)z(z-1)(z-D)(3z+D-4) \right], \quad (3.8c)$$

$$\beta_2 = -l^2(D-1)(2z-D-2) \left[6(D-2)z^2 - (D^2 - 3D + 8)z - 2(D-4) \right] / \left[2(D-2)(D-3)(D-4)z(z-1)(z-D)(3z+D-4) \right], \quad (3.8d)$$

$$\beta_3 = \frac{l^2(D-1)(2z-D-2)}{4(D-3)(D-4)z(z-D)}. \quad (3.8e)$$

As for the $z > 2 - D$ family, the lower dimensions $D = 3$ and $D = 4$ are also forbidden here. Clearly, there is no conformal limit $z = 1$ for this family.

3.3 An asymptotically Lifshitz black hole family for $z < 0$

The last family of Lifshitz black holes we describe is characterized by a negative dynamical critical exponent $z = -|z|$, whose metric reads

$$ds^2 = -\frac{l^{2|z|}}{r^{2|z|}} \left(1 - \frac{Ml^{|z|}}{r^{|z|}} \right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{Ml^{|z|}}{r^{|z|}} \right)^{-1} dr^2 + \frac{r^2}{l^2} d\vec{x}^2, \quad (3.9a)$$

while the corresponding coupling constants are parameterized as

$$\lambda = \frac{|z| [2|z|^2 - 4(D-2)|z| + (D-2)(D-3)]}{4l^2(2|z| - D + 2)}, \quad (3.9b)$$

$$\beta_1 = \frac{l^2 [3|z|^2 - 2(D-2)|z| + 2D - 5]}{2(D-3)(D-4)|z|(2|z| - D + 2)}, \quad (3.9c)$$

$$\beta_2 = -4\beta_3, \quad (3.9d)$$

$$\beta_3 = \frac{l^2 [6|z|^2 - 4(D-2)|z| + (D-1)(D-2)]}{4(D-3)(D-4)|z|(2|z| - D + 2)}. \quad (3.9e)$$

The associated Lagrangian describes a fine-tuned R^2 -corrected Einstein-Gauss-Bonnet theory

$$R - 2\lambda - \beta_3 \mathcal{L}_{\text{GB}} - \frac{l^2 R^2}{4|z|(2|z| - D + 2)}. \quad (3.10)$$

This family of black holes is again defined only in higher dimensions $D \geq 5$. Being defined only for negative dynamical critical exponents, it has no conformal analog $z = 1$.

In the next section, we analyze the lower-dimensional cases $D = 3$ and $D = 4$.

4. Critical lower dimensional Lifshitz black holes

The families of Lifshitz black holes given by (3.4), (3.8) and (3.9) are generically forbidden in dimensions lower than 5. This is due to the fact that the use of a nontrivial value for the coupling constant β_3 is artificial in these dimensions. Concretely, if one consider theories with $\beta_3 \neq 0$, and due to the fact that the Gauss-Bonnet combination \mathcal{L}_{GB} vanishes in $D = 3$ and is a total derivative in $D = 4$, it turns out that it is always possible to shift the coupling constants and to end with $\beta_3 = 0$. This shifting reads

$$(\beta_1, \beta_2, \beta_3) \mapsto (\beta_1 - \beta_3, \beta_2 + 4\beta_3, 0). \quad (4.1)$$

Despite families (3.4), (3.8) and (3.9) are formally defined for higher-dimensions, the possibility that new critical solutions exist in lower dimensions for some particular values of the dynamical exponent z is not excluded. A natural way to explore this possibility is to consider a dimensional continuation of the D -dimensional expressions and study whether some potential cancellation of the divergences of the coupling constants appears when one expands around $D = 3$ and $D = 4$. That is what we will do in this section. Using the results of this analysis as an indication, we will explicitly confirm the existence of critical solutions that, indeed, represent Lifshitz black holes in $D = 3$ and in $D = 4$.

4.1 The $z = 3$ three-dimensional asymptotically Lifshitz black hole

Let us start with the dimensional continuation and expansion of the coupling constants of the family (3.4) around $D = 3$. The cosmological constant is regular, $\lambda = O((D - 3)^0)$, but the coupling constants exhibit the following singular behavior

$$\beta_1 = -\frac{1}{4}\beta_2 = \beta_3 = -\frac{(z-3)(3z^2+5)(9z^2-12z+11)l^2}{2(z+1)(27z^4-144z^3+202z^2-144z-5)} \times \frac{1}{D-3} + O((D-3)^0). \quad (4.2)$$

This indicates that the only possibility for having a potentially regular behavior for this family at $D = 3$ appears for $z = 3$. Considering that there is in fact no continuity in the number of dimensions, one can choose the element $z = 3$ of the family (3.4). Evaluating after that for $D = 3$, the resulting Lifshitz black hole is

$$ds^2 = -\frac{r^6}{l^6} \left(1 - \frac{Ml^2}{r^2}\right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{Ml^2}{r^2}\right)^{-1} dr^2 + \frac{r^2}{l^2} dx^2, \quad (4.3a)$$

with $\lambda = -13/(2l^2)$, and surprisingly the meaningful coupling constants (i.e. after the shifting (4.1)) are those of New Massive Gravity [22]

$$\beta_2 = -(8/3)\beta_1 = 2l^2, \quad (4.3b)$$

which gives rise to the three-dimensional Lifshitz black hole previously found by the authors in [21]. A similar analysis can be done for the family (3.8); the potential regular behavior occurs in this case for $z = 5/2$. However, the resulting solution is not new but corresponds to the case $z = 5/2$, $M^+ = 0$ ($\alpha_- = 3$) of the family (2.4) valid in generic dimension D . The family (3.9) has no regular limit in $D = 3$.

4.2 The $z = 6$ four-dimensional asymptotically Lifshitz black hole

In four dimensions, we proceed in a similar way, by doing a dimensional continuation and expanding the coupling constants of the family (3.4) around $D = 4$. Again, the cosmological constant is regular, $\lambda = O((D - 4)^0)$, and the coupling constants exhibit singular behavior

$$\beta_1 = -\frac{1}{4}\beta_2 = \beta_3 = \frac{3(z-6)(z^2+4)(3z^2-4z+4)l^2}{2(z+2)(9z^4-84z^3+128z^2-112z-16)} \times \frac{1}{D-4} + O((D-4)^0). \quad (4.4)$$

The indication here is that the only possibility potentially occurs for $z = 6$. The $z = 6$ element of the family (3.4), when is evaluated in $D = 4$, indeed gives rise to a new Lifshitz black hole

$$ds^2 = -\frac{r^{12}}{l^{12}} \left(1 - \frac{Ml^4}{r^4}\right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{Ml^4}{r^4}\right)^{-1} dr^2 + \frac{r^2}{l^2} (dx^2 + dy^2), \quad (4.5a)$$

with $\lambda = -51/(2l^2)$ and meaningful coupling constants given by

$$\beta_2 = -(25/9)\beta_1 = 25l^2/64. \tag{4.5b}$$

The corresponding analysis for the family (3.8) in $D = 4$ singles out the value $z = 3$. Again, the resulting solution is not new but corresponds to the case $z = 3$, $M^+ = 0$ ($\alpha_- = 4$) of the family (2.4), which is valid in four dimensions. As before, family (3.9) has no regular limit in $D = 4$.

5. Conclusions and open problems

In this paper we have found analytic Lifshitz black hole solutions for gravity with square-curvature corrections in arbitrary dimension. Some open questions remain:

- The computation of conserved charges of the asymptotically Lifshitz black holes of higher-curvature gravity would be needed to fully understand the thermodynamical properties of both the gravitational backgrounds and the dual systems. Some important advances in this direction have been done recently in [26].
- Stability of Lifshitz black hole solutions is another question it would be interesting to address.
- Among the family of black holes we exhibited here there are extremal solutions, see (2.7) and (2.11). An interesting question is that of studying the causal structure of these spacetimes.
- Last, the condensed matter interpretation of these backgrounds within the holographic proposal of [3] deserves to be matter for further study.

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