

Article

Interdimensionality

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Abstract: In this speculative analysis, interdimensionality is introduced as the (co)existence of universes embedded into larger ones. These interdimensional universes may be isolated or intertwined, suggesting a variety of interdimensional intrinsic phenomena that can only be understood in terms of the outer, extrinsic reality.

Keywords: intrinsic perception; Hausdorff dimension; fractal

MSC: 00A73; 28A78; 28A80; 37C45; 54F35; 54F45; 83E15

1. A Caveat: Speculation and Progress

Rule inference is the process of hypothesizing a general rule or “law” from examples or “phenomena” [1,2]. The halting problem is the task to determine, given an arbitrary computer program and an input, whether the program will eventually halt or continue to run forever. This has been proven to be unsolvable in general. As the former rule inference problem can be reduced to the latter halting problem, it is provably unsolvable in general. This constraint on induction has been coped with by the philosophy of science in a variety of ways:

Popper suggested that, instead of induction and verification, which appears to be a hopeless endeavor, falsification might be a good demarcation criterion between science on the one hand, and on the other hand ideology, sophisms, or, in a more frugal term, bullshit [3]. Lakatos responded by criticizing that, due to side assumptions and a vast ‘protective belt’ of auxiliary hypotheses, in many practical circumstances, falsification fails. As a result, contemporaries can seldom predict what might turn out to become a progressive versus a degenerative research program [4].

Kuhn observed that science may be characterized by brief iconoclastic periods of revolution, followed by longer conformist periods of consolidation [5]. Feyerabend even challenged methodology as mythology and ideology akin to religious dogmas, and suggested keeping science wide open and performing an “exhaustive search” of ideas by allowing “anything” to enter the scientific debate, thereby, imposing little methodological restrictions [6]; he also recommended a formal separation between state and science, and lay judges for the evaluation of success [7] and the allocation of scientific funding.

In any case, there seems to be no convergence of conceptual progression. Taking gravity and celestial motion, for example: the Ptolemaic system was expressed in terms of geometry. It was superseded by the Copernican revolution that later became based on Newtonian gravitational forces. Later on, Newtonian gravity was replaced by the curved geometry of space–time of Einstein’s theory of general relativity. By analogy, it appears highly likely that our contemporaries would view any model superseding the present canon as utterly speculative, if not outright nonsense.

Such a historic perspective leads to greater liberty and openness of ideas, and yet this creativity needs to be guided and stimulated by empirical findings and attempts to falsify consequences and claims. This amounts to an amalgam of the aforementioned ideas brought forward in the philosophy of science, resulting in a sort of pragmatism that is



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well balanced between wild fantasy and empirical grounding. Exactly how much of those ingredients are in order may greatly depend on the temperament and character of the individual researcher.

We, therefore, present the following considerations with a caveat to the reader, as it trespasses far beyond any empirically verifiable physics of our time; and yet some aspects of it might indicate the way to fruitful avenues of scientific modeling. We hope that the following speculations are not too weird for the realistic, critical, and sober mind. At best this could be seen as a vision of things to come.

2. Definition

Interdimensionality, or, by another naming, dimensional shadowing [8]—the “emulation” of a lowerdimensional configuration space by a fractal subset of a higherdimensional manifold—is the (co)existence and (co)habitation of parts or fragments of an “outer” space of a “higher” extrinsic Hausdorff dimension [9] by some “inner” subspace entity that has a “lower” or equal intrinsic Hausdorff dimension. One may imagine such a situation as a fractal of Hausdorff dimension d embedded in a continuum, such as the Hilbert space \mathbb{R}^n or \mathbb{C}^n , with $d \leq n$. Therefore, pointedly speaking, we might exist on a sort of Cantor set or Menger sponge-like structure—fractals obtained by self-similar elimination of proper parts—of (almost) an integer Hausdorff dimension, which is part of a high-dimensional super-verse.

Formally, the Hausdorff dimension d of a set $A \in \mathbb{R}^n$, defined via the d -dimensional Hausdorff measure, is based on its “umklapp” property—the sudden change from measure value zero to infinity if the dimension parameter is taken higher or lower than a unique value—as follows. Suppose $\cup_i F_i$ covers A , and suppose further that there exists a limit in which all individual constituents F_i of this covering become infinitesimal in diameter. Then, the Hausdorff measure μ_d , and a unique dimensional parameter d called the Hausdorff dimension is

$$\mu_\delta(A) = \lim_{\epsilon \rightarrow 0^+} \inf_{\{F_i\}} \left\{ \sum_i (\text{diam } F_i)^\delta \mid \delta \in \mathbb{R}, \delta > 0, \cup_i F_i \supset A, (\text{diam } F_i) \leq \epsilon \right\}, \quad (1)$$

where the infimum is over all countable ϵ -covers $\{F_i\}$ of A ; with the dimension d as an “umklapp” parameter of

$$\mu_\delta(A) = \begin{cases} 0 & \text{if } \delta > d, \\ \infty & \text{if } \delta < d. \end{cases} \quad (2)$$

That is, the Hausdorff dimension d is the unique dimensional parameter at which the measure μ_δ as a function of the dimensional parameter value δ smaller or larger than d is infinite or vanishes, respectively. Note that the diameter “diam” presupposes the notion of a distance defined via a metric. For self-similar fractal sets, the capacity dimension c is defined by

$$c = \lim_{\epsilon \rightarrow 0^+} \log[n(\epsilon)] / \log(\epsilon^{-1}), \quad (3)$$

where $n(\epsilon)$ is the number of segments of length ϵ , equals the Hausdorff dimension d .

An example of a set of integer dimension m embedded into an outer space \mathbb{R}^n with $n > m$ is the set whose (contravariant) coordinates with respect to some (covariant) basis \mathbb{R}^n is given by

$$\left\{ \left(x_1, x_2, \dots, x_m, r_1(x_1, x_2, \dots, x_m), \dots, r_{n-m}(x_1, x_2, \dots, x_m) \right)^T \mid x_i, r_j \in \mathbb{R} \right\}, \quad (4)$$

where $r_i(x_1, x_2, \dots, x_m)$, $1 \leq i \leq n - m$ are some total, possibly constant or random, choice functions.

For most practical operational purposes [10,11] the intrinsic perception of the dimensionality of such shadowed, interdimensional object might effectively remain that of a “solid continuum” of that intrinsic (Hausdorff) dimension. It may not be too unreasonable

to compare this to the common notion of “emptiness of space in-between point particles” constituting solid physical objects, or the “perceived continuous motion” from individual still frames [12,13].

There are some findings consistent such speculations: For instance, associated with every integer-dimensional regular rectifiable m -dimensional fractal embedded in \mathbb{R}^n , there exists a locally defined tangential m -dimensional vector subspace of \mathbb{R}^n [9,14]. Even for non-integer-dimensional fractals, integer-dimensional tangent spaces may be “good” approximations for all practical physical purposes.

Further examples for cohabitation of continua that need not involve fractals are paradoxical decompositions, such as Vitali’s partition of the unit interval and the decomposition of the sphere by Hausdorff [15]. If we relax the definition of dimension, we may also speak of (dense) “scattered” point sets “inhabiting” the continuum. The variations may be many-fold; for instance, one may consider partitions or intertwined subsets of continua. One may not even deal with extrinsic continua but with general sets that allow some form of intrinsic embedding.

Let us finally review two almost trivial examples of an arbitrary number of one-dimensional subspaces of \mathbb{R}^2 , as schematically depicted in Figure 1. The first one is a collection of parallel lines. The second one is a star-shaped configuration intertwining in the origin, spanned by respective mutually distinct unit vectors. In the latter case, the only way for “flatlanders” [16] living on different subspaces to communicate with each other is through a single point—the origin.

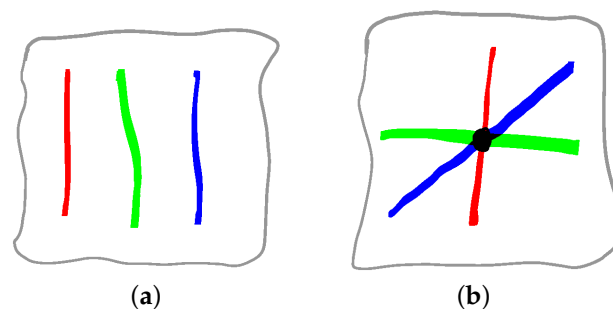


Figure 1. Schematic drawing of interdimensional configurations that are (a) isolated or (b) intertwine, as seen from some outer, embedding space.

In general, fractals need not be regular and rectifiable and of integer dimension. Rather they may be “cloud-like shapes”, with “scattered” holes and gaps. Those gaps will not be perceived intrinsically. Indeed, one may speculate that this situation gives rise to a metric that essentially mimics curvature [17].

Fractal theory has inspired and evolved into many innovative, useful, and interesting applications, especially in new materials and nanostructures. Such important developments can lead us to new views of, and physical means related to, dimensionality [18,19].

As the aim is the provision of a very general analysis that is unconstrained by the technicalities of specific models, no concrete theory is discussed. Nevertheless, it might be not too far-fetched to briefly mention some potential connections between interdimensionality and various paradigms in modern particle physics and cosmology. Some of these involve the description of a volume of space as conceptualized by holographic principles, such as the AdS/CFT correspondence related to D-branes in string theory, or the ekpyrotic models relying on string theory, branes, and extra “hidden” dimensions. Other scenarios in the context of the theory of general relativity involve traversable wormholes (aka Einstein–Rosen bridges) linking disparate points in spacetime.

3. Disjoint and Intertwining Shadows

To proceed to interdimensional motion, we need to consider intertwining areas of interdimensionality. The simplest nontrivial case is the one schematically depicted in

Figure 1b in which all universes share a single point of communication. Of greater interest might be a situation in which an entire region of space is shared. One might think also of a “small” fraction of a universe “traversing” another universe, such that, compared to the overall extension of these universes, this common share appears like the tip of an iceberg.

4. Interdimensional Motion

Interdimensional motion is the motion of some “inner” intrinsic subspace in the “outer”, extrinsic space. If two inner spaces are involved, it may happen that certain limits of motion, such as continuity or maximal speed, that are valid in one subspace, can be breached and overcome by another subspace. In what follows, some scenarios will be discussed. We shall adopt the following notation: inner “intrinsic” subspaces will be denoted by M and N .

Let us discuss this by considering a simple example of a rotating point, as schematically drawn in Figure 2a. From the point of view of M the rotation in N is observed as periodic (dis)appearances of some object rotating in M .

Another “wormhole”-like scenario schematically drawn in Figure 2b is a “bend” or “curved” (relative to the exterior “outer” continuum) reference frame M that is intermittently accessed from N . Suppose that the propagation speed limit for motion is the same $c_M = c_N$ in both frames. Then, the object appears to be traveling with a velocity greater than this limit velocity in M because of the “shortcut” access through N .

Still another scenario schematically drawn in Figure 2c is one in which N allows for faster than M -light motion—that is, $c_M \ll c_N$ —and this property is used to access regions in M through motion in N that appear space-like separated in M 's frame of reference.

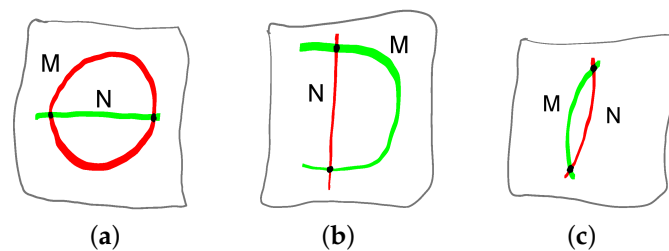


Figure 2. Schematic drawing of worldlines of interdimensional motion, as seen from the outer, embedding space: (a) periodic, (b) shortcut, and (c) coevolution.

4.1. Interdimensional Chronology Protection

In these and similar situations, no issues with respect inconsistent evolution, in particular, time paradoxes, arise. As whatever relative space–time reference frames are operationally constructed [20] in M and N , the “outer” extrinsic space, in which both M and N are embedded, regulates the phenomenology.

Indeed, from an extrinsic, “God’s eye view” of the outer space there is no consistency issue because the evolution seen from this “global” comprehensive perspective never yields or allows inconsistent phenomena. Concerns raised by intrinsic space–time frames generated with the means available in M and N are merely epistemic, and the means are relative to the devices and conventions (such as for synchronizing clocks) available to the inhabitants of M and N .

This results in an interdimensional scheme of chronology protection based on the epistemic relativity of reference frames. At the same time, from an “outer” (i.e., ontological) point of view, those frames are “bundled together” through the coembedding and cohabitation of some outer space.

There are similarities between the consistency of observable phenomena regarding the higher-dimensional bulk space and the consistent histories approach to the Many Worlds models [21]. Both involve multiple “merging” paths.

4.2. Examples of Dimensional Relativity

The following examples closely follow the scenarios schematically depicted in Figure 2b,c. They have some similarities to ballistic missiles that avoid the limitations of velocity from atmospheric drag (friction) by leaving and re-entering Earth’s atmosphere, or are analogs of supercavitation—the formation of vapor bubbles in a liquid caused by flow around an object, allowing minimal friction movement inside liquids at nearly the speed of sound.

The first example, depicted in Figure 3, shows an interdimensional dive into a dimension that allows higher velocities, or rather traversals of space per time, in M through “jump” into another dimension N , thereby, creating a shortcut from two space–time points A to B . This is different from breaking the intradimensional warp barrier by hyper-fast solitons in Einstein–Maxwell-plasma theory [22] as it employs dimensional capacities that are not bound by intradimensional motion.

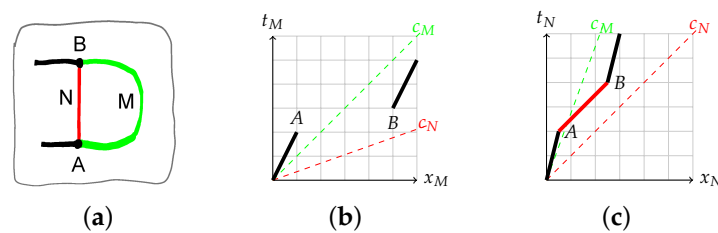


Figure 3. Schematic drawing of (a) worldlines of interdimensional “jump” motion, as seen from the outer, embedding space: (a) “dive” into N at A , reappearance at B ; (b) space–time diagram as seen from intrinsic coordinates in M ; (c) space–time diagram as seen from intrinsic coordinates in N .

The second example, depicted in Figure 4, shows an interdimensional “drag” motion that uses a dimensional motion in N whose velocity exceeds that of the normal signal velocity in M . As already mentioned, in both of these cases, consistency is guaranteed by the overall consistency in the outer embedding space.

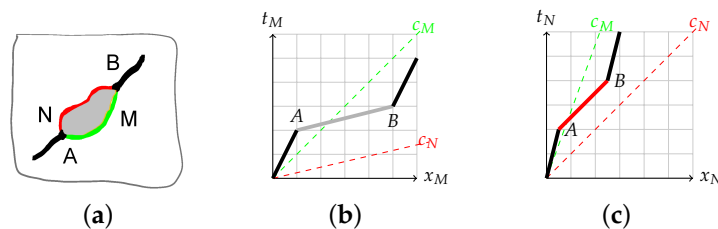


Figure 4. Schematic drawing of (a) worldlines of interdimensional forced, continuous motion, as seen from the outer, embedding space: (a) until A and from B , the motion is dominated by constraints on the velocity v_N , and between A and B , the velocity c_N dominates; (b) space–time diagram as seen from intrinsic coordinates in M ; (c) space–time diagram as seen from intrinsic coordinates in N .

5. Further Speculations

Let us conclude this article with some speculative thoughts. The first is on limits to isolating the dimensions from one another, from “keeping them apart”; in particular, in the event of some catastrophic occurrence. It may well be that the domain of dimensional intersections may increase, as such events may dominate and spread to larger parts of the “outer” space.

Secondly, interdimensionality can be compared to computer simulations, with interfaces between such universes serving as intertwining regions. The difference between virtual reality (exchanges) and (intertwining) interdimensionality is the emphasis on measure-theoretic aspects in the latter case.

The matters discussed here must be considered highly speculative, and far from a fully developed formal theory. Nevertheless, it is our conviction that, to progress, science

has to expand and explore a great variety of options, even if they appear remote to the contemporary mind.

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