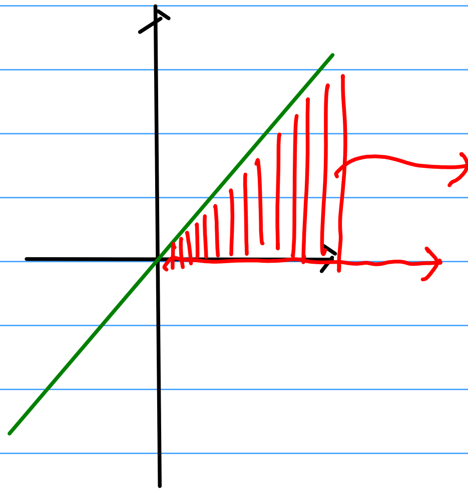
# Laplace Transform (H.1)

20160106

# Improper Integration

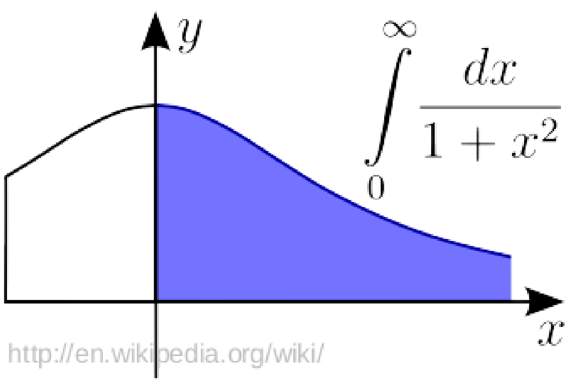
$y = x$

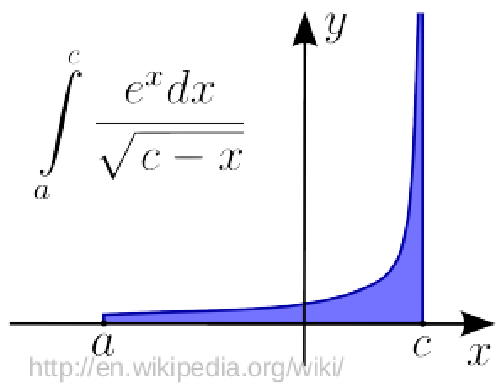


$$\int_0^{\infty} x\,dx \to \infty$$

most functions diverge

but some converge




$$\lim_{b \to +\infty} \int_a^b f(x)\,dx \Rightarrow \int_a^{+\infty} f(x)\,dx$$

$$\lim_{b \to c^-} \int_a^b f(x)\,dx \Rightarrow \int_a^{c^-} f(x)\,dx$$

$$\int_1^{+\infty} \frac{1}{x^2}\,dx = \lim_{b \to \infty} \int_1^b \frac{1}{x^2}\,dx = \lim_{b \to \infty} \left[-\frac{1}{x}\right]_1^b$$

$$= \lim_{b \to \infty} \left(-\frac{1}{b} + \frac{1}{1}\right) = 1 \quad \text{converge}$$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a \to 0^+} \int_a^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a \to 0^+} \left[2\sqrt{x}\right]_a^1$$

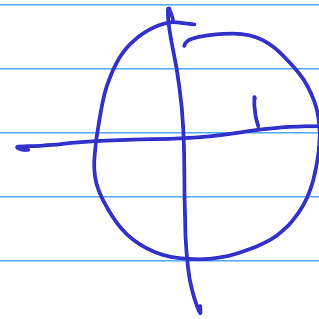
$$= \lim_{a \to 0^+} \left(2 - 2\sqrt{b}\right) = 2 \quad \text{converge}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{\sqrt{x}} = \infty \qquad f(0)$$

$s > 0$

$e^{-st}$

$s = \sigma + i\omega$




$e^{-(\sigma + i\omega)t}$

$= e^{-\sigma t} e^{-i\omega t}$

$\cos(\omega b) - i \sin(\omega$

$|e^{-i\omega b}| = 1$

# Sec 4.1 Laplace Transform Definition

ex1) $\mathcal{L}\{1\}$ $\quad 1 \longleftrightarrow \frac{1}{s}$ $\quad (s>0)$

ex2) $\mathcal{L}\{t\}$ $\quad t \longleftrightarrow \frac{1}{s^2}$ $\quad (s>0)$

ex3) (a) $\mathcal{L}\{e^{-3t}\}$ $\quad e^{-3t} \longleftrightarrow \frac{1}{s+3}$ $\quad (s>-3)$

(b) $\mathcal{L}\{e^{6t}\}$ $\quad e^{6t} \longleftrightarrow \frac{1}{s-6}$ $\quad (s>6)$

ex4) $\mathcal{L}\{\sin 2t\}$ $\quad \sin 2t \longleftrightarrow \frac{2}{s+2^2}$ $\quad (s>0)$

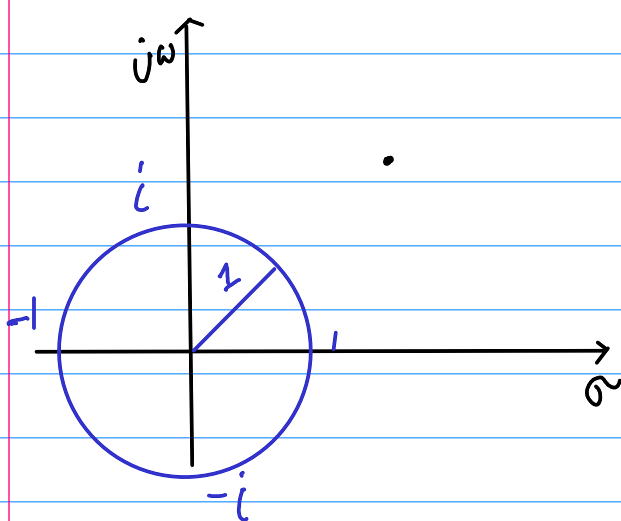
(ROC

Region of Convergence)

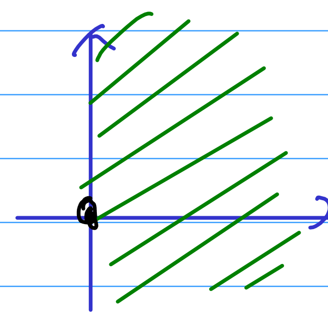
$$F(s) = \int_0^{\infty} 1 \cdot e^{-st}\, dt = \lim_{b\to\infty}\left[-\frac{1}{s}e^{-st}\right]_0^b = \lim_{b\to\infty}\left[-\frac{1}{s}e^{-sb} + \frac{1}{s}e^{-s0}\right]$$

$$-s < 0 \Rightarrow \lim_{b\to\infty} e^{-sb} = 0 \qquad \boxed{s > 0} \Rightarrow F(s) = \frac{1}{s}$$

$s = \sigma + j\omega$ complex no




$s > 0$



$$e^{-st} = e^{-(\sigma + j\omega)t}$$

$$= e^{-\sigma t} \cdot \boxed{e^{-j\omega t}}$$

$$= e^{-\sigma t} \cdot \boxed{\cos(\omega t) + i \sin(\omega t)}$$

$$e^{-s\infty} = \lim_{b\to\infty} e^{-sb} =$$

$$= \lim_{b\to\infty} \frac{1}{e^{sb}} \to 0$$

$$|e^{-st}| = |e^{\sigma t}|\, \underbrace{|e^{-j\omega t}|}_{1} = |e^{-\sigma t}|$$

$$f(t) \longleftrightarrow F(s)$$

도함수 $f'(t) \longleftrightarrow \boxed{?}$

$$f(t) \longleftrightarrow F(s)$$

$\boxed{?} \longleftrightarrow F'(s)$ 변환의 도함수

$$e^{+at} \Longleftrightarrow \frac{1}{s-a}$$

$$e^{-at} \Longleftrightarrow \frac{1}{s+a}$$

$$e^{+jkt} \Longleftrightarrow \frac{1}{s-jk}$$

$$e^{-jkt} \Longleftrightarrow \frac{1}{s+jk}$$

$$\frac{1}{2}\left(e^{+jkt} + e^{-jkt}\right) \Longleftrightarrow \frac{1}{2}\left(\frac{1}{s-jk} + \frac{1}{s+jk}\right)$$

$$\int_0^\infty \frac{1}{2}\left(e^{+jkt} + e^{-jkt}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\left(\frac{1}{s-jk} + \frac{1}{s+jk}\right)$$

$$= \frac{1}{2}\left(\frac{s+\cancel{jk} + s-\cancel{jk}}{(s-jk)(s+jk)}\right) = \frac{s}{s^2+k^2}$$

$$(s+jk)(s-jk) = \boxed{s}^2 - \boxed{jk}^2$$

$$= s^2 - \boxed{-k^2}$$

$$= s^2 + k^2$$

$$1 \Longleftrightarrow \frac{1}{s}$$

$$e^{-at} \Longleftrightarrow \frac{1}{s+a}$$

$$\cos(kt) \Longleftrightarrow \frac{s}{s^2+k^2}$$

$$\sin(kt) \Longleftrightarrow \frac{k}{s^2+k^2}$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a} \qquad \mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$$

$$\mathcal{L}\{\cos(kt)\} = \frac{s}{s^2+k^2} \qquad \mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos kt$$

$$\mathcal{L}\{\sin(kt)\} = \frac{k}{s^2+k^2} \qquad \mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin kt$$

$$\cos(kt) \longleftrightarrow \frac{s}{s^2 + k^2}$$

$$\sin(kt) \longleftrightarrow \frac{k}{s^2 + k^2}$$

$$\cosh(kt) \longleftrightarrow \frac{s}{s^2 - k^2}$$

$$\sinh(kt) \longleftrightarrow \frac{k}{s^2 - k^2}$$

$$e^{at}\cos(kt) \longleftrightarrow \frac{(s-a)}{(s-a)^2 + k^2}$$

$$e^{at}\sin(kt) \longleftrightarrow \frac{k}{(s-a)^2 + k^2}$$

$$e^{at}\cosh(kt) \longleftrightarrow \frac{(s-a)}{(s-a)^2 - k^2}$$

$$e^{at}\sinh(kt) \longleftrightarrow \frac{k}{(s-a)^2 - k^2}$$

$$f(t) \Longleftrightarrow F(s)$$

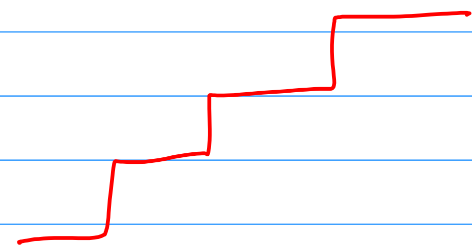
$$f'(t) \Longleftrightarrow \boxed{?}$$

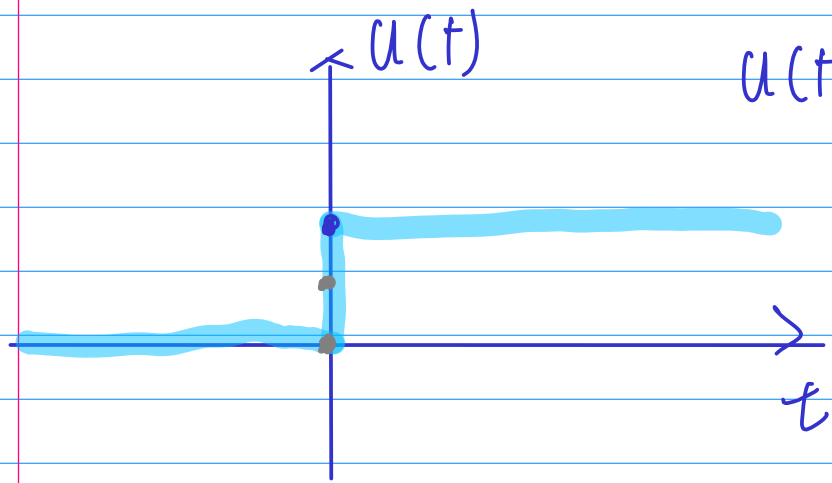
# Unit Step Function $\underline{u(t)}$

$u(t)$ unit step function

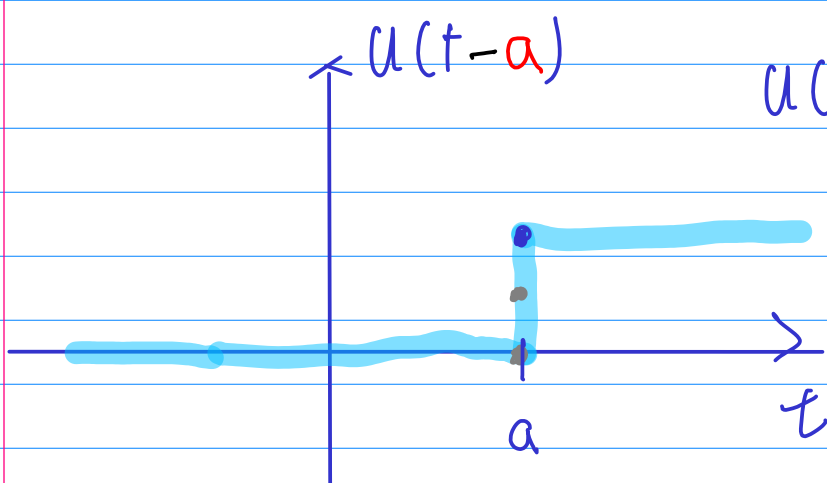
unit $\rightarrow$ 1

step $\leftarrow$ 계단

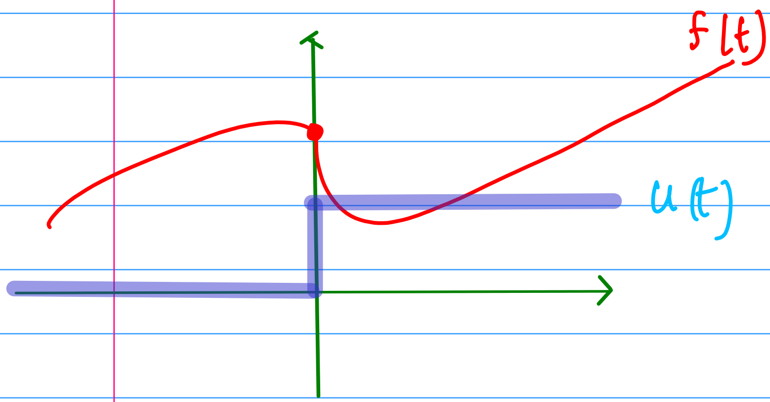


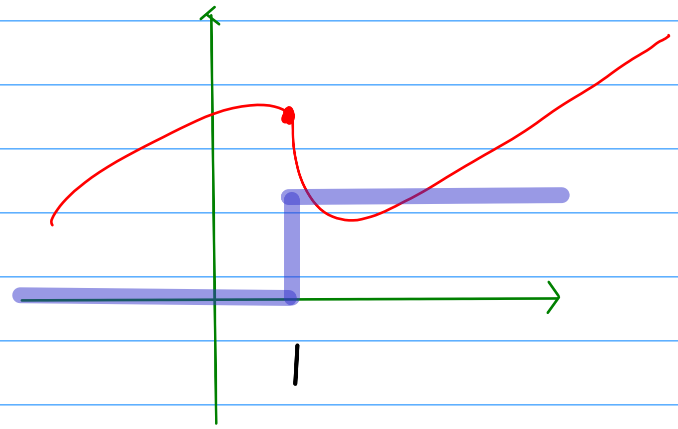



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

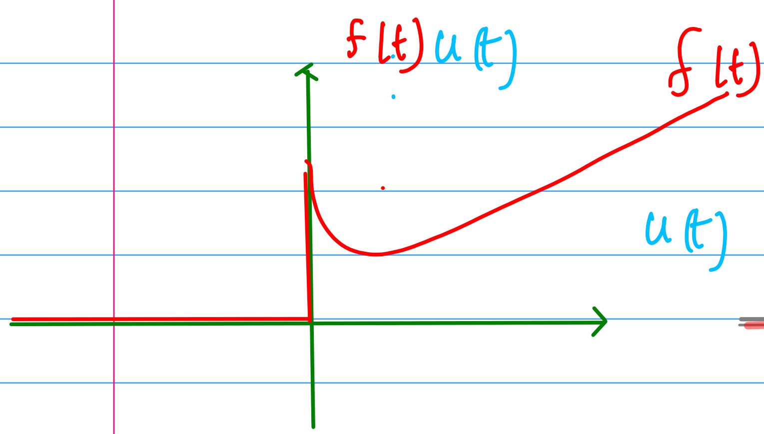


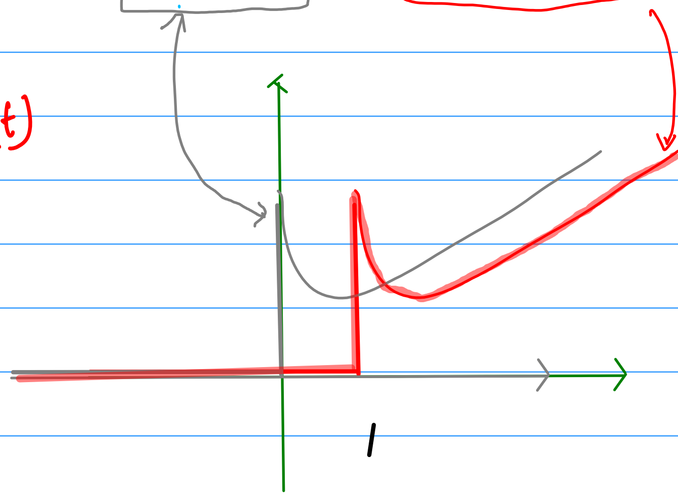

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$




$\boxed{f(t)\,u(t)} \longrightarrow \boxed{f(t-1)\,u(t-1)}$




$f(t) \cdot u(t) = ?$

$f(t) \cdot u(t) = ?$

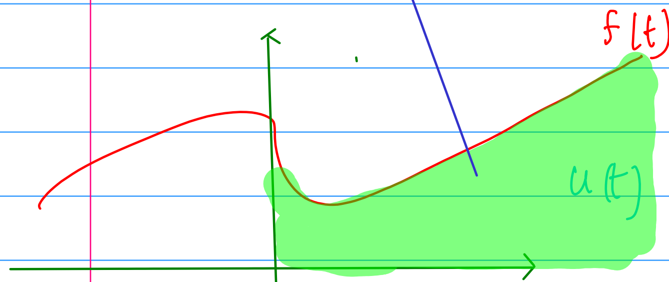
$\boxed{f(t-1)\,u(t-1)}$

shif to the right by 1

$\boxed{f(t)\,u(t)}$

$t \Leftarrow t-1$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

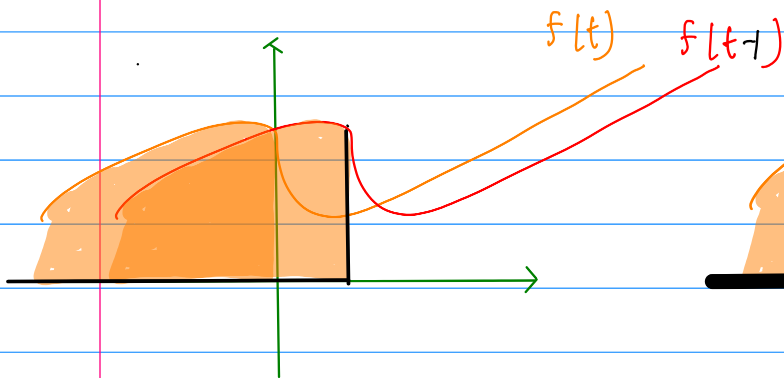


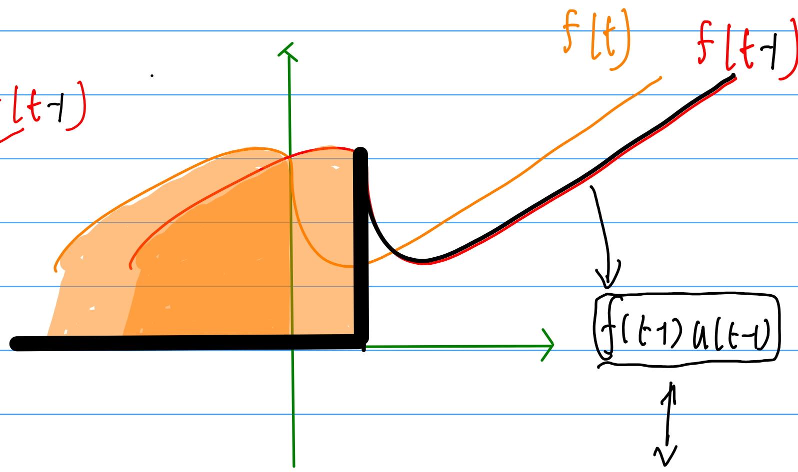

$e^{-s} F(s)$

$(-1)$ : time shift $f(t)\,u(t)$ to the right by $(+1)$

$\cancel{f(t-1)}$

$\underline{f(t-1)}\ \underline{u(t-1)}$




$\boxed{f(t-1)\,u(t-1)}$

$\updownarrow$

$e^{-s} F(s)$

$$f(t) \longleftrightarrow F(s)$$

$$f(t-a) \not\longleftrightarrow e^{-as} F(s)$$

$$f(t-a)\,\underline{u(t-a)} \longleftrightarrow e^{-as} F(s)$$

$$e^{+at} f(t) \longleftrightarrow F(s-a)$$

$$f(t) \longleftrightarrow F(s)$$

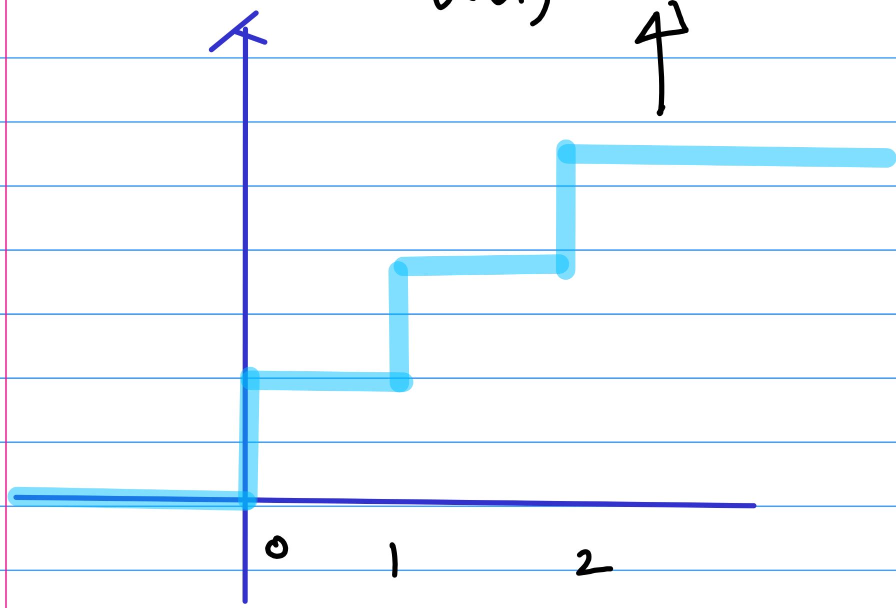
translation in
s domain

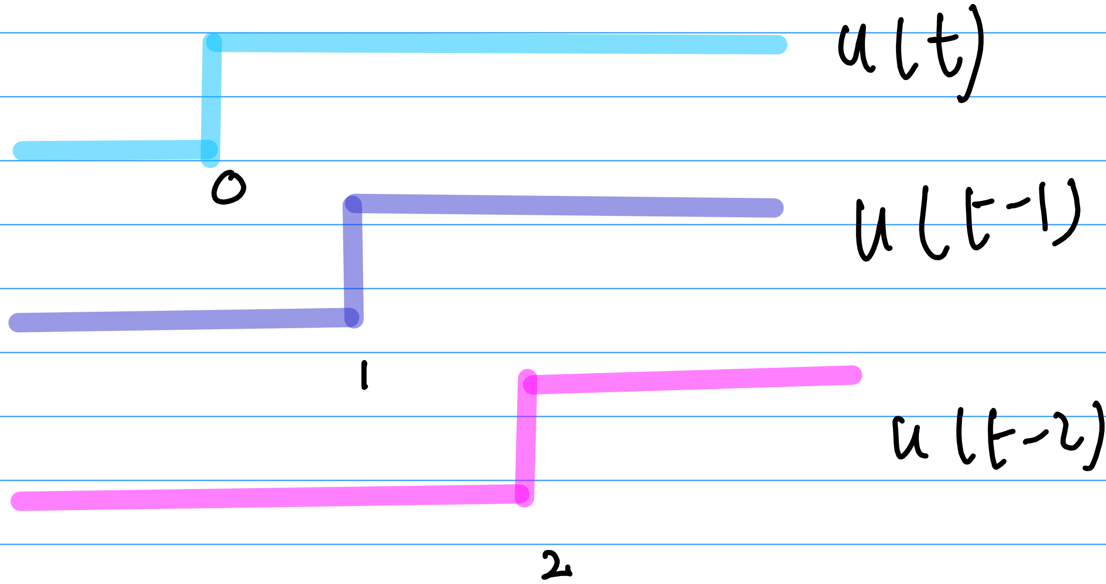
$$e^{at} f(t) \longleftrightarrow F(s-a) \qquad s \Leftarrow s-a$$

$$t \Leftarrow t-a \qquad f(t-a)\,u(t-a) \longleftrightarrow e^{-as} F(s)$$

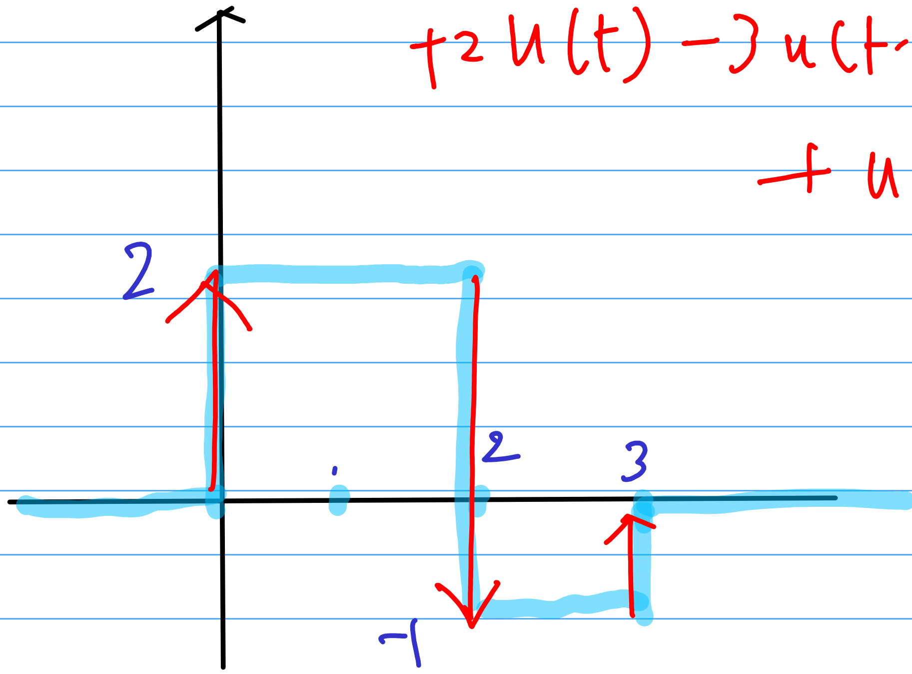
translation
in t-domain

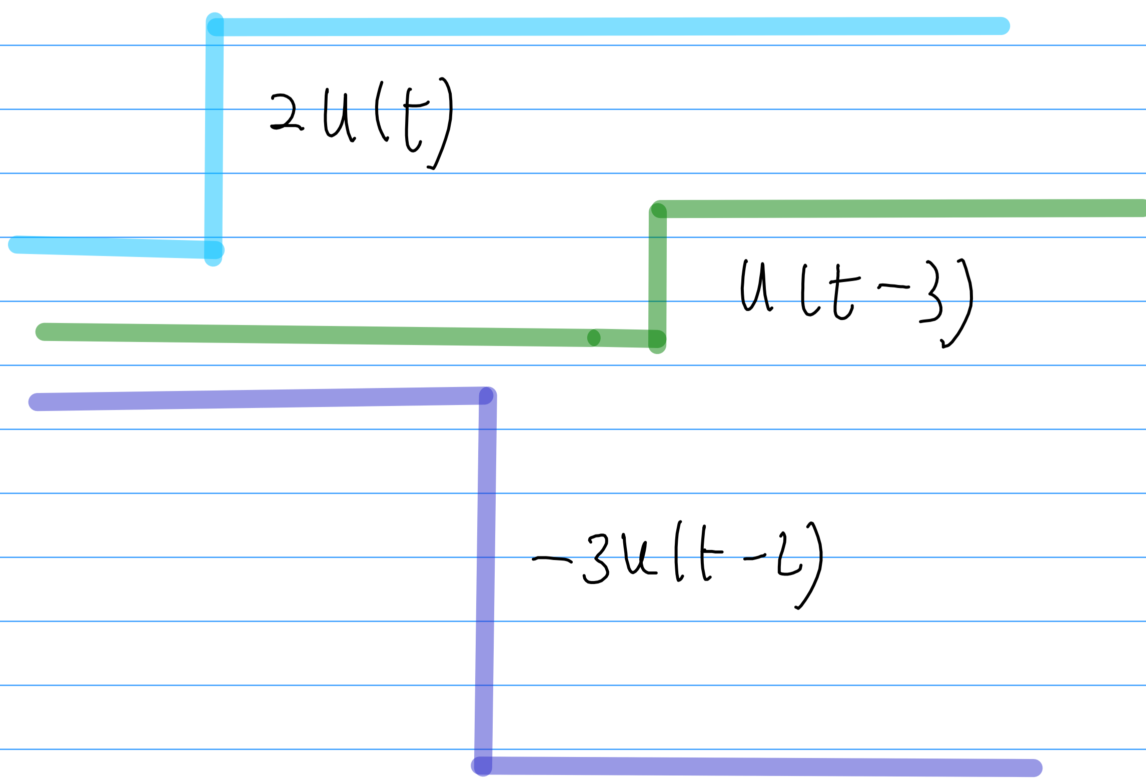
$u(t)+u(t-1)+u(t-2)$



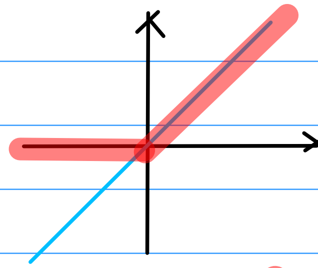



$+2u(t) - 3u(t-2)$
$+ u(t-3)$



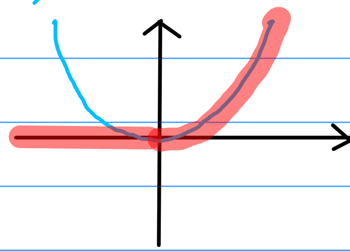



$t \cdot u(t)$



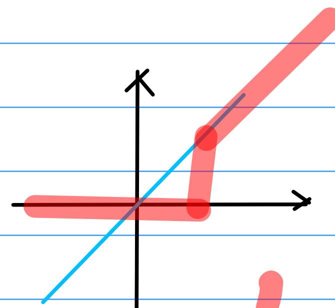
$\dfrac{1}{s^2}$

$t^2 \cdot u(t)$

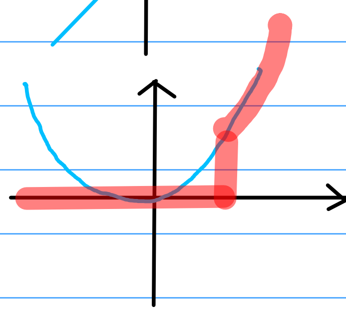


$\dfrac{2}{s^3}$

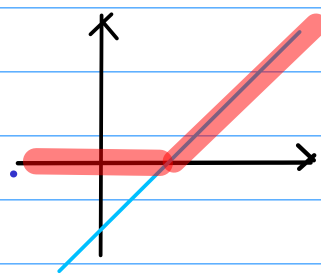
$t \; u(t-1)$



$t^2 \; u(t-1)$

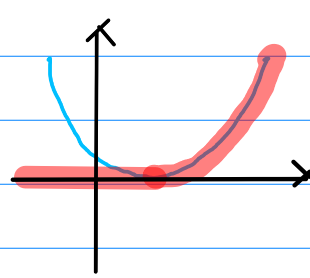


$(t-1) \cdot u(t-1)$



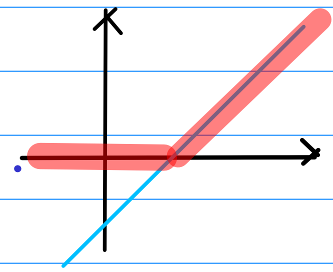
$e^{-s} \dfrac{1}{s^2}$

$(t-1)^2 \cdot u(t-1)$



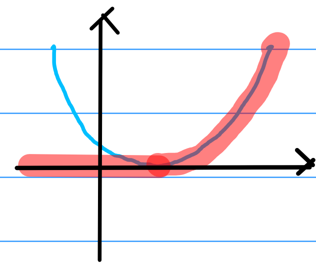
$e^{-s} \dfrac{2}{s^3}$

$(t-1)\cdot u(t-1)$



$e^{-s}\dfrac{1}{s^2}$

$(t-1)^2\cdot u(t-1)$

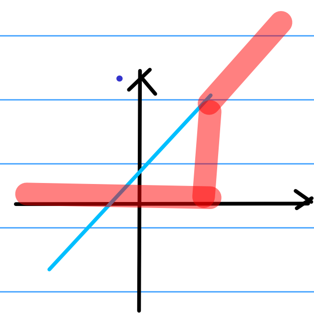


$e^{-s}\dfrac{2}{s^3}$

$t\cdot u(t-1)$

$= (t-1)\cdot u(t-1)$

$+ u(t-1)$



$e^{-s}\dfrac{1}{s^2} +$

$e^{-s}\dfrac{1}{s}$

$t^2 u(t-1)$

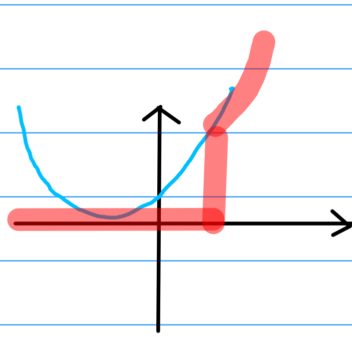
$= (t-1)^2 u(t-1)$

$+2t\,u(t-1) - u(t-1)$

$= (t-1)^2 u(t-1)$

$+2(t-1)u(t-1)$

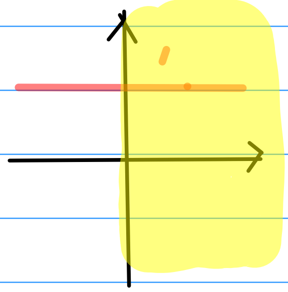
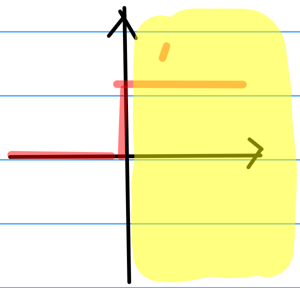
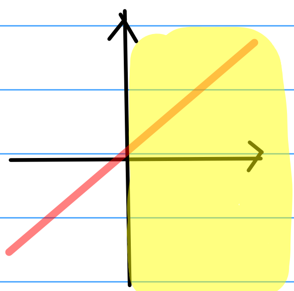
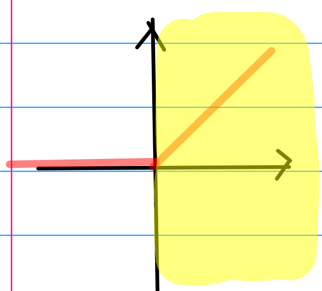
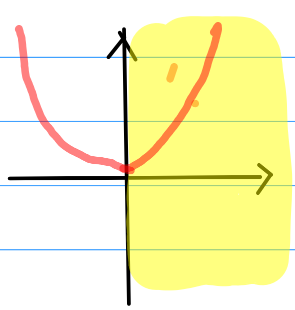
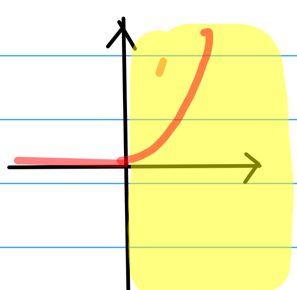
$+ u(t-1)$



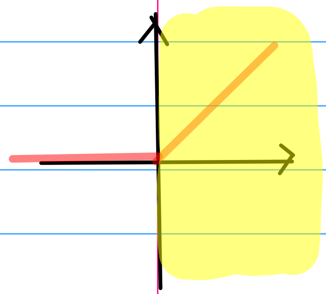
$e^{-s}\dfrac{2}{s^3} +$

$e^{-s}\dfrac{2}{s^2} +$

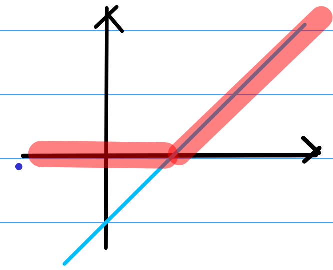
$e^{-s}\dfrac{1}{s}$

| | $f(t)$ | $F(s)$ |
|---|---|---|
|  | $1$ | $\frac{1}{s}$ |
|  | $u(t)$ | $\frac{1}{s}$ |
|  | $t$ | $\frac{1}{s^2}$ |
|  | $t\,u(t)$ | $\frac{1}{s^2}$ |
|  | $t^2$ | $\frac{2}{s^3}$ |
|  | $t^2 u(t)$ | $\frac{2}{s^3}$ |

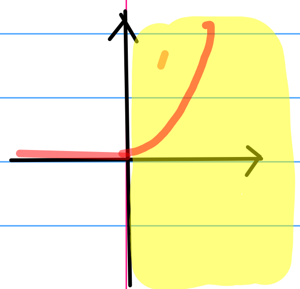
$t \leftarrow t-1$ time shift by $+1$



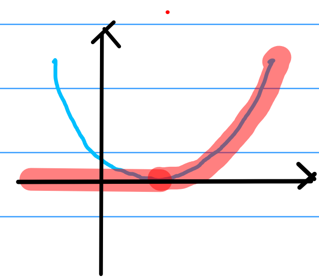
$t\,u(t)$ $\quad \frac{1}{s^2}$



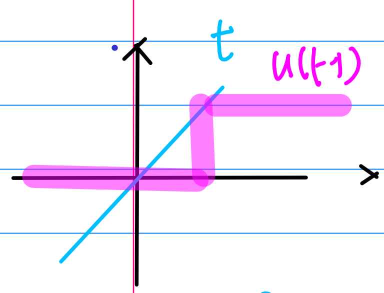
$(t-1)\,u(t-1)$ $\quad e^{-s}\frac{1}{s^2}$

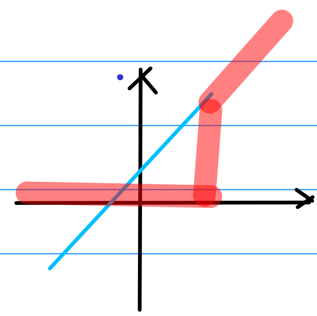


$t^2 u(t)$ $\quad \frac{2}{s^3}$

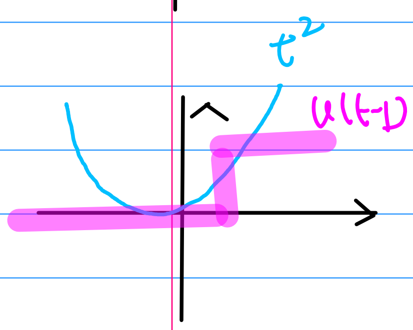


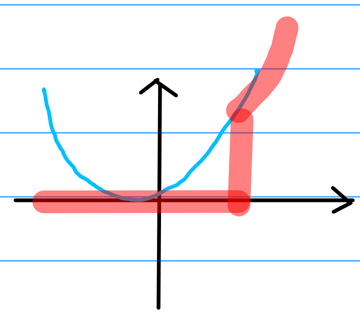
$(t-1)^2\,u(t-1)$ $\quad e^{-s}\frac{2}{s^3}$



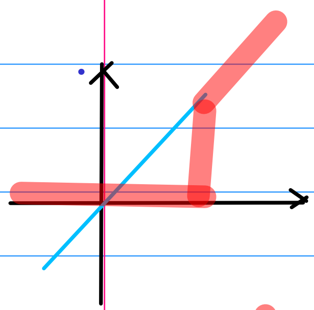



$t\,u(t-1)$



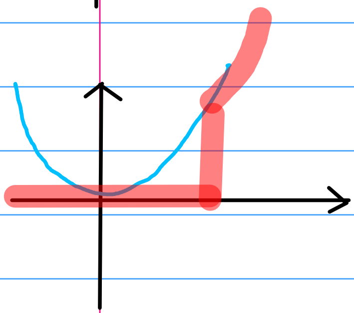



$t^2 u(t-1)$



$$t\,u(t-1) \Rightarrow (t-1)u(t-1) + u(t-1)$$

$$e^{-s}\frac{1}{s^2} + \frac{1}{s}$$

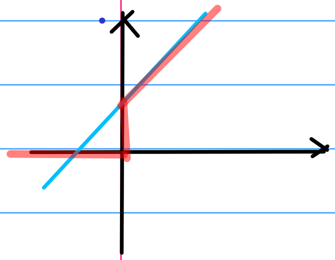


$$t^2 u(t-1) \Rightarrow (t-1)^2 u(t-1) + (2t-1)u(t-1)$$

$$\Rightarrow (t-1)^2 u(t-1) + \qquad e^{-s}\frac{2}{s^3} +$$

$$2(t-1)\,u(t-1) + \qquad e^{-s}\frac{2}{s^2} +$$

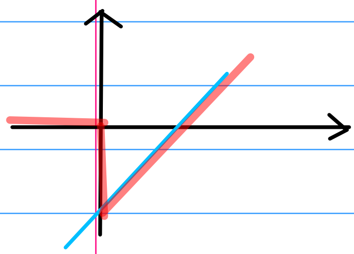
$$u(t-1) \qquad e^{-s}\frac{1}{s}$$



$(t+1)u(t)$

$(t+1)$

$\frac{1}{s^2} + \frac{1}{s}$



$(t-1)u(t)$

$(t-1)$

$\frac{1}{s^2} - \frac{1}{s}$

$$\frac{s+4}{s^2-3s-10} = \frac{s+7}{s^2-3s+\frac{9}{4}-\frac{9}{4}-10} = \frac{s-\frac{3}{2}+\frac{3}{2}+7}{(s-\frac{3}{2})^2-\frac{49}{4}}$$

$$= \frac{(s-\frac{3}{2})+\frac{17}{2}}{(s-\frac{3}{2})^2-(\frac{7}{2})^2}$$

$$\frac{s}{s^2-(\frac{7}{2})^2} \Longleftrightarrow \cosh(\tfrac{7}{2}t)$$

$$\frac{(\frac{7}{2})}{s^2-(\frac{7}{2})^2} \Longleftrightarrow \sinh(\tfrac{7}{2}t)$$

$$\frac{(s-\frac{3}{2})}{(s-\frac{3}{2})^2-(\frac{7}{2})^2} \Longleftrightarrow e^{\frac{3}{2}t}\cdot\cosh(\tfrac{7}{2}t)$$

$$\frac{\frac{17}{7}\,\frac{7}{2}}{(s-\frac{3}{2})^2-(\frac{7}{2})^2} \Longleftrightarrow e^{\frac{3}{2}t}\cdot\sinh(\tfrac{7}{2}t)$$

$$e^{\frac{3}{2}t}\cdot\cosh(\tfrac{7}{2}t) = e^{\frac{3}{2}t}\cdot\frac{1}{2}\left(e^{\frac{7}{2}t}+e^{-\frac{7}{2}t}\right)$$

$$e^{\frac{3}{2}t}\cdot\sinh(\tfrac{7}{2}t) = e^{\frac{3}{2}t}\cdot\frac{1}{2}\left(e^{\frac{7}{2}t}-e^{-\frac{7}{2}t}\right)$$

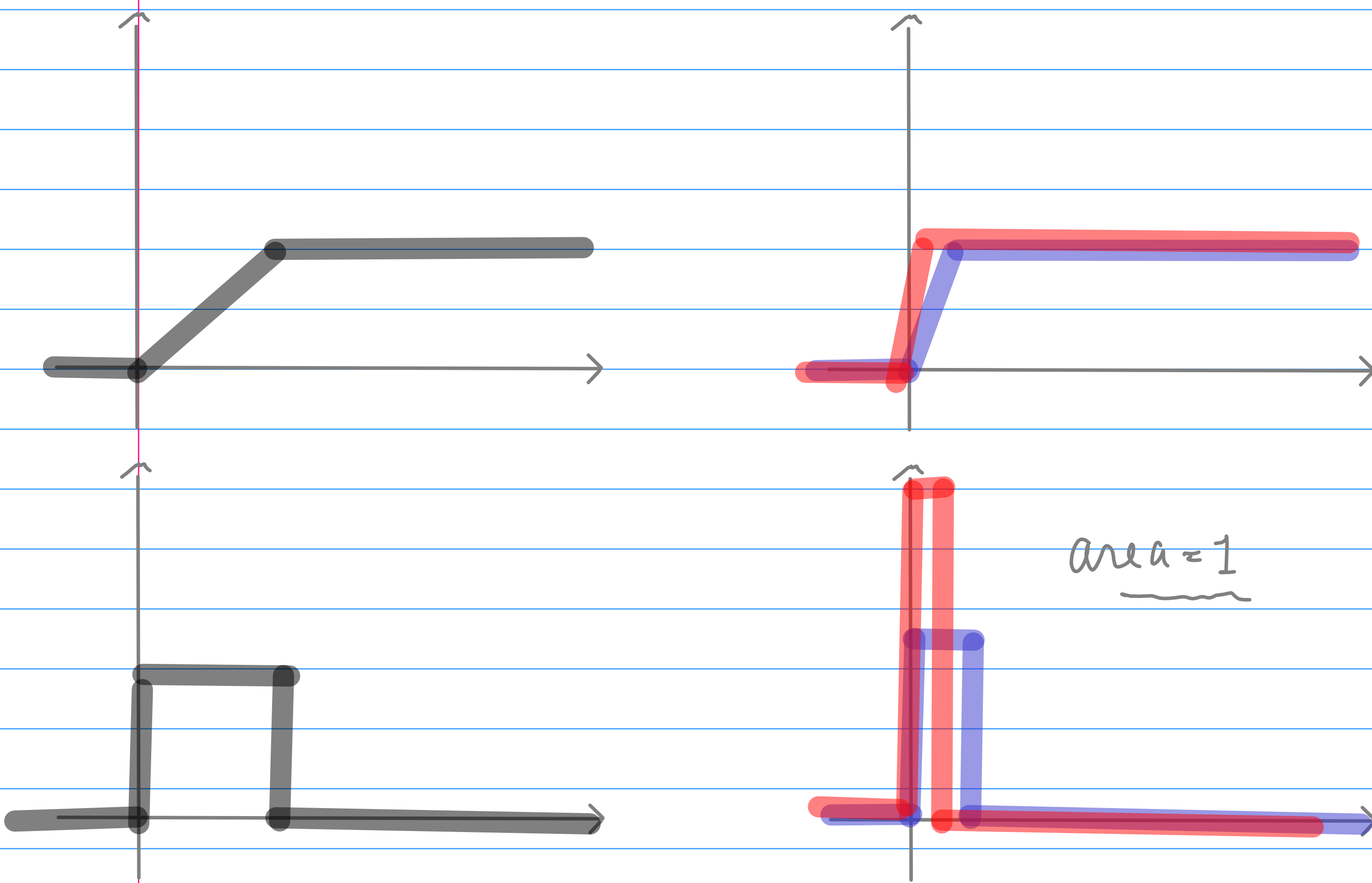
$-\frac{5}{7}e^{-2t} + \frac{12}{7}e^{5t}$

$$e^{\frac{3}{2}t} \cdot \frac{1}{2}\left(e^{\frac{7}{2}t} + e^{-\frac{7}{2}t}\right) \Rightarrow \frac{1}{2}\left(e^{5t} + e^{-2t}\right)$$

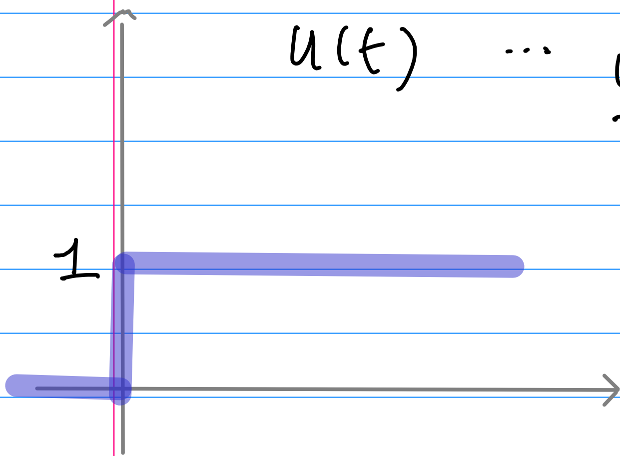
$$e^{\frac{3}{2}t} \cdot \frac{1}{2}\left(e^{\frac{7}{2}t} - e^{-\frac{7}{2}t}\right) \Rightarrow \frac{1}{2}\left(e^{5t} - e^{-2t}\right) \times \frac{17}{7}$$

$$\frac{1}{2}\left(\left(1+\frac{17}{7}\right) \qquad \left(1-\frac{17}{7}\right)\right.$$

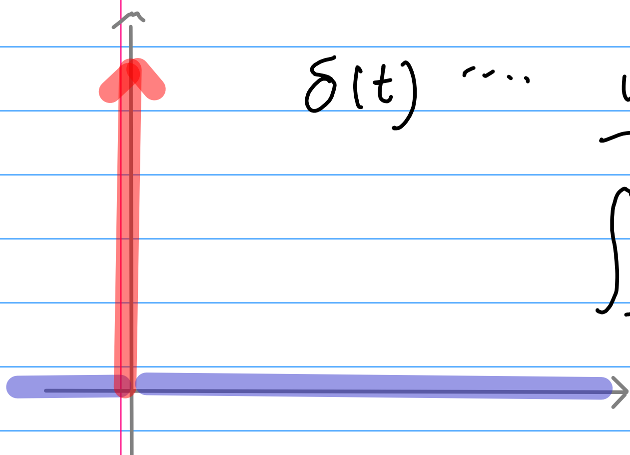
$$\frac{24}{7} \qquad \frac{-10}{7}$$


area = 1

$u(t)$ ... unit step fn




$\delta(t)$ ... unit impulse fn

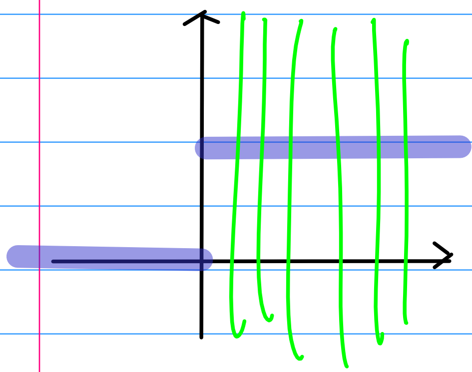


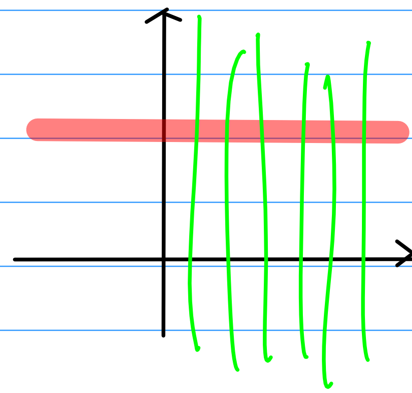
$$\int_{-\infty}^{+\infty} \delta(t)\,dt = \int_{0-}^{0+} \delta(t)\,dt = 1$$

$$u(t) \xrightarrow{\frac{d}{dt}} \delta(t)$$

$$u(t) \xleftarrow[\int_{-\infty}^{t} \cdot \, dt]{} \delta(t)$$

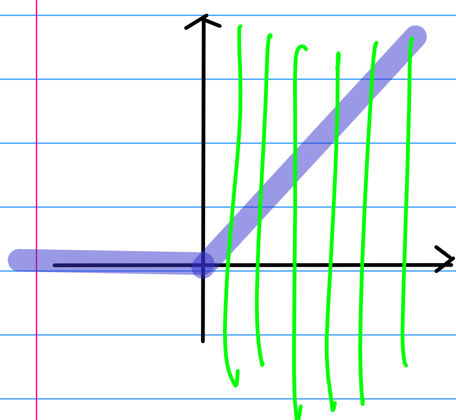
$f(t) = u(t) \neq f(t) = 1$





$F(s) = \frac{1}{s}$

$f(t) = t \cdot u(t) \neq f(t) = t$



$F(s) = \frac{1}{s^2}$

$2(t-6)^3 u(t-6) \quad -(7-e^{12-3t})u(t-4)$

$$t^3 \cdot u(t) \Longleftrightarrow \frac{3!}{s^4}$$

$$(t-6)^3 u(t-6) \Longleftrightarrow e^{-6s} \cdot \frac{3!}{s^4}$$

$$-(7-e^{12-3t})u(t-4) \qquad -3(t-4)$$

$$-(7-e^{-3(t-4)})u(t-4)$$

$$-7u(t-4) + e^{-3(t-4)} \cdot u(t-4)$$

$$e^{-3t}u(t) \Longleftrightarrow \frac{1}{s+3}$$

$$\frac{1}{s+3} e^{-4s}$$

$\times\ \boxed{y(3),\quad y'(3)}$

$$t\ \cancel{f(t)} \iff -\frac{d}{ds}\ \cancel{F(s)}$$

$$f(t) \iff F(s)$$

$$-t\ f(t) \iff \frac{d}{ds}\left(F(s)\right)$$

$$\| \qquad\qquad\qquad \Downarrow$$

$$g'(t) \qquad\qquad \boxed{\ \cdot\ }$$

$$g'(t) \iff \boxed{sG(s) - g(0)}$$

$$g(t) \iff G(s)$$

Definition

$$\int_0^{\infty} f(t) e^{-st} \, dt$$

$\left( f(t) \quad t \geqq 0 \right.$ is used $\left. \right)$

$\left( f(t) \quad t < 0 \right.$ is not used $\left. \right)$

Improper Integration $\displaystyle\int_0^{\infty} dt < \infty$

$\boxed{s > a}$ ←···· Region of Convergence

ROC

Definite Integration literal $t$ is vanished

literal $s$

$s$'s expression ····→ $F(s)$

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

$$\boxed{F(s)} = \int_0^\infty \boxed{f(t)}e^{-st}\,dt$$

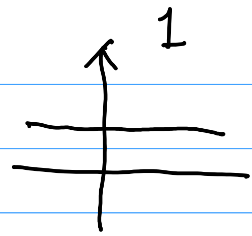
$\boxed{f(t)} \rightarrow \boxed{F(s)}$ Forward Laplace Transform

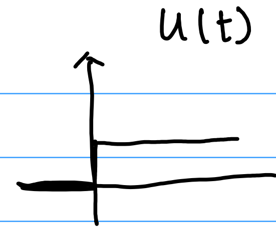
$$\mathcal{L}\{\boxed{f(t)}\} = \boxed{F(s)}$$

$\boxed{f(t)} \leftarrow \boxed{F(s)}$ Inverse Laplace Transform

$$\mathcal{L}^{-1}\{\boxed{F(s)}\} = \boxed{f(t)}$$

$f(t) = 1$




$$\int_0^\infty 1\cdot e^{-st}\,dt = \lim_{b\to\infty}\left[-\frac{1}{s}e^{-st}\right]_0^b$$

$$= \lim_{b\to\infty}\left[-\frac{1}{s}\left(e^{-sb} - e^{-s0}\right)\right]$$

$$= \lim_{b\to\infty}\left[-\frac{1}{s}\left(e^{-sb} - 1\right)\right]$$

converge

$$\frac{1}{s}$$

$|e^{-s}| < 1$ $\qquad |e^{-s}|^b \ll 1$ $\qquad \lim_{b\to\infty} e^{-sb} = 0$

$$|e^{-(x+iy)}| < 1$$

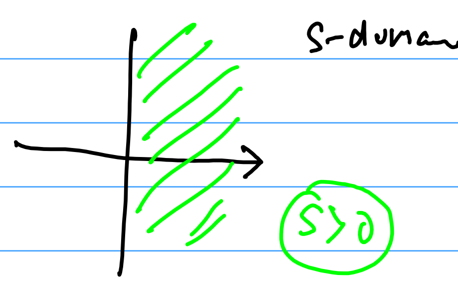
$$|e^{-x}|\,\underbrace{|e^{-iy}|}_{=1} < 1$$

$$e^{-iy} = \cos(y) - i\sin(y)$$

$$|e^{-iy}| = \sqrt{\cos^2 y + \sin^2 y} = 1$$

$$|e^{-x}| < 1$$

$$x > 0 \equiv s > 0$$



$$\boxed{F(s)} = \int_0^\infty \boxed{f(t)}\, e^{-st}\, dt$$

S-domain translation $\Downarrow$

$$F(s-a) = \int_0^\infty f(t)\, e^{-(s-a)t}\, dt$$

$$\boxed{F(s-a)} = \int_0^\infty \boxed{f(t)\, e^{+at}}\, e^{-st}\, dt$$

$$\boxed{f(t)} \longleftrightarrow \boxed{F(s)} \qquad 1 \longleftrightarrow \frac{1}{s}$$

$$\boxed{f(t)\, e^{+at}} \longleftrightarrow \boxed{F(s-a)} \qquad e^{+at} \longleftrightarrow \frac{1}{s-a}$$

$$e^{-at} \longleftrightarrow \frac{1}{s+a}$$

$$\boxed{F(s)} = \int_0^\infty \boxed{f(t)} e^{-st}\, dt$$

S-domain differentiation $\Downarrow$

$\frac{d}{ds} F(s)$ $\boxed{F'(s)}$ $= \frac{\partial}{\partial s} \int_0^\infty f(t) e^{-st}\, dt$

$$= \int_0^\infty f(t) \frac{\partial}{\partial s}\left[e^{-st}\right] dt$$

$$\boxed{F'(s)} = \int_0^\infty \boxed{f(t)\cdot(-t)} \cdot e^{-st}\, dt$$

$\boxed{f(t)} \longleftrightarrow \boxed{F(s)}$

$\boxed{f(t)\cdot(-t)} \longleftrightarrow \boxed{F'(s)}$

$1 \longleftrightarrow \frac{1}{s}$

$(-t)\cdot 1 \longleftrightarrow -\frac{1}{s^2}$

$(-t)^2\cdot 1 \longleftrightarrow \frac{2}{s^3}$

$(-t)^2\cdot 1 \longleftrightarrow \frac{6}{s^4}$

$t \longleftrightarrow \frac{1}{s^2}$

$t^2 \longleftrightarrow \frac{2}{s^3}$

$t^? \longleftrightarrow \frac{6}{s^4}$

t-domain differentiation

$\boxed{f(t)} \longleftrightarrow \boxed{F(s)}$

$\boxed{f'(t)} \longleftrightarrow \boxed{s\,F(s) - f(0)}$

$\boxed{f''(t)} \longleftrightarrow \boxed{s^2 F(s) - s\,f(0) - f'(0)}$

t-domain

translation

$\boxed{f(t)} \longleftrightarrow \boxed{F(s)}$

$\boxed{f(t-a)\,u(t-a)} \longleftrightarrow \boxed{e^{-as}F(s)}$

$$\cos(kt) = \frac{1}{2}\left(e^{+jkt} + e^{-jkt}\right)$$

$$\updownarrow \qquad \updownarrow$$

$$\frac{1}{2}\left(\frac{1}{s-jk} + \frac{1}{s+jk}\right) = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}s}{(s-jk)(s+jk)}\right)$$

$$= \frac{s}{s^2+k^2}$$

$$\cos(kt) \longleftrightarrow \frac{s}{s^2+k^2}$$

$$\sin(kt) \longleftrightarrow \frac{k}{s^2+k^2}$$

$$\cosh(kt) \longleftrightarrow \frac{s}{s^2-k^2}$$

$$\sinh(kt) \longleftrightarrow \frac{k}{s^2-k^2}$$

Convolution

$$y(t) = x * h = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$x(t) = 0 \qquad t \leq 0$

$h(t) = 0 \qquad t \leq 0$

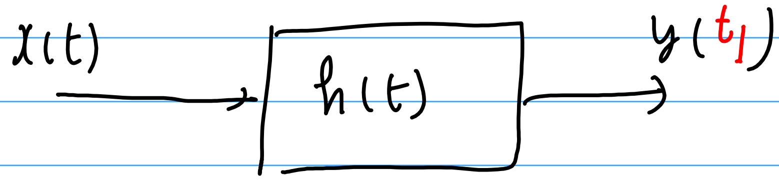
$$y(t) = x * h = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau) = 0, \qquad \tau < 0$

$$= \int_{0}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$h(t-\tau) = 0 \qquad t-\tau < 0 \qquad t < \tau$

$$y(t) = x * h = \int_{0}^{t} x(\tau)\, h(t-\tau)\, d\tau$$




$$y(t_1) = x * h = \int_0^{t_1} x(\tau)\, h(t_1 - \tau)\, d\tau$$

$$x(t) \longleftrightarrow X(s)$$

$$h(t) \longleftrightarrow H(s)$$

$$x(t) * h(t) \longleftrightarrow X(s) \cdot H(s)$$