Laurent Series and z-Transform
- Geometric Series
Time Shift B

20181029 Tue

(Causal / Anti-Causal) forms of $\left(\frac{1}{z-p} , \frac{1}{z^{-1}-p} \right) * p$

2 formulas Ⓐ

$$\boxed{\frac{1}{z-p}} \xleftrightarrow{z^{-1}} \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each

$\boxed{\frac{1}{z-p}}$

- Causal: $\boxed{-\frac{p^{-1}}{1-p^{-1}z}}$
- Anti-Causal: $\boxed{\frac{z^{-1}}{1-pz^{-1}}}$

$\boxed{\frac{1}{z^{-1}-p}}$

- Anti-Causal: $\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$
- Causal: $\boxed{\frac{z}{1-pz}}$

Simple Pole Form | Geometric Series Form

2 formulas Ⓑ

$$\boxed{\frac{p}{z-p}} \xleftrightarrow{z^{-1}} \boxed{\frac{p}{z^{-1}-p}}$$

2 representations each

$\boxed{\frac{p}{z-p}}$

- Causal: $\boxed{-\frac{1}{1-p^{-1}z}}$
- Anti-Causal: $\boxed{\frac{pz^{-1}}{1-pz^{-1}}}$

$\boxed{\frac{p}{z^{-1}-p}}$

- Anti-Causal: $\boxed{-\frac{1}{1-p^{-1}z^{-1}}}$
- Causal: $\boxed{\frac{pz}{1-pz}}$

Simple Pole Form | Geometric Series Form

$$\xrightarrow{p}$$

$$\xleftarrow[1/p]{}$$

Causal signal a_n $n \geq 0$

anti-causal signal a_n $n < 0$

Laurent Series $f(z)$

z - Transform $X(z)$

$$a_n \begin{pmatrix} \cdots & -3 & -2 & -1 & 0 & +1 & +2 & +3 & \cdots \\ \cdots & -3 & -2 & -1 & 0 & +1 & +2 & +3 & \cdots \\ \cdots & -3 & -2 & -1 & 0 & +1 & +2 & +3 & \cdots \end{pmatrix} \qquad f(z) \qquad X(z) \qquad \begin{pmatrix} p=2 \\ p=0.5 \end{pmatrix}$$

① $(n \geq 0)$ $\quad a_n = \left(\frac{1}{2}\right)^n \qquad f(z) = \frac{2}{2-z} \qquad X(z) = \frac{z}{z-0.5}$

② $(n \geq 0)$ $\quad a_n = (2)^n \qquad f(z) = \frac{0.5}{0.5-z} \qquad X(z) = \frac{z}{z-2}$

③ $(n < 0)$ $\quad a_n = \left(\frac{1}{2}\right)^n \qquad f(z) = -\frac{2}{2-z} \qquad X(z) = -\frac{z}{z-0.5}$

④ $(n < 0)$ $\quad a_n = (2)^n \qquad f(z) = -\frac{0.5}{0.5-z} \qquad X(z) = -\frac{z}{z-2}$

⑤ $(n \geq 1)$ $\quad a_{n-1} = \left(\frac{1}{2}\right)^{n-1} \qquad f(z) = \frac{2z}{2-z} \qquad X(z) = \frac{1}{z-0.5}$

⑥ $(n \geq 1)$ $\quad a_{n-1} = (2)^{n-1} \qquad f(z) = \frac{0.5z}{0.5-z} \qquad X(z) = \frac{1}{z-2}$

⑦ $(n < 1)$ $\quad a_{n-1} = \left(\frac{1}{2}\right)^{n-1} \qquad f(z) = -\frac{2z}{2-z} \qquad X(z) = -\frac{1}{z-0.5}$

⑧ $(n < 1)$ $\quad a_{n-1} = (2)^{n-1} \qquad f(z) = -\frac{0.5z}{0.5-z} \qquad X(z) = -\frac{1}{z-2}$

⑨ $(n \geq -1)$ $\quad a_{n+1} = \left(\frac{1}{2}\right)^{n+1} \qquad f(z) = \frac{2}{(2-z)z} \qquad X(z) = \frac{z^2}{z-0.5}$

⑩ $(n \geq -1)$ $\quad a_{n+1} = (2)^{n+1} \qquad f(z) = \frac{0.5}{(0.5-z)z} \qquad X(z) = \frac{z^2}{z-2}$

⑪ $(n < -1)$ $\quad a_{n+1} = \left(\frac{1}{2}\right)^{n+1} \qquad f(z) = -\frac{2}{(2-z)z} \qquad X(z) = -\frac{z^2}{z-0.5}$

⑫ $(n < -1)$ $\quad a_{n+1} = (2)^{n+1} \qquad f(z) = -\frac{0.5}{(0.5-z)z} \qquad X(z) = -\frac{z^2}{z-2}$

1

Causal $a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$f(z) = \frac{2}{2-z} \quad (|z| < 2) \quad p=2$

$X(z) = \frac{z}{z-0.5} \quad (|z| > 0.5) \quad p=0.5$

$a_n : \left(\frac{1}{2}\right)^0, \quad \left(\frac{1}{2}\right)^1, \quad \left(\frac{1}{2}\right)^2, \quad \cdots \quad (n \geq 0)$

$n=0 \quad n=1 \quad n=2$

$$f(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \cdots = \frac{1}{1-\frac{z}{2}} = \frac{2}{2-z}$$

$\frac{|z|}{2} < 1 \quad |z| < 2$

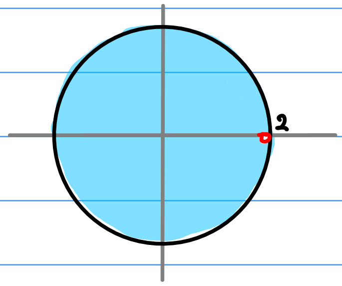
$$X(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots = \frac{1}{1-\frac{1}{2z}} = \frac{z}{z-0.5}$$

$\frac{1}{2|z|} < 1 \quad |z| > 0.5$

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$

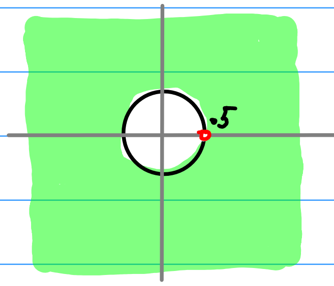
$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$



$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$X(z) = \frac{1}{1-\frac{1}{2z}} \quad |z| > 0.5$

$= \frac{z}{z-0.5}$



② Causal $a_n = (2)^n$ $(n \geq 0)$

$f(z) = \frac{0.5}{0.5 - z}$ $(|z| < 0.5)$ $p = 0.5$

$X(z) = \frac{z}{z - 2}$ $(|z| < 2)$ $p = 2$

$a_n : \quad (2)^0, \quad (2)^1, \quad (2)^2, \quad \cdots \quad (n \geq 0)$

$n=0 \qquad n=1 \qquad n=2$

$$f(z) = (2)^0 z^0 + (2)^1 z^1 + (2)^2 z^2 + \cdots = \frac{1}{1-2z} = \frac{0.5}{0.5 - z}$$

$2|z| < 1 \qquad |z| < 0.5$

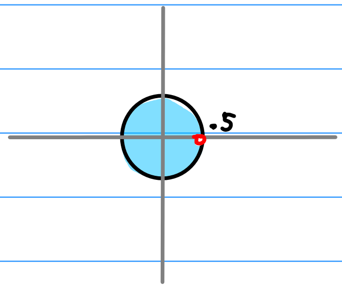
$$X(z) = (2)^0 z^0 + (2)^1 z^{-1} + (2)^2 z^{-2} + \cdots = \frac{1}{1 - \frac{2}{z}} = \frac{z}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

$a_n = (2)^n$ $(n \geq 0)$

$f(z) = \frac{1}{1-2z}$ $|z| < 0.5$

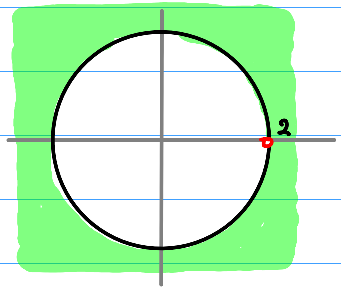
$= \frac{0.5}{0.5 - z}$



$a_n = (2)^n$ $(n \geq 0)$

$X(z) = \frac{1}{1 - \frac{2}{z}}$ $|z| > 2$

$= \frac{z}{z - 2}$



③

Anti-causal $a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$

$f(z) = -\frac{2}{2-z} \quad (|z| > 2) \quad p = 2$

$X(z) = -\frac{z}{z-0.5} \quad (|z| < 0.5) \quad p = 0.5$

$a_n: \quad \left(\frac{1}{2}\right)^{-1}, \quad \left(\frac{1}{2}\right)^{-2}, \quad \left(\frac{1}{2}\right)^{-3}, \quad \cdots \qquad (n < 0)$

$n=-1 \qquad n=-2 \qquad n=-3$

$$f(z) = (2)^1 z^{-1} + (2)^2 z^{-2} + (2)^3 z^{-3} + \cdots = \frac{\frac{2}{z}}{1-\frac{2}{z}} = \frac{2}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

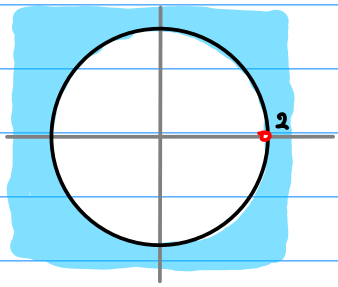
$$X(z) = (2)^1 z^1 + (2)^2 z^2 + (2)^3 z^3 + \cdots = \frac{2z}{1-2z} = \frac{z}{0.5-z}$$

$2|z| < 1 \qquad |z| < 0.5$

$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$

$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$

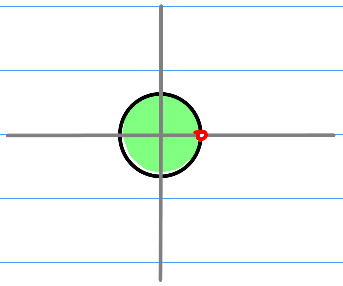
$= -\frac{2}{2-z}$



$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$

$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$

$= -\frac{z}{z-0.5}$



④ Anti-causal $a_n = (2)^n$ $(n < 0)$

$f(z) = -\frac{0.5}{0.5 - z}$ $(|z| > 0.5)$ $p = 0.5$

$X(z) = -\frac{z}{z-2}$ $(|z| < 2)$ $p = 2$

$a_n: \quad (2)^{-1}, \quad (2)^{-2}, \quad (2)^{-3}, \quad \cdots \qquad (n < 0)$

$n=-1 \qquad n=-2 \qquad n=-3$

$$f(z) = \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \cdots = \frac{\frac{1}{2z}}{1 - \frac{1}{2z}} = \frac{0.5}{z - 0.5}$$

$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$

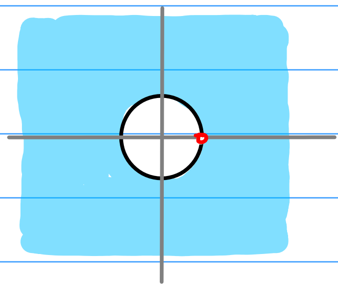
$$X(z) = \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \left(\frac{1}{2}\right)^3 z^3 + \cdots = \frac{\frac{z}{2}}{1 - \frac{z}{2}} = \frac{z}{2 - z}$$

$\frac{|z|}{2} < 1 \qquad |z| < 2$

$a_n = (2)^n \quad (n < 0)$

$$f(z) = \frac{\frac{1}{2z}}{1 - \frac{1}{2z}} \qquad |z| > 0.5$$

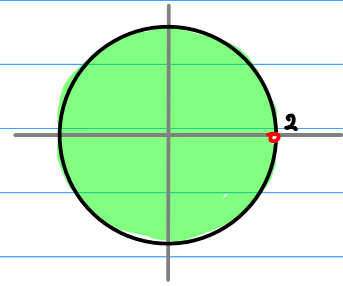
$$= -\frac{0.5}{0.5 - z}$$



$a_n = (2)^n \quad (n < 0)$

$$X(z) = \frac{\frac{z}{2}}{1 - \frac{z}{2}} \qquad |z| < 2$$

$$= -\frac{z}{z - 2}$$



⑤

Causal $a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 1)$

$f(z) = \frac{2z}{2-z}$ $(|z| < 2)$ $p = 2$

$X(z) = \frac{1}{z-0.5}$ $(|z| > 0.5)$ $p = 0.5$

$a_n :$ $\cancel{\left(\frac{1}{2}\right)^{-1}}$, $\left(\frac{1}{2}\right)^{0}$, $\left(\frac{1}{2}\right)^{1}$, $\cdots$ $(n > 0)$

$n=0$ $\quad n=1$ $\quad n=2$

$$f(z) = \left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \cdots = \frac{z}{1-\frac{z}{2}} = \frac{2z}{2-z}$$

$\frac{|z|}{2} < 1 \qquad |z| < 2$

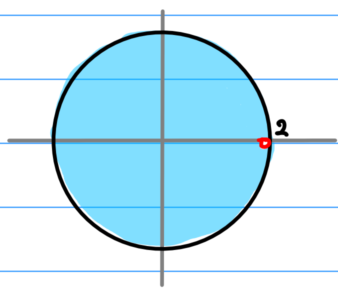
$$X(z) = \left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{1}{2z}} = \frac{1}{z-0.5}$$

$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$

$a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 0)$

$f(z) = \frac{z}{1-\frac{z}{2}}$ $\quad |z| < 2$

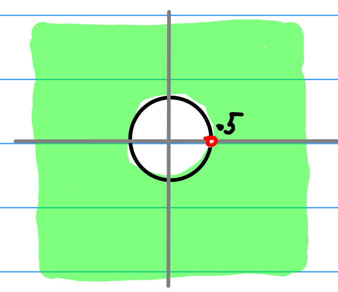
$= \frac{1}{z^{-1}-0.5} = \frac{2z}{2-z}$



$a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 0)$

$X(z) = \frac{\frac{1}{z}}{1-\frac{1}{2z}}$ $\quad |z| > 0.5$

$= \frac{1}{z-0.5}$



6

Causal $a_n = (2)^{n-1}$ $(n \geq 1)$

$f(z) = \frac{0.5z}{0.5 - z}$ $(|z| < 0.5)$ $p = 0.5$

$X(z) = \frac{1}{z-2}$ $(|z| > 2)$ $p = 2$

$a_n :$ $\cancel{(2)^{-1}}$, $(2)^0$, $(2)^1$, $\cdots$ $(n > 0)$

$n=0$ $n=1$ $n=2$

$$f(z) = (2)^0 z^1 + (2)^1 z^2 + (2)^2 z^3 + \cdots = \frac{z}{1-2z} = \frac{0.5z}{0.5-z}$$

$2|z| < 1$ $|z| < 0.5$

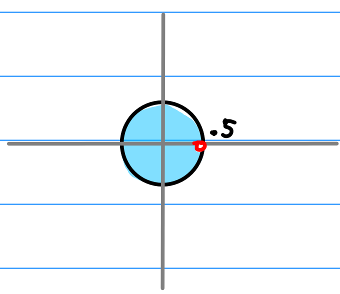
$$X(z) = (2)^0 z^{-1} + (2)^1 z^{-2} + (2)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1 - \frac{2}{z}} = \frac{1}{z-2}$$

$\frac{2}{|z|} < 1$ $|z| > 2$

$a_n = (2)^{n-1}$ $(n \geq 0)$

$f(z) = \frac{z}{1-2z}$ $|z| < 0.5$

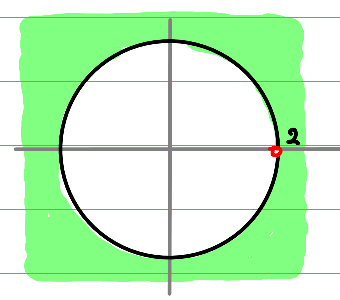
$= \frac{0.5z}{0.5-z}$



$a_n = (2)^{n-1}$ $(n \geq 0)$

$X(z) = \frac{\frac{1}{z}}{1-\frac{2}{z}}$ $|z| > 2$

$= \frac{1}{z-2}$



⑦

$$\text{Anti-causal} \quad a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n < 1)$$

$$f(z) = -\frac{2z}{2-z} \quad (|z| < 2) \quad p = 2$$

$$X(z) = -\frac{1}{z-0.5} \quad (|z| > 0.5) \quad p = 0.5$$

$$a_n: \quad \underset{n=0}{\left(\frac{1}{2}\right)^{-1}}, \quad \underset{n=-1}{\left(\frac{1}{2}\right)^{-2}}, \quad \underset{n=-2}{\left(\frac{1}{2}\right)^{-3}}, \quad \cdots \quad (n \leq 0)$$

$$f(z) = (2)^1 z^0 + (2)^2 z^{-1} + (2)^3 z^{-2} + \cdots = \frac{\frac{2}{1}}{1-\frac{2}{z}} = \frac{2z}{z-2}$$

$$\frac{2}{|z|} < 1 \qquad |z| > 2$$

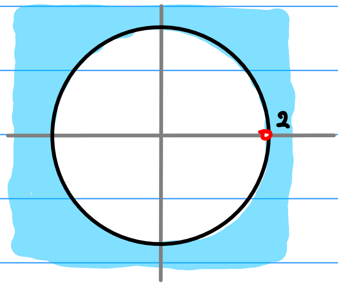
$$X(z) = (2)^1 z^0 + (2)^2 z^1 + (2)^3 z^2 + \cdots = \frac{2}{1-2z} = \frac{1}{0.5-z}$$

$$2|z| < 1 \qquad |z| < 0.5$$

$$a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n < 1)$$

$$f(z) = \frac{\frac{2}{1}}{1-\frac{2}{z}} \qquad |z| > 2$$

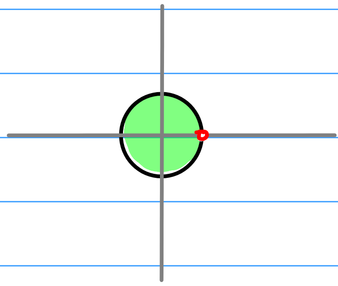
$$= -\frac{2z}{2-z}$$



$$a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n < 1)$$

$$X(z) = \frac{2}{1-2z} \qquad |z| < 0.5$$

$$= -\frac{1}{z-0.5}$$



8

Anti-causal $a_n = (2)^{n-1}$ $(n<1)$

$f(z) = -\dfrac{0.5z}{0.5-z}$ $(|z| < 0.5)$ $p = 0.5$

$X(z) = -\dfrac{1}{z-2}$ $(|z| > 2)$ $p = 2$

$a_n: \quad (2)^{-1}, \quad (2)^{-2}, \quad (2)^{-3}, \quad \cdots \qquad (n<1)$

$n=0 \qquad n=-1 \qquad n=-2$

$$f(z) = \left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots = \frac{\frac{1}{2}}{1-\frac{1}{2z}} = \frac{0.5z}{z-0.5}$$

$\dfrac{1}{2|z|} < 1 \qquad |z| > 0.5$

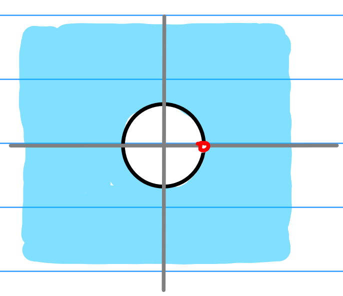
$$X(z) = \left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots = \frac{\frac{1}{2}}{1-\frac{z}{2}} = \frac{1}{2-z}$$

$\dfrac{|z|}{2} < 1 \qquad |z| < 2$

$a_n = (2)^{n-1}$ $(n<1)$

$f(z) = \dfrac{\frac{1}{2}}{1-\frac{1}{2z}}$ $\quad |z| > 0.5$

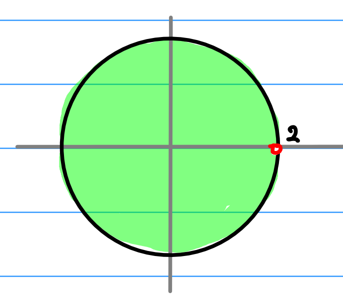
$= -\dfrac{0.5z}{0.5-z}$



$a_n = (2)^{n-1}$ $(n<1)$

$X(z) = \dfrac{\frac{1}{2}}{1-\frac{z}{2}}$ $\quad |z| < 2$

$= -\dfrac{1}{z-2}$



$$a_n \begin{pmatrix} \cdots & -3 & -2 & -1 & 0 & +1 & +2 & +3 & \cdots \\ \cdots & -3 & -2 & -1 & 0 & +1 & +2 & +3 & \cdots \\ \cdots & -3 & -2 & -1 & 0 & +1 & +2 & +3 & \cdots \end{pmatrix} \quad f(z) \quad X(z) \quad \begin{pmatrix} p = 1 \\ p = 1^{-1} \end{pmatrix}$$

1	$(n \geq 0)$	$a_n = (1)^n$	$f(z) = \dfrac{1}{1-z}$	$X(z) = \dfrac{z}{z-1}$
2	$(n \geq 0)$	$a_n = (1)^n$	$f(z) = \dfrac{1}{1-z}$	$X(z) = \dfrac{z}{z-1}$
3	$(n < 0)$	$a_n = (1)^n$	$f(z) = -\dfrac{1}{1-z}$	$X(z) = -\dfrac{z}{z-1}$
4	$(n < 0)$	$a_n = (1)^n$	$f(z) = -\dfrac{1}{1-z}$	$X(z) = -\dfrac{z}{z-1}$
5	$(n \geq 1)$	$a_{n-1} = (1)^{n-1}$	$f(z) = \dfrac{z}{1-z}$	$X(z) = \dfrac{1}{z-1}$
6	$(n \geq 1)$	$a_{n-1} = (1)^{n-1}$	$f(z) = \dfrac{z}{1-z}$	$X(z) = \dfrac{1}{z-1}$
7	$(n < 1)$	$a_{n-1} = (1)^{n-1}$	$f(z) = -\dfrac{z}{1-z}$	$X(z) = -\dfrac{1}{z-1}$
8	$(n < 1)$	$a_{n-1} = (1)^{n-1}$	$f(z) = -\dfrac{z}{1-z}$	$X(z) = -\dfrac{1}{z-1}$
9	$(n \geq -1)$	$a_{n+1} = (1)^{n+1}$	$f(z) = \dfrac{1}{(1-z)z}$	$X(z) = \dfrac{z}{z-1}$
10	$(n \geq -1)$	$a_{n+1} = (1)^{n+1}$	$f(z) = \dfrac{1}{(1-z)z}$	$X(z) = \dfrac{z}{z-1}$
11	$(n < -1)$	$a_{n+1} = (1)^{n+1}$	$f(z) = -\dfrac{1}{(1-z)z}$	$X(z) = -\dfrac{z}{z-1}$
12	$(n < -1)$	$a_{n+1} = (1)^{n+1}$	$f(z) = -\dfrac{1}{(1-z)z}$	$X(z) = -\dfrac{z}{z-1}$

① ②

Causal $a_n = (1)^n$ $(n \geq 0)$

$f(z) = \dfrac{1}{1-z}$ $(|z| > 1)$ $p = 1$

$X(z) = \dfrac{z}{z-1}$ $(|z| < 1)$ $p = 1$

$a_n: \quad (1)^0, \quad (1)^1, \quad (1)^2, \quad \cdots \qquad (n \geq 0)$

$\quad\quad n=0 \quad\quad n=1 \quad\quad n=2$

$$f(z) = (1)^0 z^0 + (1)^1 z^1 + (1)^2 z^2 + \cdots = \frac{1}{1-z} = \frac{1}{1-z}$$

$|z| < 1 \qquad |z| < 1$

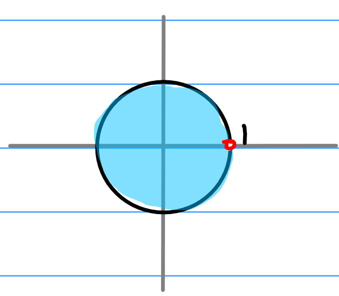
$$X(z) = (1)^0 z^0 + (1)^1 z^{-1} + (1)^2 z^{-2} + \cdots = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}$$

$\frac{1}{|z|} < 1 \qquad |z| > 1$

$a_n = (1)^n \quad (n \geq 0)$

$f(z) = \dfrac{1}{1-z} \quad |z| < 1$

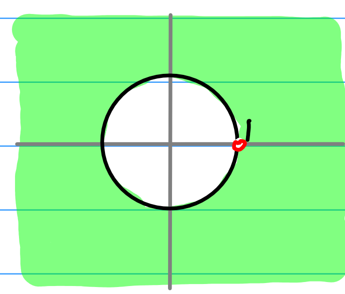
$= \dfrac{z^{-1}}{z^{-1} - 1} = \dfrac{1}{1-z}$



$a_n = (1)^n \quad (n \geq 0)$

$X(z) = \dfrac{1}{1 - \frac{1}{z}} \quad |z| > 1$

$= \dfrac{z}{z-1}$



3 4

Anti-causal $a_n = (1)^n$ $(n<0)$

$f(z) = -\frac{1}{1-z}$ $(|z|>1)$ $p=1$

$X(z) = -\frac{z}{z-1}$ $(|z|<1)$ $p=1$

$a_n: \underset{n=-1}{(1)^{-1}}, \quad \underset{n=-2}{(1)^{-2}}, \quad \underset{n=-3}{(1)^{-3}}, \quad \dots \qquad (n<0)$

$$f(z) = (1)^1 z^{-1} + (1)^2 z^{-2} + (1)^3 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{1}{z}} = \frac{1}{z-1}$$

$\frac{1}{|z|} < 1 \qquad |z| > 1$

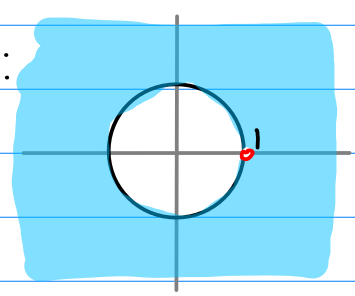
$$X(z) = (1)^1 z^1 + (1)^2 z^2 + (1)^3 z^3 + \cdots = \frac{z}{1-z} = \frac{z}{1-z}$$

$|z| < 1$

$a_n = (1)^n$ $(n<0)$

$f(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}}$ $\quad |z| > 1$

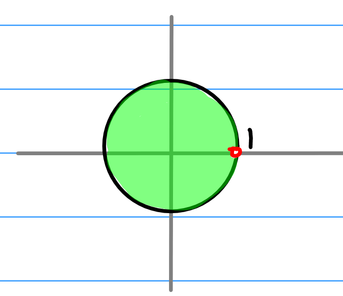
$= -\frac{1}{1-z}$



$a_n = (1)^n$ $(n<0)$

$X(z) = \frac{z}{1-z}$ $\quad |z| < 1$

$= -\frac{z}{z-1}$



5

6

Causal $a_n = (1)^{n-1}$ $(n \geq 1)$

$f(z) = \frac{z}{1-z}$ $(|z| > 1)$ $p = 1$

$X(z) = \frac{1}{z-1}$ $(|z| < 1)$ $p = 1$

a_n : $\underbrace{\cancel{(1)^{-1}}}_{n=0}$, $\underbrace{(1)^{0}}_{n=1}$, $\underbrace{(1)^{1}}_{n=2}$, $\cdots$ $(n > 0)$

$$f(z) = (1)^0 z^1 + (1)^1 z^2 + (1)^2 z^3 + \cdots = \frac{z}{1-z} = \frac{z}{1-z}$$

$|z| < 1$

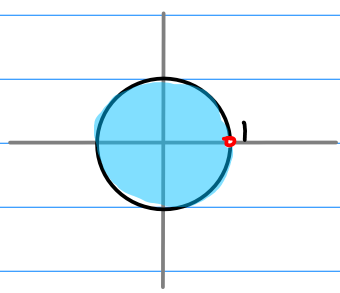
$$X(z) = (1)^0 z^{-1} + (1)^1 z^{-2} + (1)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{1}{z}} = \frac{1}{z-1}$$

$\frac{1}{|z|} < 1$ $|z| > 1$

$a_n = (1)^{n-1}$ $(n \geq 0)$

$f(z) = \frac{z}{1-z}$ $|z| < 1$

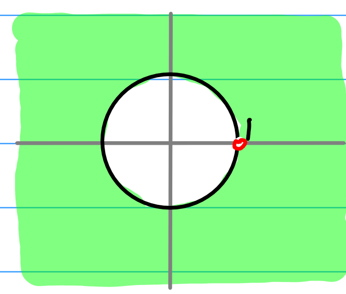
$= \frac{1}{z^{-1} - 1} = \frac{z}{1-z}$



$a_n = (1)^{n-1}$ $(n \geq 0)$

$X(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}}$ $|z| > 1$

$= \frac{1}{z-1}$



⑦
⑧

$\boxed{\text{Anti-causal } a_n = (1)^{n-1} \quad (n<1)}$

$\boxed{\begin{array}{l} f(z) = -\frac{z}{1-z} \ (|z|>1) \quad p=1 \\ X(z) = -\frac{1}{z-1} \ (|z|<1) \quad p=1 \end{array}}$

$a_n : \underset{n=0}{(1)^{-1}}, \quad \underset{n=-1}{(1)^{-2}}, \quad \underset{n=-2}{(1)^{-3}}, \quad \cdots \qquad (n \leq 0)$

$$f(z) = (1)^{1} z^{0} + (1)^{2} z^{-1} + (1)^{3} z^{-2} + \cdots = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}$$

$$\frac{1}{|z|} < 1 \qquad |z| > 1$$

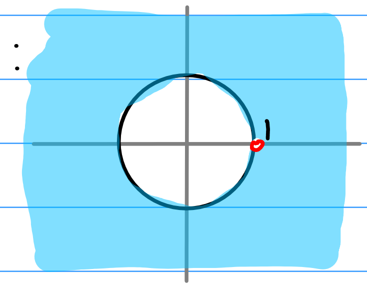
$$X(z) = (1)^{1} z^{0} + (1)^{2} z^{1} + (1)^{3} z^{2} + \cdots = \frac{1}{1-z} = \frac{1}{1-z}$$

$$|z| < 1$$

$a_n = (1)^{n-1} \quad (n<1)$

$f(z) = \frac{1}{1-\frac{1}{z}} \quad |z|>1$

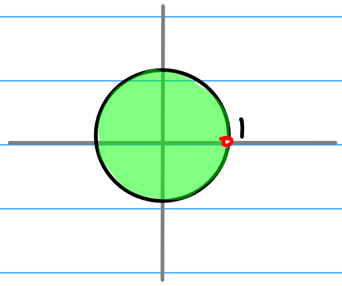
$= -\frac{z}{1-z}$



$a_n = (1)^{n-1} \quad (n<1)$

$X(z) = \frac{1}{1-z} \quad |z| < 1$

$= -\frac{1}{z-1}$



$\boxed{A}$ Causal

$\left(\frac{1}{2}\right)^n$	$\left(\frac{1}{2}\right)^n$
$(2)^n$	$(2)^n$

$\frac{2}{2-z}$	$\frac{z}{z-0.5}$
$\frac{0.5}{0.5-z}$	$\frac{z}{z-2}$

$f(z)$	$X(z)$
$f(z)$	$X(z)$

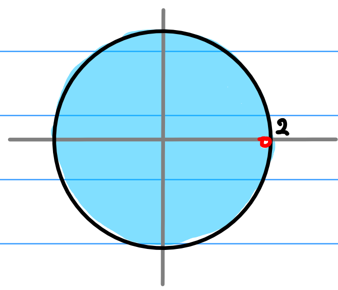
$f(z)$ | $X(z)$

$\left(\frac{1}{2}\right)^n$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$

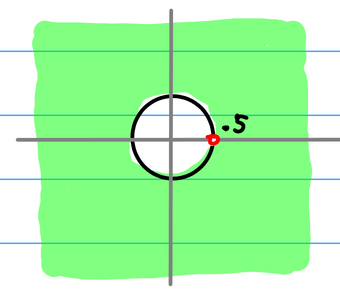


$\overset{z^{-1}}{\longleftrightarrow}$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \quad |z| > 0.5$$

$$= \frac{z}{z-0.5}$$



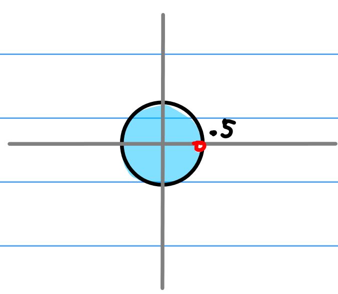
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

$$a_n = (2)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$

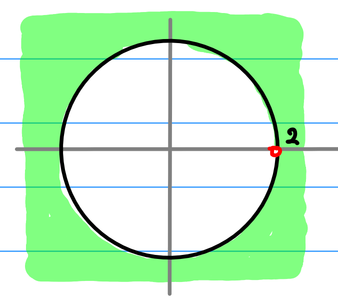


$\overset{z^{-1}}{\longleftrightarrow}$

$$a_n = (2)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-\frac{2}{z}} \quad |z| > 2$$

$$= \frac{z}{z-2}$$



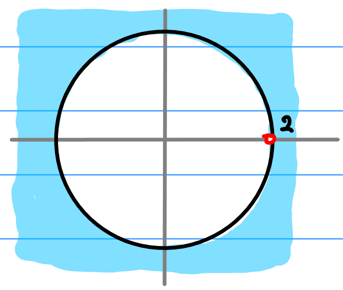
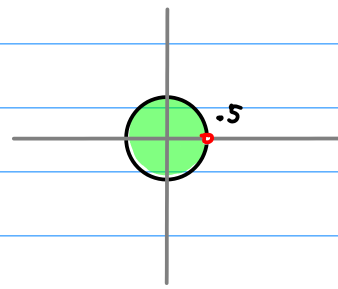
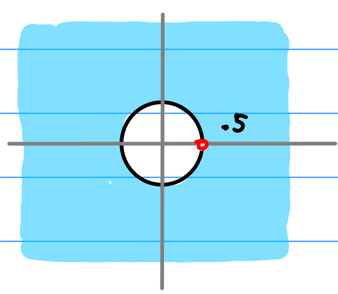
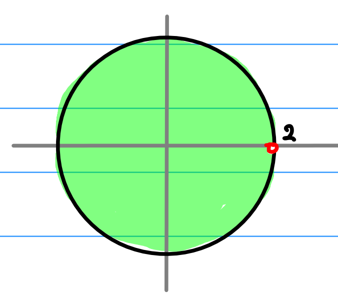
$\boxed{B}$

Anti-causal

$\left(\frac{1}{2}\right)^n$	$\left(\frac{1}{2}\right)^n$
$(2)^n$	$(2)^n$

$-\frac{2}{2-z}$	$-\frac{z}{z-0.5}$
$-\frac{0.5}{0.5-z}$	$-\frac{z}{z-2}$

$f(z)$	$X(z)$
$f(z)$	$X(z)$

	$f(z)$		$X(z)$
$\left(\frac{1}{2}\right)^n$	$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$ $f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \quad \lvert z\rvert > 2$ $= -\frac{2}{2-z}$  	$\overset{z^{-1}}{\longleftrightarrow}$	$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$ $X(z) = \frac{2z}{1-2z} \quad \lvert z\rvert < 0.5$ $= -\frac{z}{z-0.5}$ 
	$\updownarrow p^{-1}$		$\updownarrow p^{-1}$
$(2)^n$	$a_n = (2)^n \quad (n < 0)$ $f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \quad \lvert z\rvert > 0.5$ $= -\frac{0.5}{0.5-z}$  	$\overset{z^{-1}}{\longleftrightarrow}$	$a_n = (2)^n \quad (n < 0)$ $X(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \quad \lvert z\rvert < 2$ $= -\frac{z}{z-2}$ 

$\boxed{C}$

Causal

Anti-causal

$\left(\frac{1}{2}\right)^n$	$\left(\frac{1}{2}\right)^n$
$(2)^n$	$(2)^n$

$\frac{2}{2-z}$	$-\frac{2}{2-z}$
$\frac{0.5}{0.5-z}$	$-\frac{0.5}{0.5-z}$

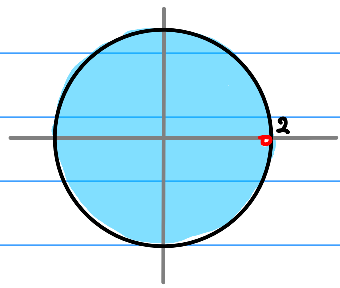
$f(z)$	$f(z)$
$f(z)$	$f(z)$

$\left(\frac{1}{2}\right)^n$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$

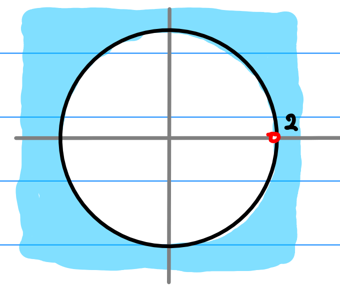


$\xleftrightarrow{-1}$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \quad |z| > 2$$

$$= -\frac{2}{2-z}$$



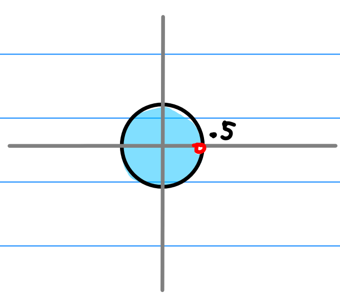
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

$$a_n = (2)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$

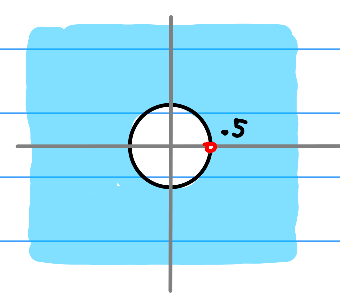


$\xleftrightarrow{-1}$

$$a_n = (2)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \quad |z| > 0.5$$

$$= -\frac{0.5}{0.5-z}$$



$\boxed{D}$

Causal

Anti-causal

$\left(\frac{1}{2}\right)^n$	$\left(\frac{1}{2}\right)^n$
$(2)^n$	$(2)^n$

$\frac{z}{z-0.5}$	$-\frac{z}{z-0.5}$
$\frac{z}{z-2}$	$-\frac{z}{z-2}$

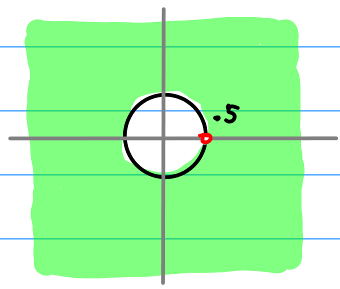
$X(z)$	$X(z)$
$X(z)$	$X(z)$

$\left(\frac{1}{2}\right)^n$

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

$$= \frac{z}{z-0.5}$$

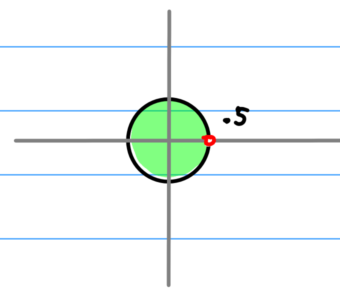


$\xleftrightarrow{-1}$

$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$

$$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$$

$$= -\frac{z}{z-0.5}$$



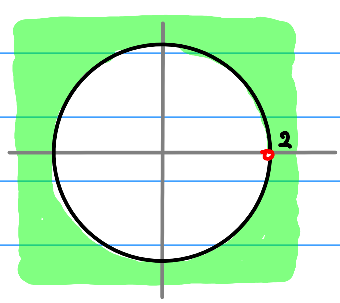
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

$a_n = (2)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= \frac{z}{z-2}$$

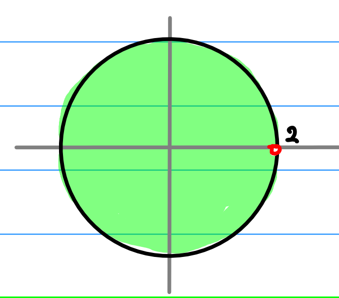


$\xleftrightarrow{-1}$

$a_n = (2)^n \quad (n < 0)$

$$X(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$



Causal b^n | Anti-causal b^n

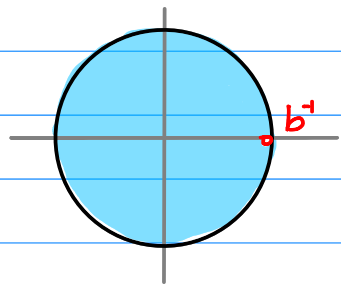
f(z)

Causal:

$$a_n = (b)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1 - bz} \qquad |z| < b^{-1}$$

$$= \frac{b^{-1}}{b^{-1} - z}$$

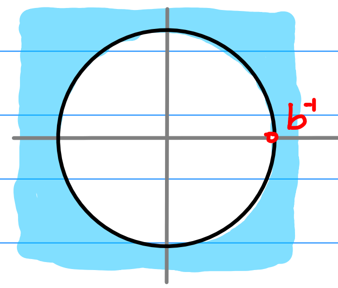


Anti-causal:

$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1}z^{-1}}{1 - b^{-1}z^{-1}} \qquad |z| > b^{-1}$$

$$= -\frac{b^{-1}}{b^{-1} - z}$$



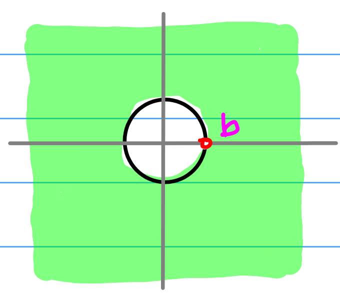
X(z)

Causal:

$$a_n = (b)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - bz^{-1}} \qquad |z| > b$$

$$= \frac{z}{z - b}$$

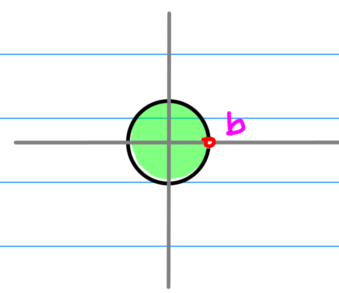


Anti-causal:

$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1}z}{1 - b^{-1}z} \qquad |z| < b$$

$$= -\frac{z}{z - b}$$



$\boxed{E}$ Causal

$\left(1\right)^n$	$\left(1\right)^n$
$\left(1\right)^n$	$\left(1\right)^n$

$\frac{1}{1-z}$	$\frac{z}{z-1}$
$\frac{1}{1-z}$	$\frac{z}{z-1}$

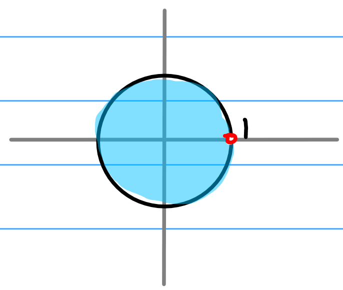
$f(z)$	$X(z)$
$f(z)$	$X(z)$

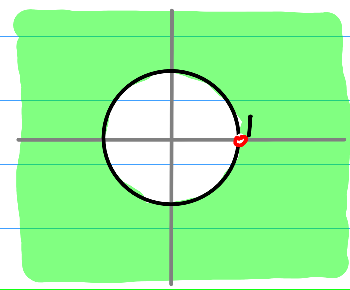
	$f(z)$		$X(z)$
$\left(\frac{1}{1}\right)^n$	$a_n = \left(\frac{1}{1}\right)^n \quad (n \geqslant 0)$		$a_n = \left(\frac{1}{1}\right)^n \quad (n \geqslant 0)$

$$f(z) = \frac{1}{1-\frac{z}{1}} \quad |z| < 1$$
$$= \frac{z^{-1}}{z^{-1}-1} = \frac{1}{1-z}$$

$\overset{z^{-1}}{\longleftrightarrow}$

$$X(z) = \frac{1}{1-\frac{1}{z}} \quad |z| > 1$$
$$= \frac{z}{z-1}$$





$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$\left(1\right)^n$

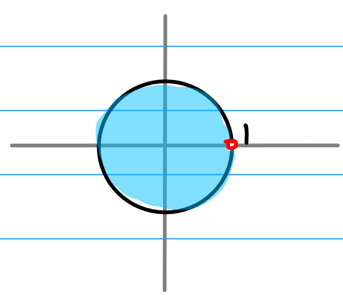
$a_n = \left(1\right)^n \quad (n \geqslant 0)$

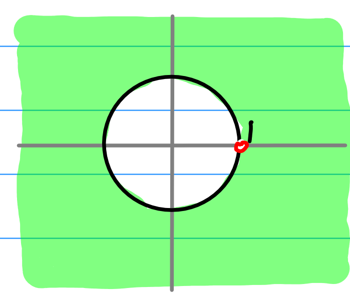
$$f(z) = \frac{1}{1-z} \quad |z| < 1$$
$$= \frac{z^{-1}}{z^{-1}-1} = \frac{1}{1-z}$$

$\overset{z^{-1}}{\longleftrightarrow}$

$a_n = \left(1\right)^n \quad (n \geqslant 0)$

$$X(z) = \frac{1}{1-\frac{1}{z}} \quad |z| > 1$$
$$= \frac{z}{z-1}$$





$\boxed{F}$ Anti-causal

$(1)^n$	$(1)^n$
$(1)^n$	$(1)^n$

$-\frac{1}{1-z}$	$-\frac{z}{z-1}$
$-\frac{1}{1-z}$	$-\frac{z}{z-1}$

$f(z)$	$X(z)$
$f(z)$	$X(z)$

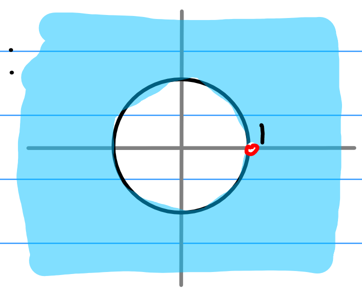
$f(z)$ — $X(z)$

$(\frac{1}{2})^n$

$a_n = (\frac{1}{1})^n \quad (n<0)$

$f(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \quad |z|>1$

$= -\frac{1}{1-z}$

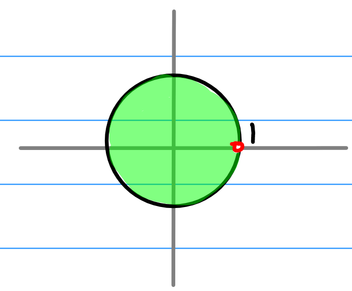


$\xleftrightarrow{z^{-1}}$

$a_n = (\frac{1}{1})^n \quad (n<0)$

$X(z) = \frac{z}{1-z} \quad |z|<1$

$= -\frac{z}{z-1}$



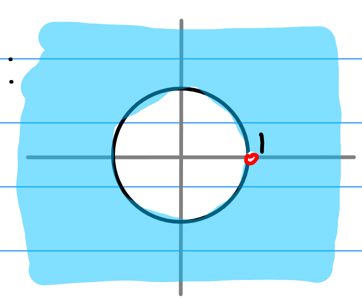
$\updownarrow p^{-1}$ $\updownarrow p^{-1}$

$(1)^n$

$a_n = (1)^n \quad (n<0)$

$f(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \quad |z|>1$

$= -\frac{1}{1-z}$

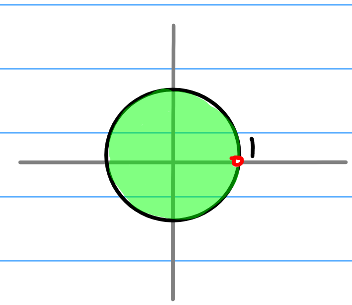


$\xleftrightarrow{z^{-1}}$

$a_n = (1)^n \quad (n<0)$

$X(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \quad |z|<1$

$= -\frac{z}{z-1}$



$\boxed{G}$

Causal

Anti-causal

$(1)^n$	$(1)^n$
$(1)^n$	$(1)^n$

$\frac{1}{1-z}$	$-\frac{1}{1-z}$
$\frac{1}{1-z}$	$-\frac{1}{1-z}$

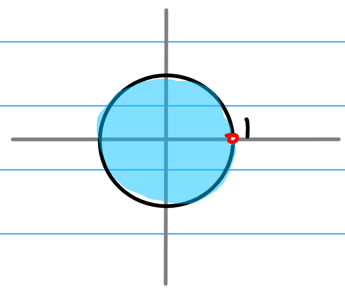
$f(z)$	$f(z)$
$f(z)$	$f(z)$

$\left(\frac{1}{1}\right)^n$

$$a_n = \left(\frac{1}{1}\right)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 1$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{1}{1-z}$$

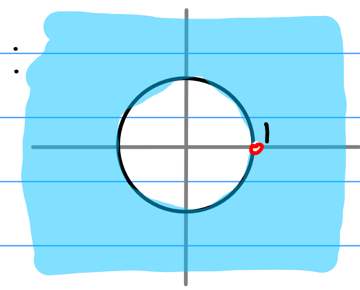


$\xleftrightarrow{-1}$

$$a_n = \left(\frac{1}{1}\right)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \quad |z| > 1$$

$$= -\frac{1}{1-z}$$



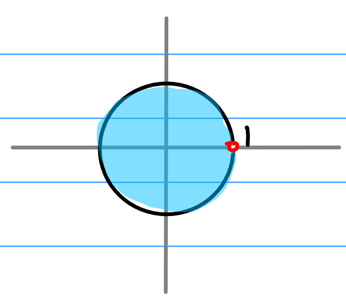
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(1)^n$

$$a_n = (1)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-z} \quad |z| < 1$$

$$= \frac{z^{-1}}{z^{-1}-1} = \frac{1}{1-z}$$

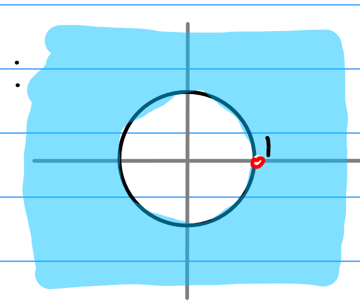


$\xleftrightarrow{-1}$

$$a_n = (2)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \quad |z| > 1$$

$$= -\frac{1}{1-z}$$



$\boxed{H}$

Causal

Anti-causal

$(1)^n$	$(1)^n$
$(1)^n$	$(1)^n$

$\frac{z}{z-1}$	$-\frac{z}{z-1}$
$\frac{z}{z-1}$	$-\frac{z}{z-1}$

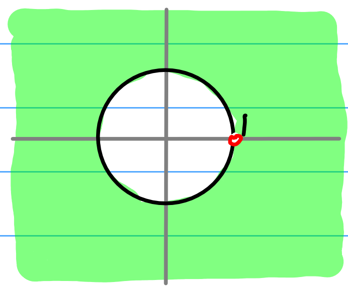
$X(z)$	$X(z)$
$X(z)$	$X(z)$

$\left(\frac{1}{1}\right)^n$

$a_n = \left(\frac{1}{1}\right)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{z}} \qquad |z| > 1$$

$$= \frac{z}{z-1}$$

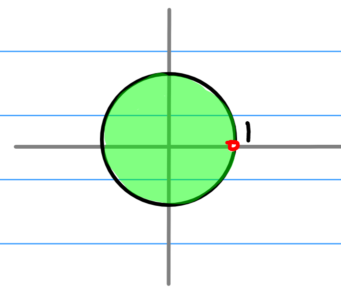


$\xleftrightarrow{-1}$

$a_n = \left(\frac{1}{1}\right)^n \quad (n < 0)$

$$X(z) = \frac{z}{1-z} \qquad |z| < 1$$

$$= -\frac{z}{z-1}$$



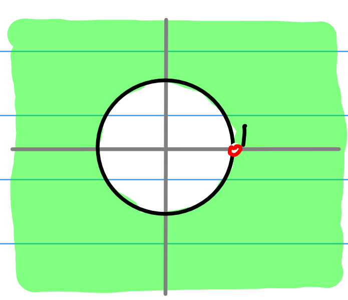
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(1)^n$

$a_n = (1)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{z}} \qquad |z| > 1$$

$$= \frac{z}{z-1}$$

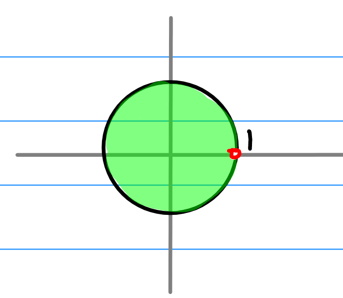


$\xleftrightarrow{-1}$

$a_n = (1)^n \quad (n < 0)$

$$X(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \qquad |z| < 1$$

$$= -\frac{z}{z-1}$$



Causal b^n Anti-causal b^n

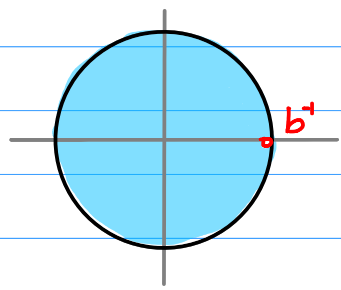
$f(z)$

Causal

$$a_n = (b)^n \quad (n \geqslant 0)$$

$$f(z) = \frac{1}{1 - bz} \qquad |z| < b^{-1}$$

$$= \frac{b^{-1}}{b^{-1} - z}$$

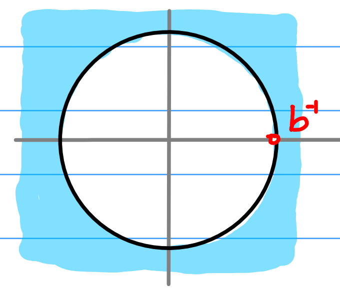


Anti-causal

$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1} z^{-1}}{1 - b^{-1} z^{-1}} \qquad |z| > b^{-1}$$

$$= - \frac{b^{-1}}{b^{-1} - z}$$



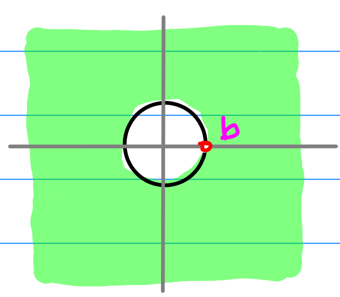
$X(z)$

Causal

$$a_n = (b)^n \quad (n \geqslant 0)$$

$$X(z) = \frac{1}{1 - bz^{-1}} \qquad |z| > b$$

$$= \frac{z}{z - b}$$



Anti-causal

$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1} z}{1 - b^{-1} z} \qquad |z| < b$$

$$= - \frac{z}{z - b}$$