

Laurent Series and z-Transform Examples case 4.B

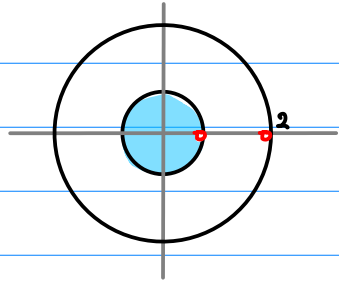
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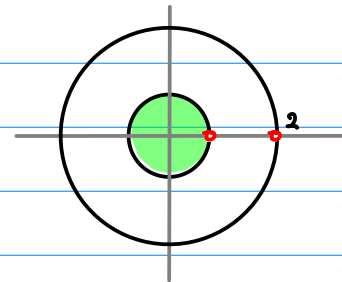
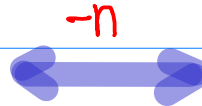
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4.B

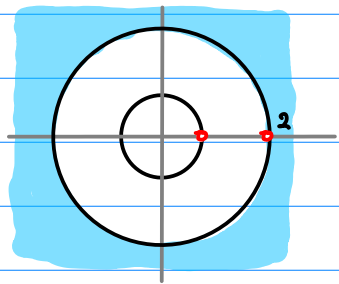
$$f(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



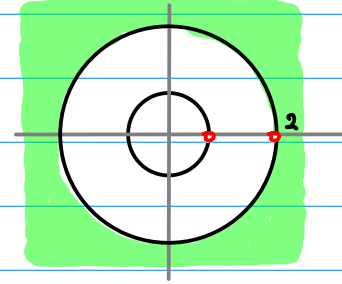
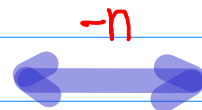
$$\sum_{n=-1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$



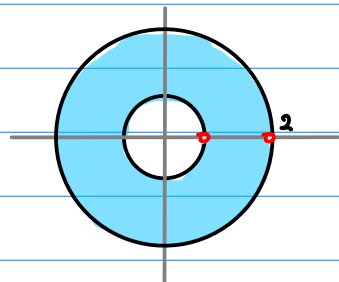
$$\sum_{n=-1}^{\infty} \left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$



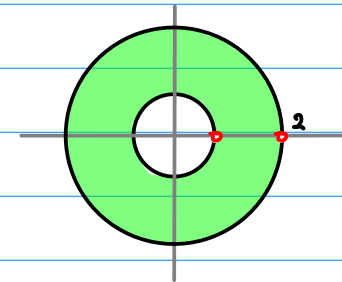
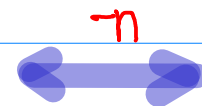
$$\sum_{n=0}^{\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^n$$



$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$



$$\sum_{n=0}^{\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

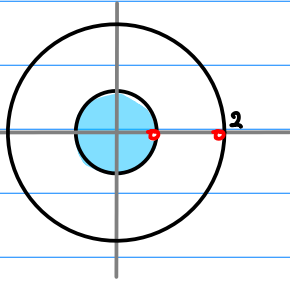


$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty} 2^{n+1} z^{-n}$$

4.B

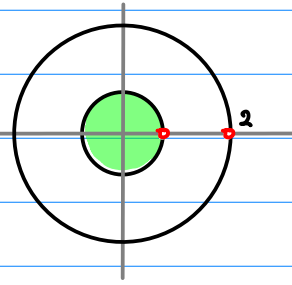
$$f(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{3}{2} \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

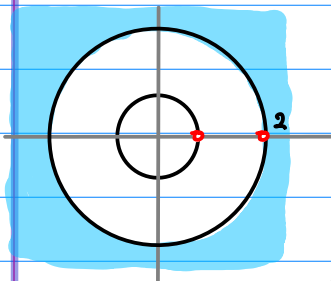
$$f(z) = \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

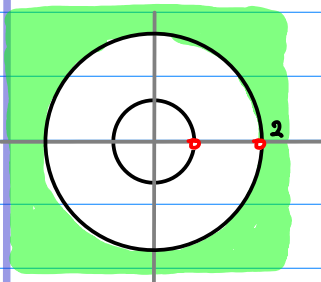
$$X(z) = \sum_{n=-1}^{-\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

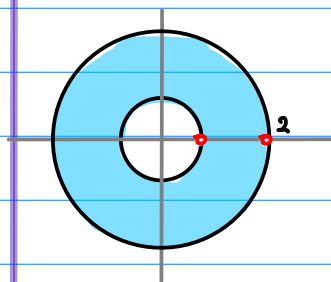
$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

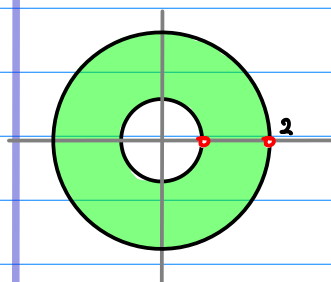
$$X(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

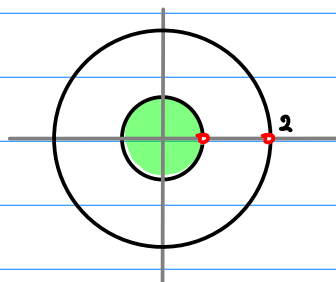
$$f(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} + \sum_{n=-1}^{-\infty} 2^{n+1} z^{-n}$$

$$X(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right)$$



$$|z| < 0.5$$

$$|z| < 2$$

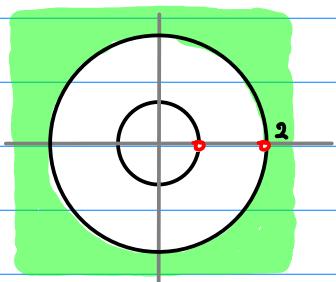
$$\sum_{n=-1}^{\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^{-n}$$

$$- \frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})}$$

$$= - \sum_{n=0}^{\infty} (z)(2z)^n + \sum_{n=0}^{\infty} (z)(\frac{z}{2})^n$$

$$= - \sum_{n=0}^{\infty} 2^n z^{n+1} + \sum_{n=0}^{\infty} (\frac{1}{2})^n z^{n+1}$$

$$= \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^n$$



$$0.5 < |z|$$

$$2 < |z|$$

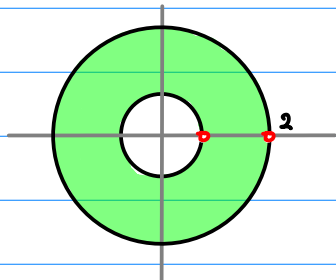
$$\sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^{-n}$$

$$+ \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(z)}{1-(\frac{z}{2})}$$

$$= + \sum_{n=0}^{\infty} (\frac{1}{2})(\frac{1}{2z})^n - \sum_{n=0}^{\infty} (z)(\frac{z}{2})^n$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} - \sum_{n=0}^{\infty} 2^{n+1} z^{-n}$$

$$= \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^{-n}$$



$$0.5 < |z|$$

$$|z| < 2$$

$$\sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} + \sum_{n=1}^{\infty} 2^{n+1} z^{-n}$$

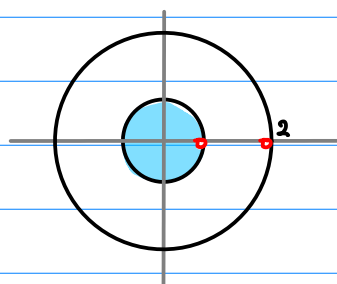
$$+ \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} + \frac{(z)}{1-(\frac{z}{2})}$$

$$= + \sum_{n=0}^{\infty} (\frac{1}{2})(\frac{1}{2z})^n + \sum_{n=0}^{\infty} (z)(\frac{z}{2})^n$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^n z^{n+1}$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n$$

$$f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right)$$



$$|z| < 0.5$$

$$|z| < 2$$

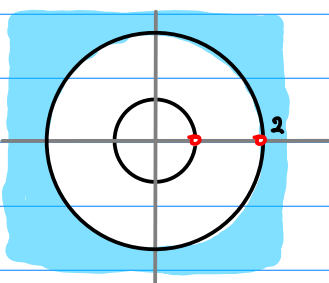
$$\sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$

$$-\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (z)(2z)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} 2^n z^{n+1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$



$$0.5 < |z|$$

$$2 < |z|$$

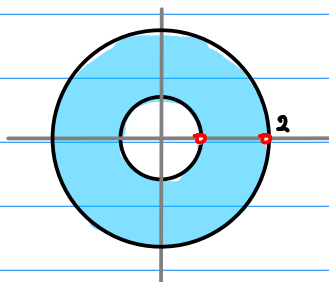
$$\sum_{n=0}^{\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^n$$

$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} - \sum_{n=0}^{\infty} 2^{n+1} z^{n+1}$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$



$$0.5 < |z|$$

$$|z| < 2$$

$$\sum_{n=0}^{\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

