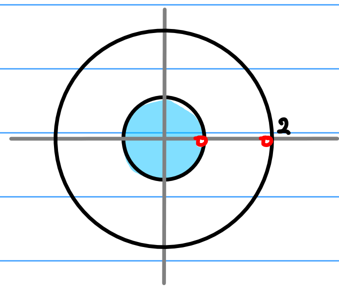
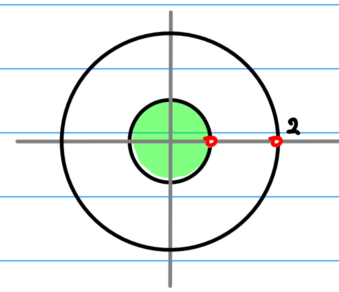
Laurent Series and z-Transform Examples case 4.B

20171228

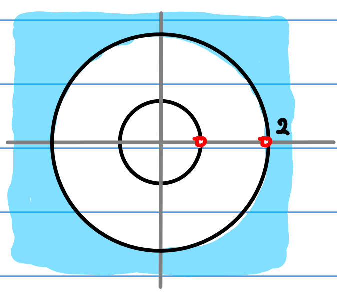
4. B

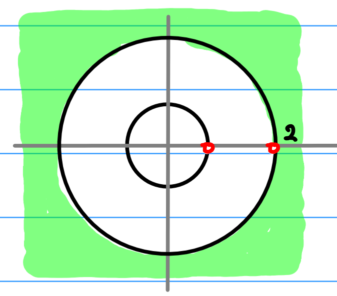
$$f(z) = \frac{3}{2}\,\frac{-0.5\,z^2}{(z-1)(z-0.5)} = X(z) = \frac{3}{2}\,\frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



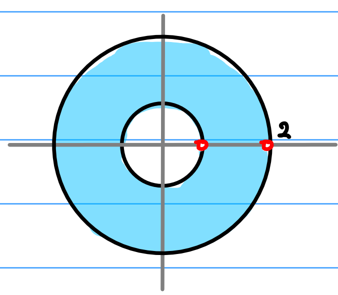


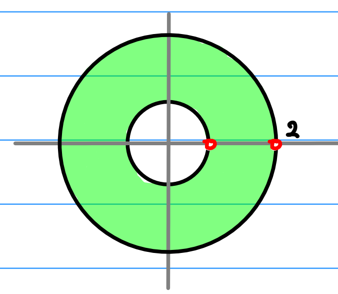
$$\sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^{n} \quad \xleftrightarrow{-n} \quad \sum_{n=-1}^{-\infty}\left[2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$





$$\sum_{n=0}^{-\infty}\left[2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}\right] z^{n} \quad \xleftrightarrow{-n} \quad \sum_{n=0}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^{-n}$$



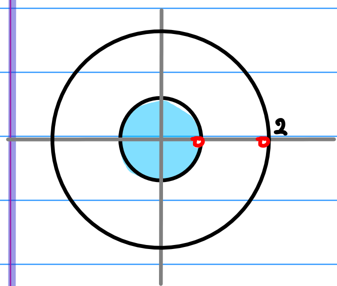


$$\sum_{n=0}^{-\infty} 2^{n-1} z^{n} + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1} z^{n} \quad \xleftrightarrow{-n} \quad \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=-1}^{-\infty} 2^{n+1} z^{-n}$$

4.B

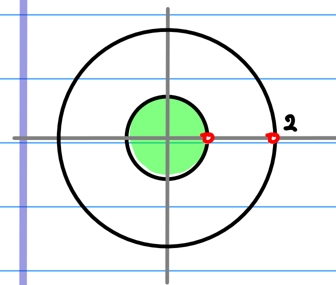
$$f(z) = \frac{3}{2}\,\frac{-0.5\,z^2}{(z-1)(z-0.5)} \quad = \quad X(z) = \frac{3}{2}\,\frac{-0.5\,z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

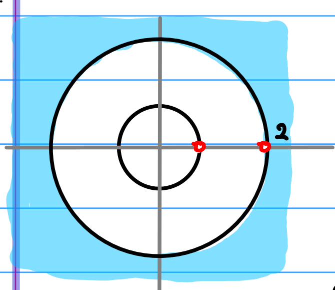
$$f(z) = \sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

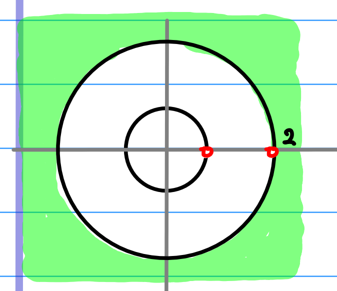
$$X(z) = \sum_{n=-1}^{-\infty} \left[2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - \left(\frac{1}{2}\right)^{n-1} & (n \leq 0) \end{cases}$$

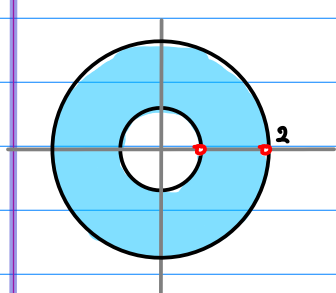
$$f(z) = \sum_{n=0}^{-\infty} \left[2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}\right] z^n$$



$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

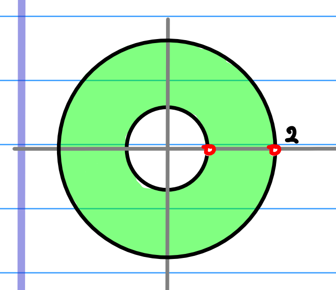
$$X(z) = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^{-n}$$

III



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} z^n$$



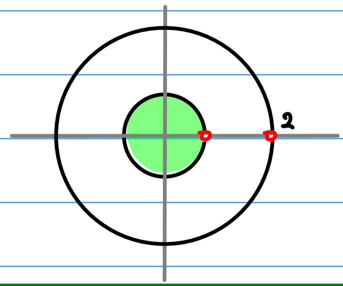
$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=-1}^{-\infty} 2^{n+1} z^{-n}$$

$$X(z) = \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$

$|z| < 2$



$$-\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty}(z)(2z)^n + \sum_{n=0}^{\infty}(z)\left(\frac{z}{2}\right)^n$$

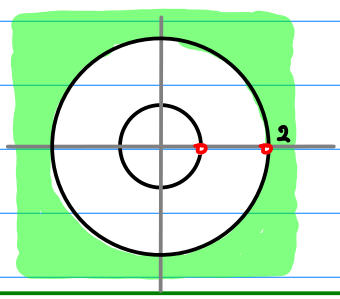
$$= -\sum_{n=0}^{\infty} 2^n z^{n+1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=1}^{\infty}\left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^n$$

$$\boxed{\sum_{n=-1}^{-\infty}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] z^{-n}}$$

$0.5 < |z|$

$2 < |z|$



$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty}(2)\left(\frac{2}{z}\right)^n$$

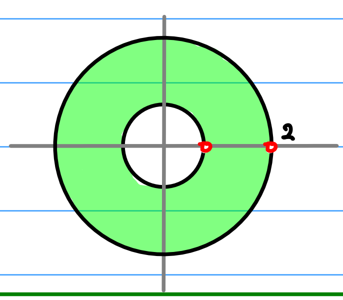
$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n} - \sum_{n=0}^{\infty} 2^{n+1} z^{-n}$$

$$= \sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^{-n}$$

$$\boxed{\sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^{-n}}$$

$0.5 < |z|$

$|z| < 2$



$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

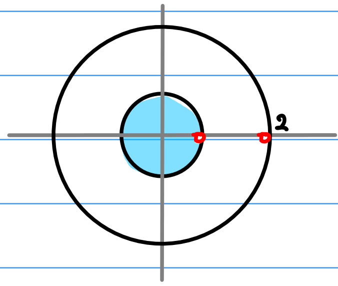
$$= +\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty}(z)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^n$$

$$\boxed{\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=-1}^{-\infty} 2^{n+1} z^{-n}}$$

$$f(z) = \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$



$|z| < 0.5$

$|z| < 2$

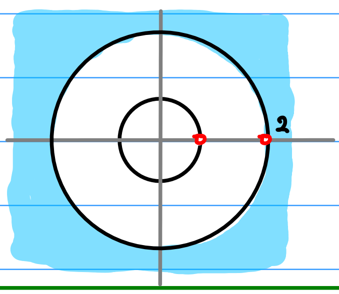
$$-\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (z)(2z)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} 2^n z^{n+1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^n$$

$$\boxed{\sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^n}$$



$0.5 < |z|$

$2 < |z|$

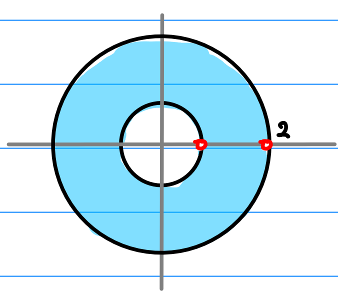
$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty} (2)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} - \sum_{n=0}^{\infty} 2^{n+1} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^{-n}$$

$$\boxed{\sum_{n=0}^{-\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] z^n}$$



$0.5 < |z|$

$|z| < 2$

$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

$$\boxed{\sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n}$$