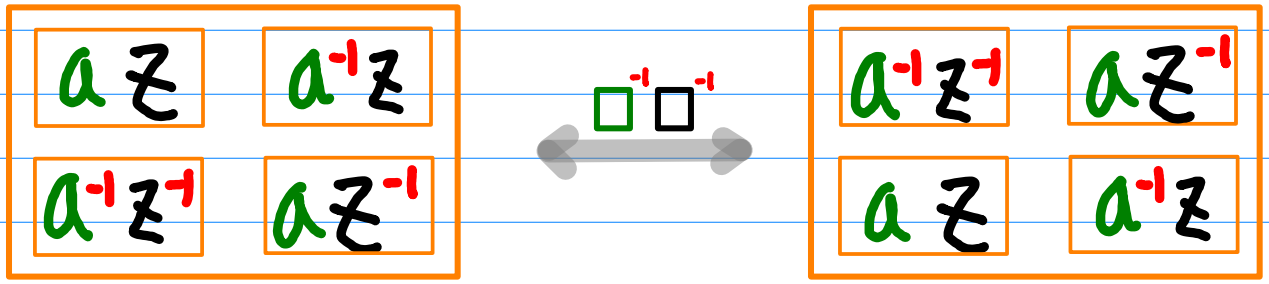
Laurent Series and z-Transform - Geometric Series Permutations A

20230711 Tue

Common ratios in geometric series

Assume $a \geq 1$



considered geometric series forms

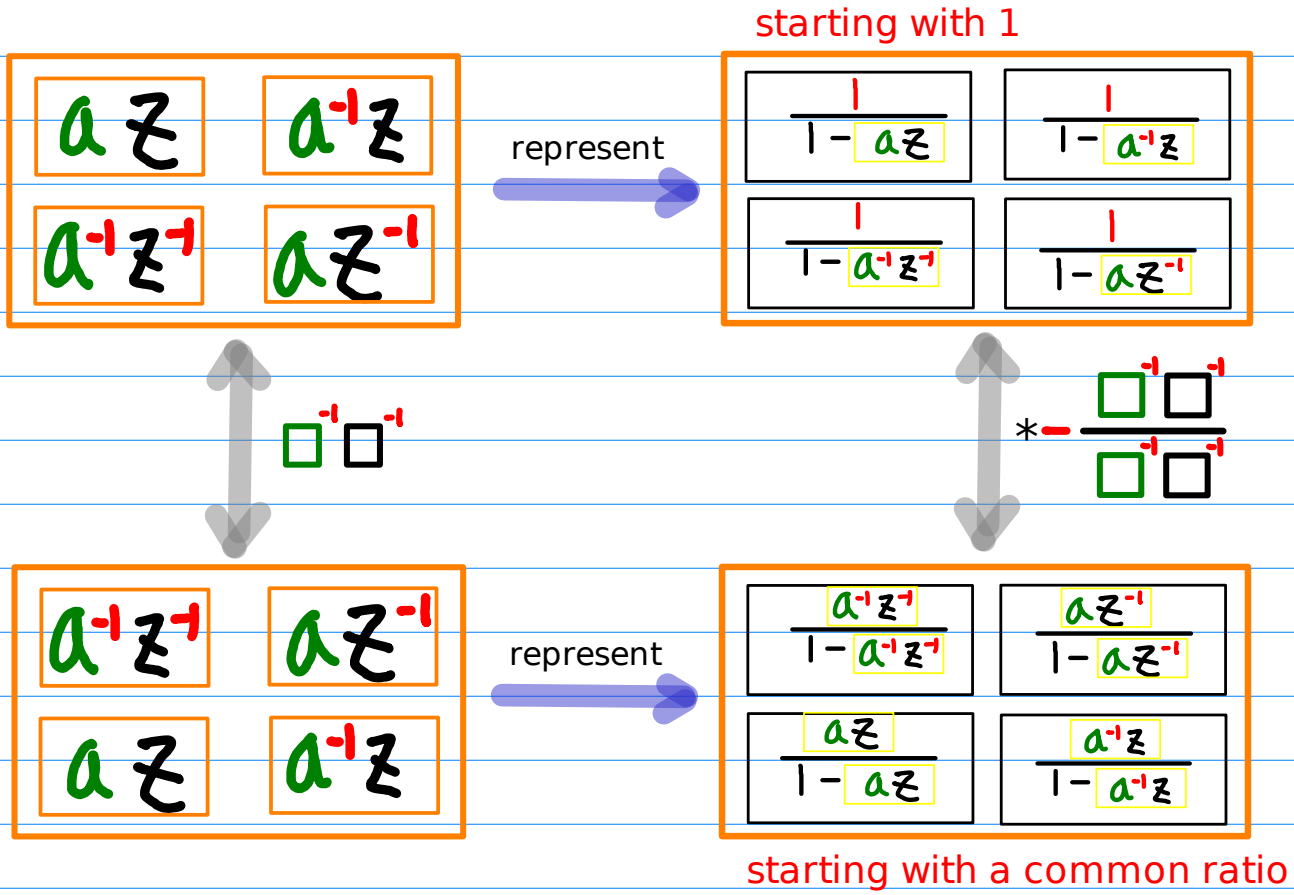
unit starting

$$\frac{1}{1-\text{C.R.}}$$

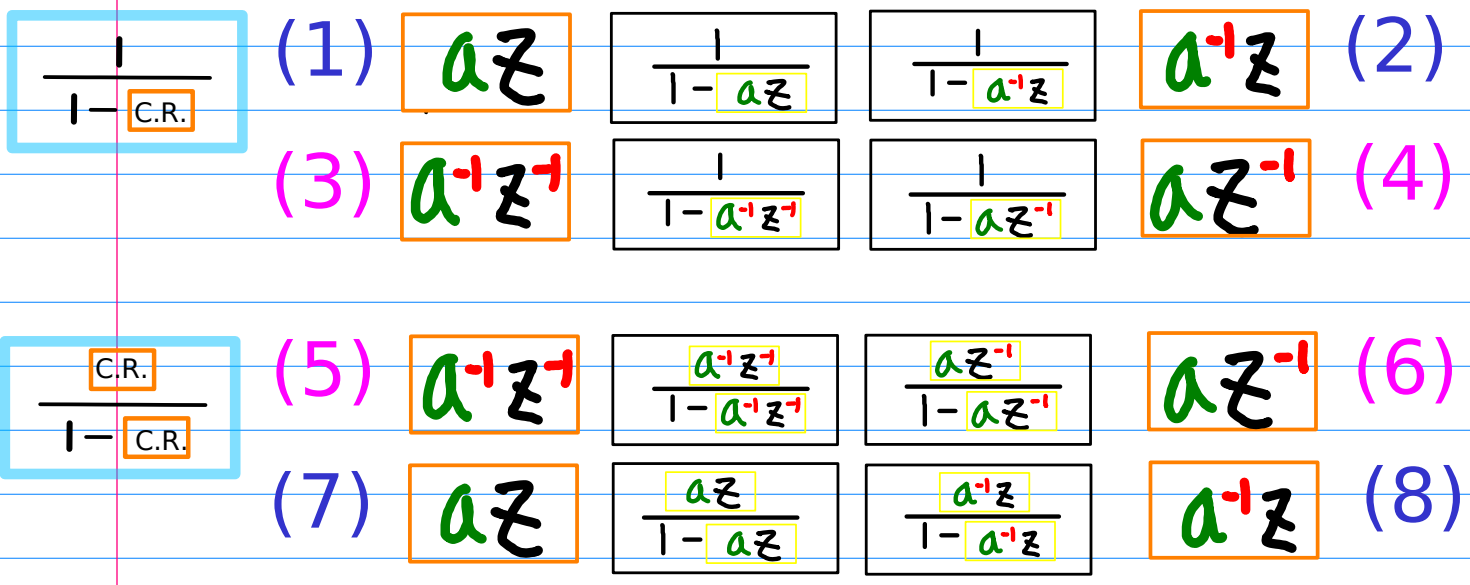
C.R. starting

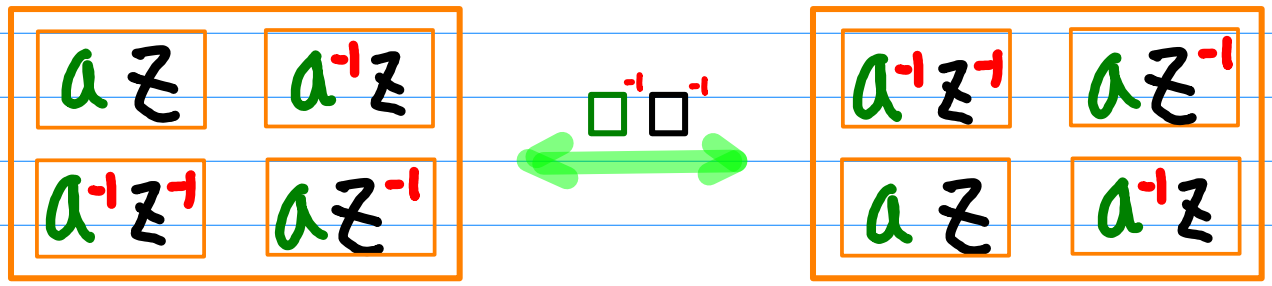
$$\frac{\text{C.R.}}{1-\text{C.R.}}$$

Representing geometric series



Numbering combinations





unit starting

(1), (2)

(3), (4)

C.R. starting

(5), (6)

(7), (8)

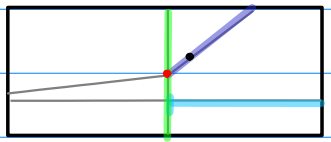
Unit starting **origin including**

(1) $\frac{1}{1 - az}$ a^n u(n)

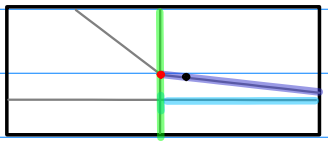
(2) $\frac{1}{1 - a^{-1}z}$ a^{-n} u(n)

(3) $\frac{1}{1 - a^{-1}z^{-1}}$ a^n u(-n)

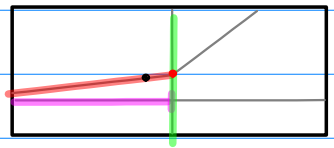
(4) $\frac{1}{1 - az^{-1}}$ a^{-n} u(-n)



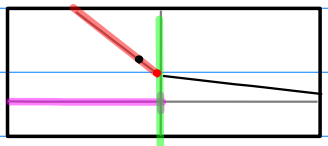
a^n u(n)



a^{-n} u(n)



a^n u(-n)



a^{-n} u(-n)

C.R. starting

origin excluding

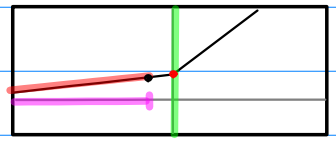
(Common Ratio)

(5) $\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $a^n\,u(-n-1)$

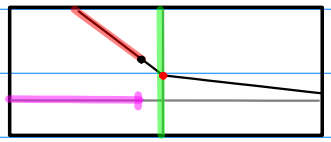
(6) $\dfrac{az^{-1}}{1-az^{-1}}$ $a^{-n}u(-n-1)$

(7) $\dfrac{az}{1-az}$ $a^n\,u(n-1)$

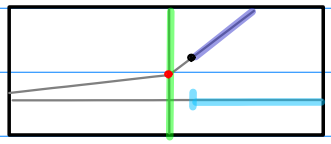
(8) $\dfrac{a^{-1}z}{1-a^{-1}z}$ $a^{-n}\,u(n-1)$



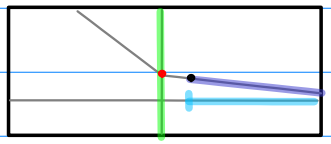
$a^n\,u(-n-1)$



$a^{-n}\,u(-n-1)$



$a^n\,u(n-1)$



$a^{-n}\,u(n-1)$

Decoding Geometric Series

Positive Exponent

$a^n z^n, a^{-n} z^{-n} \Rightarrow a^n$

even number of -1 exponent

Negative Exponent

$a^{-n} z^n, a^n z^{-n} \Rightarrow a^{-n}$

odd number of -1 exponent

Causal

$\square z \Rightarrow u(n), u(n-1)$

$\frac{1}{1-\square z} \Rightarrow u(n)$

$\frac{\square z}{1-\square z} \Rightarrow u(n-1)$

Anti-causal

$\square z^{-1} \Rightarrow u(-n), u(-n-1)$

$\frac{1}{1-\square z^{-1}} \Rightarrow u(-n)$

$\frac{\square z^{-1}}{1-\square z^{-1}} \Rightarrow u(-n-1)$

Decoding examples

Positive Exponent

even number of -1 exponent

Negative Exponent

odd number of -1 exponent

starting with 1 **o-including** u(n) u(-n)

(1) **o-including**

$\frac{1}{1-az}$ $|z|<a^{-1}$

$az \Rightarrow a^n$

$1, az \Rightarrow u(n)$

(2) **o-including**

$\frac{1}{1-a^{-1}z}$ $|z|<a$

$a^{-1}z \Rightarrow a^{-n}$

$1, a^{-1}z \Rightarrow u(n)$

(3) **o-including**

$\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$a^{-1}z^{-1} \Rightarrow a^n$

$1, a^{-1}z^{-1} \Rightarrow u(-n)$

(4) **o-including**

$\frac{1}{1-az^{-1}}$ $|z|>a$

$az^{-1} \Rightarrow a^{-n}$

$1, az^{-1} \Rightarrow u(-n)$

starting with a common ratio **o-execluding** u(n-1) u(-n-1)

(5) **o-execluding**

$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$a^{-1}z^{-1} \Rightarrow a^n$

$a^{-1}z^{-1}, a^{-1}z^{-1} \Rightarrow u(-n-1)$

(6) **o-excluding**

$\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$

$az^{-1} \Rightarrow a^{-n}$

$az^{-1}, az^{-1} \Rightarrow u(-n-1)$

(7) **o-excluding**

$\frac{az}{1-az}$ $|z|<a^{-1}$

$az \Rightarrow a^n$

$az, az \Rightarrow u(n-1)$

(8) **o-excluding**

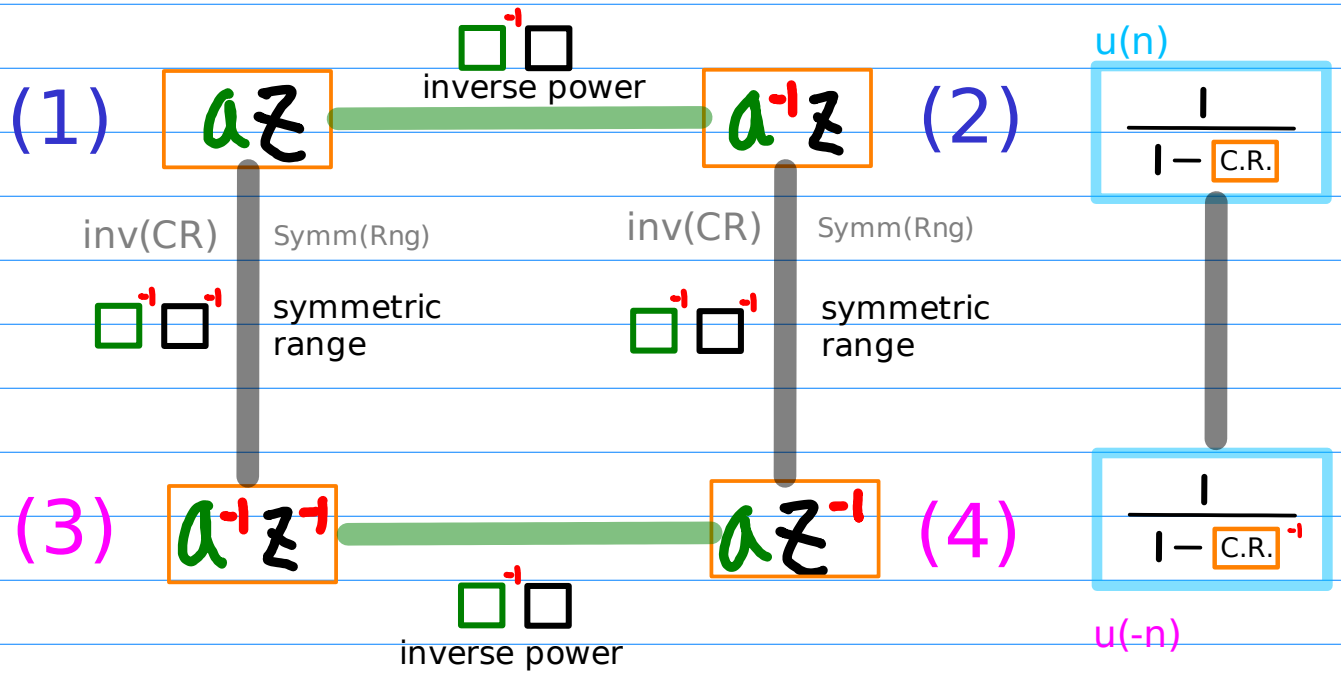
$\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$

$a^{-1}z \Rightarrow a^{-n}$

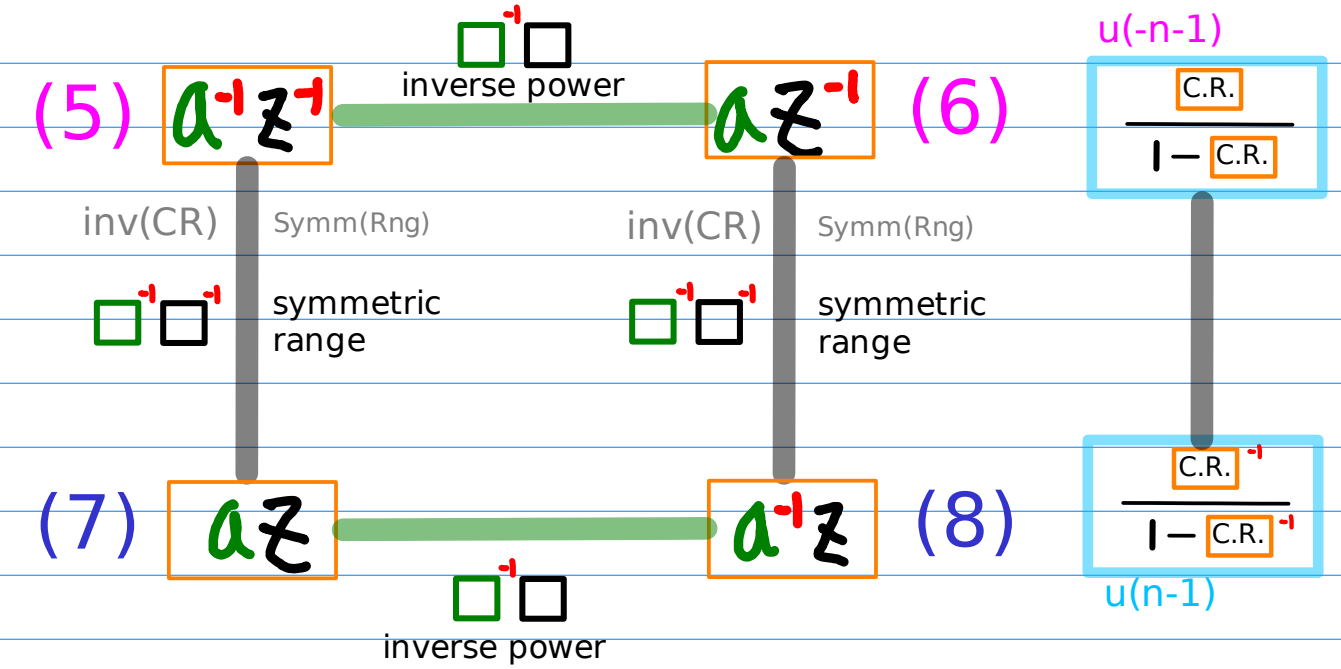
$a^{-1}z, a^{-1}z \Rightarrow u(n-1)$

1. Symmetric range relations

origin including geometric sequences

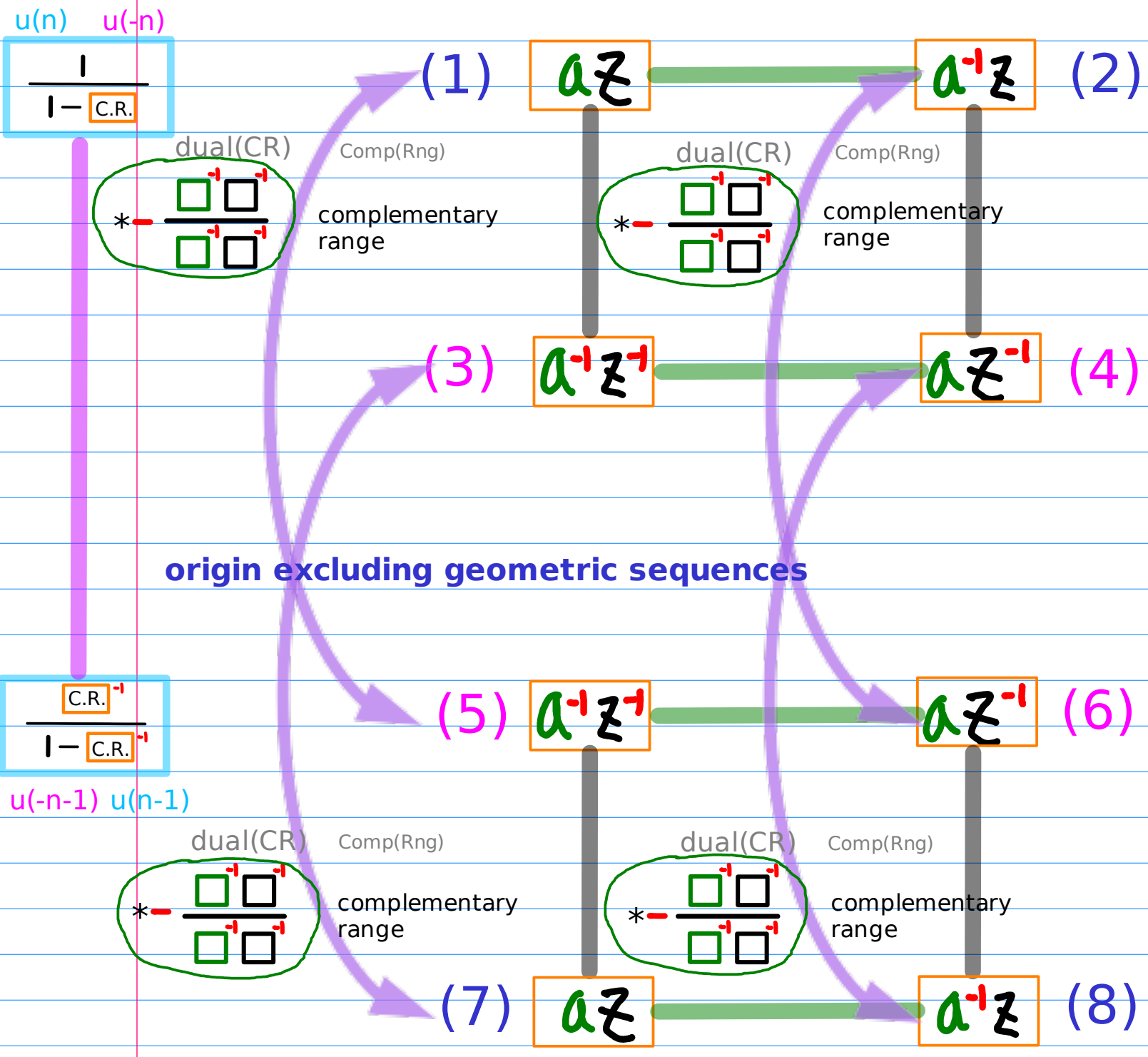


origin excluding geometric sequences



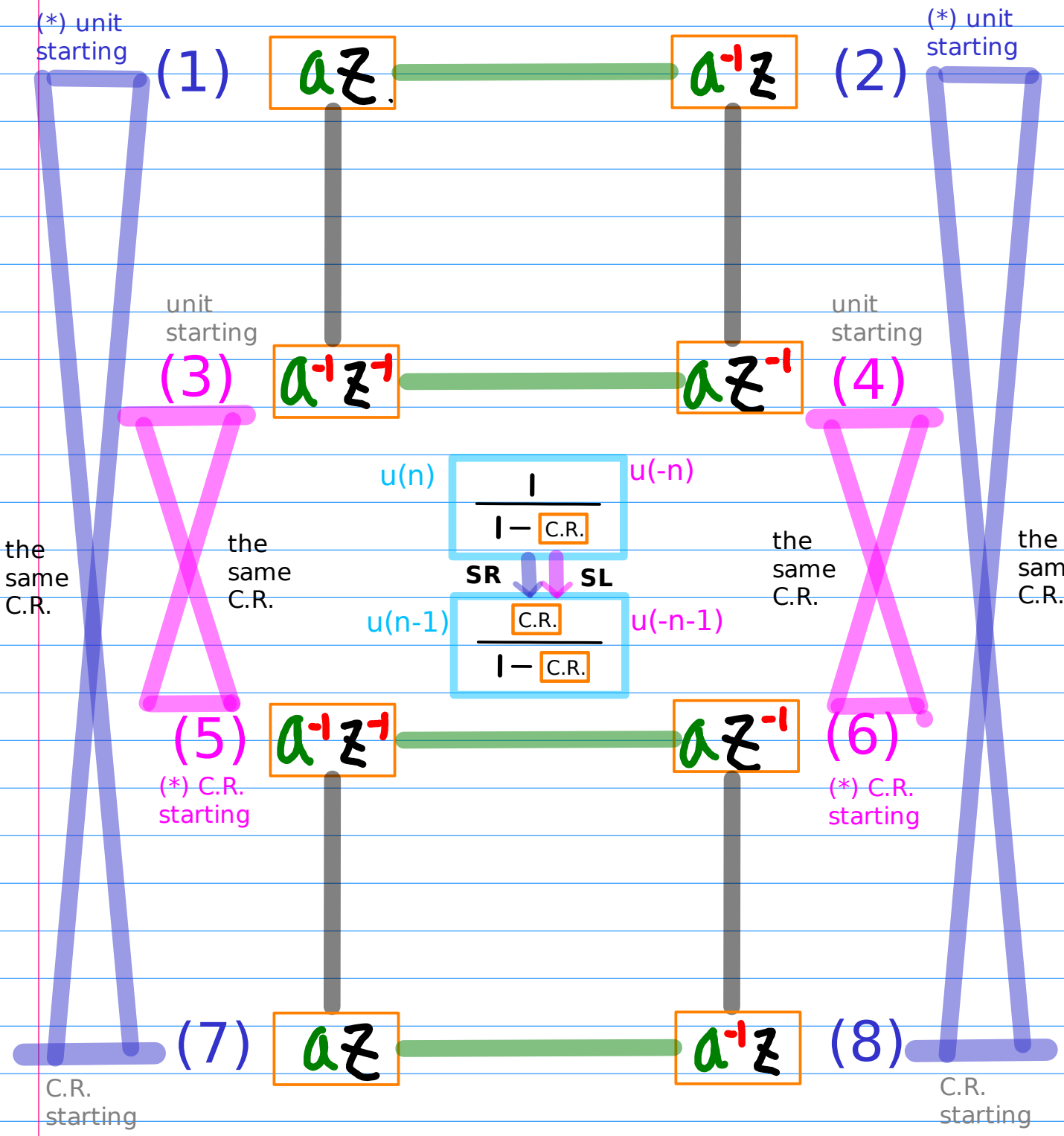
2. Complementary range relations

origin including geometric sequences



(1) (2)
(7)
(5) (6)
(3) (4)

3. Shifted range relations

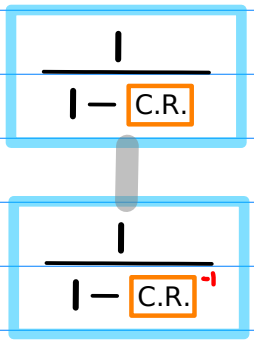


Range combinations in unshifted sequence (1)

1. Symmetric Range

(1) (2)

(3) (4)

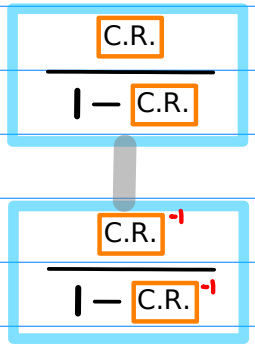


u(n)

u(-n)

(5) (6)

(7) (8)

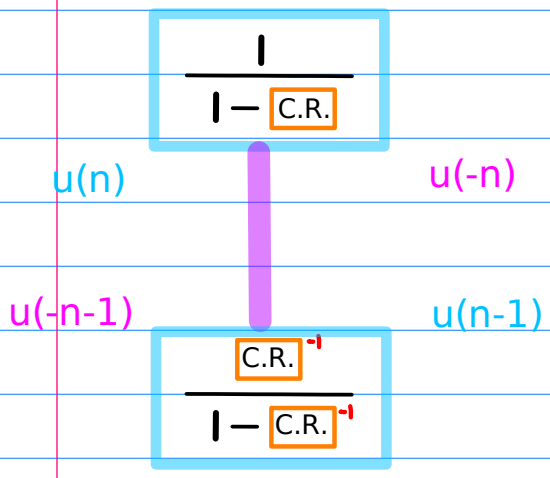


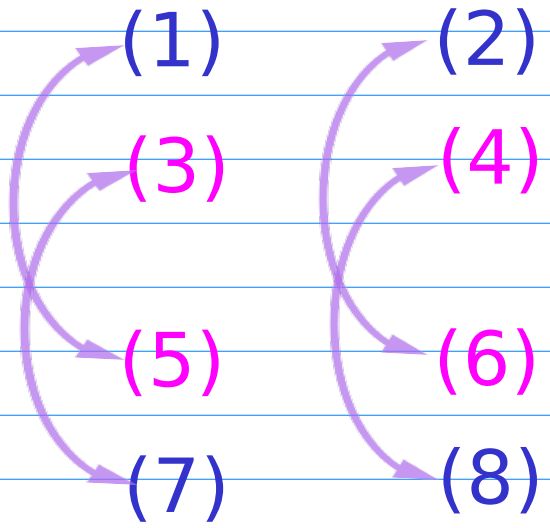
u(-n-1)

u(n-1)

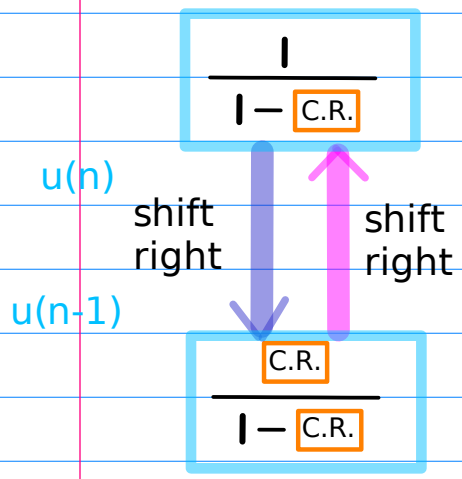
Range combinations in unshifted sequence (2)

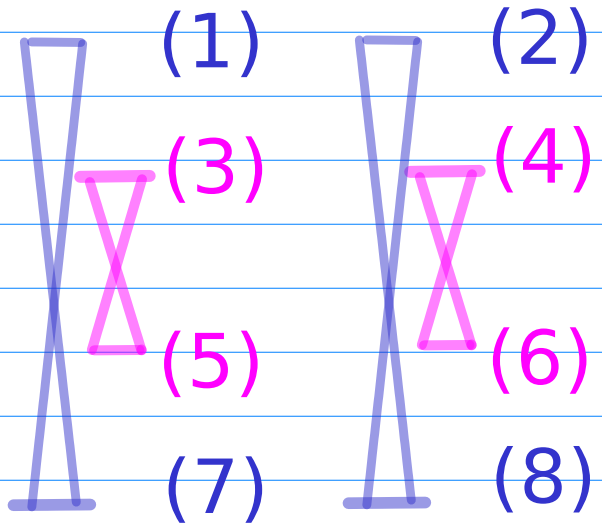
2. Complementary Range





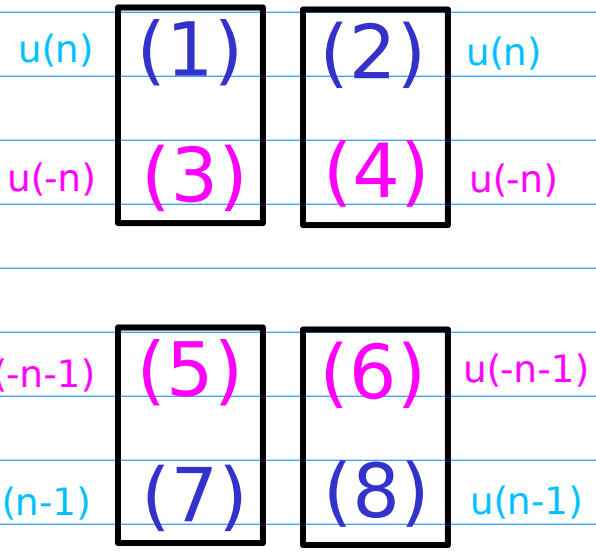
3. Shifted Range



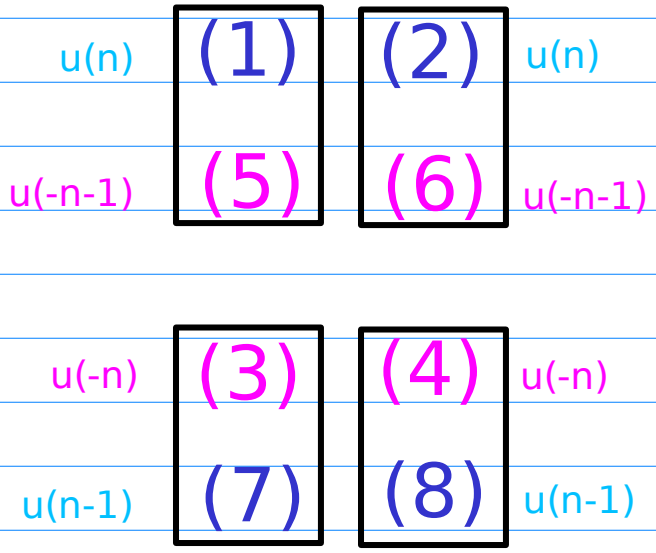


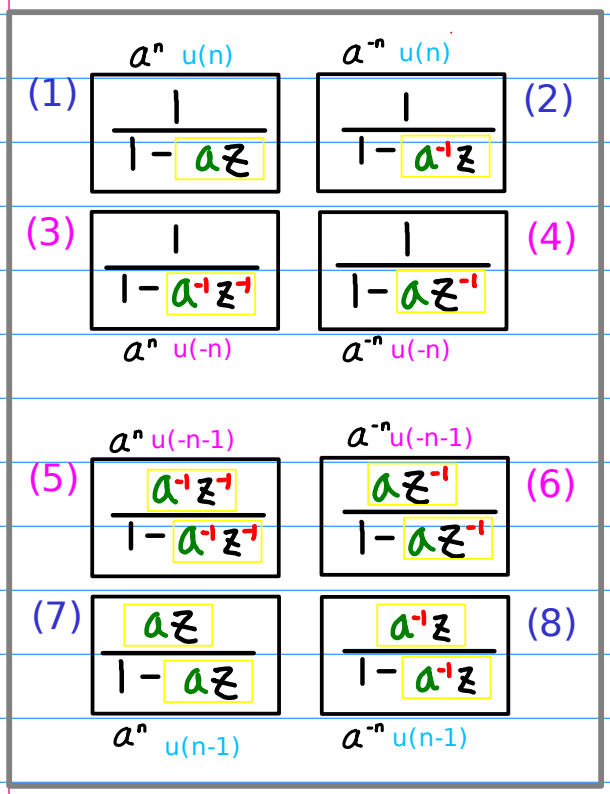
Range combinations in unshifted sequence (3)

1.Symmetric Range

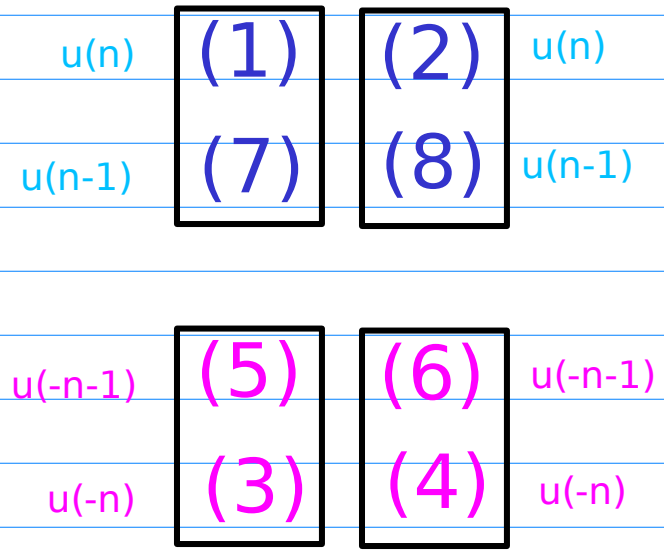


2. Complementary Range





3. Shifted Range



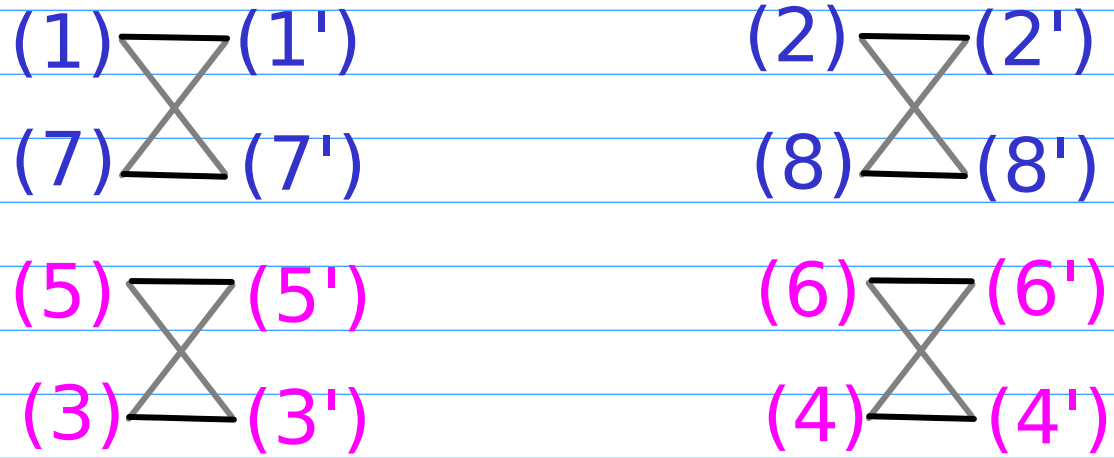
Unshifted Sequence **Shifted Sequence**

(1) (2) (1') (2')

(3) (4) (3') (4')

(5) (6) (5') (6')

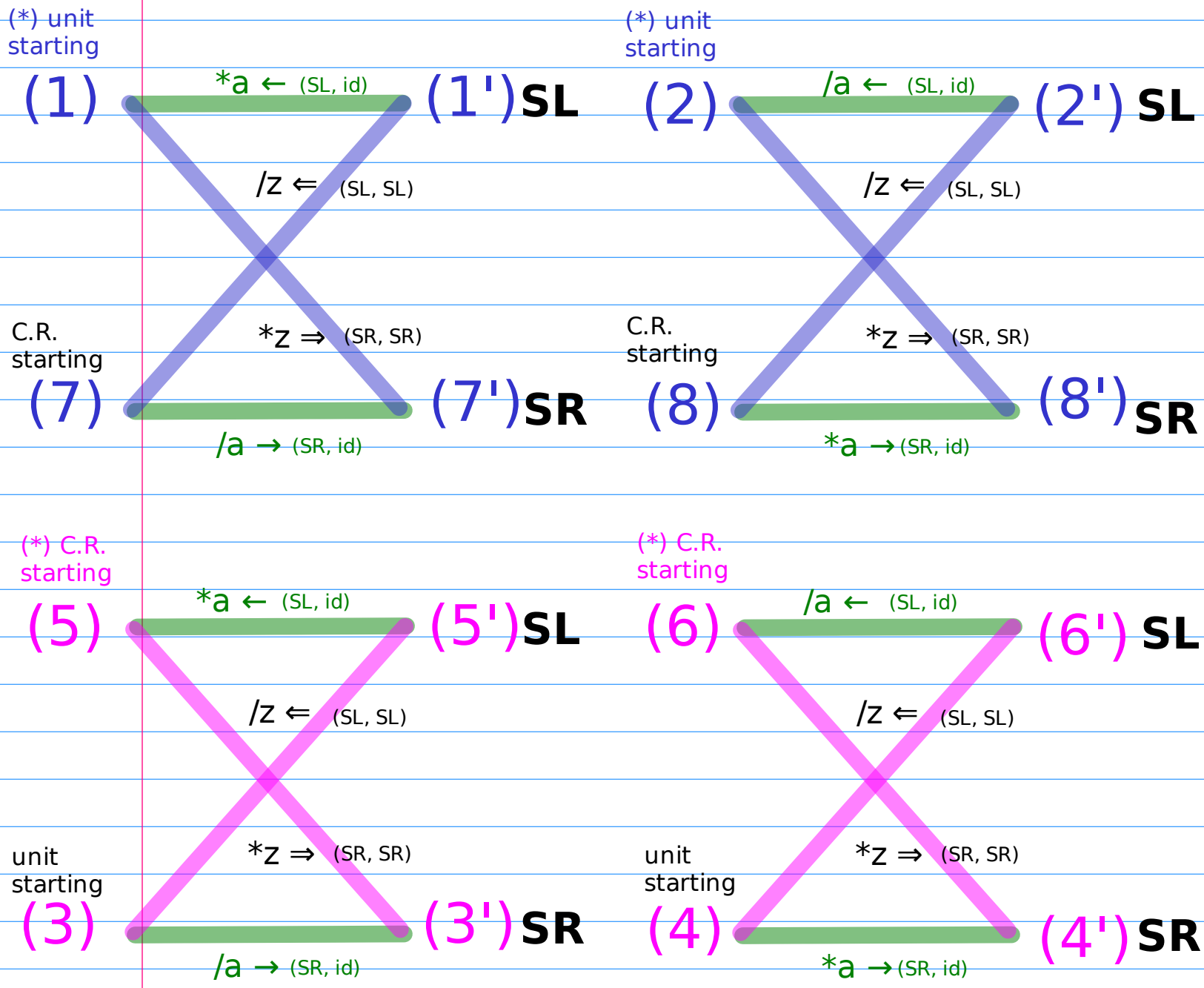
(7) (8) (7') (8')



Unshifted & Shifted Sequences (1)

(exponent, range)

{SL, SR} {SL, SR, id}



1. Symmetric Range

in unshifed and shifted sequences

u(n) u(-n-1)

u(-n) u(n-1)

Original Sequences

(1) $\frac{1}{1-az}$ $\frac{1}{1-a^{-1}z}$ (2)

(3) $\frac{1}{1-a^{-1}z^{-1}}$ $\frac{1}{1-az^{-1}}$ (4)

(5) $\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\frac{az^{-1}}{1-az^{-1}}$ (6)

(7) $\frac{az}{1-az}$ $\frac{a^{-1}z}{1-a^{-1}z}$ (8)

Shifted Sequences

(1') $\frac{a}{1-az}$ $\frac{a^{-1}}{1-a^{-1}z}$ (2')

(3') $\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\frac{a}{1-az^{-1}}$ (4')

(5') $\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\frac{z^{-1}}{1-az^{-1}}$ (6')

(7') $\frac{z}{1-az}$ $\frac{z}{1-a^{-1}z}$ (8')

(1) $a^n\,u(n)$ $a^{-n}\,u(n)$ (2)

(3) $a^n\,u(-n)$ $a^{-n}\,u(-n)$ (4)

(5) $a^n u(-n-1)$ $a^{-n}u(-n-1)$ (6)

(7) $a^n u(n-1)$ $a^{-n}u(n-1)$ (8)

(1') $a^{n+1}u(n)$ $a^{-n-1}u(n)$ (2')

(3') $a^{n-1}u(-n)$ $a^{-n+1}u(-n)$ (4')

(5') $a^{n+1}u(-n-1)$ $a^{-n-1}u(-n-1)$ (6')

(7') $a^{n-1}u(n-1)$ $a^{-n+1}u(n-1)$ (8')

2. Complementary Range

in unshifed and shifted sequences

u(n) — u(-n)

u(-n-1) — u(n-1)

Original Sequences

(1) $\frac{1}{1-az}$ $\frac{1}{1-a^{-1}z}$ (2)

(5) $\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\frac{az^{-1}}{1-az^{-1}}$ (6)

(3) $\frac{1}{1-a^{-1}z^{-1}}$ $\frac{1}{1-az^{-1}}$ (4)

(7) $\frac{az}{1-az}$ $\frac{a^{-1}z}{1-a^{-1}z}$ (8)

Shifted Sequences

(1') $\frac{a}{1-az}$ $\frac{a^{-1}}{1-a^{-1}z}$ (2')

(5') $\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\frac{z^{-1}}{1-az^{-1}}$ (6')

(3') $\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\frac{a}{1-az^{-1}}$ (4')

(7') $\frac{z}{1-az}$ $\frac{z}{1-a^{-1}z}$ (8')

(1) $a^n\,u(n)$ $a^{-n}\,u(n)$ (2)

(5) $a^n u(-n-1)$ $a^{-n}u(-n-1)$ (6)

(3) $a^n\,u(-n)$ $a^{-n}\,u(-n)$ (4)

(7) $a^n u(n-1)$ $a^{-n}u(n-1)$ (8)

(1') $a^{n+1}u(n)$ $a^{-n-1}u(n)$ (2')

(5') $a^{n+1}u(-n-1)$ $a^{-n-1}u(-n-1)$ (6')

(3') $a^{n-1}u(-n)$ $a^{-n+1}u(-n)$ (4')

(7') $a^{n-1}u(n-1)$ $a^{-n+1}u(n-1)$ (8')

3. Shifted Range

in unshifed and shifted sequences

u(n) u(-n)

u(n-1) u(-n-1)

Original Sequences

(1) $\frac{1}{1-az}$ $\frac{1}{1-a^{-1}z}$ (2)

(7) $\frac{az}{1-az}$ $\frac{a^{-1}z}{1-a^{-1}z}$ (8)

(5) $\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\frac{az^{-1}}{1-az^{-1}}$ (6)

(3) $\frac{1}{1-a^{-1}z^{-1}}$ $\frac{1}{1-az^{-1}}$ (4)

Shifted Sequences

(1') $\frac{a}{1-az}$ $\frac{a^{-1}}{1-a^{-1}z}$ (2')

(7') $\frac{z}{1-az}$ $\frac{z}{1-a^{-1}z}$ (8')

(5') $\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\frac{z^{-1}}{1-az^{-1}}$ (6')

(3') $\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\frac{a}{1-az^{-1}}$ (4')

(1) $a^n\,u(n)$ $a^{-n}\,u(n)$ (2)

(7) $a^n u(n-1)$ $a^{-n}u(n-1)$ (8)

(5) $a^n u(-n-1)$ $a^{-n}u(-n-1)$ (6)

(3) $a^n\,u(-n)$ $a^{-n}\,u(-n)$ (4)

(1') $a^{n+1}u(n)$ $a^{-n-1}u(n)$ (2')

(7') $a^{n-1}u(n-1)$ $a^{-n+1}u(n-1)$ (8')

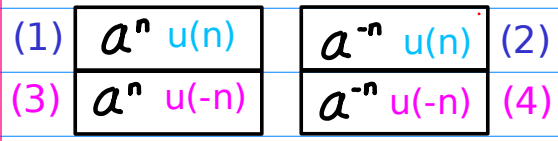
(5') $a^{n+1}u(-n-1)$ $a^{-n-1}u(-n-1)$ (6')

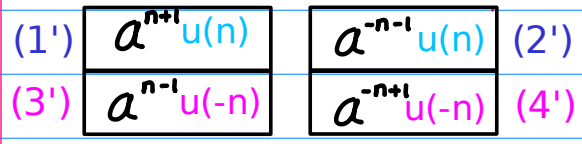
(3') $a^{n-1}u(-n)$ $a^{-n+1}u(-n)$ (4')

Range combinations

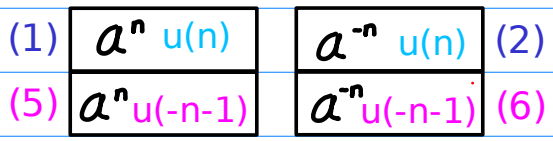
in unshifed and shifted sequences

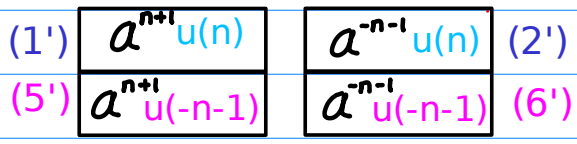
1. Symmetric Range



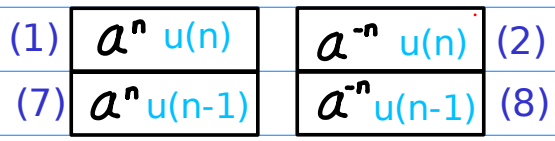


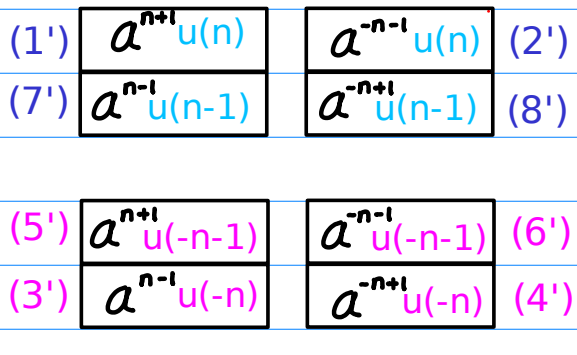
2. Complementary Range



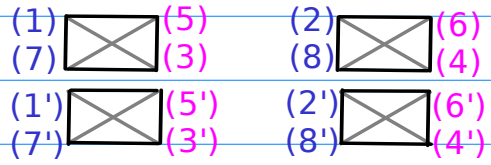


3. Shifted Range



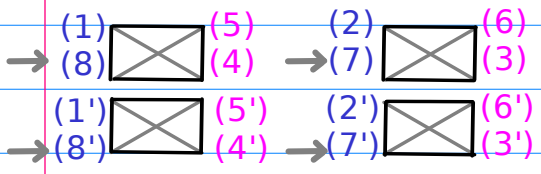


Case 1 Shifted, Complementary, & Symmetric Ranges



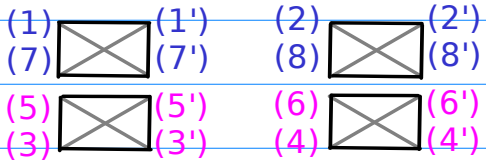
u(n) u(-n-1)	a^{n} a^{n}	a^{-n} a^{-n}
u(n-1) u(-n)	a^{n} a^{n}	a^{-n} a^{-n}
u(n) u(-n-1)	a^{n+1} a^{n+1}	a^{-n-1} a^{-n-1}
u(n-1) u(-n)	a^{n-1} a^{n-1}	a^{-n+1} a^{-n+1}

Case 2 Shifted, Complementary, & Symmetric Ranges - Flipped



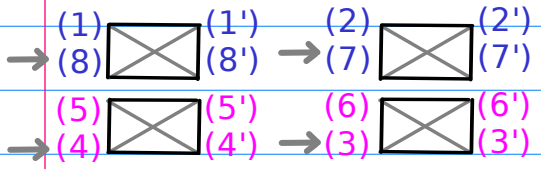
u(n) u(-n-1)	a^{n} a^{n}	a^{-n} a^{-n}
u(n-1) u(-n)	a^{-n} a^{-n}	a^{n} a^{n}
u(n) u(-n-1)	a^{n+1} a^{n+1}	a^{-n-1} a^{-n-1}
u(n-1) u(-n)	a^{-n+1} a^{-n+1}	a^{n-1} a^{n-1}

Case 3 Shifted, Id, & Shifted Ranges



u(n) u(n)	a^{n} a^{n+1}	a^{-n} a^{-n-1}
u(n-1) u(n-1)	a^{n} a^{n-1}	a^{-n} a^{-n+1}
u(-n-1) u(-n-1)	a^{n} a^{n+1}	a^{-n} a^{-n-1}
u(-n) u(-n)	a^{n} a^{n-1}	a^{-n} a^{-n+1}

Case 4 Shifted, Id, & Shifted Ranges - Flipped



u(n) u(n)	a^{n} a^{n+1}	a^{-n} a^{-n-1}
u(n-1) u(n-1)	a^{-n} a^{-n+1}	a^{n} a^{n-1}
u(-n-1) u(-n-1)	a^{n} a^{n+1}	a^{-n} a^{-n-1}
u(-n) u(-n)	a^{-n} a^{-n+1}	a^{n} a^{n-1}

Case 1
exp

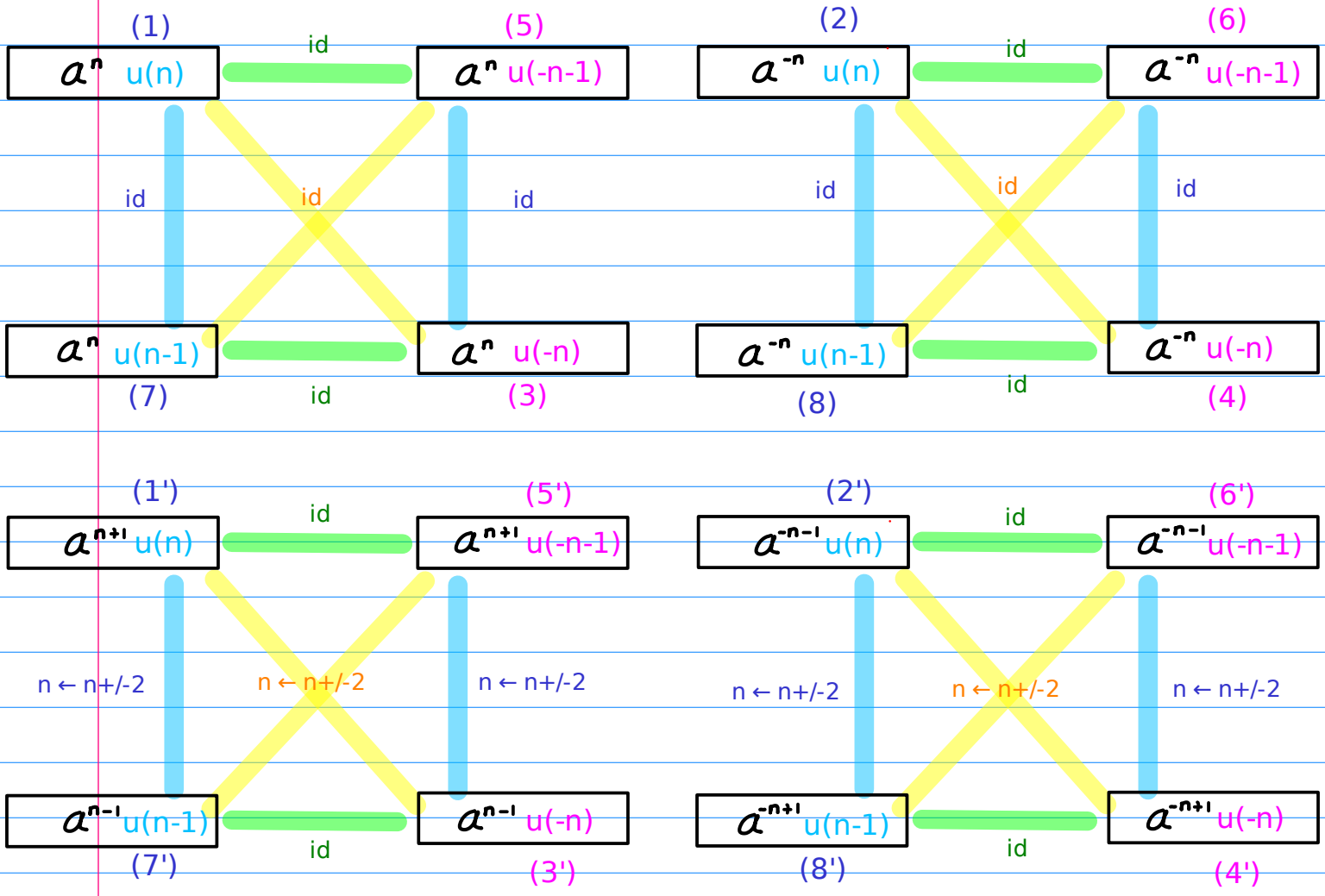
(1) (5)
(7) (3)

(2) (6)
(8) (4)

(1') (5')
(7') (3')

(2') (6')
(8') (4')

Shifted, Complementary, & Symmetric Ranges



Case 1 Shifted, Complementary, & Symmetric Ranges

(1) (5)
(7) (3)

(2) (6)
(8) (4)

(1') (5')
(7') (3')

(2') (6')
(8') (4')

a^n a^n
a^n a^n
a^{n+1} a^{n+1}
a^{n-1} a^{n-1}

a^{-n} a^{-n}
a^{-n} a^{-n}
a^{-n-1} a^{-n-1}
a^{-n+1} a^{-n+1}

Case 1 range

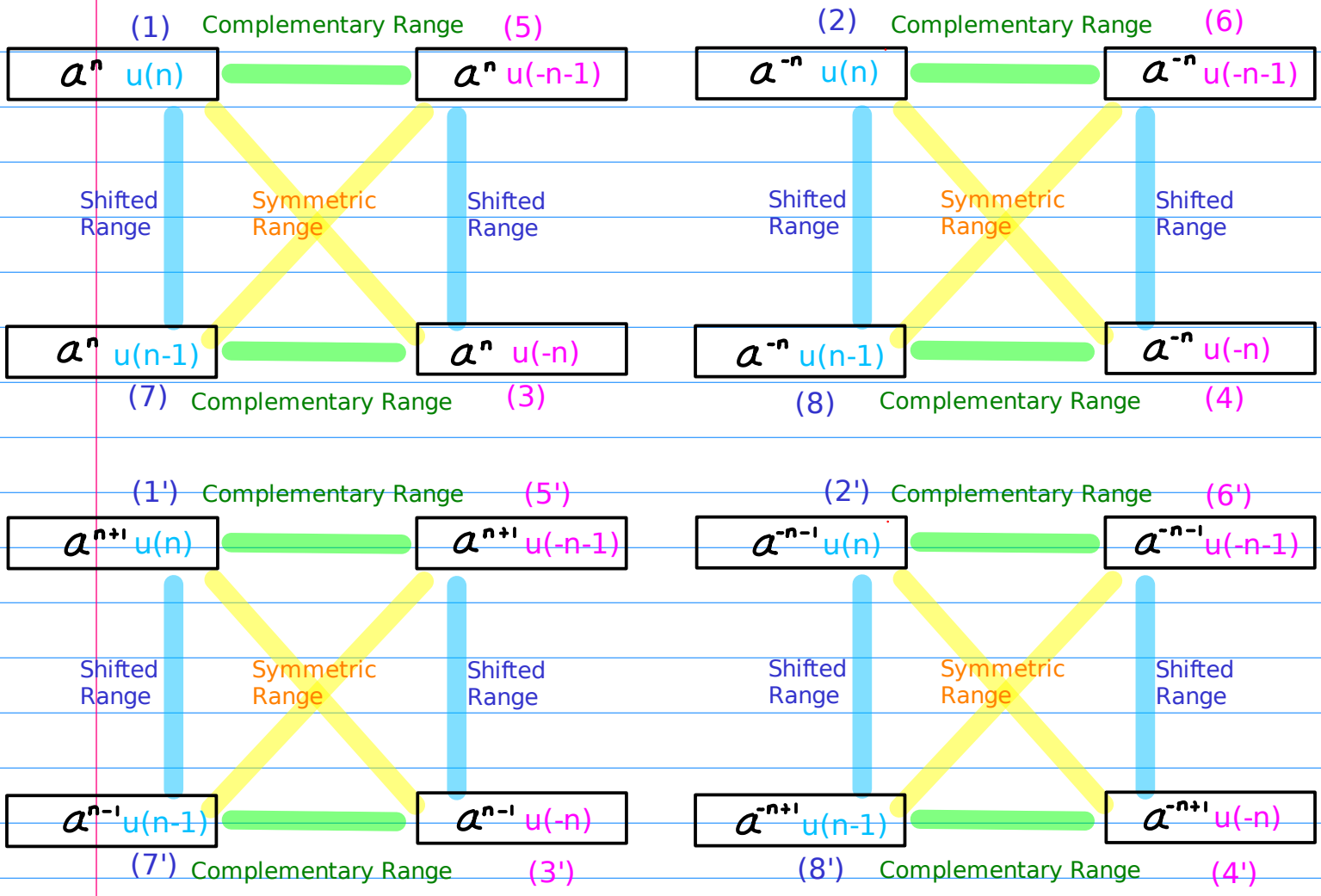
(1) (5)
(7) (3)

(2) (6)
(8) (4)

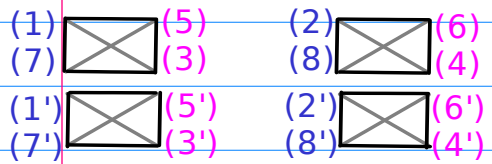
(1') (5')
(7') (3')

(2') (6')
(8') (4')

Shifted, Complementary, & Symmetric Ranges



Case 1 Shifted, Complementary, & Symmetric Ranges



u(n) u(-n-1)
u(n-1) u(-n)
u(n) u(-n-1)
u(n-1) u(-n)

u(n) u(-n-1)
u(n-1) u(-n)
u(n) u(-n-1)
u(n-1) u(-n)

Case 2
exp

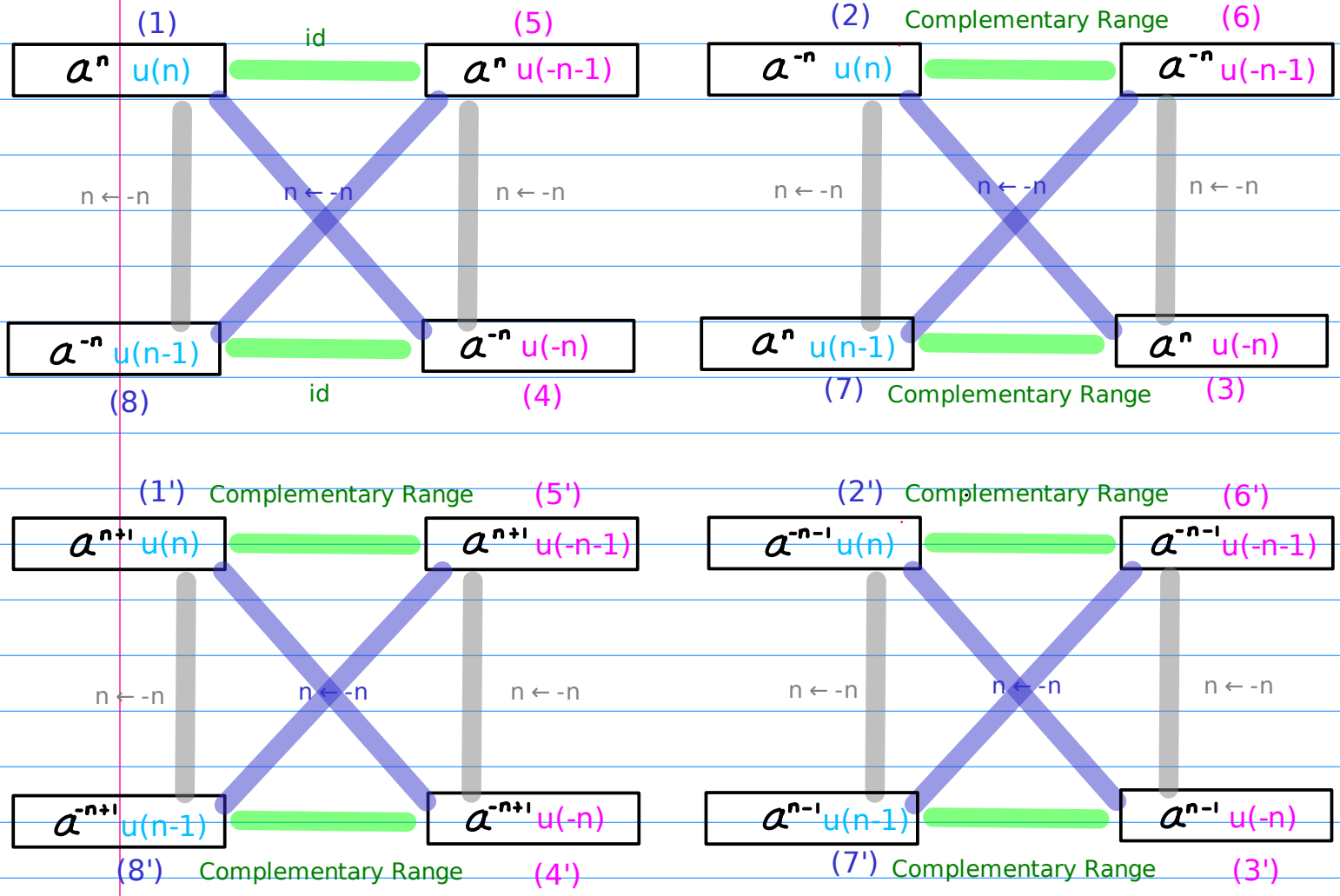
(1) (5)
(8) (4)

(2) (6)
(7) (3)

(1') (5')
(8') (4')

(2') (6')
(7') (3')

Shifted, Complementary, & Symmetric Ranges - Flipped



Case 2 Shifted, Complementary, & Symmetric Ranges - Flipped

(1) (5)
(8) (4)

(2) (6)
(7) (3)

(1') (5')
(8') (4')

(2') (6')
(7') (3')

a^{n} a^{n}
a^{-n} a^{-n}
a^{n+1} a^{n+1}
a^{-n+1} a^{-n+1}

a^{-n} a^{-n}
a^{n} a^{n}
a^{-n-1} a^{-n-1}
a^{n-1} a^{n-1}

Case 2 range

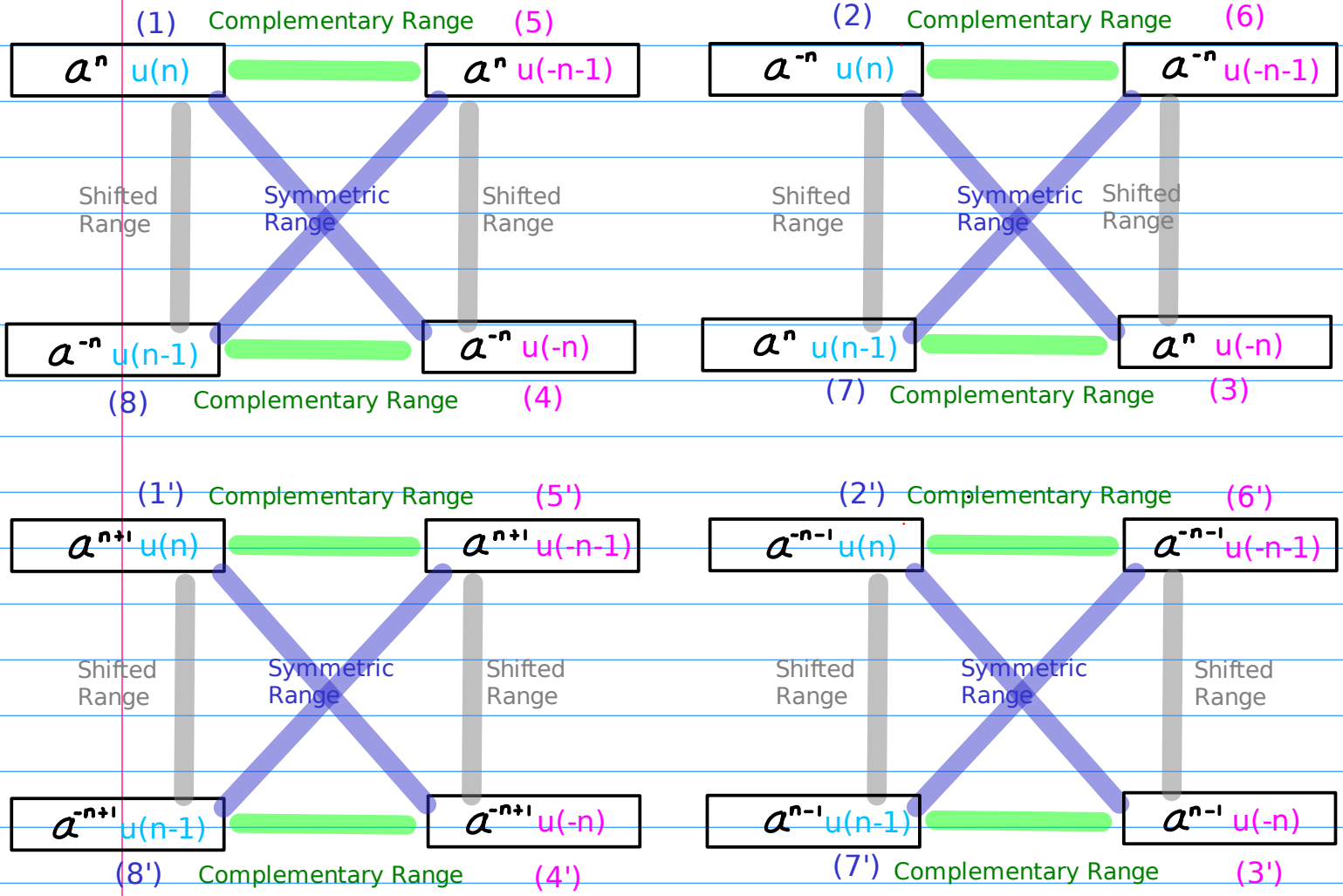
(1) (5)
(8) (4)

(2) (6)
(7) (3)

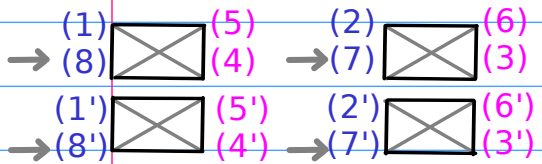
(1') (5')
(8') (4')

(2') (6')
(7') (3')

Shifted, Complementary, & Symmetric Ranges - Flipped



Case 2 Shifted, Complementary, & Symmetric Ranges - Flipped



u(n) u(-n-1)
u(n-1) u(-n)
u(n) u(-n-1)
u(n-1) u(-n)

u(n) u(-n-1)
u(n-1) u(-n)
u(n) u(-n-1)
u(n-1) u(-n)

Case 3
exp

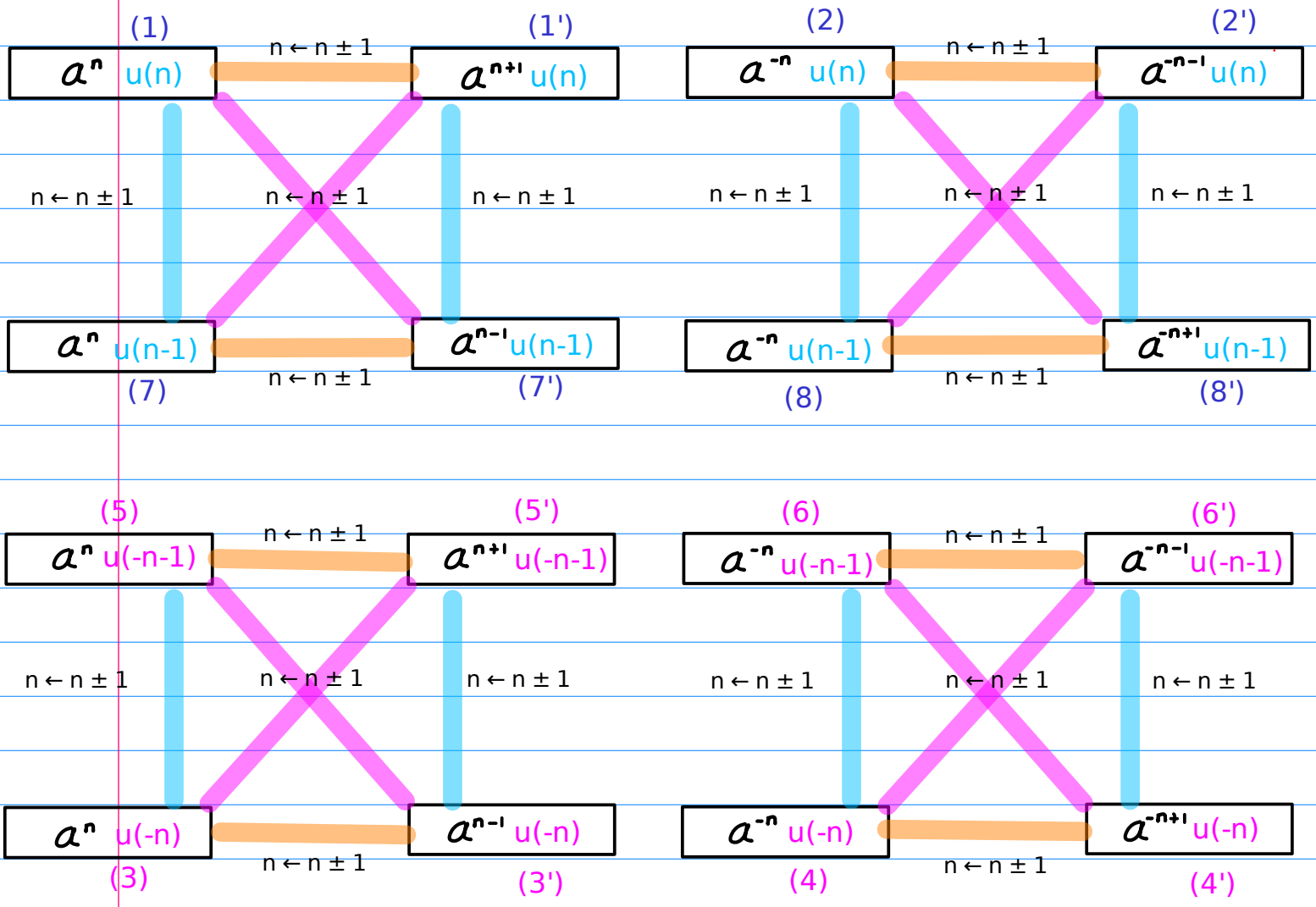
(1) (1')
(7) (7')

(2) (2')
(8) (8')

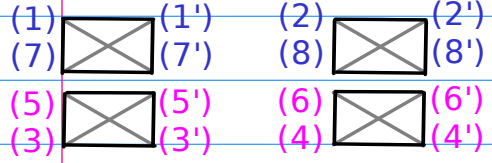
(5) (5')
(3) (3')

(6) (6')
(4) (4')

Shifted, Id, & Shifted Ranges



Case 3 Shifted, Id, & Shifted Ranges



a^{n} a^{n+1}
a^{n} a^{n-1}
a^{n} a^{n+1}
a^{n} a^{n-1}

a^{-n} a^{-n-1}
a^{-n} a^{-n+1}
a^{-n} a^{-n-1}
a^{-n} a^{-n+1}

Case 3 range

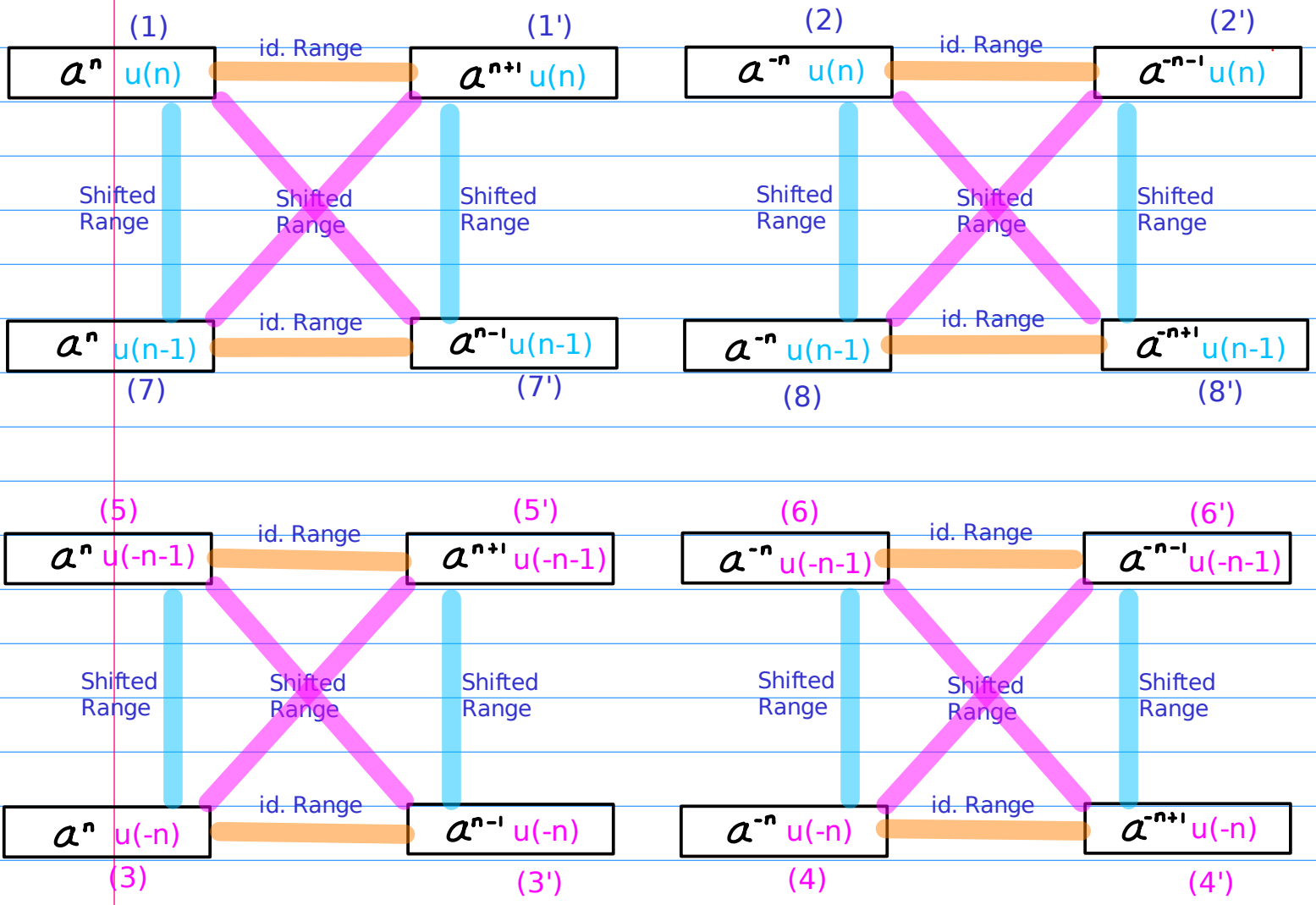
(1) (1') (7) (7')

(2) (2') (8) (8')

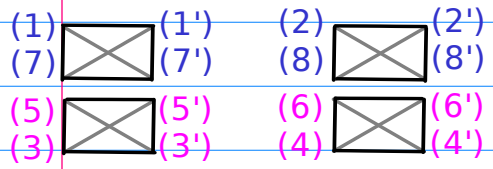
(5) (5') (3) (3')

(6) (6') (4) (4')

Shifted, Id, & Shifted Ranges



Case 3 Shifted, Id, & Shifted Ranges



u(n) u(n)
u(n-1) u(n-1)
u(-n-1) u(-n-1)
u(-n) u(-n)

u(n) u(n)
u(n-1) u(n-1)
u(-n-1) u(-n-1)
u(-n) u(-n)

Case 4
exp

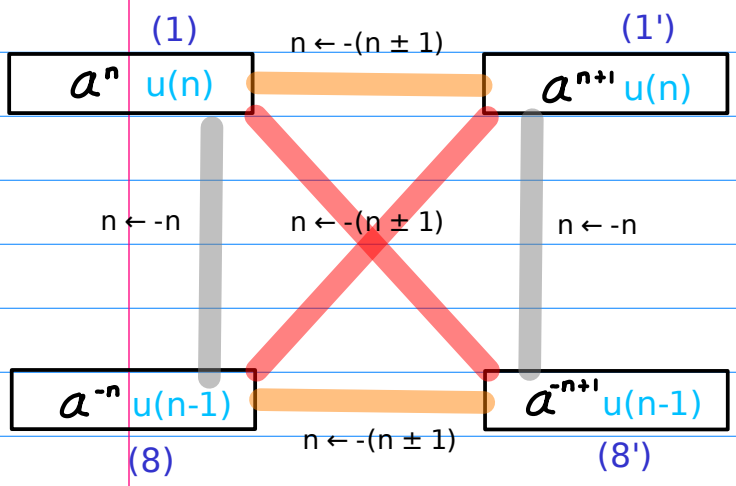
(1) (1')
(8) (8')

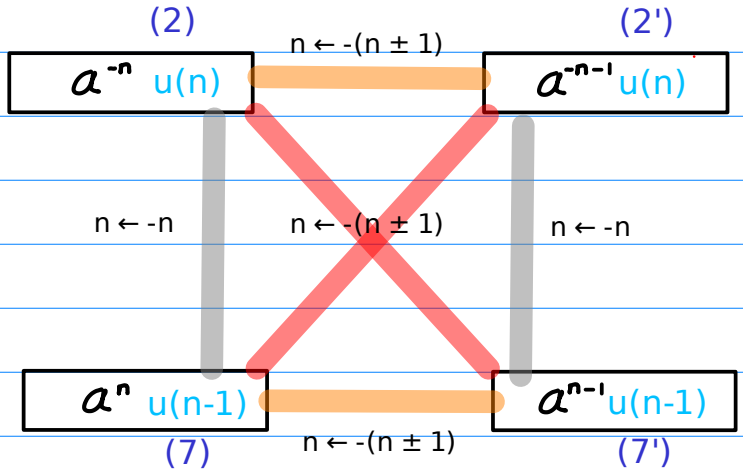
(2) (2')
(7) (7')

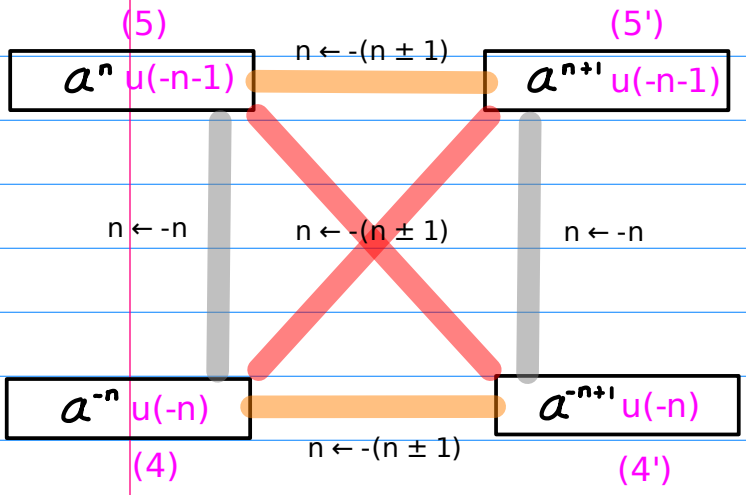
(5) (5')
(4) (4')

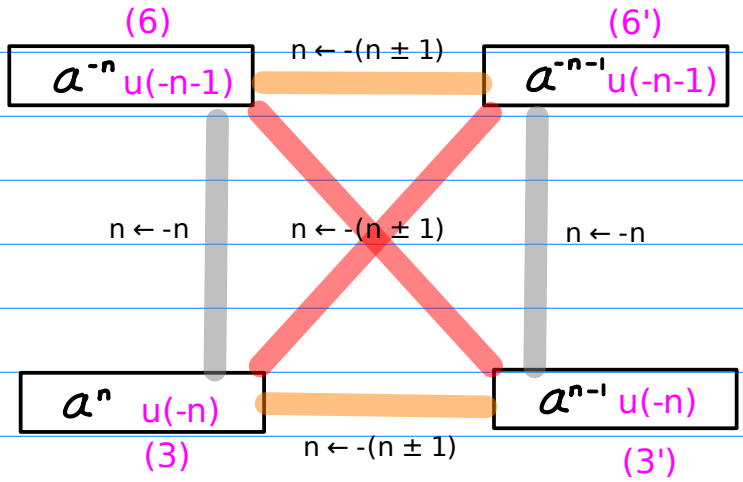
(6) (6')
(3) (3')

Shifted, Id, & Shifted Ranges - Flipped

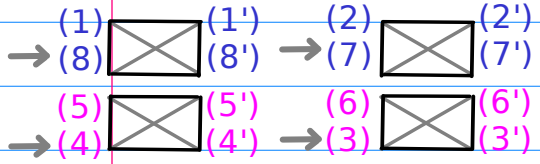








Case 4 Shifted, Id, & Shifted Ranges - Flipped



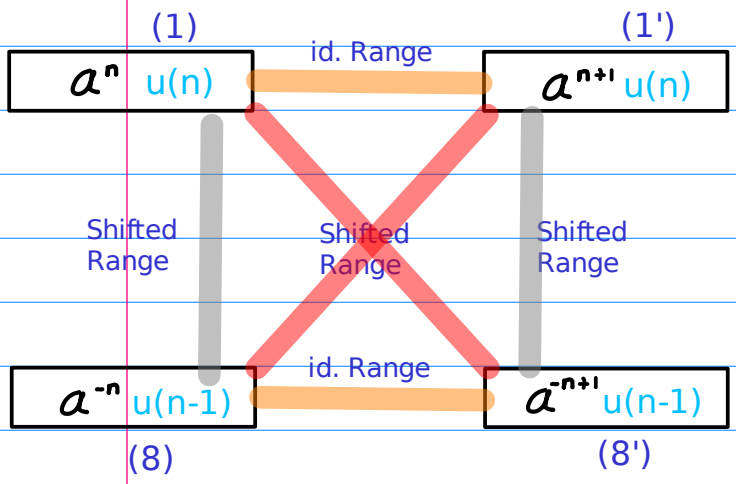
a^{n} a^{n+1}
a^{-n} a^{-n+1}
a^{n} a^{n+1}
a^{-n} a^{-n+1}

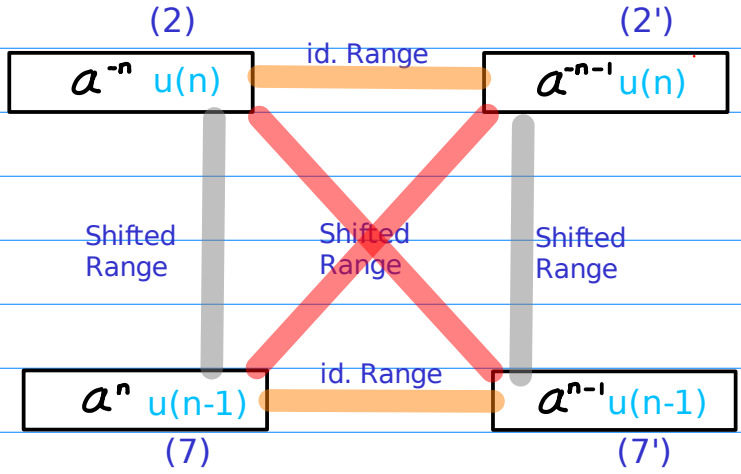
a^{-n} a^{-n-1}
a^{n} a^{n-1}
a^{-n} a^{-n-1}
a^{n} a^{n-1}

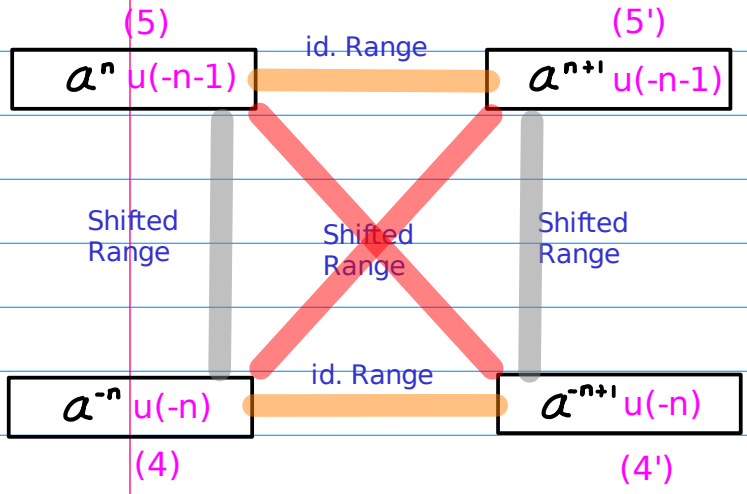
Case 4 range → (1) (1') (8) (8') ← (2) (2') (7) (7')

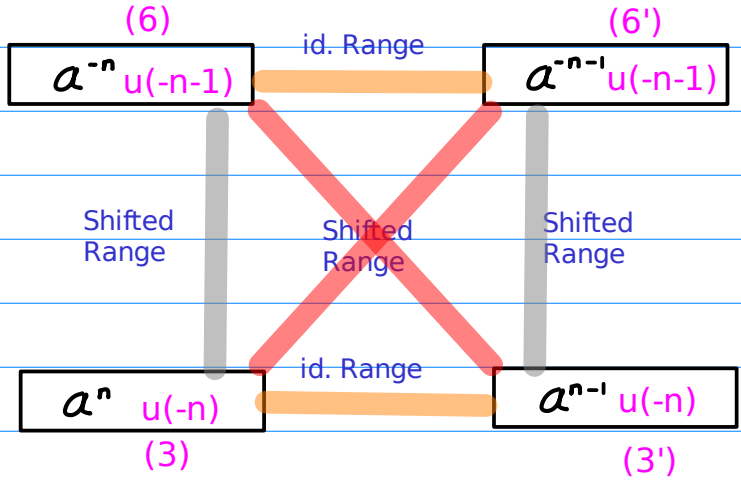
→ (5) (5') (4) (4') ← (6) (6') (3) (3')

Shifted, Id, & Shifted Ranges - Flipped

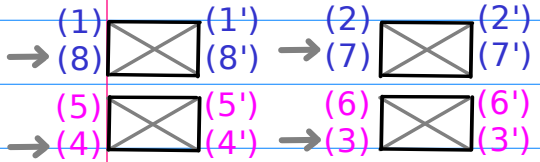








Case 4 Shifted, Id, & Shifted Ranges - Flipped



u(n) u(n)
u(n-1) u(n-1)
u(-n-1) u(-n-1)
u(-n) u(-n)

u(n) u(n)
u(n-1) u(n-1)
u(-n-1) u(-n-1)
u(-n) u(-n)

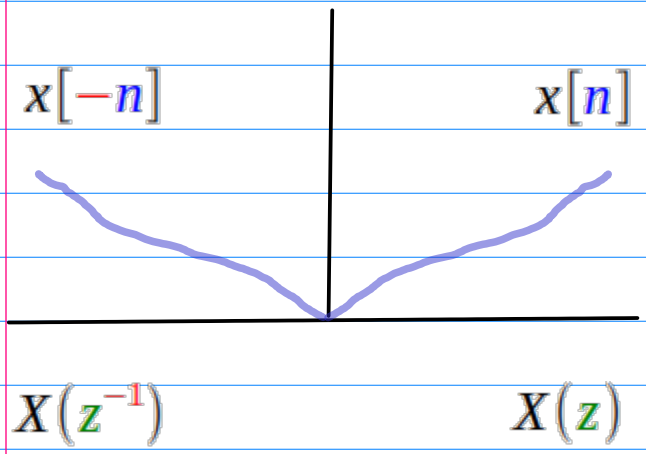
Time Reversal in z-Transform

Time reversal

$$x[n] \quad \Longleftrightarrow \quad x[-n]$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$X(z) \quad \Longleftrightarrow \quad X(z^{-1})$$

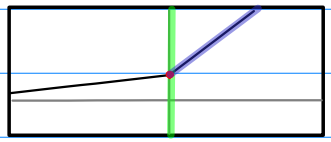


$$\mathcal{Z}\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n)\, z^{-n}$$

$$= \sum_{m=-\infty}^{\infty} x(m)\, z^{m}$$

$$= \sum_{m=-\infty}^{\infty} x(m)\left(z^{-1}\right)^{-m}$$

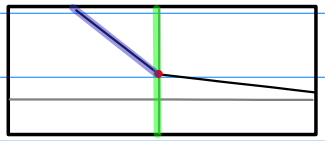
$$= X(z^{-1})$$



$2^n u(n)$

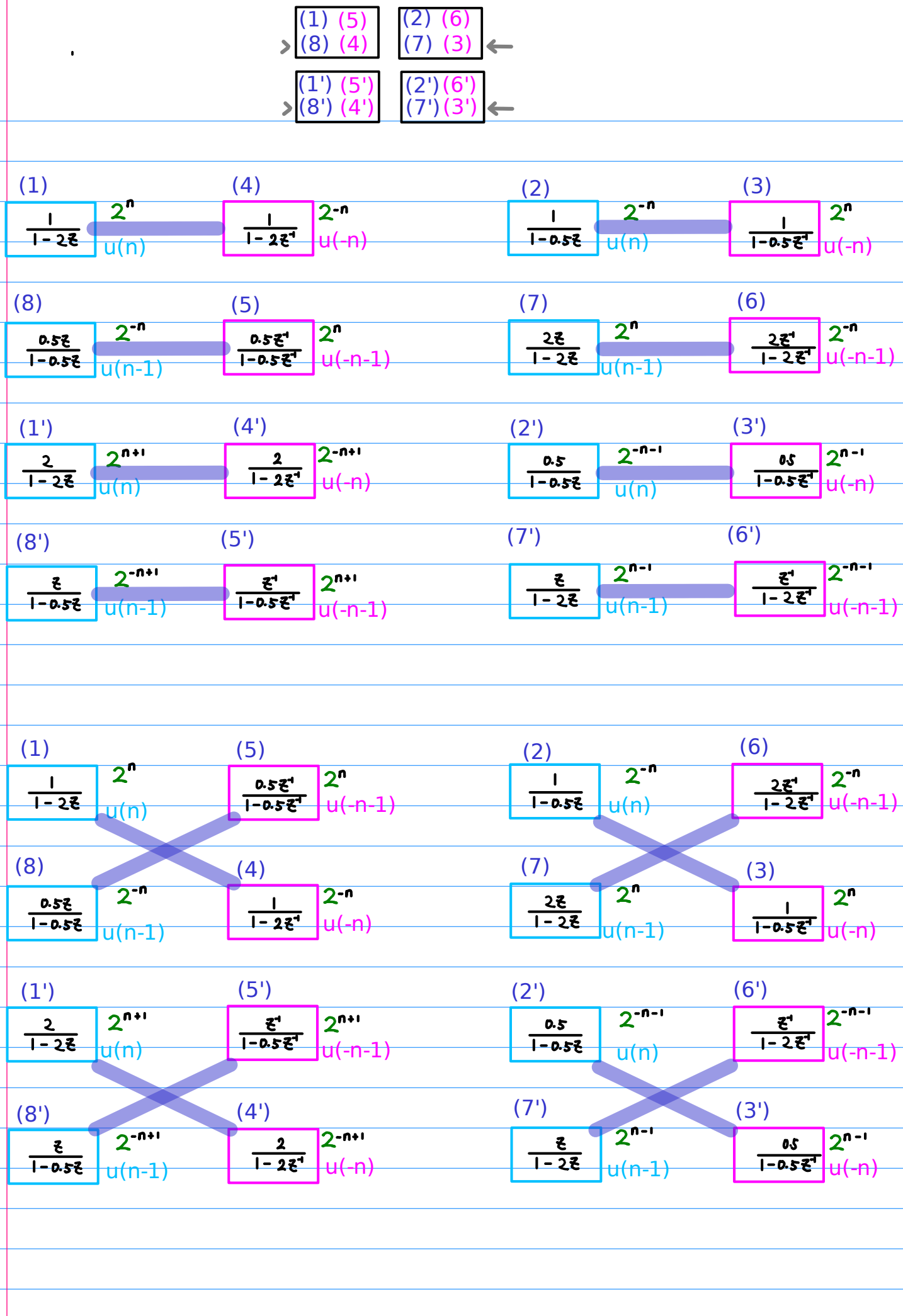
(1) $\frac{1}{1-2z}$ $2^n u(n)$

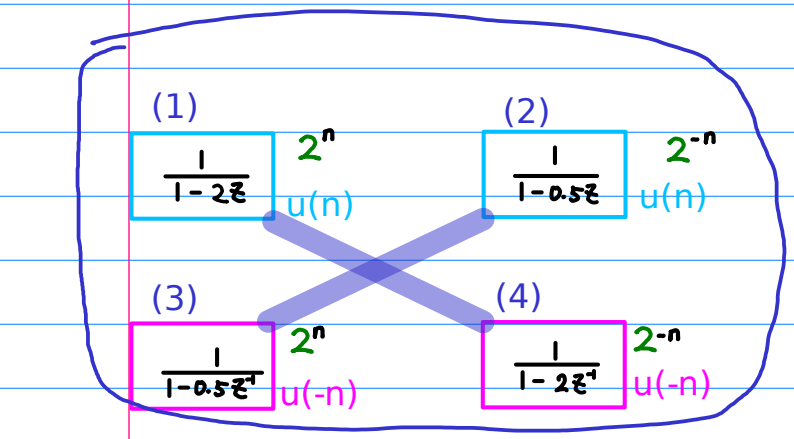
$z \Longleftrightarrow z^{-1}$ $\qquad$ $n \Longleftrightarrow -n$

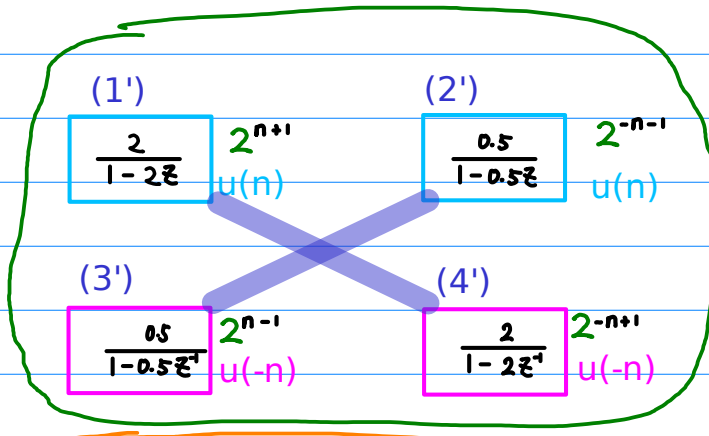


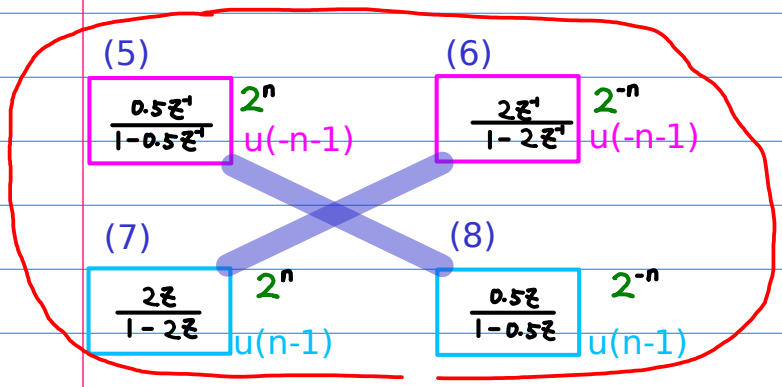
$2^{-n} u(-n)$

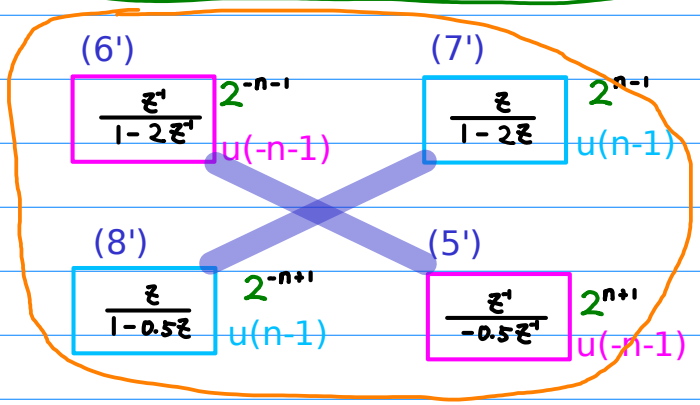
(4) $\frac{1}{1-2z^{-1}}$ $2^{-n} u(-n)$

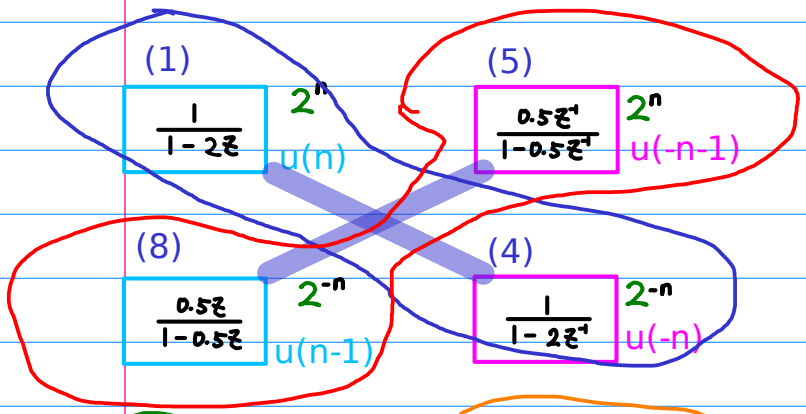

(1) (5)
(8) (4)
(2) (6)
(7) (3)
(1') (5')
(8') (4')
(2') (6')
(7') (3')
(1)
$\frac{1}{1-2z}$
2^n
u(n)
(4)
$\frac{1}{1-2z^{-1}}$
2^{-n}
u(-n)
(2)
$\frac{1}{1-0.5z}$
2^{-n}
u(n)
(3)
$\frac{1}{1-0.5z^{-1}}$
2^n
u(-n)
(8)
$\frac{0.5z}{1-0.5z}$
2^{-n}
u(n-1)
(5)
$\frac{0.5z^{-1}}{1-0.5z^{-1}}$
2^n
u(-n-1)
(7)
$\frac{2z}{1-2z}$
2^n
u(n-1)
(6)
$\frac{2z^{-1}}{1-2z^{-1}}$
2^{-n}
u(-n-1)
(1')
$\frac{2}{1-2z}$
2^{n+1}
u(n)
(4')
$\frac{2}{1-2z^{-1}}$
2^{-n+1}
u(-n)
(2')
$\frac{0.5}{1-0.5z}$
2^{-n-1}
u(n)
(3')
$\frac{0.5}{1-0.5z^{-1}}$
2^{n-1}
u(-n)
(8')
$\frac{z}{1-0.5z}$
2^{-n+1}
u(n-1)
(5')
$\frac{z^{-1}}{1-0.5z^{-1}}$
2^{n+1}
u(-n-1)
(7')
$\frac{z}{1-2z}$
2^{n-1}
u(n-1)
(6')
$\frac{z^{-1}}{1-2z^{-1}}$
2^{-n-1}
u(-n-1)
(1)
$\frac{1}{1-2z}$
2^n
u(n)
(5)
$\frac{0.5z^{-1}}{1-0.5z^{-1}}$
2^n
u(-n-1)
(8)
$\frac{0.5z}{1-0.5z}$
2^{-n}
u(n-1)
(4)
$\frac{1}{1-2z^{-1}}$
2^{-n}
u(-n)
(2)
$\frac{1}{1-0.5z}$
2^{-n}
u(n)
(6)
$\frac{2z^{-1}}{1-2z^{-1}}$
2^{-n}
u(-n-1)
(7)
$\frac{2z}{1-2z}$
2^n
u(n-1)
(3)
$\frac{1}{1-0.5z^{-1}}$
2^n
u(-n)
(1')
$\frac{2}{1-2z}$
2^{n+1}
u(n)
(5')
$\frac{z^{-1}}{1-0.5z^{-1}}$
2^{n+1}
u(-n-1)
(8')
$\frac{z}{1-0.5z}$
2^{-n+1}
u(n-1)
(4')
$\frac{2}{1-2z^{-1}}$
2^{-n+1}
u(-n)
(2')
$\frac{0.5}{1-0.5z}$
2^{-n-1}
u(n)
(6')
$\frac{z^{-1}}{1-2z^{-1}}$
2^{-n-1}
u(-n-1)
(7')
$\frac{z}{1-2z}$
2^{n-1}
u(n-1)
(3')
$\frac{0.5}{1-0.5z^{-1}}$
2^{n-1}
u(-n)

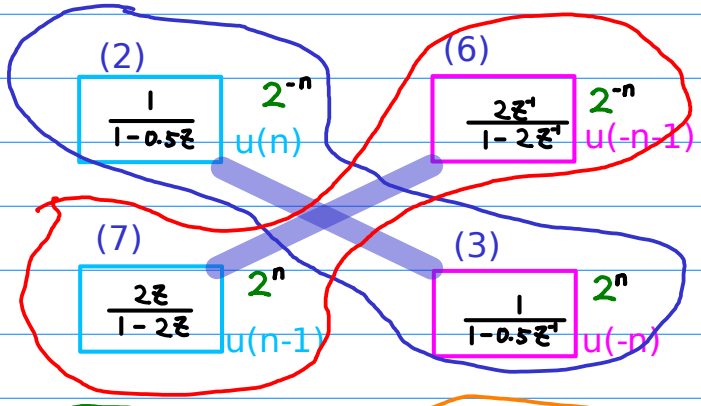

(1)
$\frac{1}{1-2z}$
2^n
u(n)
(2)
$\frac{1}{1-0.5z}$
2^{-n}
u(n)
(3)
$\frac{1}{1-0.5z^{-1}}$
2^n
u(-n)
(4)
$\frac{1}{1-2z^{-1}}$
2^{-n}
u(-n)

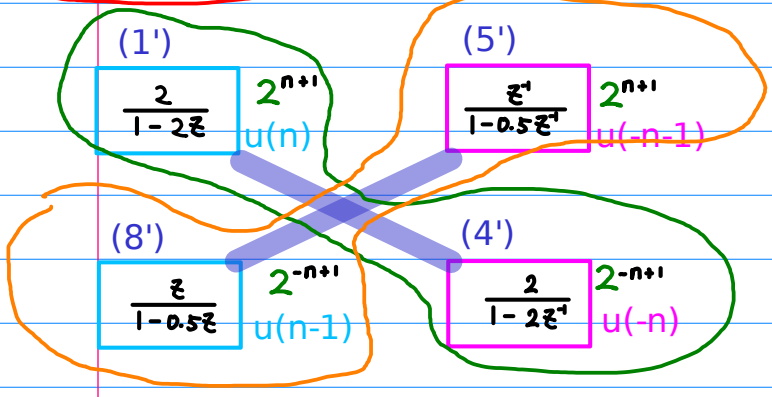

(1')
$\frac{2}{1-2z}$
2^{n+1}
u(n)
(2')
$\frac{0.5}{1-0.5z}$
2^{-n-1}
u(n)
(3')
$\frac{0.5}{1-0.5z^{-1}}$
2^{n-1}
u(-n)
(4')
$\frac{2}{1-2z^{-1}}$
2^{-n+1}
u(-n)

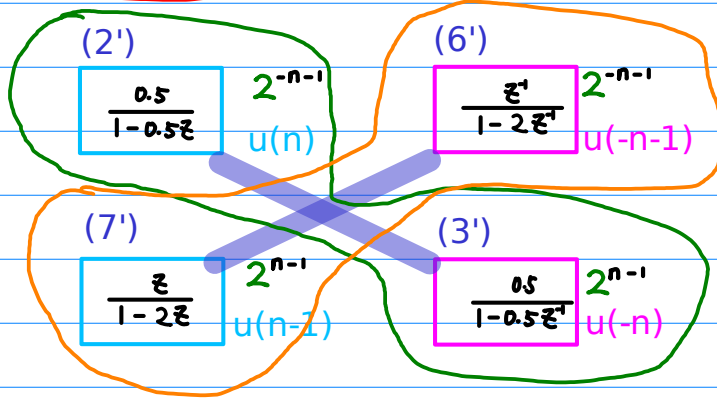

(5)
$\frac{0.5z^{-1}}{1-0.5z^{-1}}$
2^n
u(-n-1)
(6)
$\frac{2z^{-1}}{1-2z^{-1}}$
2^{-n}
u(-n-1)
(7)
$\frac{2z}{1-2z}$
2^n
u(n-1)
(8)
$\frac{0.5z}{1-0.5z}$
2^{-n}
u(n-1)


(6')
$\frac{z^{-1}}{1-2z^{-1}}$
2^{-n-1}
u(-n-1)
(7')
$\frac{z}{1-2z}$
2^{n-1}
u(n-1)
(8')
$\frac{z}{1-0.5z}$
2^{-n+1}
u(n-1)
(5')
$\frac{z^{-1}}{-0.5z^{-1}}$
2^{n+1}
u(-n-1)

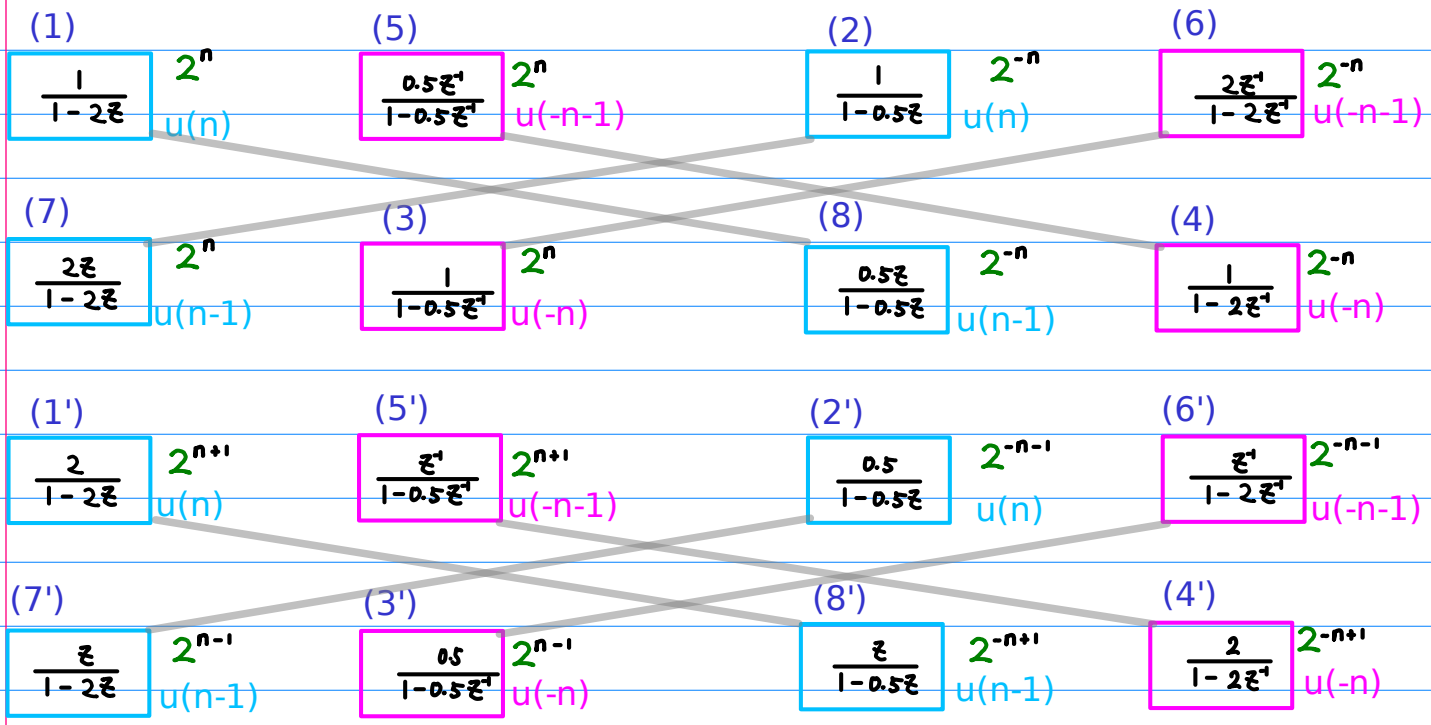

(1)
$\frac{1}{1-2z}$
2^n
u(n)
(5)
$\frac{0.5z^{-1}}{1-0.5z^{-1}}$
2^n
u(-n-1)
(8)
$\frac{0.5z}{1-0.5z}$
2^{-n}
u(n-1)
(4)
$\frac{1}{1-2z^{-1}}$
2^{-n}
u(-n)


(2)
$\frac{1}{1-0.5z}$
2^{-n}
u(n)
(6)
$\frac{2z^{-1}}{1-2z^{-1}}$
2^{-n}
u(-n-1)
(7)
$\frac{2z}{1-2z}$
2^n
u(n-1)
(3)
$\frac{1}{1-0.5z^{-1}}$
2^n
u(-n)

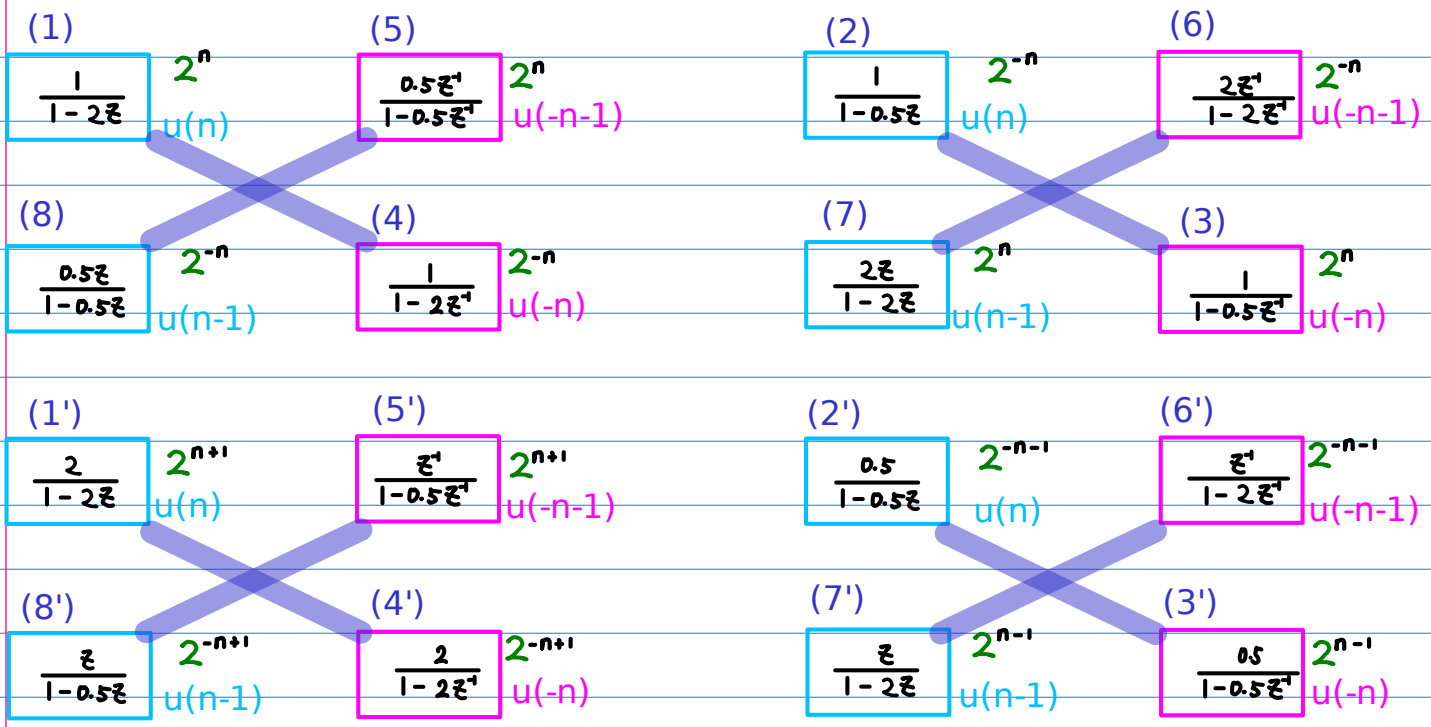

(1')
$\frac{2}{1-2z}$
2^{n+1}
u(n)
(5')
$\frac{z^{-1}}{1-0.5z^{-1}}$
2^{n+1}
u(-n-1)
(8')
$\frac{z}{1-0.5z}$
2^{-n+1}
u(n-1)
(4')
$\frac{2}{1-2z^{-1}}$
2^{-n+1}
u(-n)


(2')
$\frac{0.5}{1-0.5z}$
2^{-n-1}
u(n)
(6')
$\frac{z^{-1}}{1-2z^{-1}}$
2^{-n-1}
u(-n-1)
(7')
$\frac{z}{1-2z}$
2^{n-1}
u(n-1)
(3')
$\frac{0.5}{1-0.5z^{-1}}$
2^{n-1}
u(-n)

(1) (5) (2) (6)
(7) (3) (8) (4)



(1) (5) (2) (6)
(8) (4) (7) (3)



Case 1

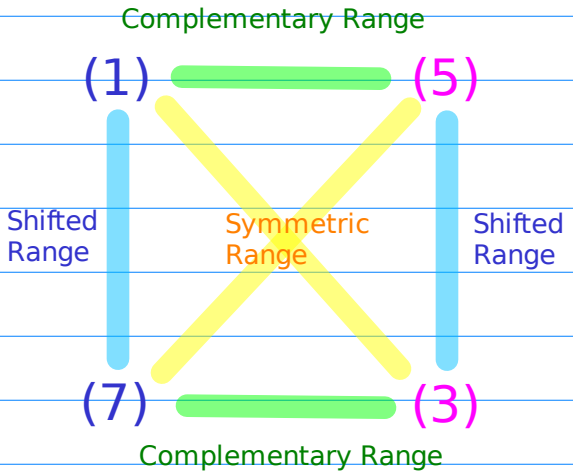
(1) (5)
(7) (3)

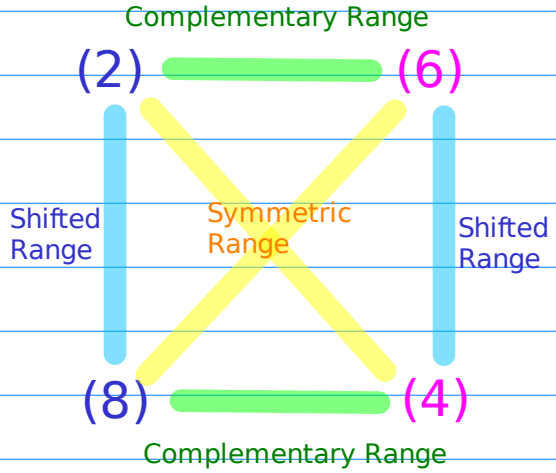
(2) (6)
(8) (4)

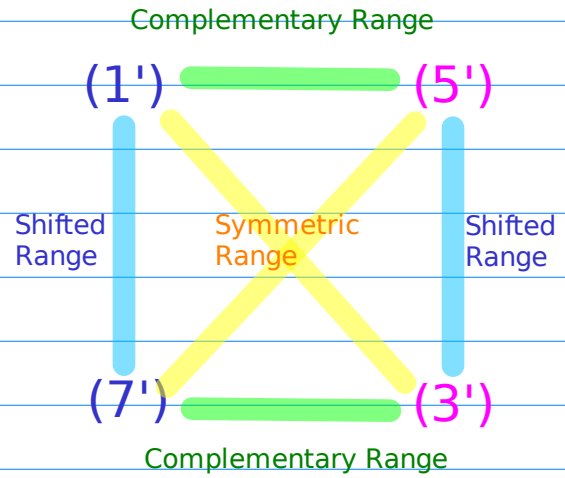
(1') (5')
(7') (3')

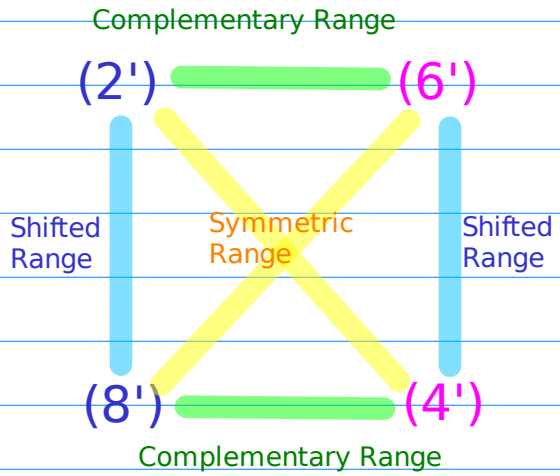
(2') (6')
(8') (4')

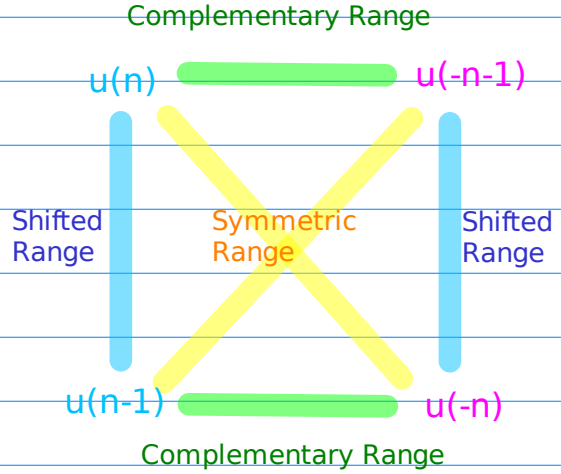
Shifted, Complementary, & Symmetric Ranges

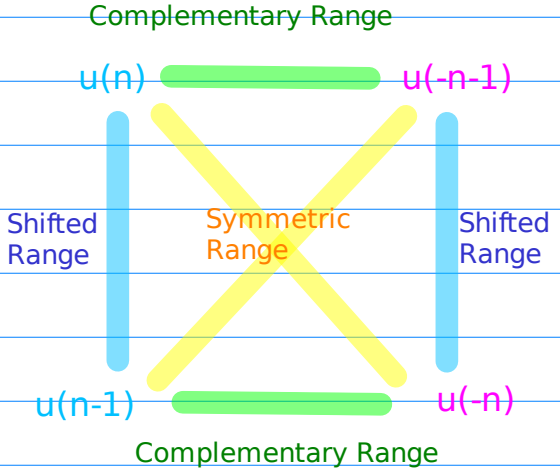












Case 1 (1) (5) (7) (3) (2) (6) (8) (4) (1') (5') (7') (3') (2') (6') (8') (4')

Partial fractions and geometric power series

$P^{-1} = 0.5$

$P = 2$

-(1) $-\frac{1}{1-2z}$ $-2^n\ u(n)$

(5) $\frac{0.5z^{-1}}{1-0.5z^{-1}}$ $2^n\ u(-n-1)$

-(2) $-\frac{1}{1-0.5z}$ $-2^{-n}\ u(n)$

(6) $\frac{2z^{-1}}{1-2z^{-1}}$ $2^{-n}\ u(-n-1)$

-(7) $-\frac{2z}{1-2z}$ $-2^n\ u(n-1)$

(3) $\frac{1}{1-0.5z^{-1}}$ $2^n\ u(-n)$

-(8) $-\frac{0.5z}{1-0.5z}$ $2^{-n}\ u(n-1)$

(4) $\frac{1}{1-2z^{-1}}$ $-2^{-n}\ u(-n)$

-(1') $-\frac{2}{1-2z}$ $-2^{n+1}\ u(n)$

(5') $\frac{z^{-1}}{1-0.5z^{-1}}$ $2^{n+1}\ u(-n-1)$

-(2') $-\frac{0.5}{1-0.5z}$ $-2^{-n-1}\ u(n)$

(6') $\frac{z^{-1}}{1-2z^{-1}}$ $2^{-n-1}\ u(-n-1)$

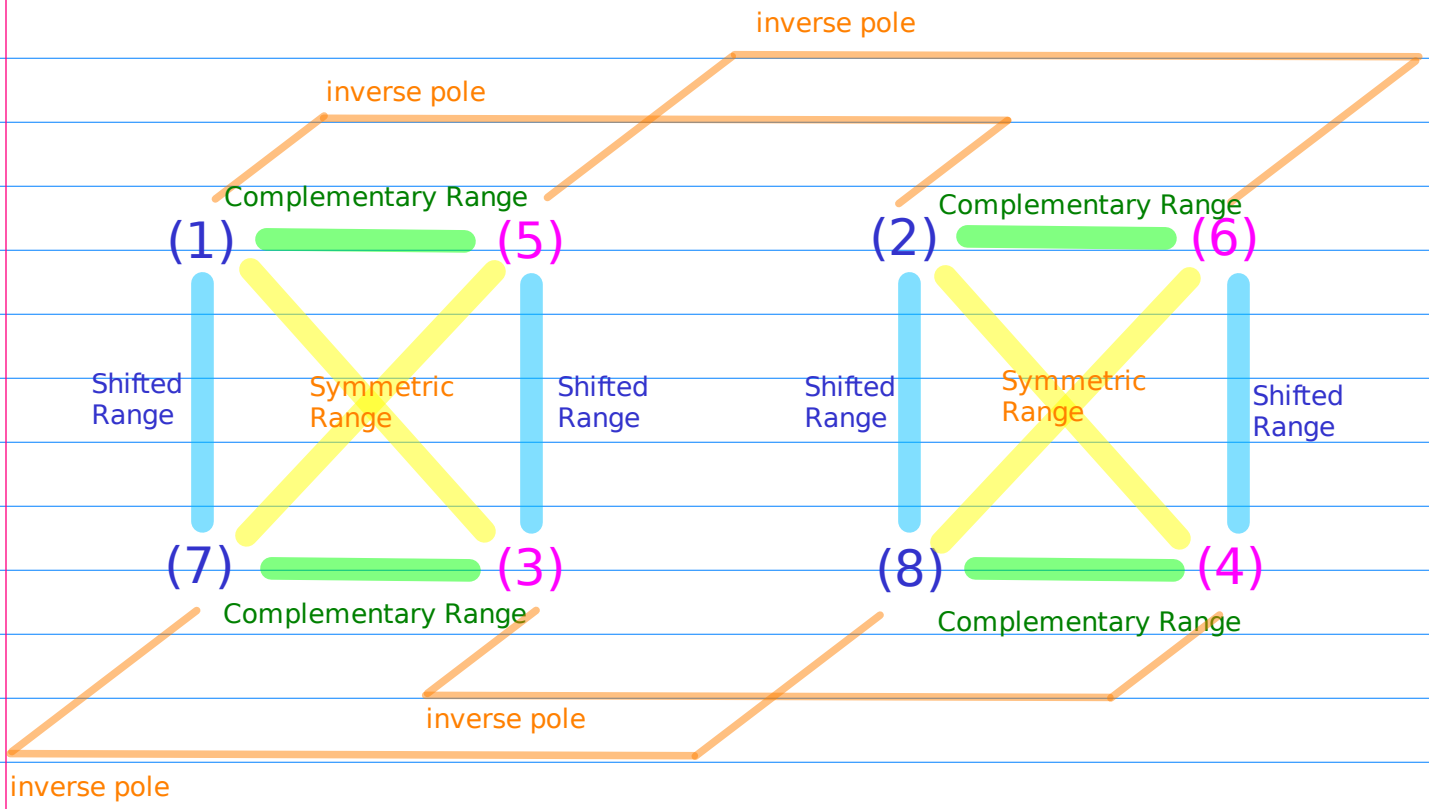
-(7') $-\frac{z}{1-2z}$ $-2^{n-1}\ u(n-1)$

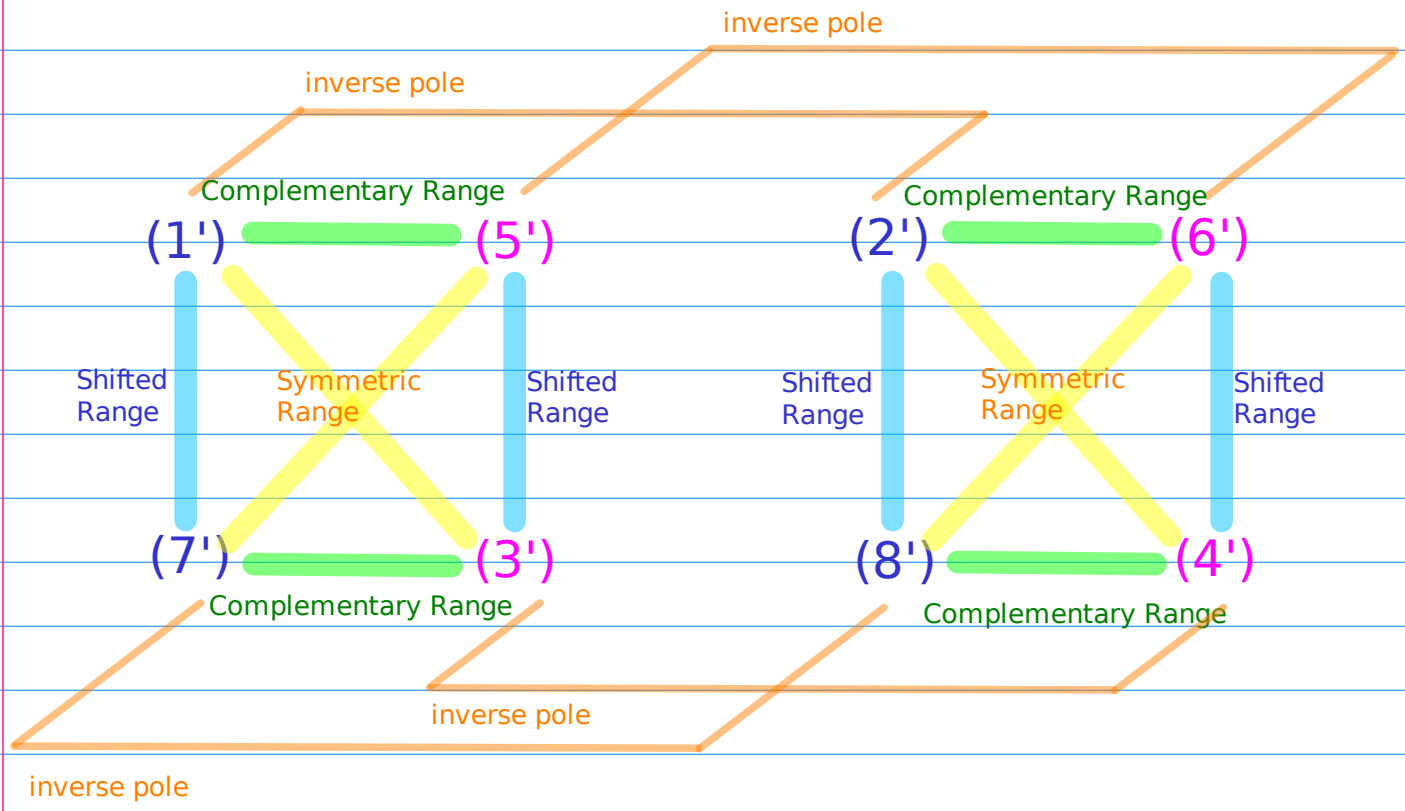
(3') $\frac{0.5}{1-0.5z^{-1}}$ $2^{n-1}\ u(-n)$

-(8') $-\frac{z}{1-0.5z}$ $-2^{-n+1}\ u(n-1)$

(4') $\frac{2}{1-2z^{-1}}$ $2^{-n+1}\ u(-n)$

Case 1


inverse pole
inverse pole
Complementary Range
(1)
(5)
Shifted Range
Symmetric Range
Shifted Range
(7)
(3)
Complementary Range
inverse pole
inverse pole
Complementary Range
(2)
(6)
Shifted Range
Symmetric Range
Shifted Range
(8)
(4)
Complementary Range


inverse pole
inverse pole
Complementary Range
(1')
(5')
Shifted Range
Symmetric Range
Shifted Range
(7')
(3')
Complementary Range
inverse pole
inverse pole
Complementary Range
(2')
(6')
Shifted Range
Symmetric Range
Shifted Range
(8')
(4')
Complementary Range

Case 1

a^n ← a^-n

u(n) ↔ u(n-1)

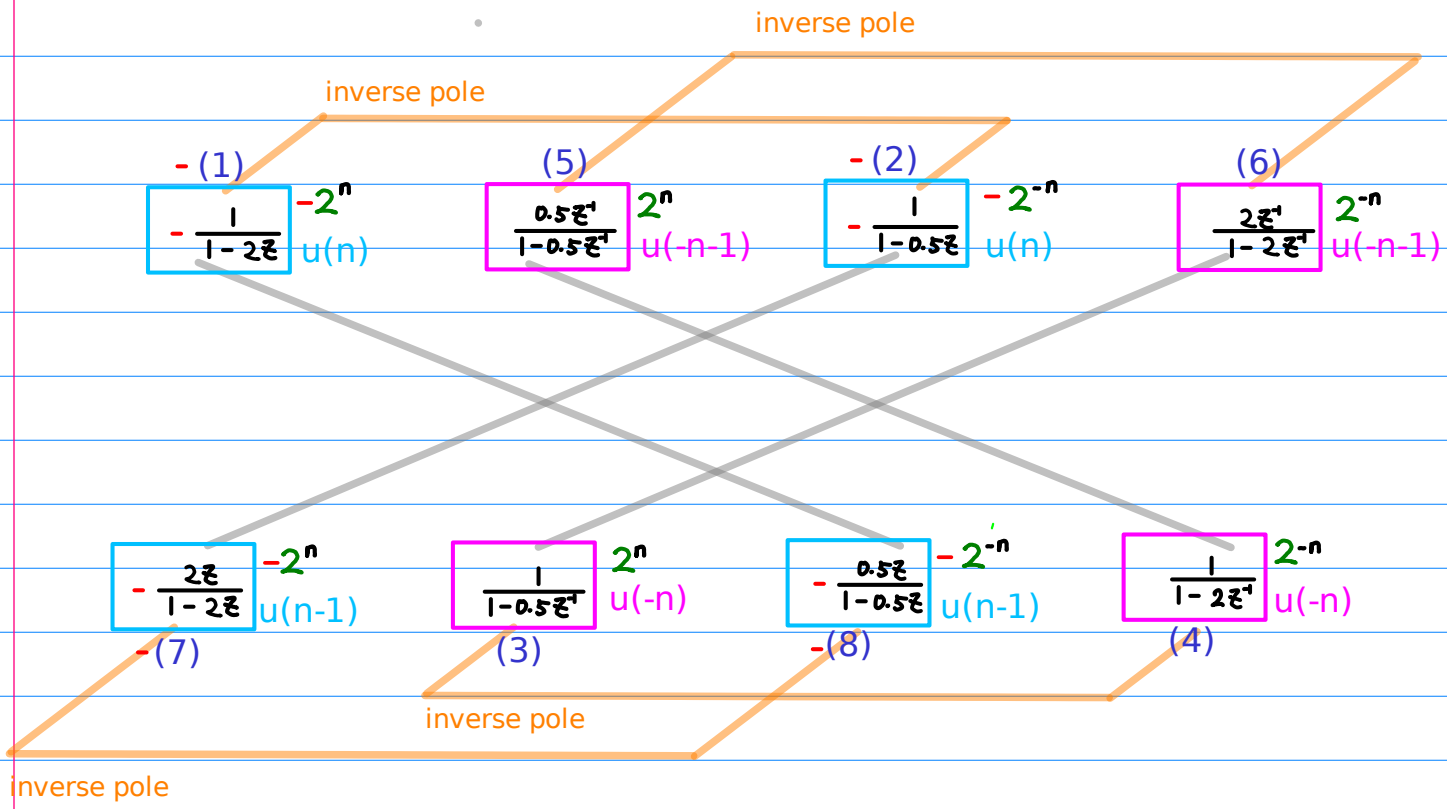
u(-n) ↔ u(-n-1)

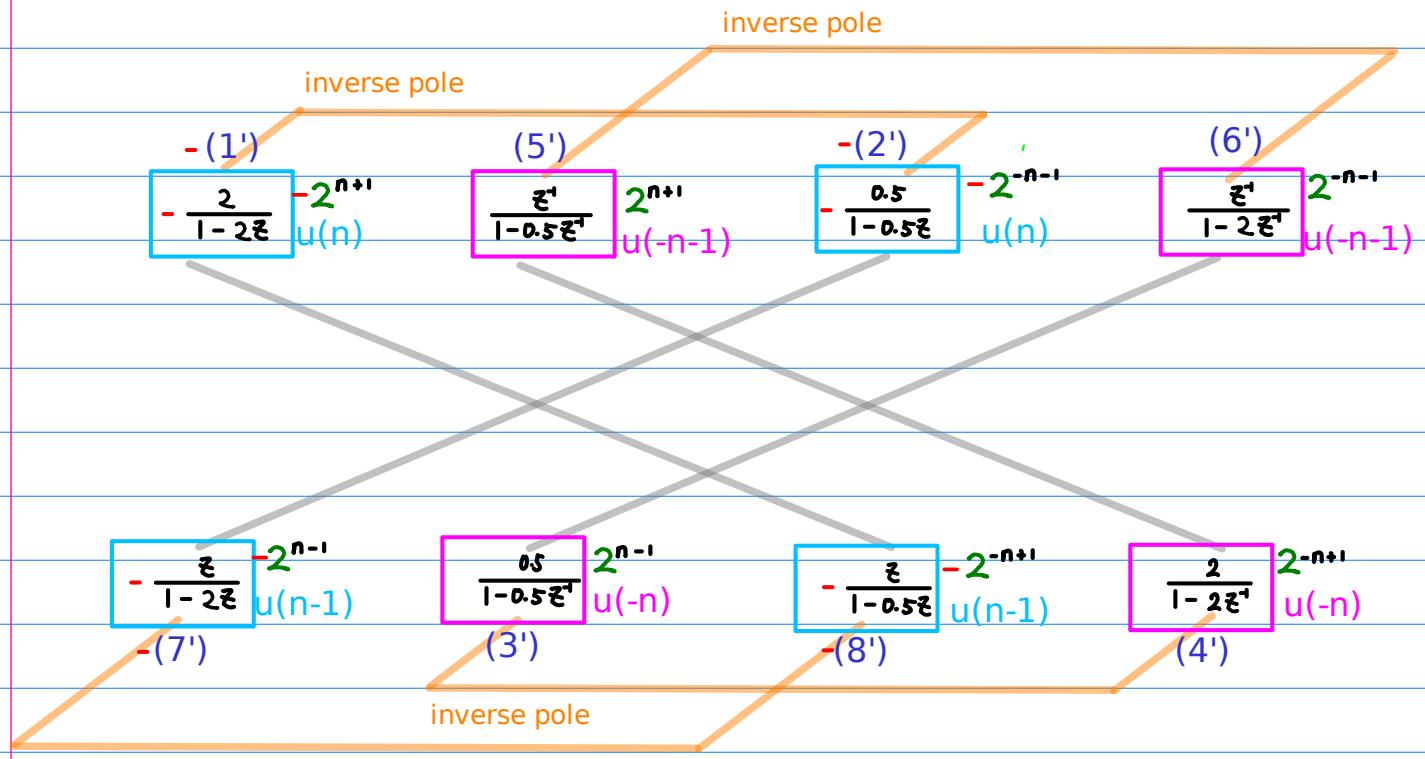
reciprocal power

shift of a range

(1) (5)
(7) (3)

(2) (6)
(8) (4)



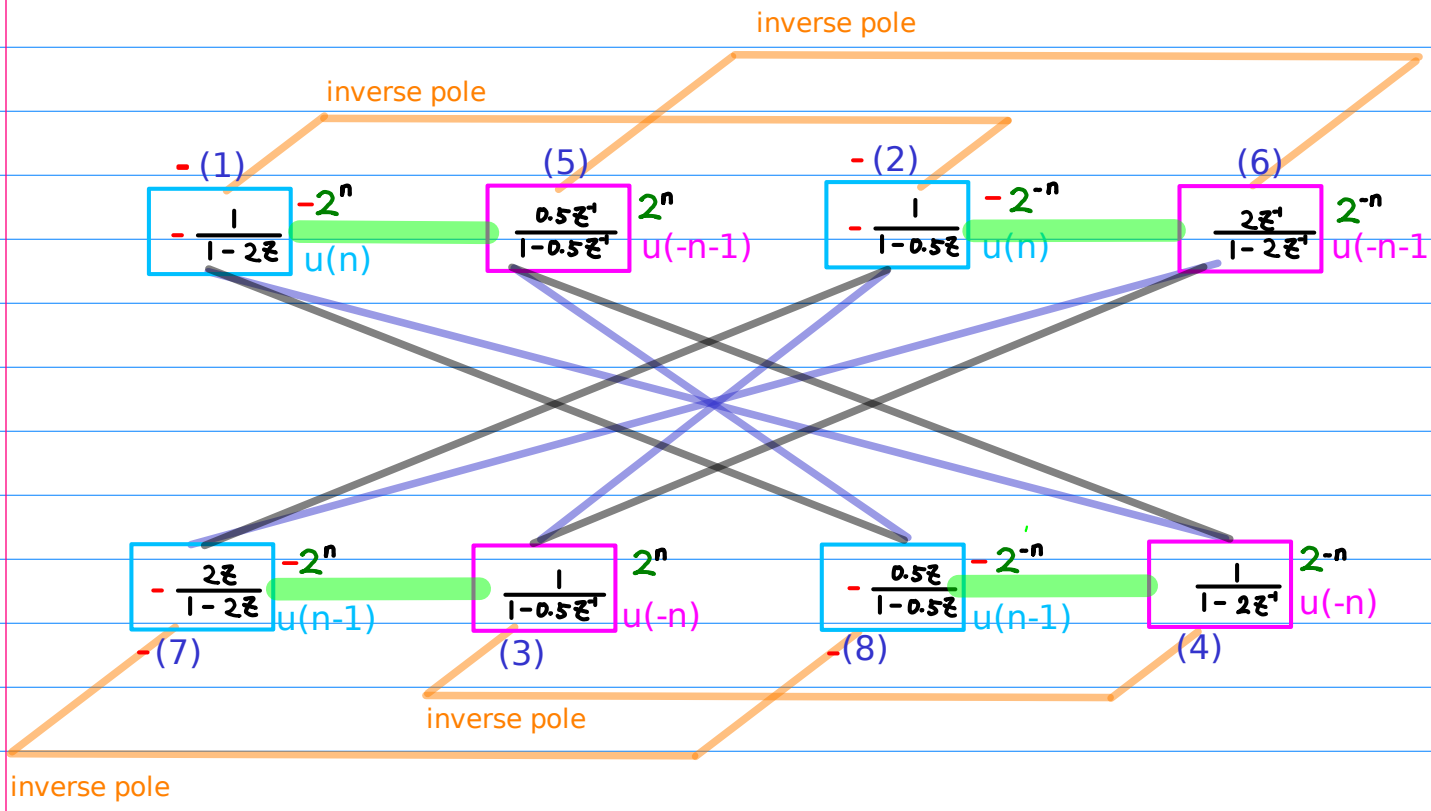


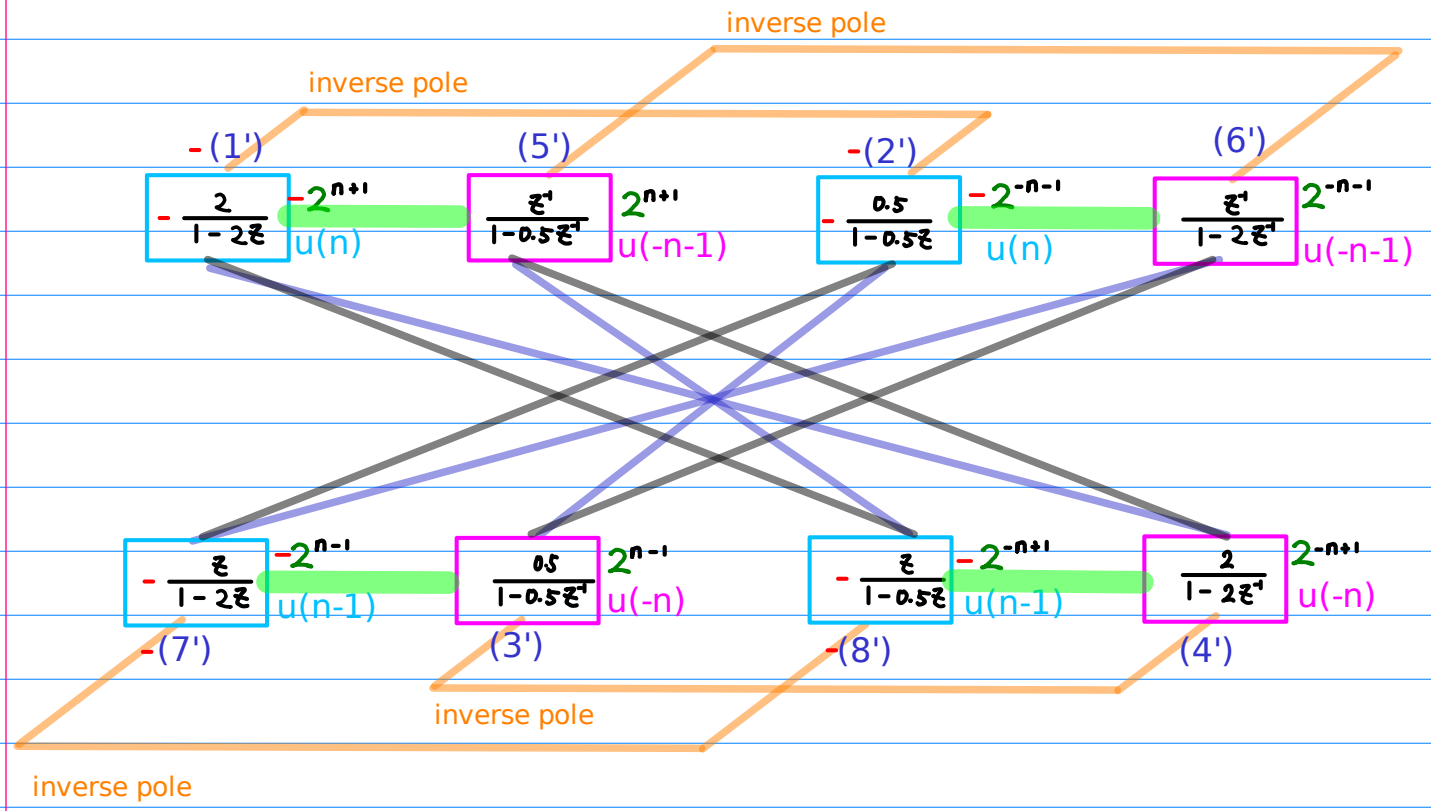
Case 1

Time Reversal $-1, z^{-1}$

(1) (5)
(7) (3)

(2) (6)
(8) (4)

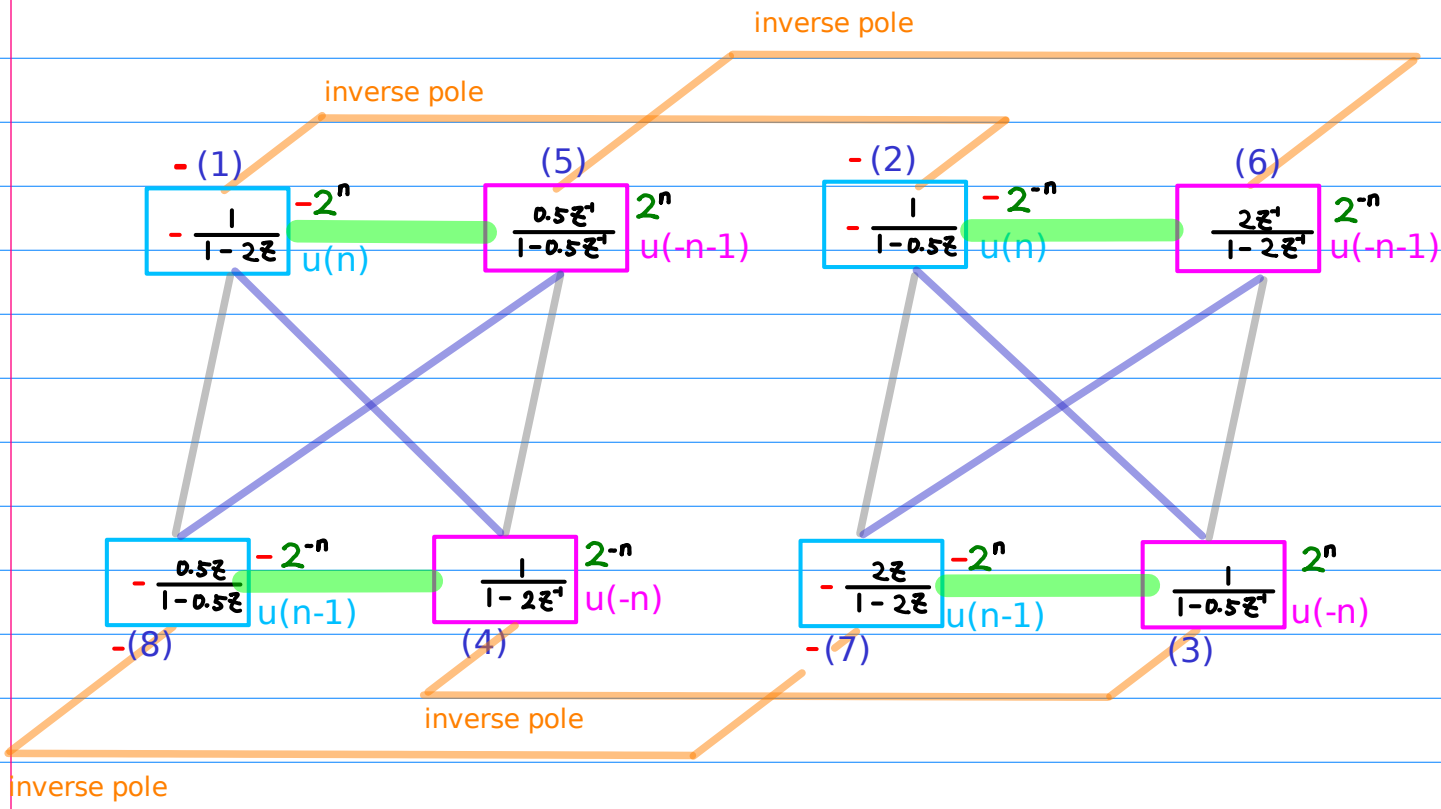


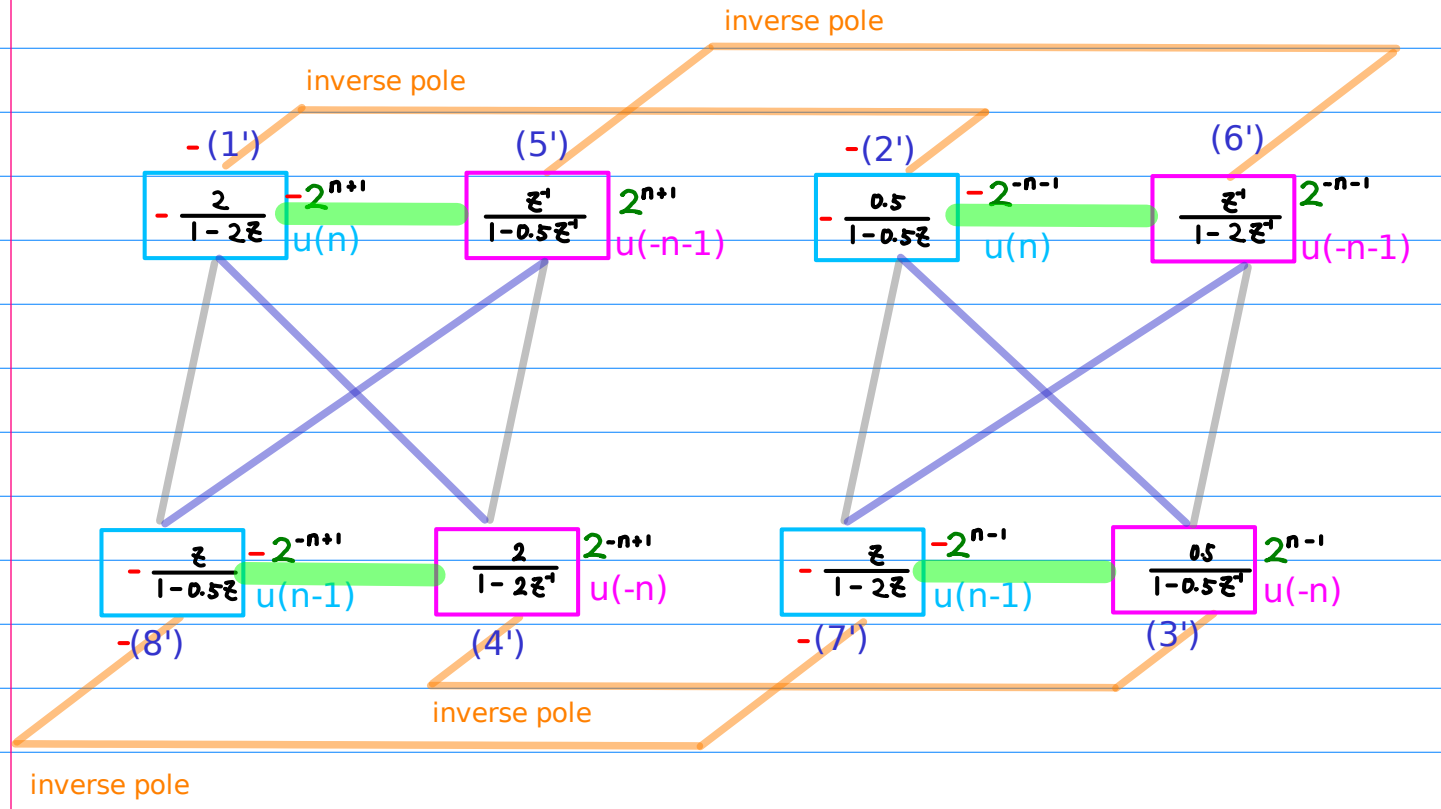


Case 1

Time Reversal $-1, z^{-1}$

(1) (5) (2) (6)
(8) (4) (7) (3)





Case 2

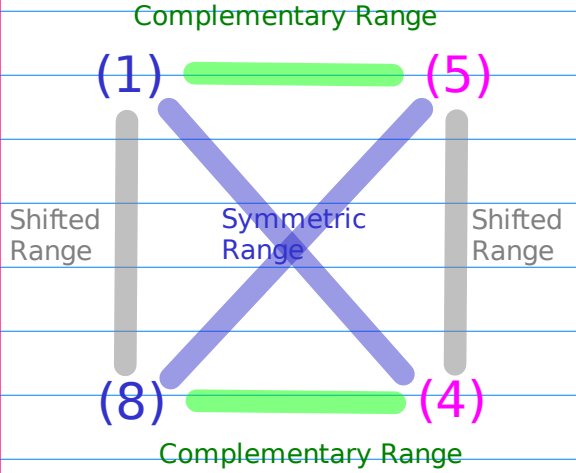
(1) (5)
(8) (4)

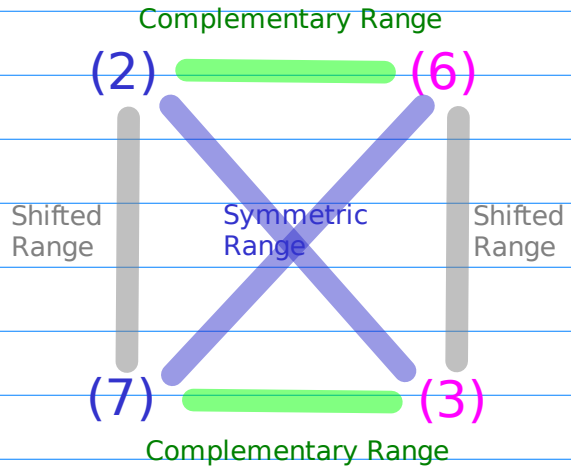
(2) (6)
(7) (3)

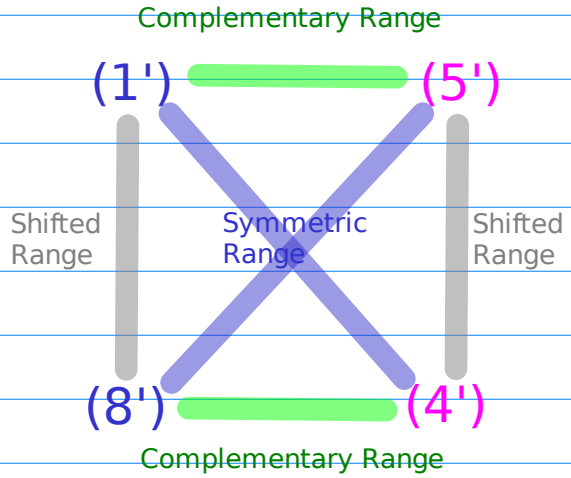
(1') (5')
(8') (4')

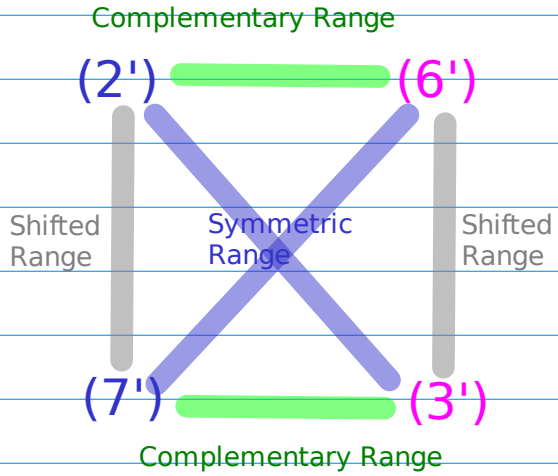
(2') (6')
(7') (3')

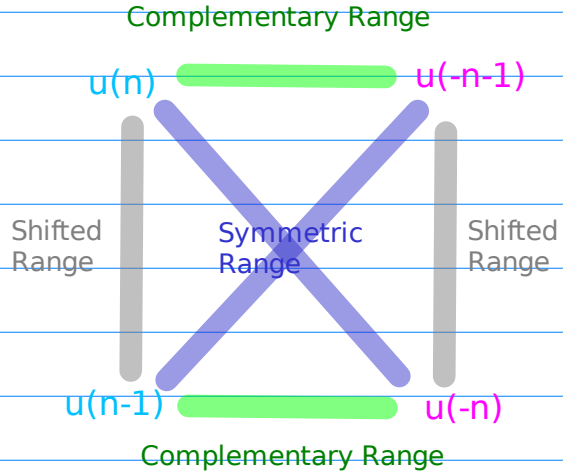
Shifted, Complementary, & Symmetric Ranges - Flipped

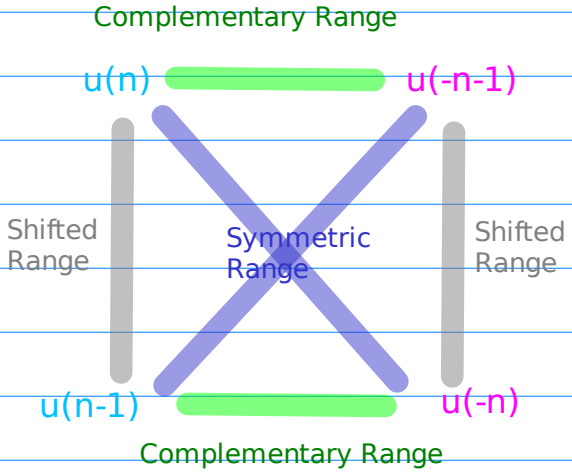












Case 2

(1) (5)
(8) (4)

(2) (6)
(7) (3)

(1') (5')
(8') (4')

(2')(6')
(7')(3')

Partial fractions and geometric power series

$P^{-1} = 0.5$

$P = 2$

-(1) $-\frac{1}{1-2z}$ -2^n u(n)

(5) $\frac{0.5z^{-1}}{1-0.5z^{-1}}$ 2^n u(-n-1)

-(2) $-\frac{1}{1-0.5z}$ -2^{-n} u(n)

(6) $\frac{2z^{-1}}{1-2z^{-1}}$ 2^{-n} u(-n-1)

-(8) $-\frac{0.5z}{1-0.5z}$ -2^n u(n-1)

(4) $\frac{1}{1-2z^{-1}}$ 2^n u(-n)

-(7) $-\frac{2z}{1-2z}$ -2^{-n} u(n-1)

(3) $\frac{1}{1-0.5z^{-1}}$ 2^{-n} u(-n)

-(1') $-\frac{2}{1-2z}$ -2^{n+1} u(n)

(5') $\frac{z^{-1}}{1-0.5z^{-1}}$ 2^{n+1} u(-n-1)

-(2') $-\frac{0.5}{1-0.5z}$ -2^{-n-1} u(n)

(6') $\frac{z^{-1}}{1-2z^{-1}}$ 2^{-n-1} u(-n-1)

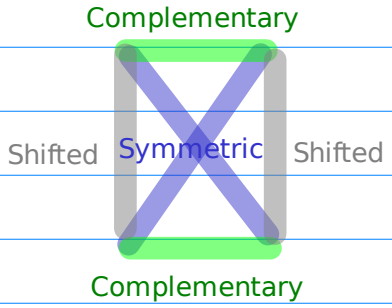
-(8') $-\frac{z}{1-0.5z}$ -2^{-n+1}u(n-1)

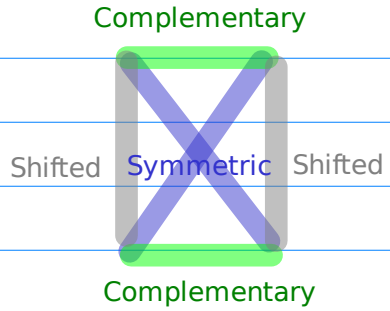
(4') $\frac{2}{1-2z^{-1}}$ 2^{-n+1} u(-n)

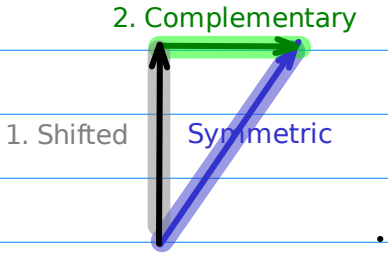
-(7') $-\frac{z}{1-2z}$ -2^{n-1} u(n-1)

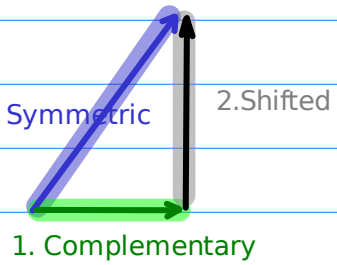
(3') $\frac{0.5}{1-0.5z^{-1}}$ 2^{n-1} u(-n)

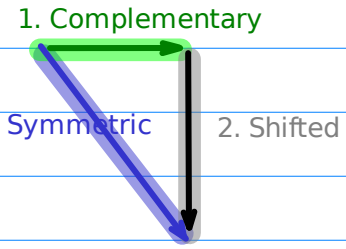
Case 2

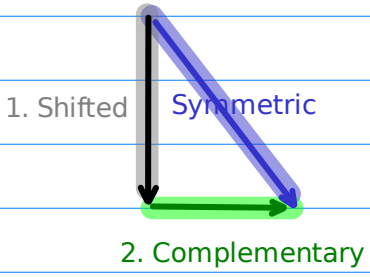

Complementary
Shifted
Symmetric
Shifted
Complementary


Complementary
Shifted
Symmetric
Shifted
Complementary


2. Complementary
1. Shifted
Symmetric


Symmetric
2.Shifted
1. Complementary


1. Complementary
Symmetric
2. Shifted

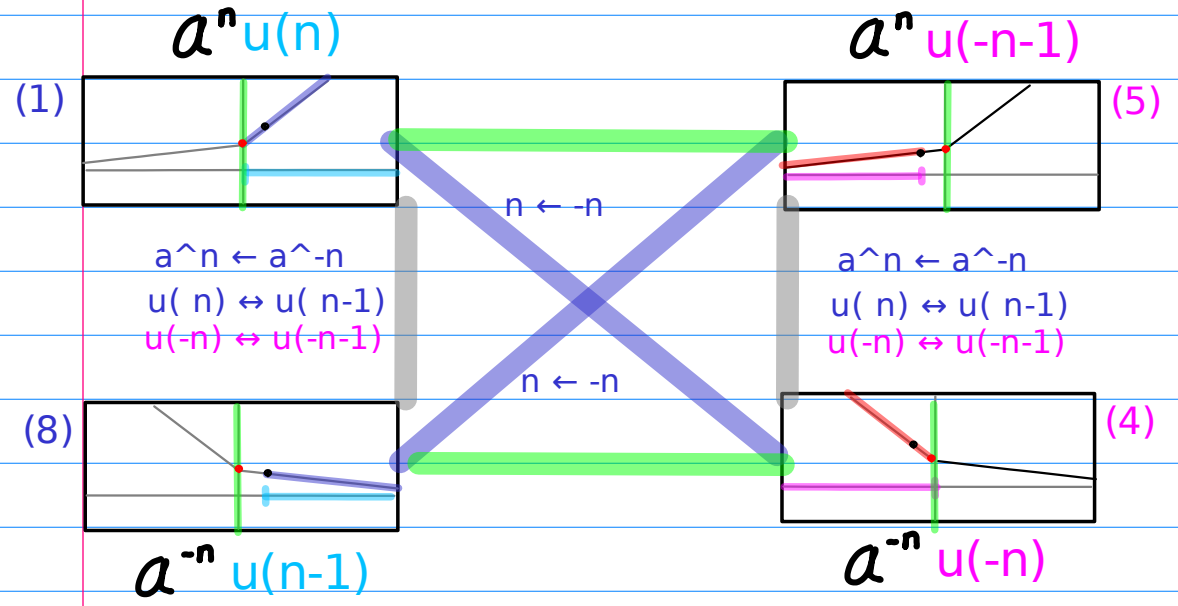

1. Shifted
Symmetric
2. Complementary

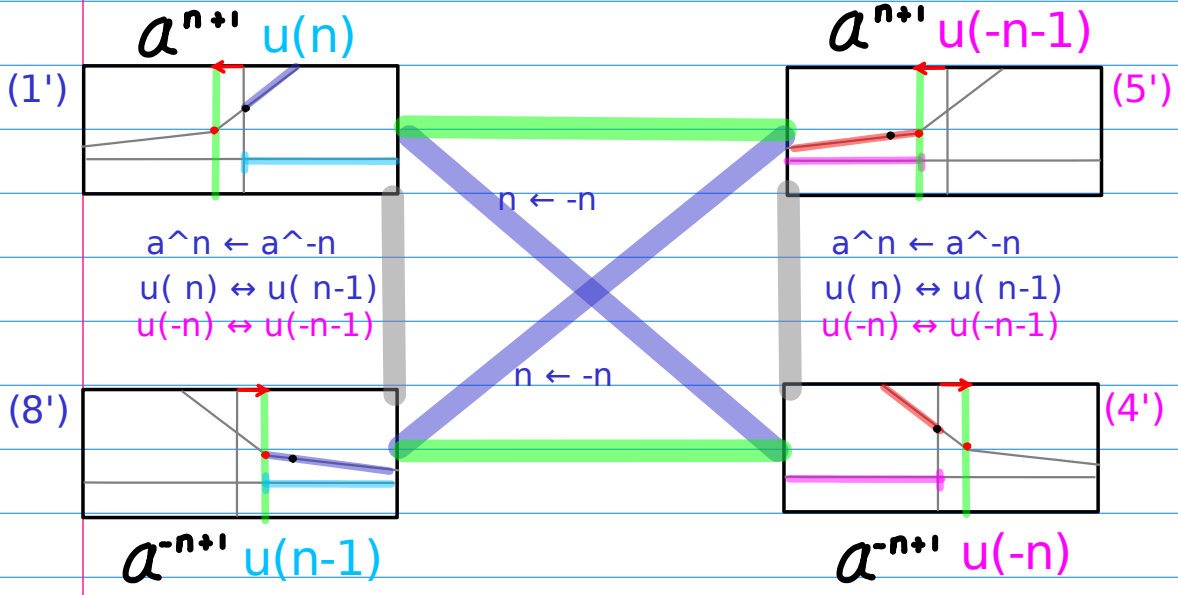
Case 2

(1) (5)
(8) (4)

(1') (5')
(8') (4')

Shifted, Complementary, & Symmetric Ranges - Flipped



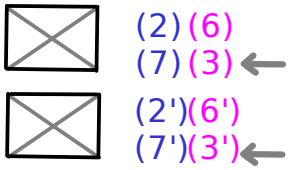


Time Reversal

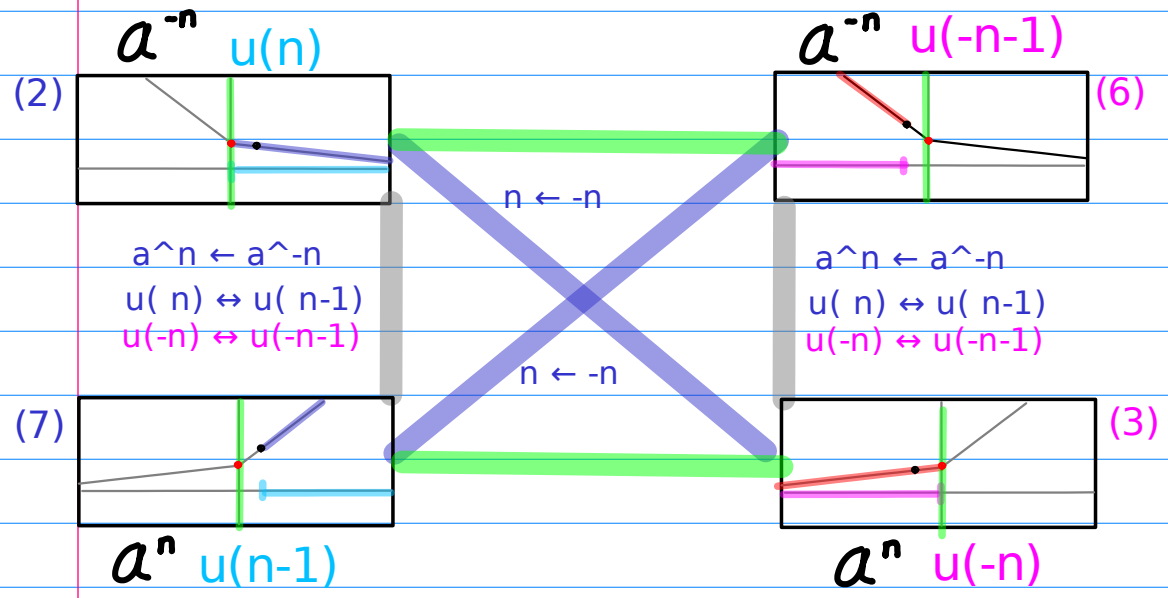
reciprocal power

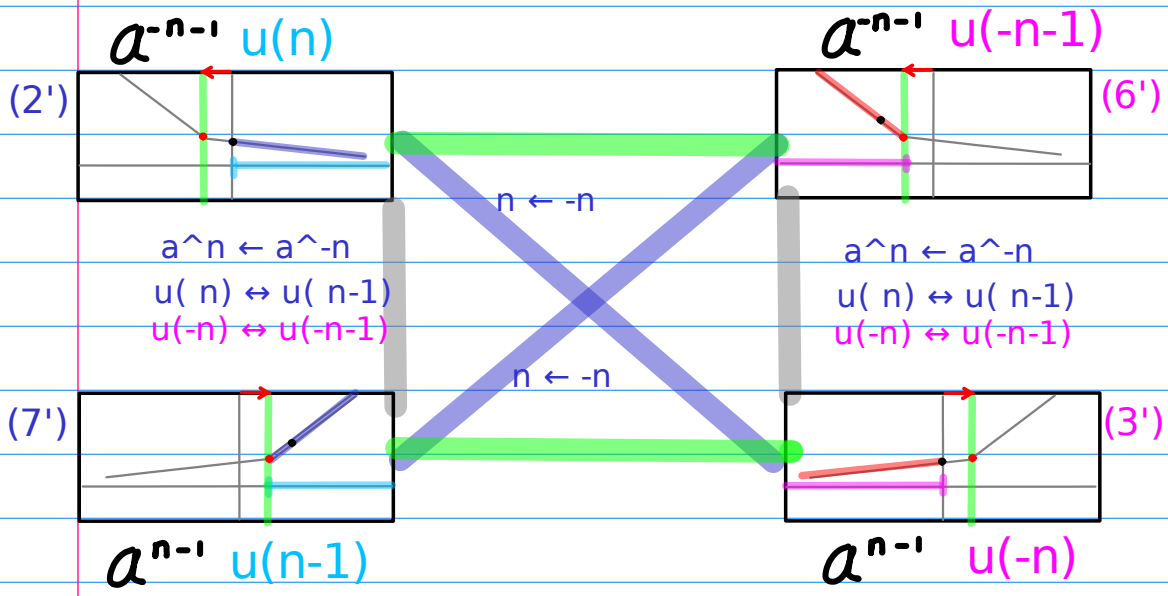
shift of a range

Case 2



Shifted, Complementary, & Symmetric Ranges - Flipped



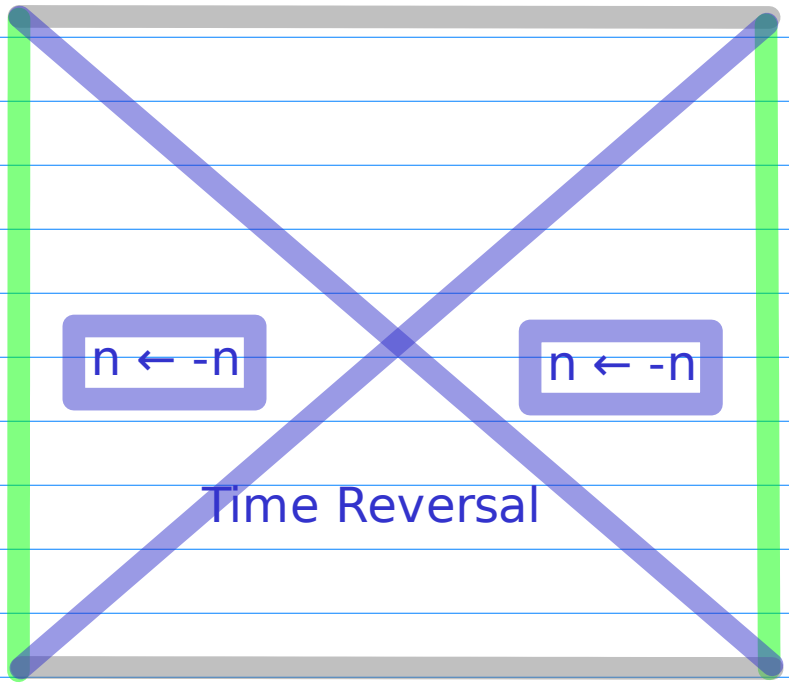


Time Reversal

reciprocal power
shift of a range

reciprocal power,
range shifting

a^n ← a^-n
u(n) ↔ u(n-1)
u(-n) ↔ u(-n-1)



Complementary
Region

Complementary
Region

u(n) ↔ u(-n-1)
u(-n) ↔ u(n-1)

Complementary
Region

a^n ← a^-n
u(n) ↔ u(n-1)
u(-n) ↔ u(-n-1)

reciprocal power,
range shifting

u(n) ↔ u(-n-1)
u(-n) ↔ u(n-1)

Complementary
Region

u(n) → u(-n-1) → u(-n)
u(-n) → u(n-1) → u(n)
u(-n-1) → u(n) → u(n-1)
u(n-1) → u(-n) → u(-n-1)

Time Reversal

a^n ← a^-n
u(n) ↔ u(-n)
u(n-1) ↔ u(-n-1)

Case 1

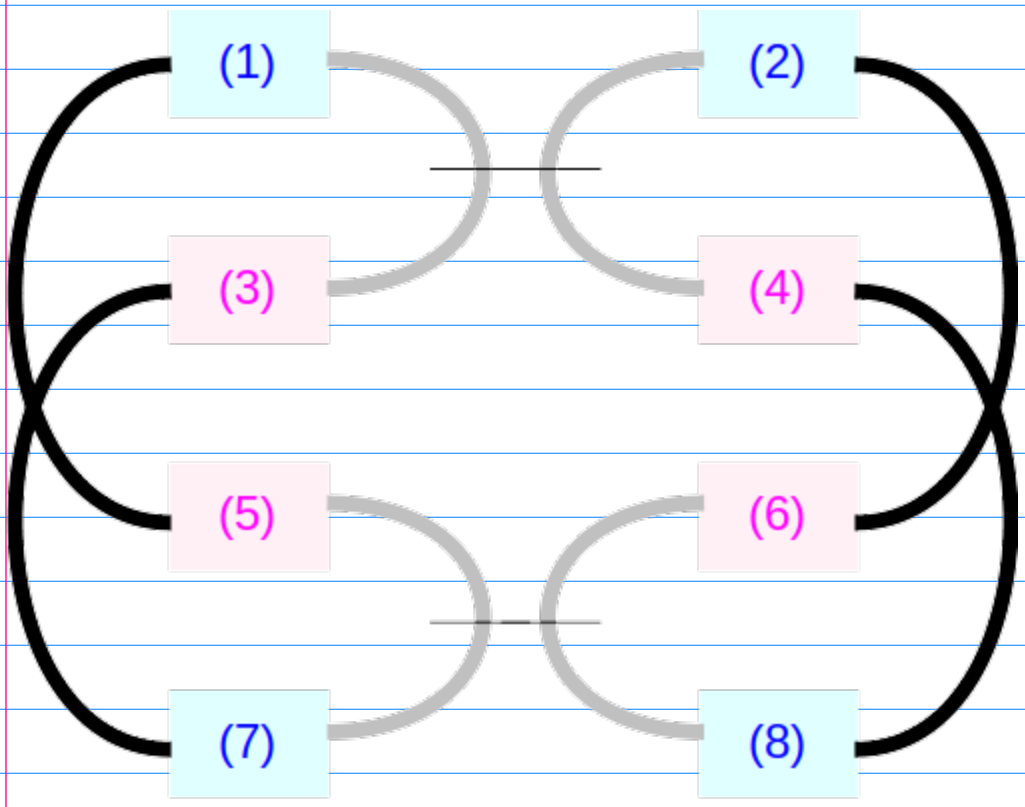
(1) (5)
(7) (3)

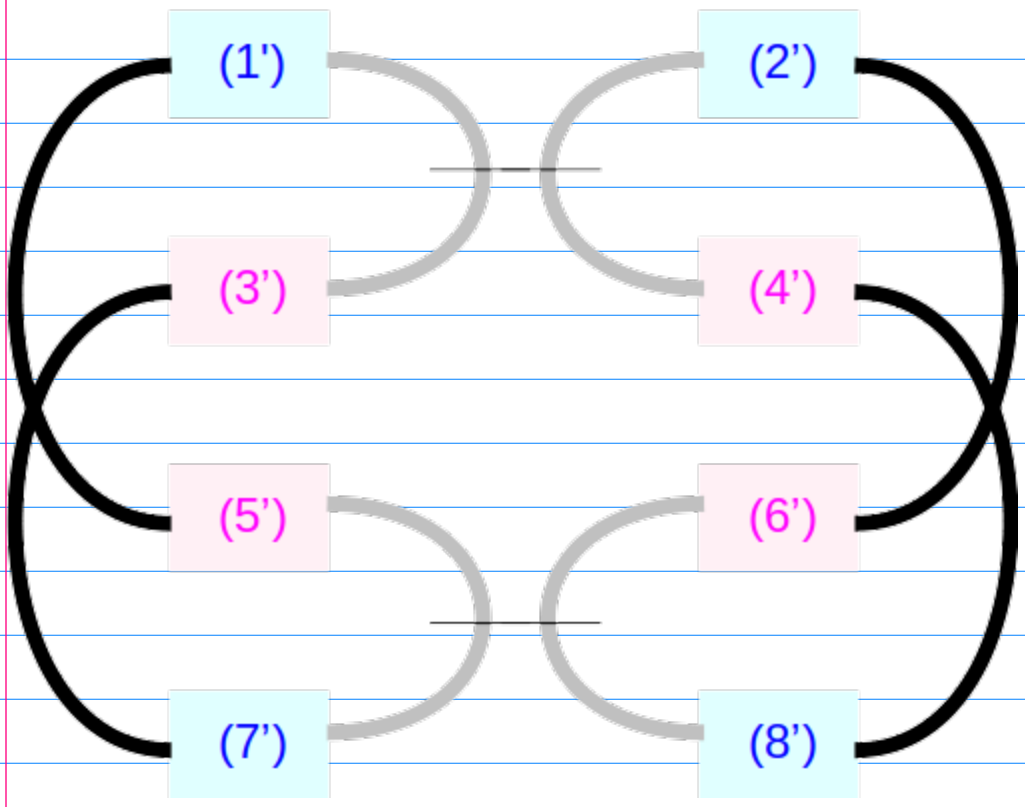
(2) (6)
(8) (4)

(1') (5')
(7') (3')

(2') (6')
(8') (4')

Shifted, Complementary, & Symmetric Ranges



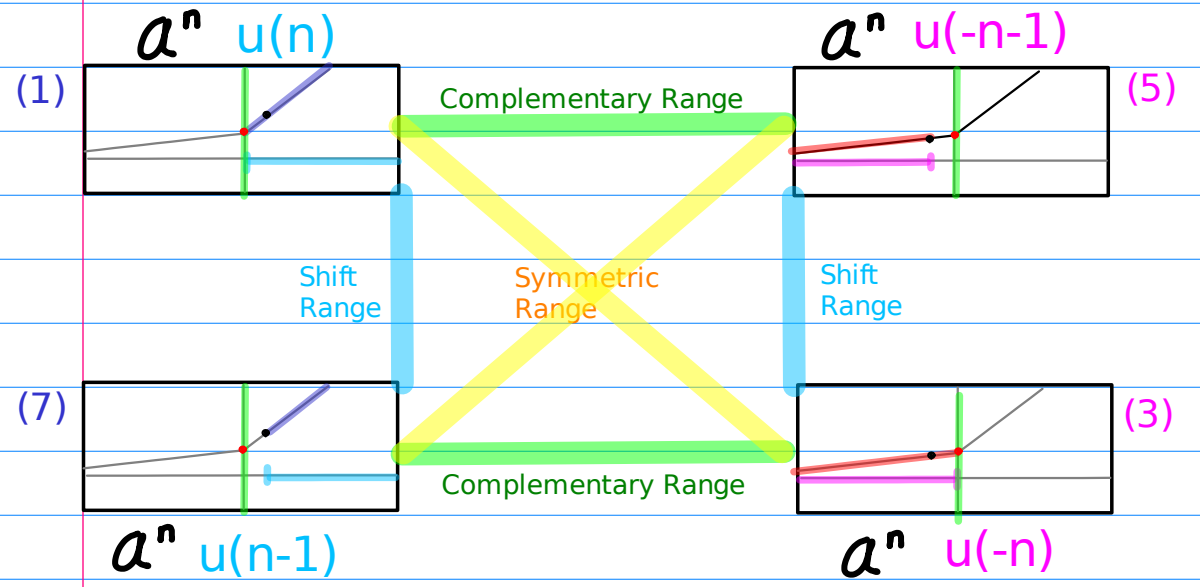


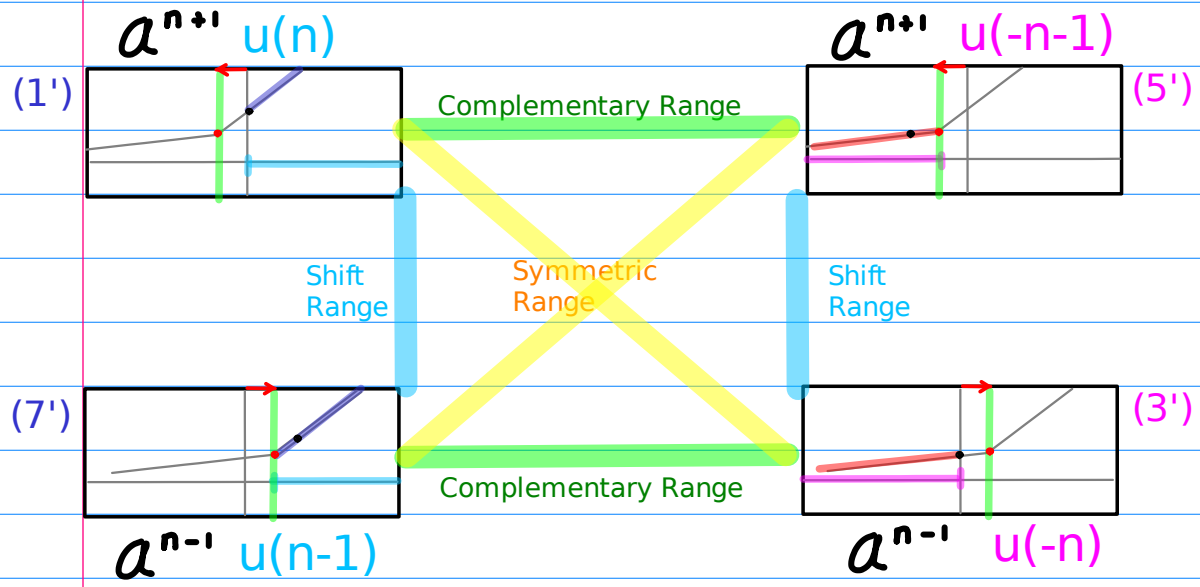
Case 1a

(1) (5) u(n) u(-n-1)
(7) (3) u(n-1) u(-n)

(1') (5') u(n) u(-n-1)
(7') (3') u(n-1) u(-n)

Shifted, Complementary, & Symmetric Ranges



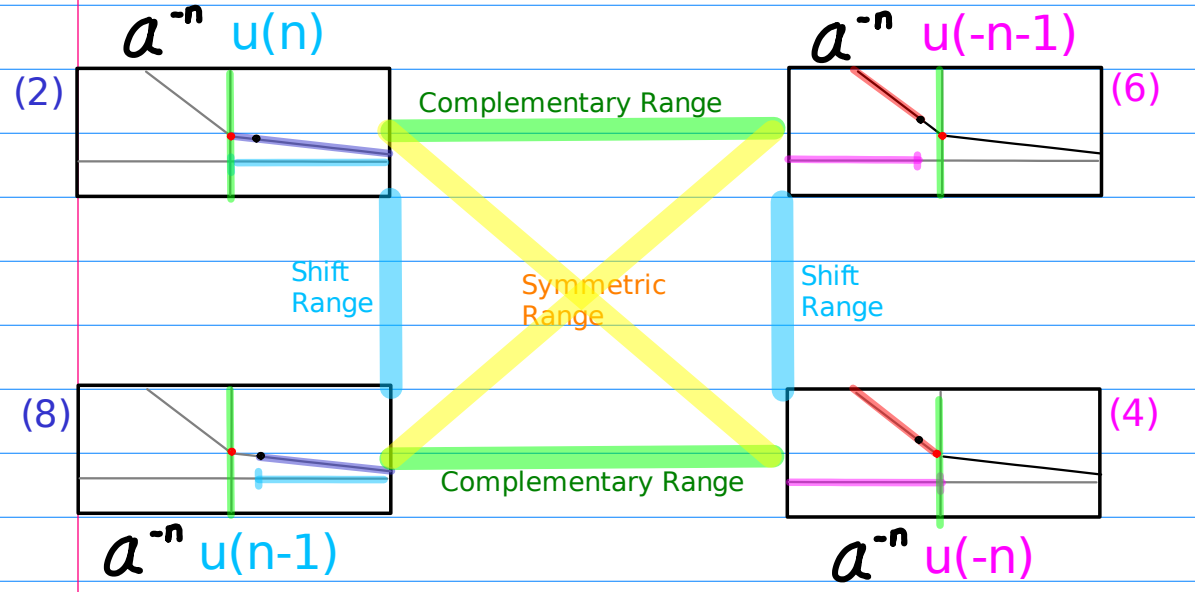


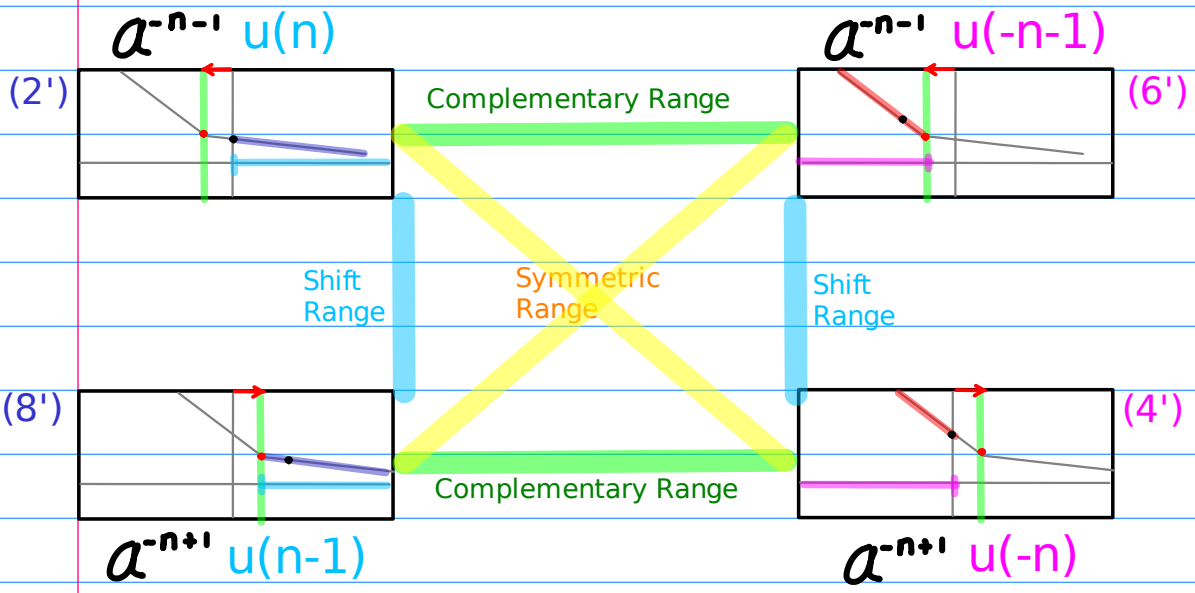
Case 1b

(2) (6) u(n) u(-n-1)
(8) (4) u(n-1) u(-n)

(2') (6') u(n) u(-n-1)
(8') (4') u(n-1) u(-n)

Shifted, Complementary, & Symmetric Ranges





Case 2

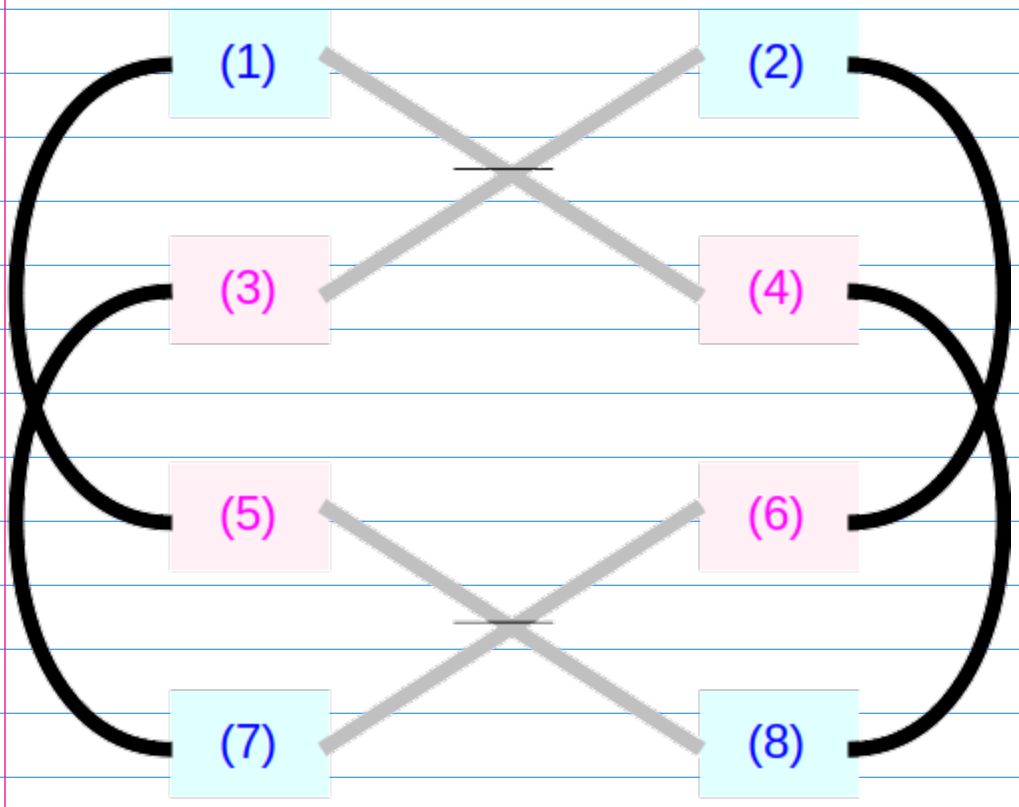
(1)(5)
(8)(4)

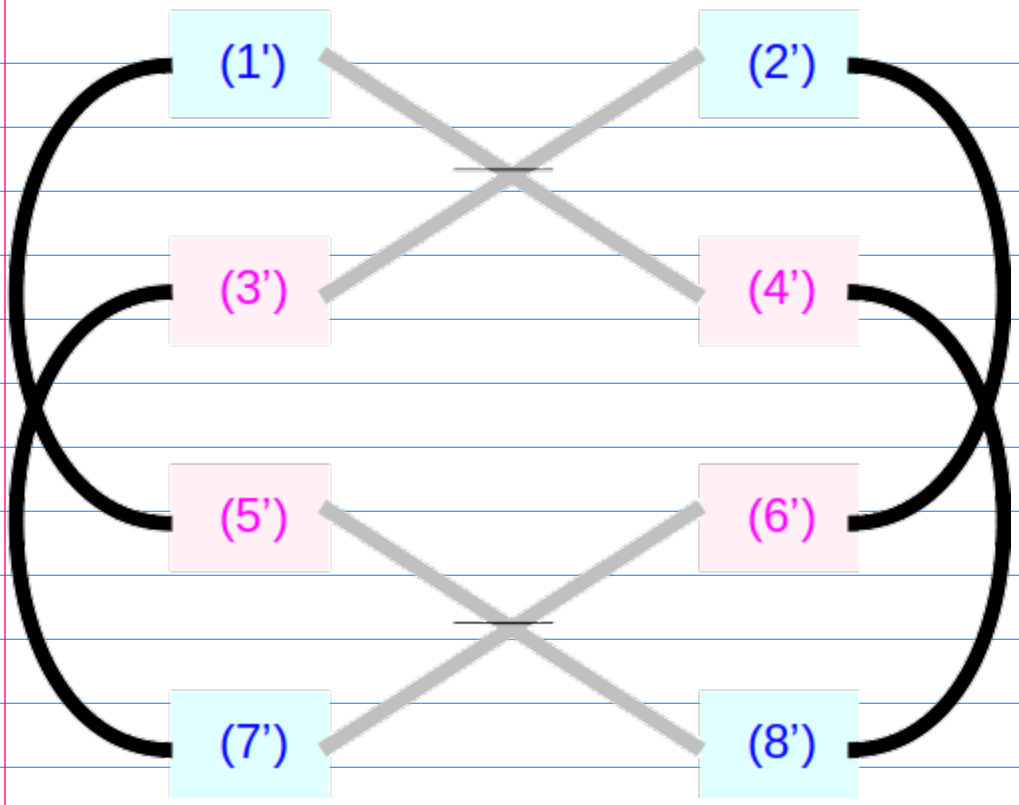
(2)(6)
(7)(3)

(1')(5')
(8')(4')

(2')(6')
(7')(3')

Shifted, Complementary, & Symmetric Ranges - Flipped



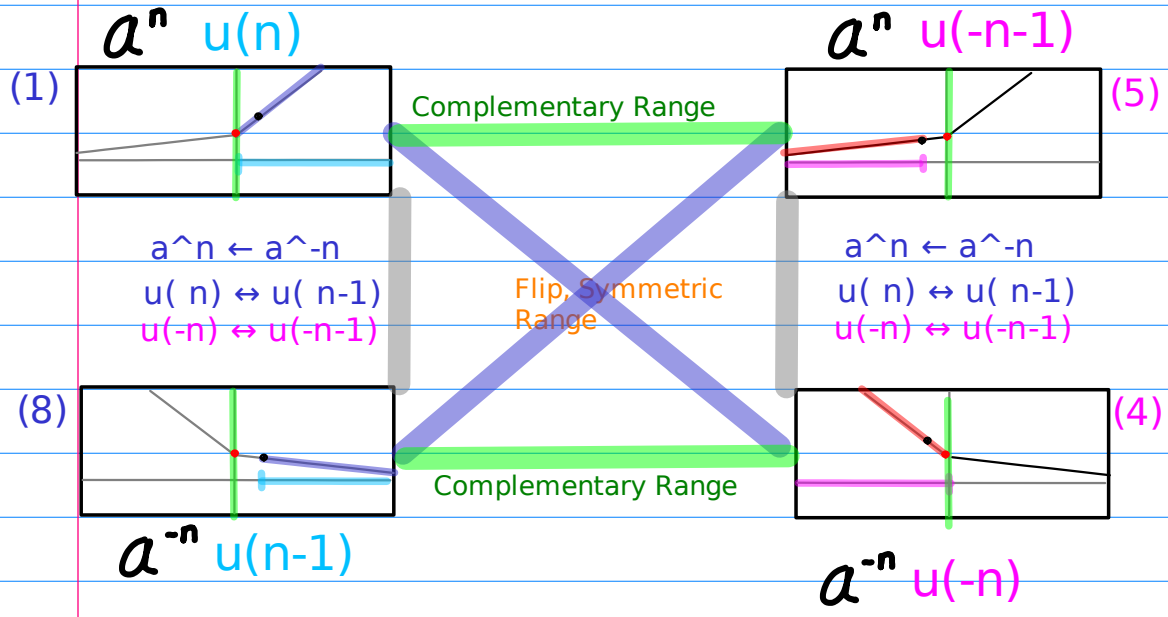


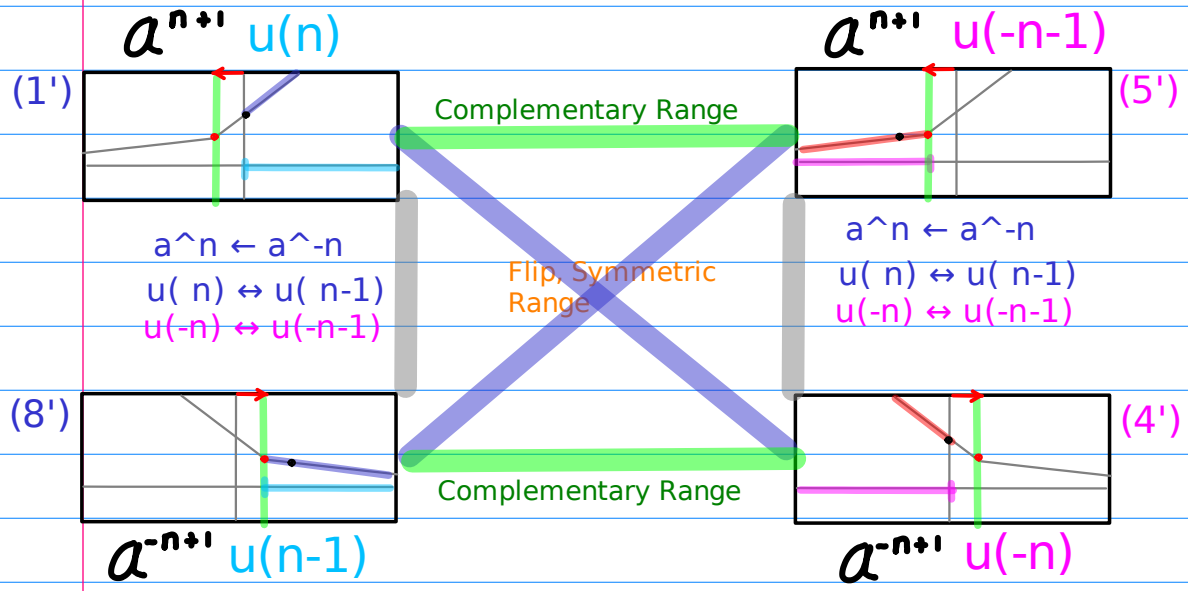
Case 2a

(1) (5) u(n) u(-n-1)
(8) (4) u(n-1) u(-n)

(1') (5') u(n) u(-n-1)
(8') (4') u(n-1) u(-n)

Shifted, Complementary, & Symmetric Ranges - Flipped



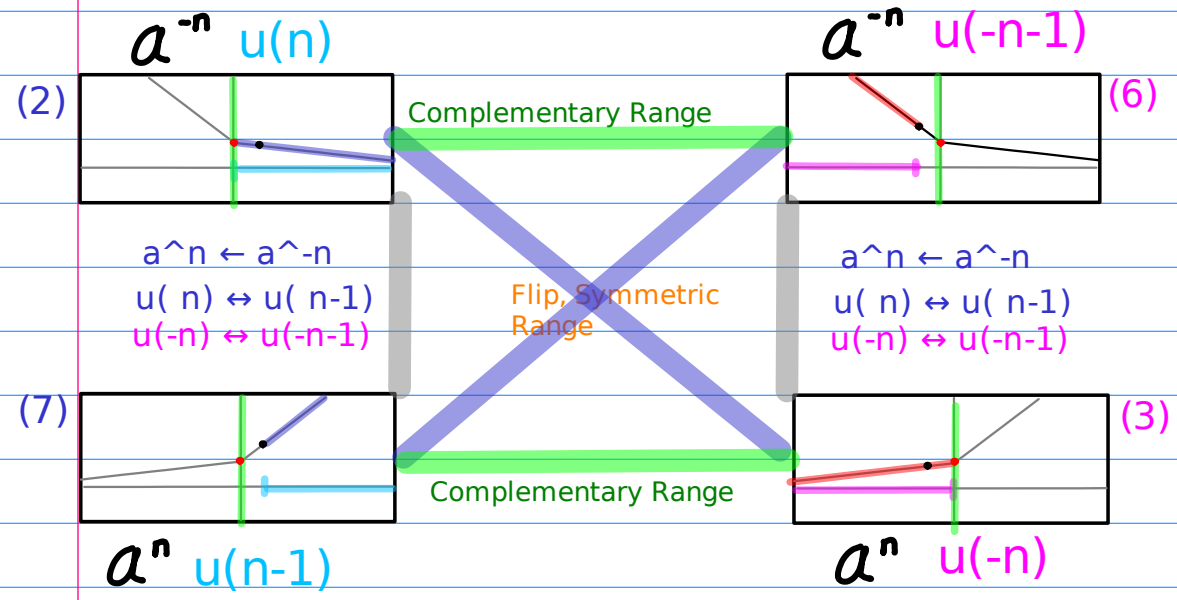


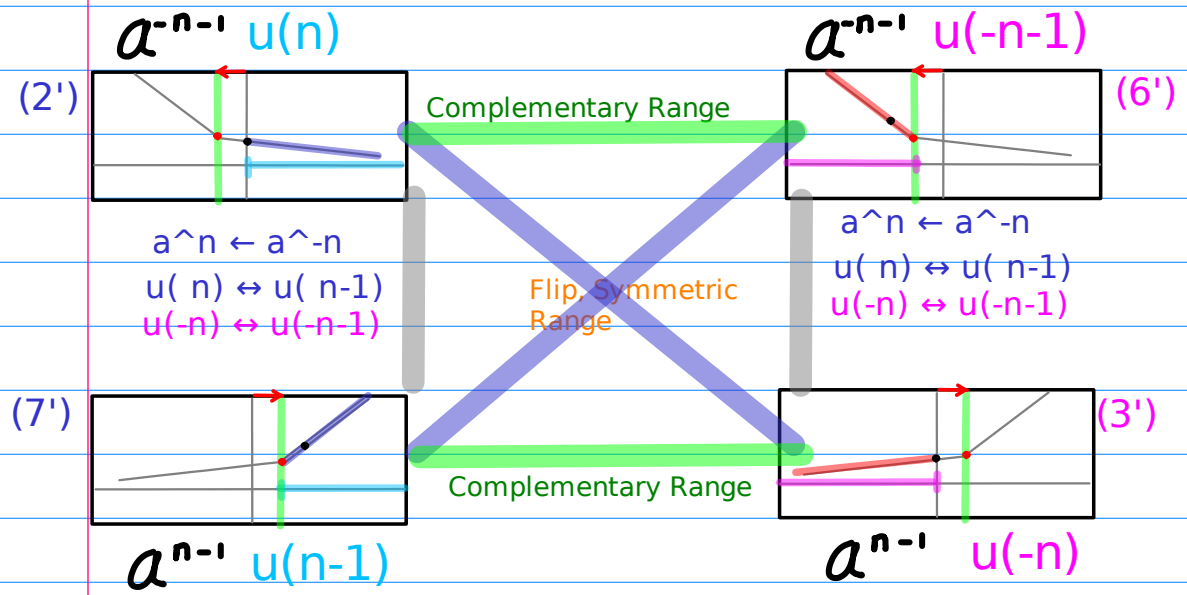
Case 2b

(2) (6) u(n) u(-n-1)
(7) (3) u(n-1) u(-n)

(2') (6') u(n) u(-n-1)
(7') (3') u(n-1) u(-n)

Shifted, Complementary, & Symmetric Ranges - Flipped





Case 3

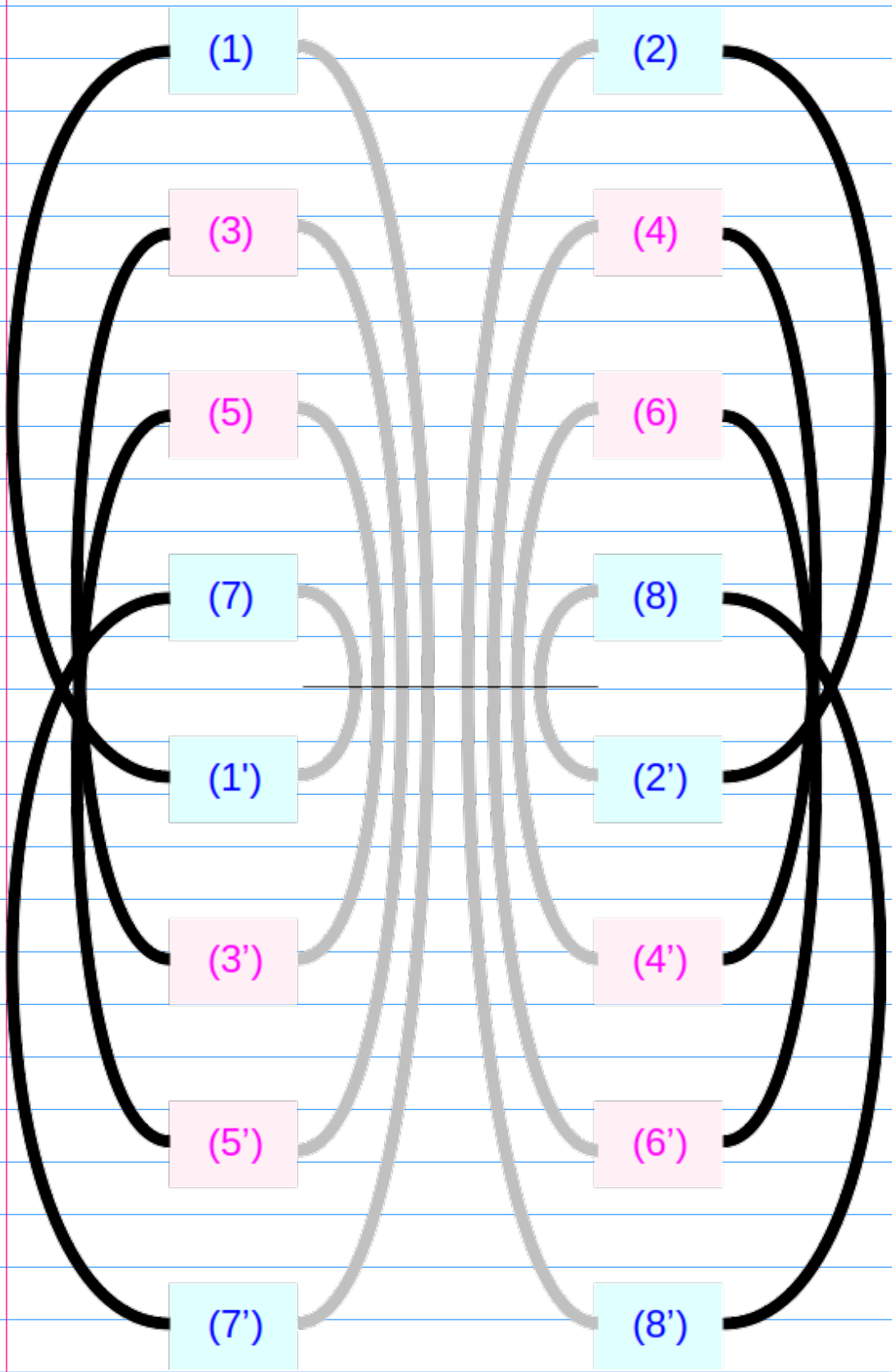
(1) (1') (7) (7') 

(2) (2') (8) (8') 

(5) (5') (3) (3') 

(6) (6') (4) (4') 

Shifted, Id, & Shifted Ranges

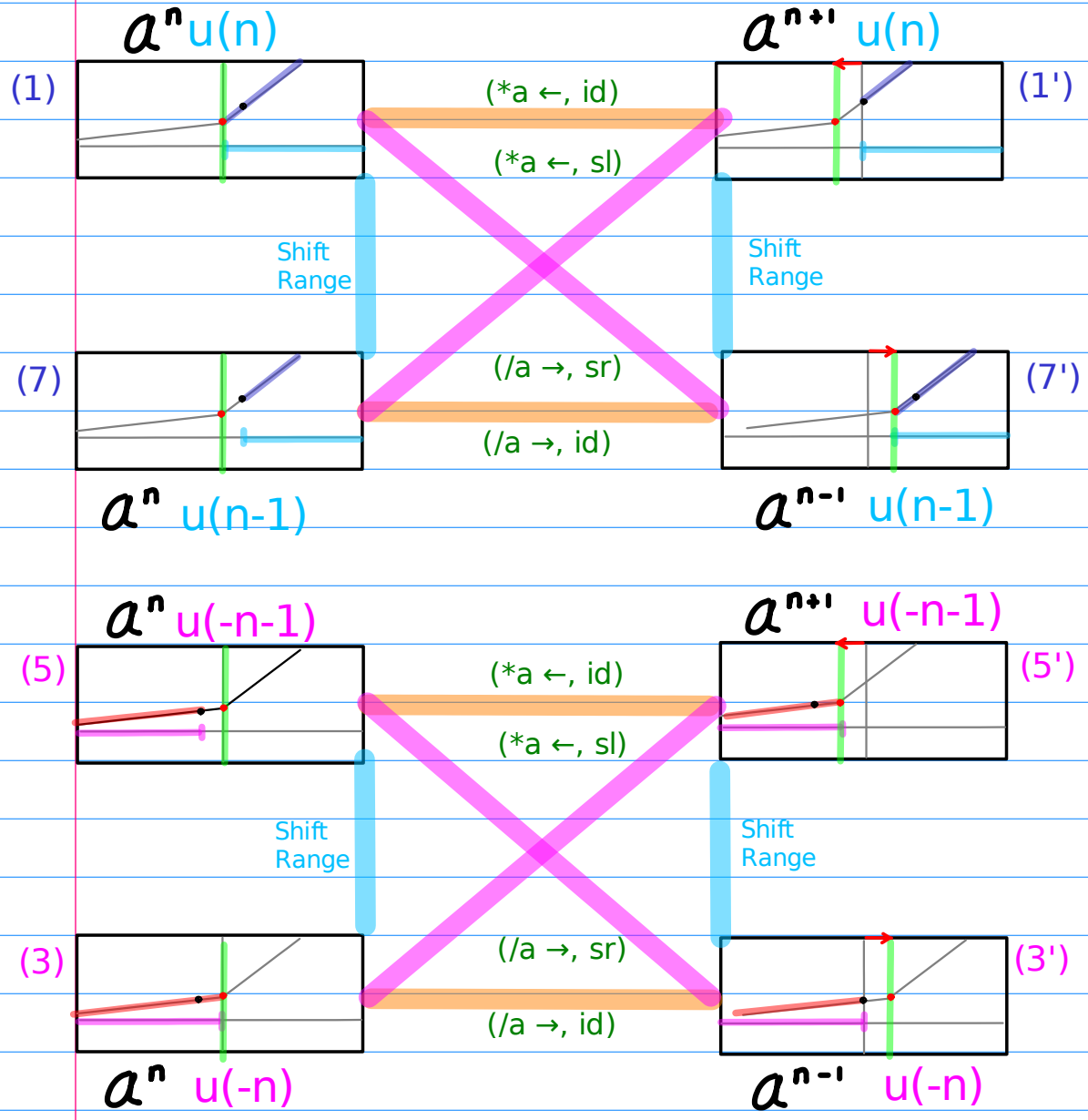


Case 3a

(1) (1')
(7) (7')

(5) (5')
(3) (3')

Shifted, Id, & Shifted Ranges

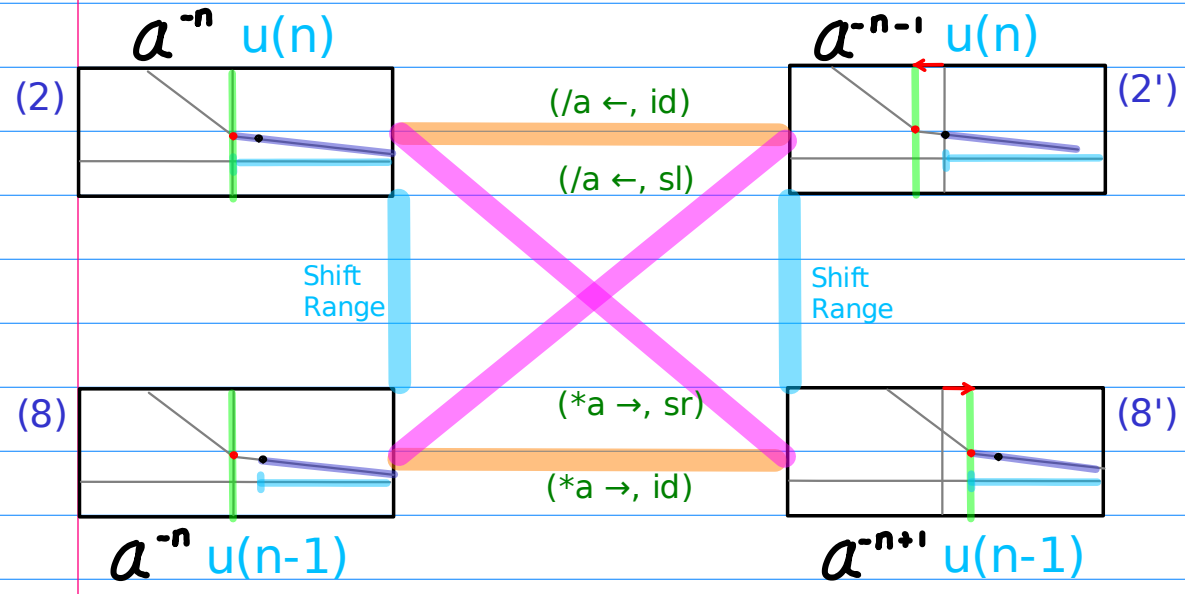


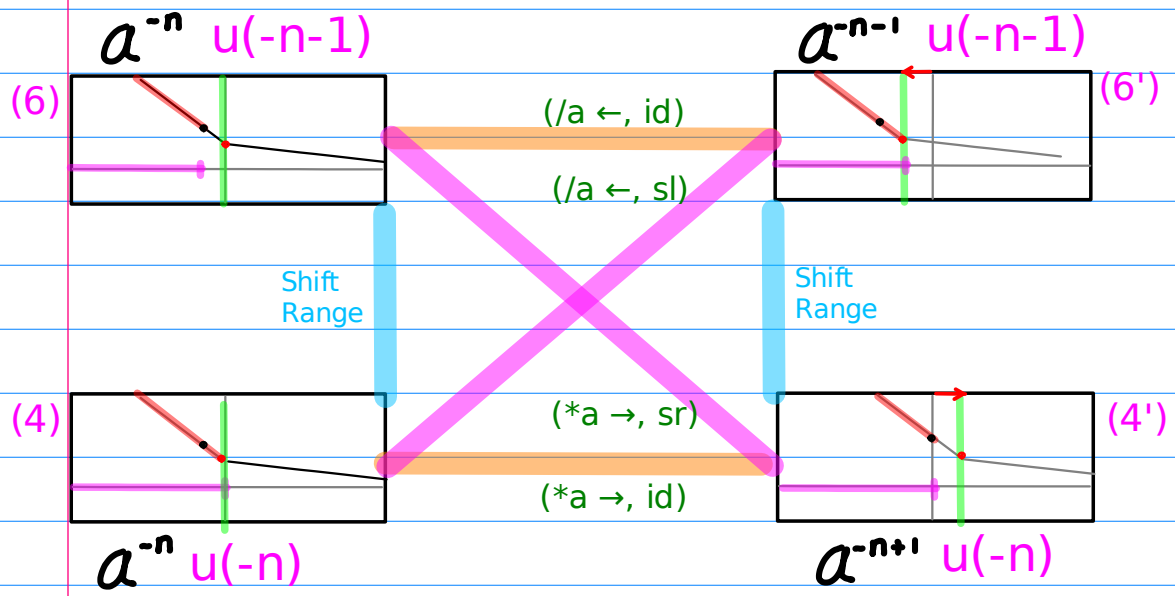
Case 3b

(2) (2')
(8) (8')

(6) (6')
(4) (4')

Shifted, Id, & Shifted Ranges





Case 4

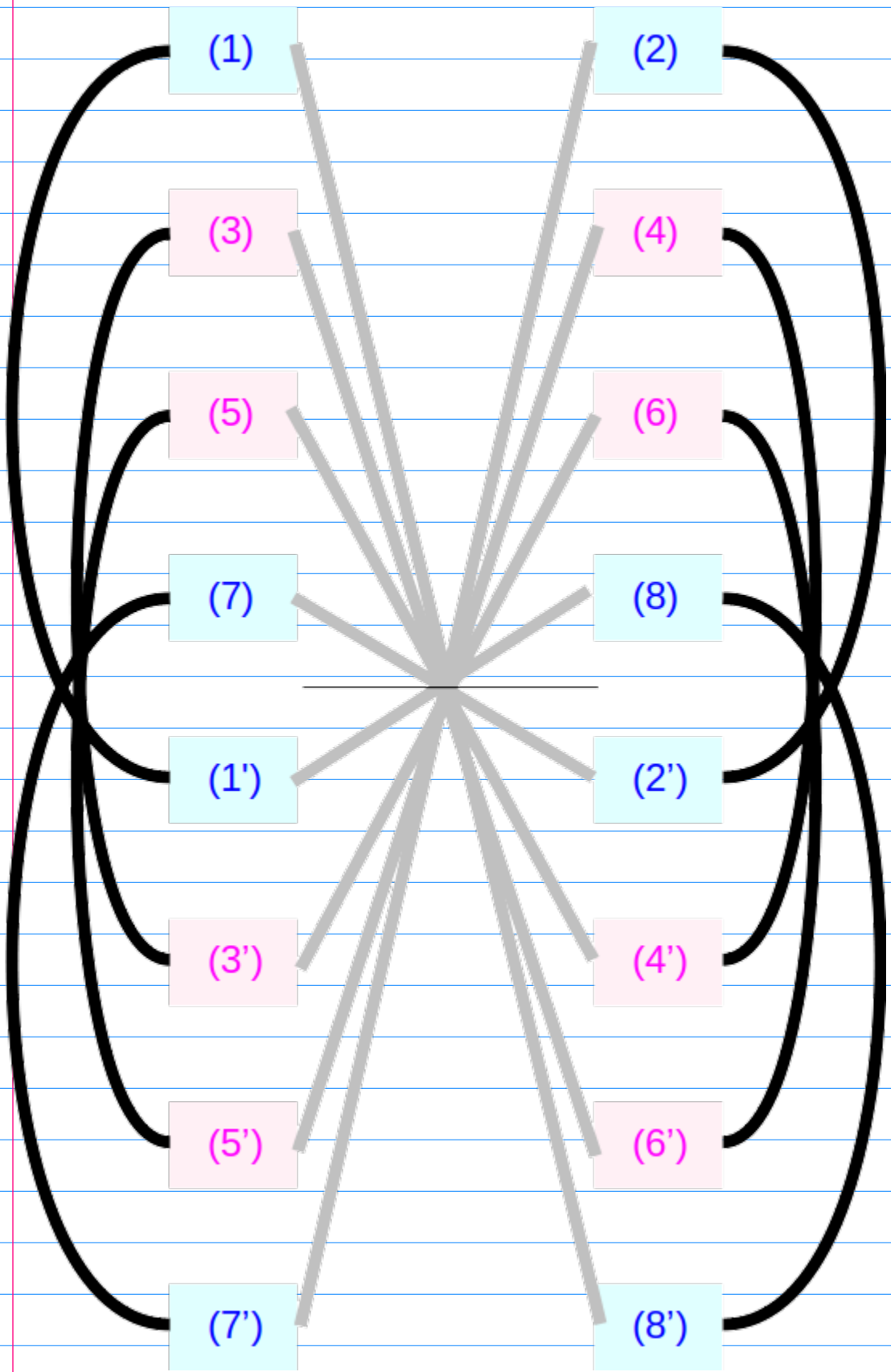
(1) (1')
(8) (8')

(2) (2')
(7) (7')

(5) (5')
(4) (4')

(6) (6')
(3) (3')

Shifted, Id, & Shifted Ranges
- Flipped

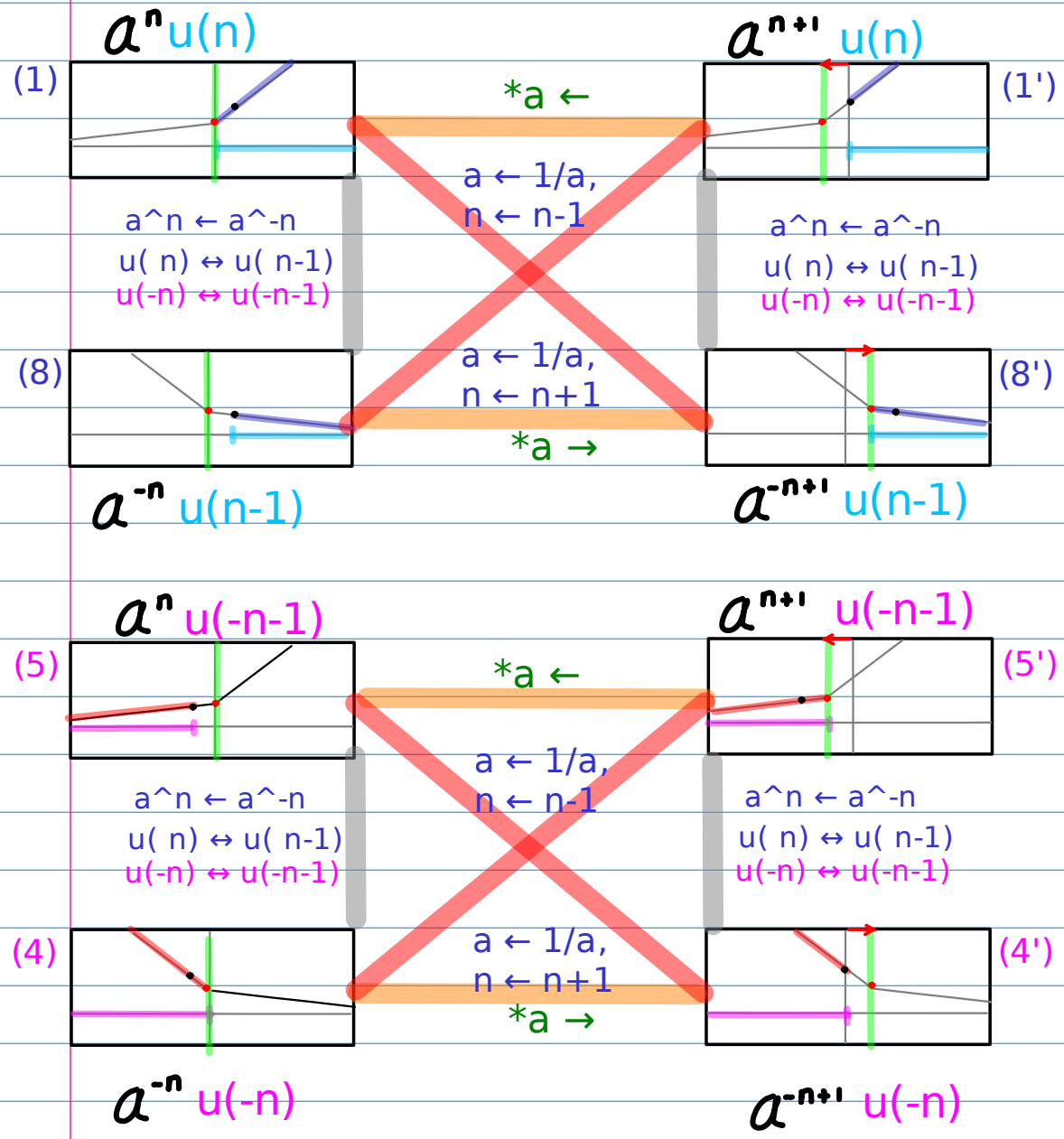


Case 4a

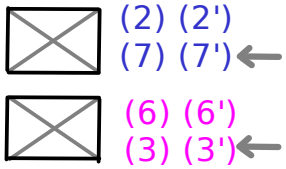
(1) (1') → (8) (8')

(5) (5') → (4) (4')

Shifted, Id, & Shifted Ranges - Flipped



Case 4b



Shifted, Id, & Shifted Ranges - Flipped