

Boundary Value Problems (1A)

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BVP

BVP

$$a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x) y = g(x)$$

$$A_1 y(a) + B_1 y'(a) = C_1$$

$$A_2 y(b) + B_2 y'(b) = C_2$$

$$a < x < b$$

$$a_2(x) y'' + a_1(x) y' + a_0(x) y = g(x)$$

$$A_2 y(b) + B_2 y'(b) = C_2$$

$$A_2 y(b) + B_2 y'(b) = C_2$$

$$a < x < b$$

BVP Example

$$\frac{d^2 y}{dx^2} + \lambda y = 0$$

$$y'' + \lambda y = 0$$

$$\lambda = 0$$

$$m^2 = 0$$

$$y(x) = c_1 x + c_2$$

$$y''(x) = 0$$

$$\lambda = -\alpha^2$$

$$m^2 - \alpha^2 = 0$$

$$\begin{aligned} y(x) &= c_1 e^{+\alpha x} + c_2 e^{-\alpha x} \\ &= c_3 \cosh(\alpha x) + c_4 \sinh(\alpha x) \end{aligned}$$

$$m_1 = +\alpha$$

$$m_2 = -\alpha$$

$$\lambda = +\alpha^2$$

$$m^2 + \alpha^2 = 0$$

$$\begin{aligned} y(x) &= c_1 e^{+i\alpha x} + c_2 e^{-i\alpha x} \\ &= c_3 \cos(\alpha x) + c_4 \sin(\alpha x) \end{aligned}$$

$$m_1 = +i\alpha$$

$$m_2 = -i\alpha$$

BVP Example

$$\frac{d^2 y}{dx^2} + \lambda y = 0 \quad 0 < x < L$$

$$y(0) = 0$$

$$y(L) = 0$$

$$y'' + \lambda y = 0 \quad 0 < x < L$$

$$y(0) = 0$$

$$y(L) = 0$$

$$\lambda = 0 \quad y(x) = c_1 x + c_2$$

$$y(0) = c_2 = 0$$

$$y(L) = c_1 L = 0$$

$$y(x) = 0$$

$$\lambda = -\alpha^2 \quad y(x) = c_1 \cosh \alpha x + c_2 \sinh \alpha x$$

$$y(0) = c_1 \cdot 1 + c_2 \cdot 0 = 0$$

$$y(L) = c_2 \sinh \alpha L = 0$$

$$y(x) = 0$$

$$\lambda = +\alpha^2 \quad y(x) = c_1 \cos \alpha x + c_2 \sin \alpha x$$

$$y(0) = c_1 \cdot 1 + c_2 \cdot 0 = 0$$

$$y(L) = c_2 \sin \alpha L = 0$$

$$y(x) = c_2 \sin(\alpha x)$$

$$\alpha = \frac{n\pi}{L}$$

BVP Example

$$\lambda = +\alpha^2 \quad y(x) = c_1 \cos \alpha x + c_2 \sin \alpha x$$

$$y(0) = c_1 \cdot 1 + c_2 \cdot 0 = 0$$

$$y(L) = c_2 \sin \alpha L = 0$$

$$\alpha L = n\pi \quad \alpha = \frac{n\pi}{L}$$

$$\lambda_1 = \left(\frac{1\pi}{L}\right)^2$$

$$y_1(x) = c_2 \sin\left(\frac{1\pi}{L}x\right)$$

$$\lambda_2 = \left(\frac{2\pi}{L}\right)^2$$

$$y_2(x) = c_2 \sin\left(\frac{2\pi}{L}x\right)$$

$$\lambda_3 = \left(\frac{3\pi}{L}\right)^2$$

$$y_3(x) = c_2 \sin\left(\frac{3\pi}{L}x\right)$$

BVP Example

References

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