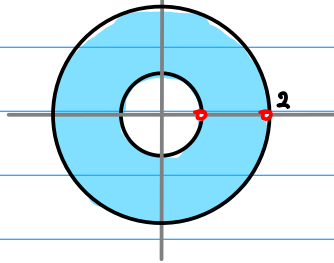
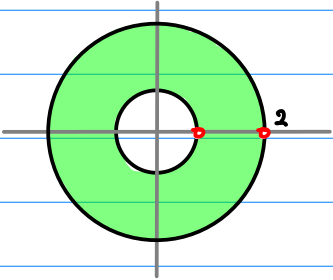
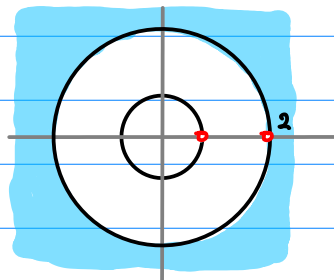
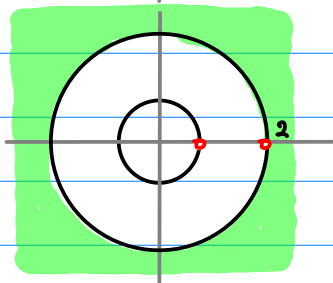
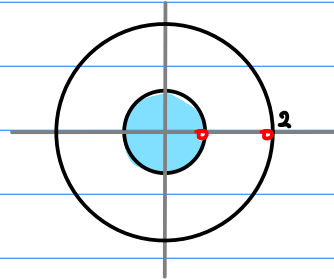
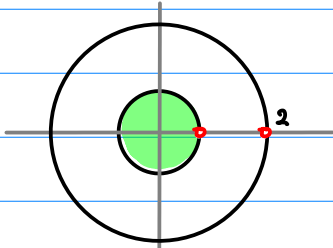


Laurent Series and z-Transform Examples case 3.B

20171226

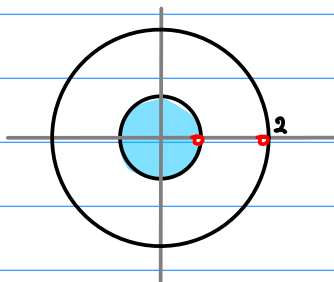
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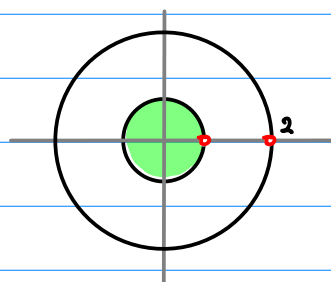
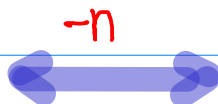


3.B

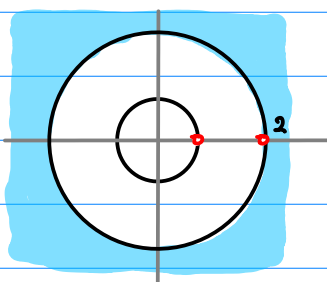
$$f(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)} = X(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)}$$



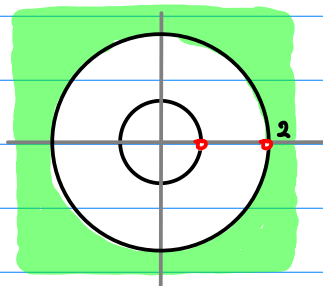
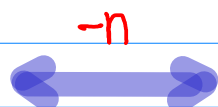
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$



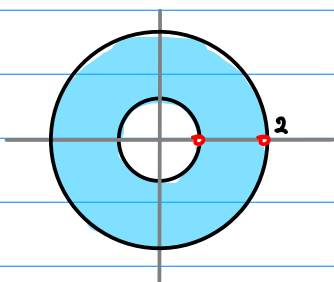
$$\sum_{n=0}^{\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^{-n}$$



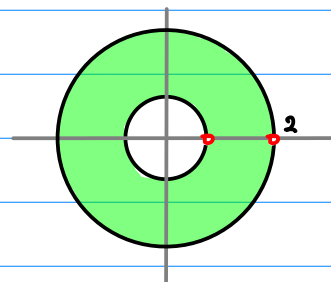
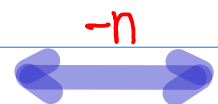
$$\sum_{n=-1}^{\infty} \left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$



$$\sum_{n=-1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^{-n}$$



$$\sum_{n=-1}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

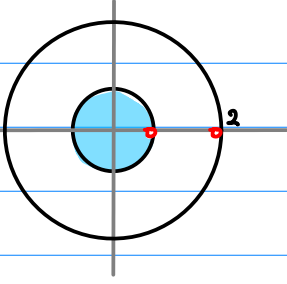


$$\sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

3.B

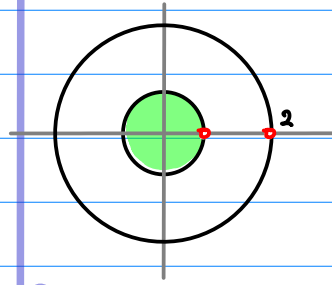
$$f(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)} = X(z) = \frac{3}{2} \frac{-1}{(z-1)(z-2)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 2^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

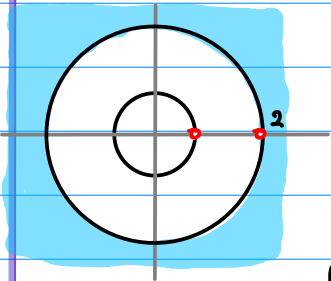
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - (\frac{1}{2})^{n-1} & (n \leq 0) \end{cases}$$

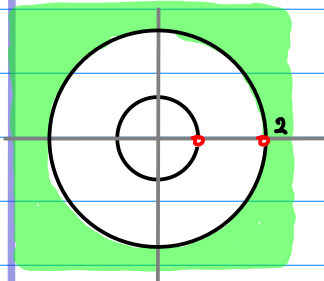
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 2^{n+1} - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

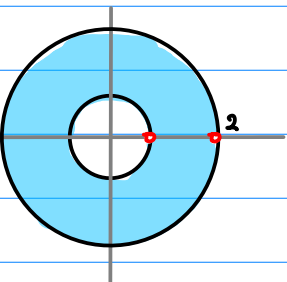
$$f(z) = \sum_{n=-1}^{\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n-1} - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

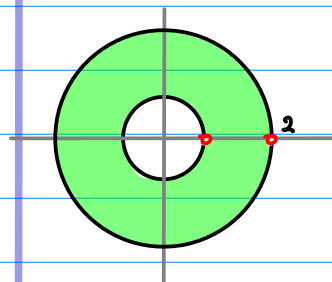
$$X(z) = \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 2^{n+1} & (n < 0) \end{cases}$$

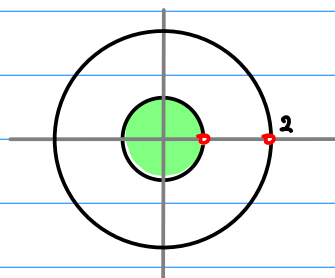
$$f(z) = \sum_{n=-1}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

$$X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$



$$|z| < 0.5$$

$$|z| < 2$$

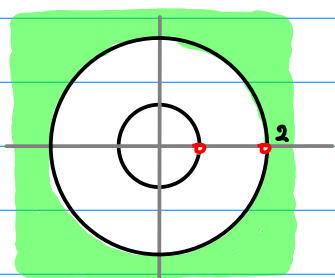
$$\sum_{n=0}^{-\infty} [2^{n+1} - (\frac{1}{2})^{n+1}] z^{-n}$$

$$-\frac{(2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})}$$

$$= -\sum_{n=0}^{\infty} (2)(2z)^n + \sum_{n=0}^{\infty} (\frac{1}{2})(\frac{z}{2})^n$$

$$= -\sum_{n=0}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$

$$= \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 2^{n+1}] z^n$$



$$0.5 < |z|$$

$$2 < |z|$$

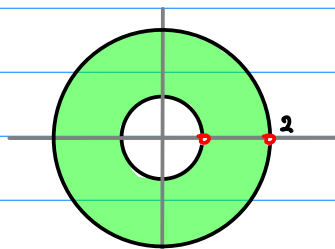
$$\sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^{-n}$$

$$\frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{z}{2})}$$

$$= +\sum_{n=0}^{\infty} (\frac{1}{z})(\frac{1}{2z})^n - \sum_{n=0}^{\infty} (\frac{1}{z})(\frac{z}{2})^n$$

$$= +\sum_{n=0}^{\infty} (\frac{1}{2})^n z^{-n-1} - \sum_{n=0}^{\infty} (2)^n z^{-n-1}$$

$$= \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^{-n}$$



$$0.5 < |z|$$

$$|z| < 2$$

$$\sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

$$+\frac{(\frac{1}{z})}{1-(\frac{1}{2z})} + \frac{(\frac{1}{z})}{1-(\frac{z}{2})}$$

$$= +\sum_{n=0}^{\infty} (\frac{1}{z})(\frac{1}{2z})^n + \sum_{n=0}^{\infty} (\frac{1}{z})(\frac{z}{2})^n$$

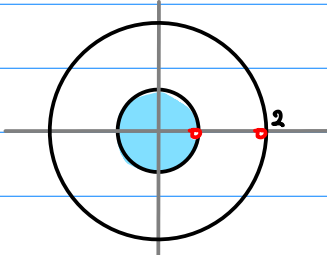
$$= +\sum_{n=0}^{\infty} (\frac{1}{2})^n z^{-n-1} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$

$$= \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$

$$X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$$f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$|z| < 0.5$
 $|z| < 2$



$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$

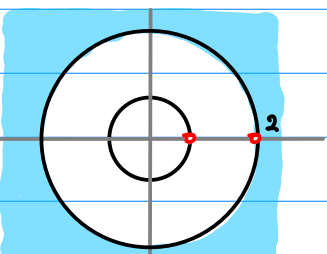
$$-\frac{(2)}{1-(2z)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (2)(2z)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$

$0.5 < |z|$
 $2 < |z|$



$$\sum_{n=-1}^{\infty} \left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$

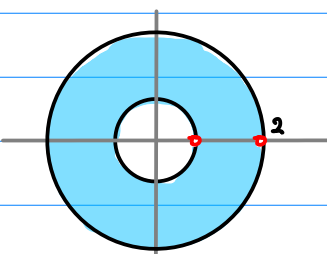
$$\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{z}{2}\right)^n$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n-1} - \sum_{n=0}^{\infty} (2)^n z^{-n-1}$$

$$= \sum_{n=-1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$

$0.5 < |z|$
 $|z| < 2$



$$\sum_{n=-1}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{z}{2}\right)^n$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n-1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

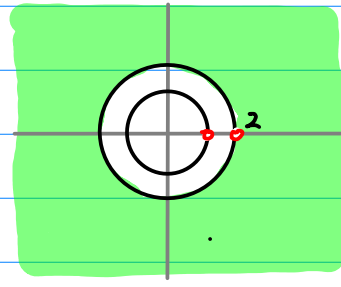
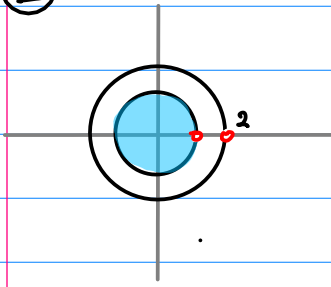
$$= \sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n + \sum_{n=0}^{\infty} 2^{n+1} z^n$$

$$f(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$



$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z)$$

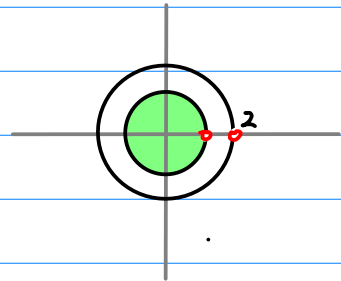
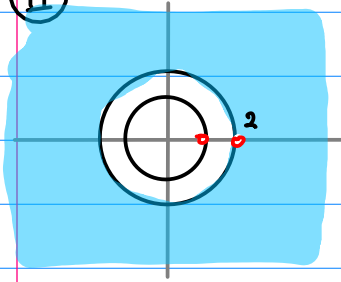
Ⓘ



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

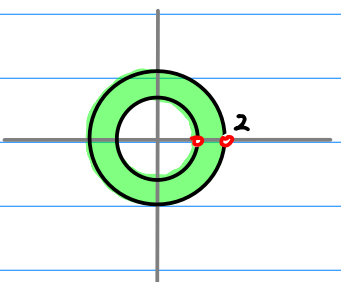
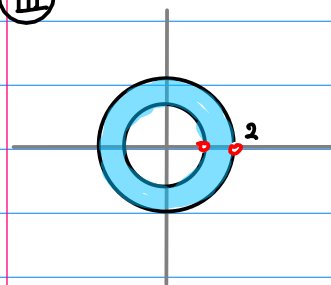
Ⓢ



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n+1} - 1 & (n \leq 0) \end{cases}$$

Ⓣ



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{-n+1} - 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

$$X(z) = \sum_{n=0}^{-\infty} \left(\frac{1}{2}\right)^{-n+1} z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

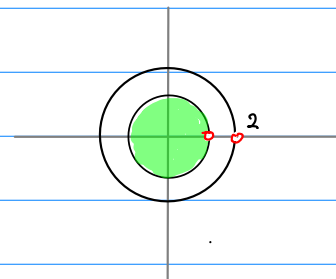
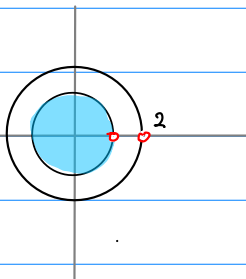
$$= \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{z}{2}\right)} - \frac{1}{1 - z}$$

$$= \frac{-1}{z-2} + \frac{1}{z-1}$$

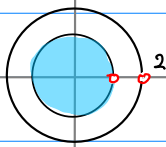
$$= \frac{-z+1+z-2}{(z-1)(z-2)}$$

$$= \frac{-1}{(z-1)(z-2)}$$

$$\left|\frac{z}{2}\right| < 1 \quad \left|\frac{z}{1}\right| < 1$$

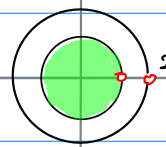


I



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

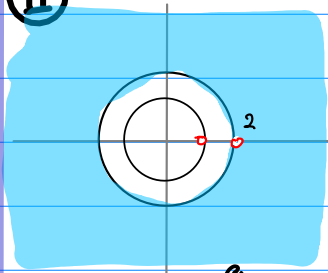
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

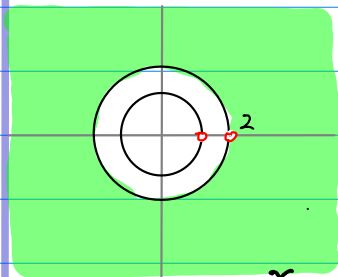
$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

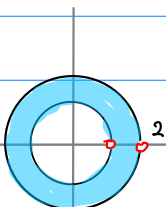
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

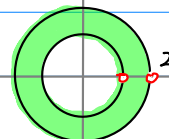
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} \cdot z^{-n}$$

III



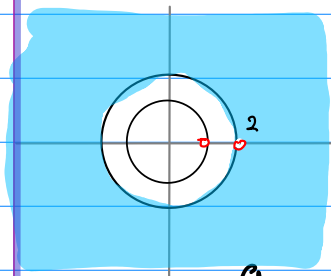
$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

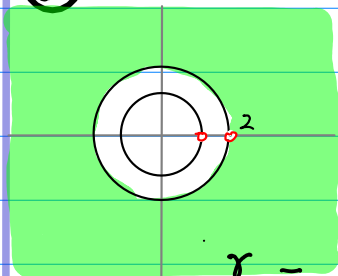
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

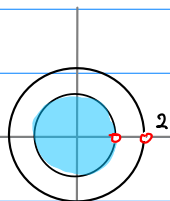
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

Ⓘ



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

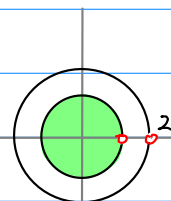
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

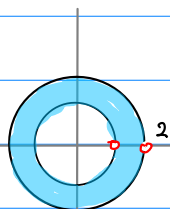
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

Ⓛ



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n+1} - 1 & (n \leq 0) \end{cases}$$

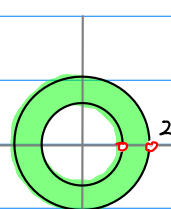
$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

Ⓜ

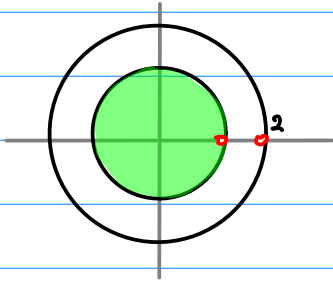


$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

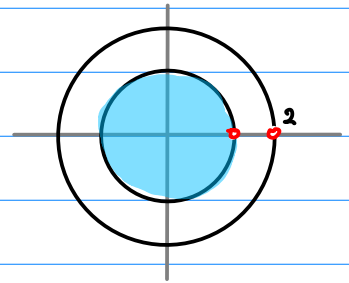
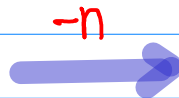
$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$

3.B

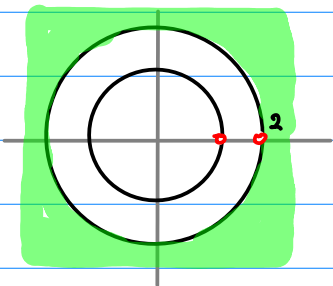
$$X(z) = \frac{-1}{(z-1)(z-2)} = f(z) = \frac{-1}{(z-1)(z-2)}$$



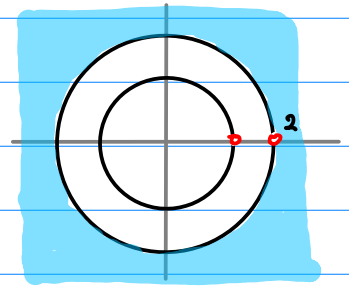
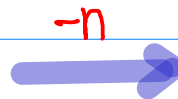
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$



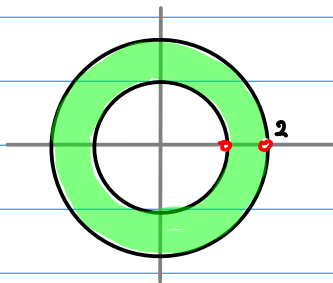
$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^{-n}$$



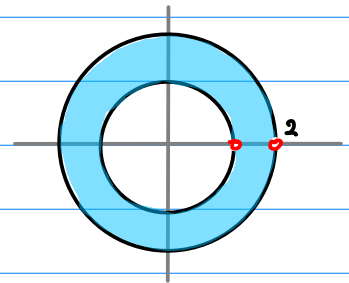
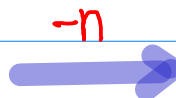
$$\sum_{n=-1}^{-\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$



$$\sum_{n=1}^{\infty} [1 - 2^{n-1}] z^{-n}$$

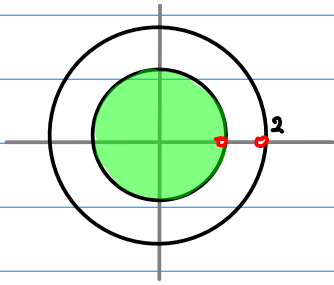


$$-\sum_{n=0}^{\infty} z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$+\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{-n}$$

$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{(z-1)} - \frac{1}{(z-2)}$$

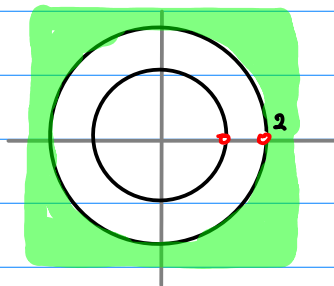


$$= -\frac{(+)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (1)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

$$= \sum_{n=0}^{\infty} \left[2^{n-1} - 1\right] z^{-n}$$

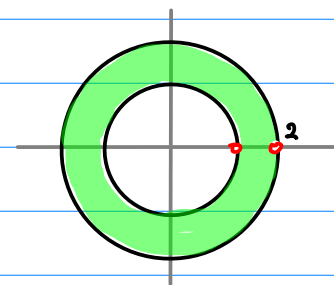


$$+ \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} \left[1 - 2^n\right] z^{-n-1}$$

$$= \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{-n}$$



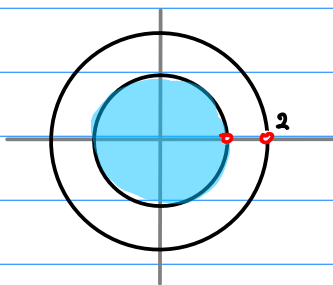
$$+ \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{z}{2}\right)^n$$

$$= + \sum_{n=0}^{\infty} z^{-n-1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$= + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

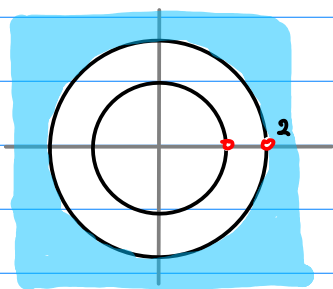
$$f(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{(z-1)} - \frac{1}{(z-2)}$$



$$= -\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (1)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

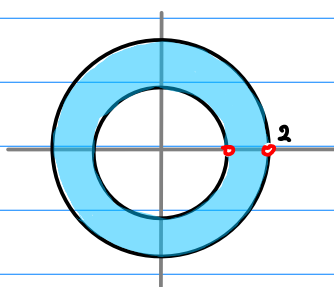


$$+ \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} [1 - 2^n] z^{-n-1}$$

$$= \sum_{n=-1}^{\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1}\right] z^n$$



$$+ \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{z}{2}\right)^n$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^{n+1} z^n$$

$$= + \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$