Complex Fourier Series (H.1)

20160102

b

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \sin(nx)\right)}$$

$\left.\begin{matrix} a_0, & a_1, & a_2, & a_3, & \cdots \\ & b_1, & b_2, & b_3 & \cdots \end{matrix}\right)$ 알 때

$f(x)$ 를 만들수 있다.

$$\boxed{\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx \end{aligned}}$$

원래 함수 $f(x)$로부터 계수구하기.

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$\left.\begin{matrix} a_0, & a_1, & a_2, & a_3, & \cdots\cdots \\ & b_1, & b_2, & b_3 & \cdots \end{matrix}\right)$ 알 때

$f(x)$ 를 만들수 있다.

$$\boxed{\begin{aligned} a_0 &= \frac{1}{p}\int_{-p}^{+p} f(x)\,dx \\ a_n &= \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx \\ b_n &= \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx \end{aligned}}$$

원래 함수 $f(x)$로부터 계수구하기.

$$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 2x + 2 = 0$$

$$. = \frac{-b \pm \sqrt{b^2 - ac}}{a}$$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm \sqrt{4} \cdot \sqrt{-1}}{2}$$

$$= -1 \pm \sqrt{-1}$$

$$= -1 \pm i$$

$$\underline{x_1 = -1 + i} \quad \text{or} \quad x_2 = -1 - i$$

$$(x - x_1)(x - x_2) = 0$$

$$(x - (-1 + i))(x - (-1 - i))$$

$$= ((x+1) - i)((x+1) + i)$$

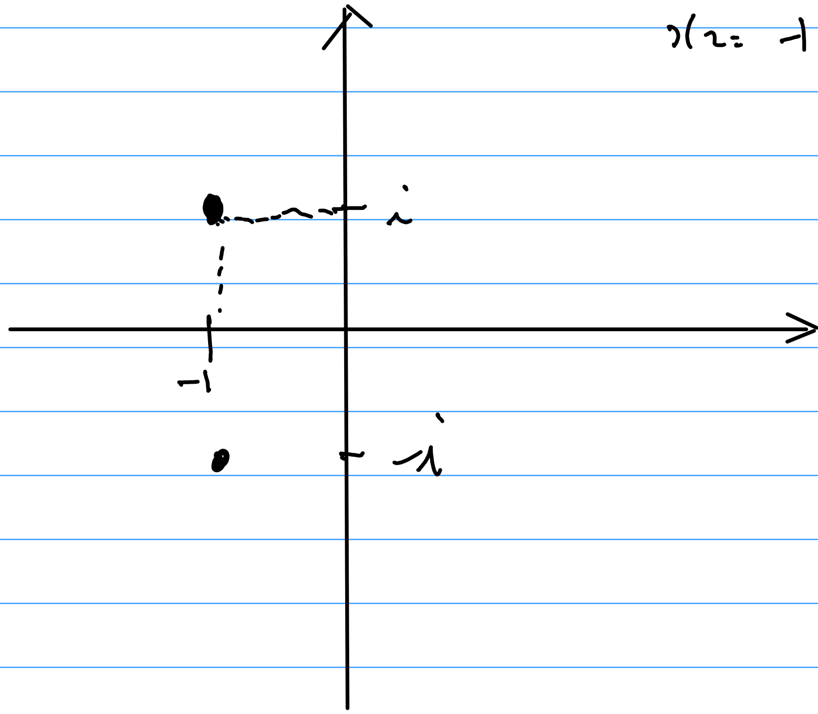
$$(A - B)(A + B) \Rightarrow A^2 - B^2$$

$$= (x-1)^2 - (i)^2$$

$$= (x-1)^2 + 1 = x^2 - 2x + 2 = 0$$

$x_1 = -1 + i$

$x_2 = -1 - i$



$\square^2 \geq 0$

$(x+1)^2 = -9$

$x \in$ 허수범위

$x^2+2x+1=-9$

$x^2+2x+10=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-4\cdot 10}}{2\cdot 1}$$

$D > 0$ 서로다른 실근

$D = 0$ 중근

$D < 0$ 허근

$$= \frac{-2 \pm \sqrt{4\cdot 9\cdot(-1)}}{2}$$

$$= \frac{-2 \pm \sqrt{(2\cdot 3)^2\cdot(-1)}}{2}$$

$$= \frac{-2 \pm \sqrt{6^2}\cdot\sqrt{-1}}{2}$$

$$= \frac{-2 \pm 6i}{2}$$

$$x = -1 \pm 3i$$

$$(x-(-1+3i))(x-(-1-3i)) = 0$$

$$z = \mathrm{Re}(z) + i\,\mathrm{Im}(z)$$
$$+\ \bar{z} = \mathrm{Re}(z) - i\,\mathrm{Im}(z)$$

$$z + \bar{z} = 2\,\mathrm{Re}(z)$$

$$\mathrm{Re}(z) = \frac{1}{2}(z + \bar{z})$$

$$z = \cancel{\mathrm{Re}(z)} + i\,\mathrm{Im}(z)$$
$$-)\ \bar{z} = \cancel{\mathrm{Re}(z)} - i\,\mathrm{Im}(z)$$

$$z - \bar{z} = 2i\,\mathrm{Im}(z)$$

$$\mathrm{Im}(z) = \frac{1}{2i}(z - \bar{z})$$

$$z = \overset{\mathrm{Re}(z)}{3} + \overset{\mathrm{Im}(z)}{4}i \qquad\qquad z = 3 + 4i$$
$$+\ \bar{z} = 3 - 4i \qquad\qquad -\ \bar{z} = 3 - 4i$$

$$z - \bar{z} = +8i$$

$$z + \bar{z} = 6$$

$$4 = \frac{1}{2i}(z - \bar{z})$$

$$3 = \frac{1}{2}(z + \bar{z})$$

Euler formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i(-\theta)} = \cos(-\theta) + i\sin(-\theta)$$

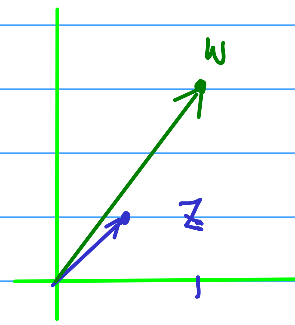
$$e^{-i\theta} = \cos\theta - i\sin\theta$$

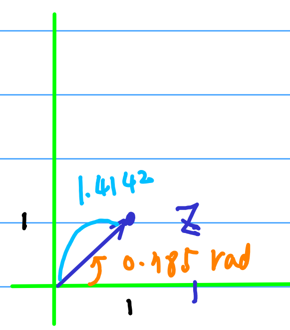
$$\begin{array}{rrcl} & e^{i\theta} & = & \cos\theta + i\sin\theta \\ + & e^{-i\theta} & = & \cos\theta - i\sin\theta \\ \hline & e^{i\theta} + e^{-i\theta} & = & 2\cos\theta \end{array}$$

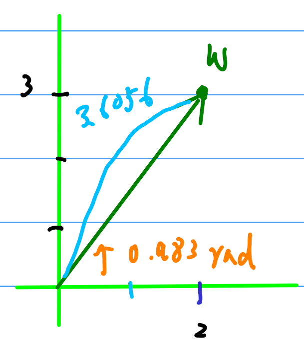
$$\begin{array}{rrcl} & e^{i\theta} & = & \cos\theta + i\sin\theta \\ - & e^{-i\theta} & = & \cos\theta - i\sin\theta \\ \hline & e^{i\theta} - e^{-i\theta} & = & 2i\sin\theta \end{array}$$

$$\boxed{\begin{aligned} \cos\theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin\theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \end{aligned}}$$

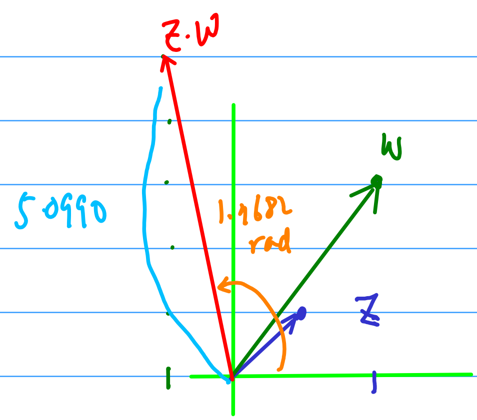
```
octave:5> z
z =  1 + 1i
octave:6>
octave:6> re(z)
error: 're' undefined near line 1 column 1
octave:6> Re(z)
error: 'Re' undefined near line 1 column 1
octave:6> real(z)
ans =  1
octave:7> imag(z)
ans =  1
octave:8> z
z =  1 + 1i
octave:9> w = 2 + 3*i
w =  2 + 3i
octave:10> z
z =  1 + 1i
octave:11> w
w =  2 + 3i
octave:12> z*w
ans = -1 + 5i
octave:13> abs(z*w)
ans =  5.0990
octave:14>
octave:14> arg(z*w)
ans =  1.7682
octave:15>
octave:15> abs(z)
ans =  1.4142
octave:16> abs(w)
ans =  3.6056
octave:17> abs(z)*abs(w)
ans =  5.0990
octave:18> arg(z)
ans =  0.78540
octave:19> arg(w)
ans =  0.98279
octave:20> arg(z)+ arg(w)
ans =  1.7682
octave:21>
```







z·w =



$$z_1 = r_1 e^{j\theta_1}$$

$$z_2 = r_2 e^{j\theta_2}$$

$$z_1 \cdot z_2 = r_1 \cdot r_2 \, e^{j(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

2015.12.23

p182 예제 1, 2, 3.

p185 연습문제 1, 3, 5, 7, 9

11, 13, 15, 17, 19

21, 23, 25.

Fourier Series

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\cos\left(\frac{n\pi}{p}x\right) = \frac{1}{2}\left[e^{i\left(\frac{n\pi}{p}x\right)} + e^{-i\left(\frac{n\pi}{p}x\right)}\right]$$

$$\sin\left(\frac{n\pi}{p}x\right) = \frac{1}{2i}\left[e^{i\left(\frac{n\pi}{p}x\right)} - e^{-i\left(\frac{n\pi}{p}x\right)}\right]$$

$$a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right) = \frac{a_n}{2}\left[e^{i\left(\frac{n\pi}{p}x\right)} + e^{-i\left(\frac{n\pi}{p}x\right)}\right]$$

$$\boxed{\frac{1}{i} = \frac{i}{i\cdot i} = -i \qquad \frac{1}{2i} = -\frac{i}{2}} \longrightarrow \frac{-i\,b_n}{2}\left[e^{i\left(\frac{n\pi}{p}x\right)} - e^{-i\left(\frac{n\pi}{p}x\right)}\right]$$

$$\left(\frac{a_n}{2} - \frac{i\,b_n}{2}\right)e^{i\left(\frac{n\pi}{p}x\right)} + \left(\frac{a_n}{2} + \frac{i\,b_n}{2}\right)e^{-i\left(\frac{n\pi}{p}x\right)}$$

$$\frac{1}{2}(a_n - i\,b_n)e^{i\left(\frac{n\pi}{p}x\right)} + \frac{1}{2}(a_n + i\,b_n)e^{-i\left(\frac{n\pi}{p}x\right)}$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

Euler Formula

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(\frac{1}{2}(a_n - i b_n) e^{i\left(\frac{n\pi}{p}x\right)} + \frac{1}{2}(a_n + i b_n) e^{-i\left(\frac{n\pi}{p}x\right)}\right)$$

$$C_0 + \sum_{n=1}^{\infty}\left(\boxed{C_n} e^{i\left(\frac{n\pi}{p}x\right)} + \boxed{C_{-n}} e^{-i\left(\frac{n\pi}{p}x\right)}\right)$$

$$\boxed{C_0} = \frac{a_0}{2}$$

$$\boxed{C_n} = \frac{1}{2}(a_n - i b_n)$$

complex conjugate

$$\boxed{C_{-n}} = \frac{1}{2}(a_n + i b_n)$$

complex number

real number

real number

$$C_0 = \frac{a_0}{2}$$

$$C_n = \frac{1}{2}(a_n - i b_n) \Leftarrow$$

$$C_{-n} = \frac{1}{2}(a_n + i b_n)$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$\boxed{C_0} = \frac{a_0}{2} = \frac{1}{2p}\int_{-p}^{+p} f(x)\,dx$$

$$\boxed{C_n} = \frac{1}{2}(a_n - i b_n) = \frac{1}{2}\left(\frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx - \frac{i}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx\right)$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$= \frac{1}{2p}\int_{-p}^{+p} f(x)\left[\cos\left(\frac{n\pi}{p}x\right) - i\sin\left(\frac{n\pi}{p}x\right)\right]dx$$

$$= \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p}x\right)}\,dx$$

$$\boxed{C_{-n}} = \frac{1}{2}(a_n + i b_n) = \frac{1}{2}\left(\frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx + \frac{i}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx\right)$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$= \frac{1}{2p}\int_{-p}^{+p} f(x)\left[\cos\left(\frac{n\pi}{p}x\right) + i\sin\left(\frac{n\pi}{p}x\right)\right]dx$$

$$= \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{+i\left(\frac{n\pi}{p}x\right)}\,dx$$

$$C_0 = \frac{a_0}{2} \qquad a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$C_n = \frac{1}{2}(a_n - i\,b_n) \qquad a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$C_{-n} = \frac{1}{2}(a_n + i\,b_n) \qquad b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$\boxed{C_0} = \frac{a_0}{2}$$

$$\boxed{C_n} = \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p}x\right)}\,dx$$

$$\boxed{C_{-n}} = \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{+i\left(\frac{n\pi}{p}x\right)}\,dx$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$\boxed{f(x) = C_0 + \sum_{n=1}^{\infty} C_n e^{i\left(\frac{n\pi}{p}x\right)} + C_{-n} e^{-i\left(\frac{n\pi}{p}x\right)}}$$

$$C_0 = \frac{a_0}{2} = \frac{1}{2p}\int_{-p}^{+} f(x)\,dx$$

$$C_n = \frac{1}{2}(a_n - i\,b_n) = \frac{1}{2p}\int_{-p}^{+p} f(x)\,e^{-i\left(\frac{n\pi}{p}x\right)}dx$$

$$C_{-n} = \frac{1}{2}(a_n + i\,b_n) = \frac{1}{2p}\int_{-p}^{+p} f(x)\,e^{+i\left(\frac{n\pi}{p}x\right)}dx$$

$$\boxed{c_0 + \sum_{n=1}^{\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)}} + c_{-n} e^{i\left(-\frac{n\pi}{p}x\right)}$$

$c_0 e^{i\left(\frac{0\pi}{p}x\right)}$

$= c_0 e^0 = c_0$

$n=1$ $\quad c_1 e^{i\left(\frac{1\pi}{p}x\right)} + c_{-1} e^{i\left(-\frac{1\pi}{p}x\right)}$

$n=2$ $\quad c_2 e^{i\left(\frac{2\pi}{p}x\right)} + c_{-2} e^{i\left(-\frac{2\pi}{p}x\right)}$

$n=3$ $\quad c_3 e^{i\left(\frac{3\pi}{p}x\right)} + c_{-3} e^{i\left(-\frac{3\pi}{p}x\right)}$

$\vdots$

$$\Rightarrow \sum_{n=0}^{\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)} + \boxed{\sum_{n=1}^{\infty} c_{-n} e^{i\left(-\frac{n\pi}{p}x\right)}} \quad n \to k$$

$$\Rightarrow \sum_{n=0}^{\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)} + \boxed{\sum_{k=-1}^{-\infty} c_k e^{i\left(\frac{k\pi}{p}x\right)}} \quad k$$

$k=-1 \quad c_{-1} e^{i\left(\frac{-1\pi}{p}x\right)}$

$k=-2 \quad c_{-2} e^{i\left(\frac{-2\pi}{p}x\right)}$

$$\Rightarrow \sum_{n=0}^{\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)} + \boxed{\sum_{n=-1}^{-\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)}} \quad n$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)}$$

Complex Fourier Series

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\left(\frac{n\pi}{p}x\right)}}$$

$$\boxed{c_n = \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p}x\right)}\, dx}$$

$n = 0, \pm 1, \pm 2, \ldots$

$n = -\ldots -1, 0, 1, 2 \ldots$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

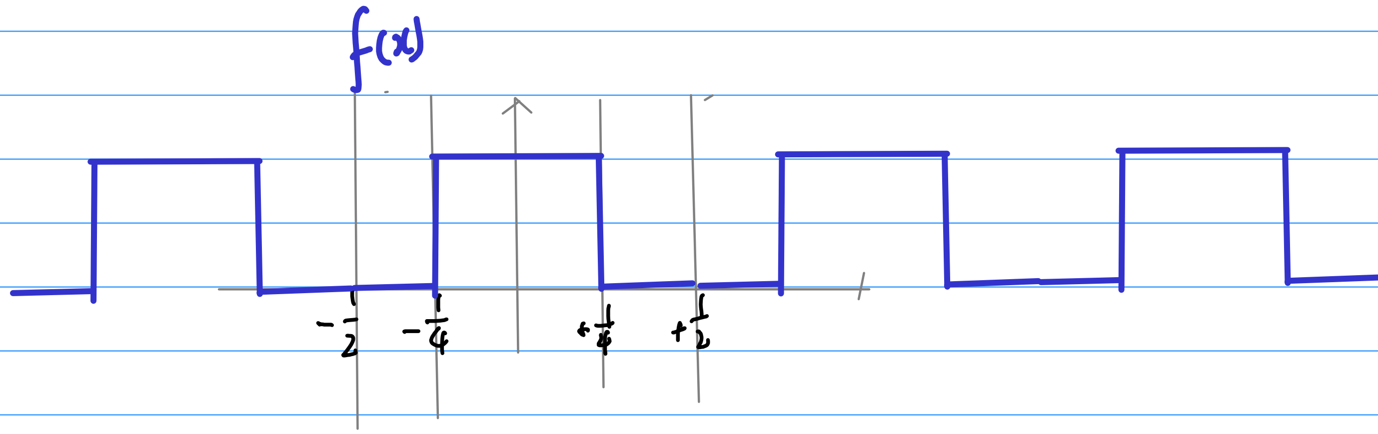
$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\, dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x) \cos\left(\frac{n\pi}{p}x\right) dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x) \sin\left(\frac{n\pi}{p}x\right) dx$$

$n = 1, 2, 3, \ldots$

12.4 Ex 3)



$$f(x) = \begin{cases} 0 & -\frac{1}{2} \leq x \leq -\frac{1}{4} \\ 1 & -\frac{1}{4} \leq x \leq \frac{1}{4} \\ 0 & \frac{1}{4} < x < \frac{1}{2} \end{cases}$$

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} C_n e^{i\left(\frac{n\pi}{p} x\right)}}$$

$$(e^{mx})' = e^{mx} \cdot m$$

$$\boxed{C_n = \frac{1}{2p} \int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p} x\right)}\, dx}$$

$$\int e^{mx}\, dx$$

$$\frac{1}{m} \int e^{mx} m\, dx$$

$$e^{mx}$$

$$C_n = \frac{1}{2 \cdot \frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} f(x)\, e^{-i\left(\frac{n\pi}{\frac{1}{2}} x\right)}\, dx$$

$$= \int_{-\frac{1}{4}}^{+\frac{1}{4}} 1 \cdot e^{-(i 2n\pi)x}\, dx = \left[\frac{1}{-(\quad)} e^{-(i2n\pi)x} \right]_{-\frac{1}{4}}^{+\frac{1}{4}}$$

$$(e^{2x})' = e^{2x} \cdot 2$$

$$(e^{x^2+1})' = e^{x^2+1} \cdot (x^2+1)' = e^{x^2+1} \cdot 2x$$

$$\frac{d}{dg}(e^{g}) = e$$

$$\frac{d}{dx} \quad g \quad .$$

$$\frac{d}{dx} e^{g} = \left(\frac{d}{dg} e^{g}\right) \cdot \left(\frac{dg}{dx}\right)$$

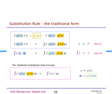
$$(e^{x^2+1})' = e^{x^2+1} \cdot (x^2+1)' = e^{x^2+1} \cdot 2x$$

$$\frac{d}{dx}(e^{g}) = \frac{g}{dg}(e^{g}) \quad \frac{d}{dx} g.$$

$$= e^{g} \cdot 2x = e^{x^2+1} \cdot 2x$$

Chain Rule

$$\frac{df}{dx} = \frac{df}{dg} \times \frac{dg}{dx}$$



Substitution Rule – the traditional form

$$f(g(x)) + C \Leftarrow \boxed{\int \cdot\, dx} \Leftarrow f'(g(x)) \cdot g'(x)$$

$$f(g(x)) + C = \int f'(g(x)) \cdot g'(x) dx \qquad f \Leftarrow f' \quad \textit{view (I)}$$

$$\int f(u)\, du = \int f(g(x)) \cdot g'(x)\, dx \qquad \int f \Leftarrow f \quad \textit{view (II)}$$

The Traditional Substitution Rule Formula

$$\boxed{\int f(g(x)) \cdot g'(x)\, dx = \int f(u)\, du}$$

$$u = g(x)$$

$$du = \boxed{g'(x) dx}$$

$$C_n = \frac{1}{2 \cdot \frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} f(x)\, e^{-i\left(\frac{n\pi}{\frac{1}{2}} x\right)} dx$$

$$= \int_{-\frac{1}{4}}^{+\frac{1}{4}} 1 \cdot e^{-(i 2n\pi)x} dx$$

$$e^{mx}$$

$$\int e^{\overset{u}{\overset{\parallel}{\boxed{mx}}}}\, \underline{m\, dx}$$

$$= \frac{1}{-(i 2n\pi)} \int_{-\frac{1}{4}}^{+\frac{1}{4}} e^{\underbrace{-(i2n\pi)x}_{u}} \underbrace{\boxed{-(i2n\pi)\, dx}}_{du} = \int e^u\, du = e^u$$

$$= \frac{1}{-(i 2n\pi)} \int_{-\frac{1}{4}}^{\frac{1}{4}} e^u\, du \qquad \left[\frac{1}{-(i2n\pi)} e^{-(i2n\pi)x}\right]_{-\frac{1}{4}}^{+\frac{1}{4}}$$

$$\left[\frac{1}{-(i2n\pi)} e^{-(i2n\pi)x}\right]_{-\frac{1}{4}}^{+\frac{1}{4}}$$

$$= \frac{1}{-i2n\pi}\left[e^{-i\frac{n}{2}\pi} - e^{+i\frac{n}{2}\pi}\right]$$

$$= -\frac{1}{n\pi}\left[\frac{e^{-in\pi/2} - e^{+in\pi/2}}{2i}\right]$$

$$= \frac{1}{n\pi}\left[\frac{e^{in\pi/2} - e^{-in\pi/2}}{2i}\right]$$

$$C_n = \frac{1}{n\pi} \sin(n\pi/2)$$

$$\cancel{C_0 = \frac{x}{0}}$$

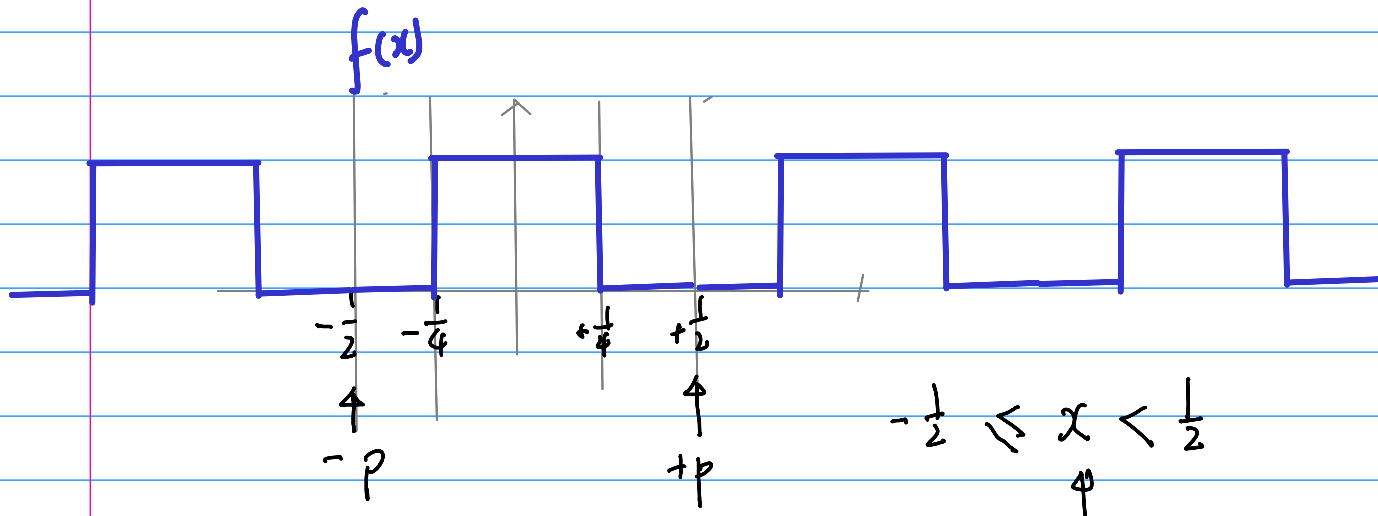
$$C_0 = \int_{-\frac{1}{4}}^{+\frac{1}{4}} 1 \cdot e^{-(i2 0\pi)x} dx = \int_{-\frac{1}{4}}^{\frac{1}{4}} 1\, dx = \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

$$C_0 = \frac{1}{2}$$

$$C_n = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

12.4 Ex 3)



$-\frac{1}{2} \leq x < \frac{1}{2}$

~~integer~~

-0.5 . 0 -1.4144

$3.14r$

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} C_n e^{i\left(\frac{n\pi}{p}x\right)}}$$

$$\boxed{C_n = \frac{1}{2p} \int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p}x\right)}\, dx}$$

$C_0 = \frac{1}{2}$

$C_n = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$

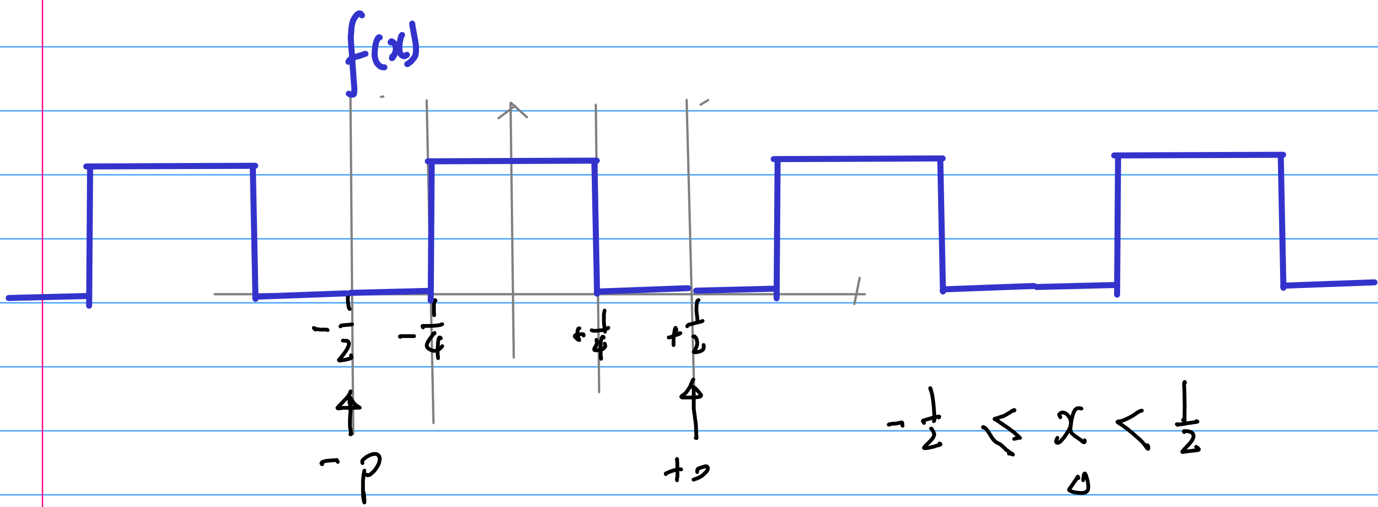
$$f(x) = C_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} C_n e^{i\left(\frac{n\pi}{p}x\right)}$$

$$= \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right) e^{i(2n\pi x)}$$

$$= \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right) \left(\overset{\neq 1}{\cos(2n\pi x)} + i \cdot \overset{\neq 0}{\sin(2n\pi x)}\right)$$

$n \Rightarrow$ integer

$x \neq$ integer real



$$-\frac{1}{2} \leq x < \frac{1}{2}$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$a_0 = \frac{1}{\frac{1}{2}}\int_{-\frac{1}{4}}^{\frac{1}{4}} 1\,dx = 2\cdot\frac{1}{2} = 1$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$= 2\int_{-\frac{1}{4}}^{\frac{1}{4}} 1\,\cos(2n\pi x)\,dx$$

$$= 2\left[\frac{1}{(2n\pi)}\sin(2n\pi x)\right]_{-\frac{1}{4}}^{\frac{1}{4}}$$

$$= \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right)\right]$$

$$= \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$= 2\int_{-\frac{1}{4}}^{\frac{1}{4}} 1\,\sin(2n\pi x)\,dx$$

$$= 2\left[\frac{-1}{(2n\pi)}\cos(2n\pi x)\right]_{-\frac{1}{4}}^{\frac{1}{4}}$$

$$= \frac{-1}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - \cos\left(-\frac{n\pi}{2}\right)\right]$$

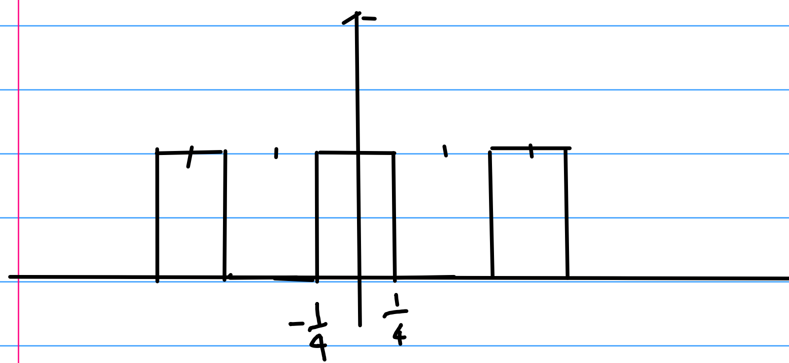
$$= 0$$

$$\boxed{\begin{aligned} C_0 &= \frac{a_0}{2} \\ C_n &= \frac{1}{2}(a_n - i b_n) \\ C_{-n} &= \frac{1}{2}(a_n + i b_n) \end{aligned}}$$

$$= \frac{1}{2}\cdot(\ )$$

$$= \frac{1}{2}\left(\frac{2}{n\pi}\cdot\sin\left(\frac{n\pi}{2}\right) - i\cdot 0\right)$$

$$= \frac{1}{2}\left(\frac{2}{n\pi}\cdot\sin\left(\frac{n\pi}{2}\right) + i\cdot 0\right)$$



$$\frac{n\pi}{P} = \frac{n\pi}{\frac{1}{2}} = 2n\pi$$

$$a_n = 2\int_{-\frac{1}{2}}^{+\frac{1}{2}} \cos(2\pi n x)\,dx = 2\int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(2\pi n x)\,dx = 2\left[\frac{1}{2\pi n}\sin(2\pi n x)\right]_{-\frac{1}{4}}^{\frac{1}{4}}$$

Fourier Series Coefficient

$$= \frac{2}{2\pi n}\left[\sin\left(\frac{\pi n}{2}\right) - \sin\left(-\frac{\pi n}{2}\right)\right]$$

$$= \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)\cos(2\pi n x)$$

$$a_n = 4\int_{0}^{+\frac{1}{2}} \cos(2\pi n x)\,dx = 4\int_{0}^{\frac{1}{4}} \cos(2\pi n x)\,dx = 4\left[\frac{1}{2\pi n}\sin(2\pi n x)\right]_{0}^{\frac{1}{4}}$$

Cosine Series Coefficient

$$= \frac{4}{2\pi n}\left[\sin\left(\frac{\pi n}{2}\right)\right]$$

$$= \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)\cos(2\pi n x)$$

$$c_n = \frac{1}{2}(a_n - i b_n) = \frac{1}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{\pi n}\sin\left(\frac{\pi n}{2}\right) e^{-i2\pi n x}$$

$$C_n = \frac{1}{n\pi} \sin \frac{n\pi}{2}$$

$$= \frac{1}{2} \frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}$$

$$C_n = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$\underline{\operatorname{Sinc}(x)} = \frac{\sin(x)}{x}$$

Laplace Transform

$$F(s) = \int_{0}^{\infty} f(t)e^{-st}\,dt$$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{+st}\,ds$$

Fourier Serie

$$\boxed{C_n = \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p}x\right)}\,dx}$$

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} C_n\, e^{i\left(\frac{n\pi}{p}x\right)}}$$