Laurent Series and z-Transform
- Geometric Series
Combinations B

20201002 Fr

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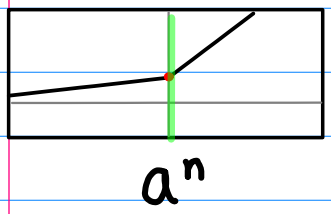
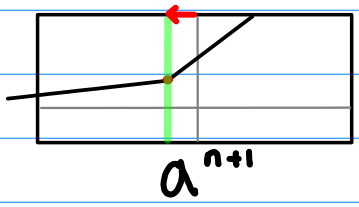
Making Shifted Sequences

Shifting Property (1) - Exponential Functions

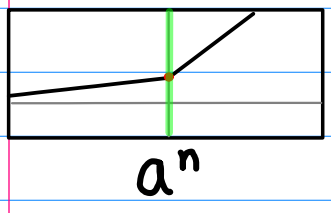


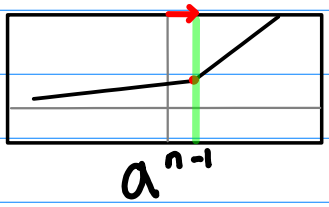


left shift

SHL.Exp

n → n+1

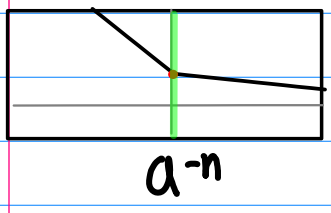


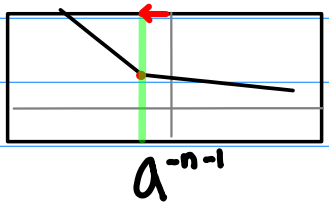


right shift

SHR.Exp

n → n-1

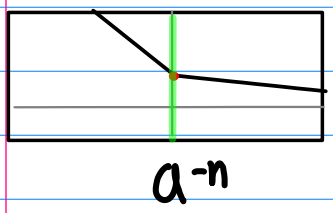


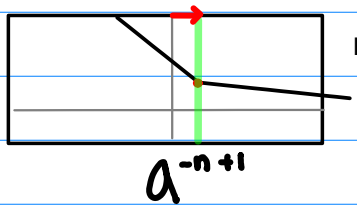


left shift

SHL.Exp

n → n+1





right shift

SHR.Exp

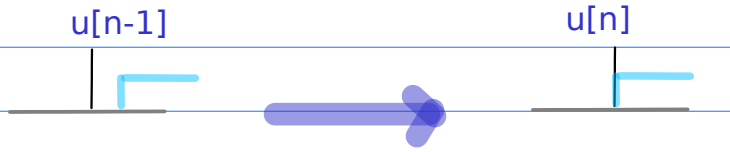
n → n-1

*a	**Left Shifted**	**SHL.Exp ... a^n**	a^n	$\longrightarrow$ a^{n+1}
	Right Shifted	**SHR.Exp ... a^-n**	a^{-n}	$\longrightarrow$ a^{-n+1}
/a	**Right Shifted**	**SHR.Exp ... a^n**	a^n	$\longrightarrow$ a^{n-1}
	Left Shifted	**SHL.Exp ... a^-n**	a^{-n}	$\longrightarrow$ a^{-n-1}

Shifting Property (2) - Ranges

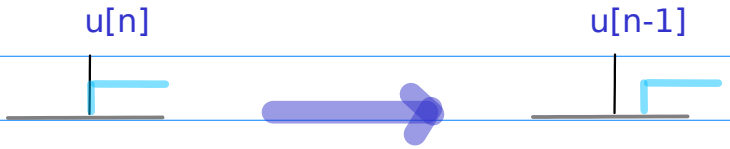
SHL.Rng

n → n+1



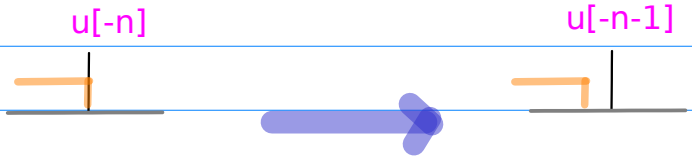
SHR.Rng

n → n-1



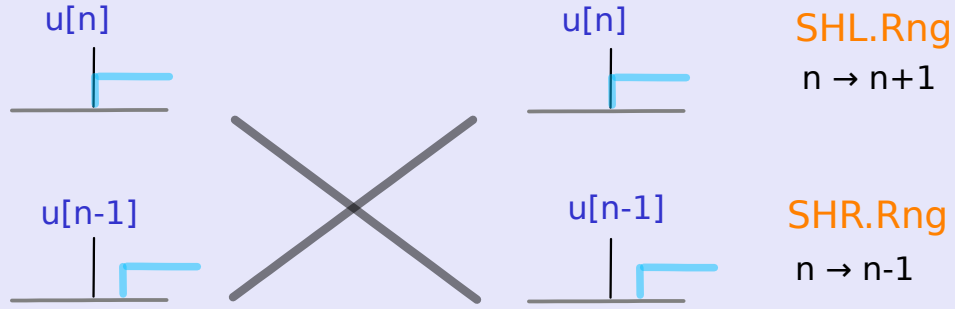
SHL.Rng

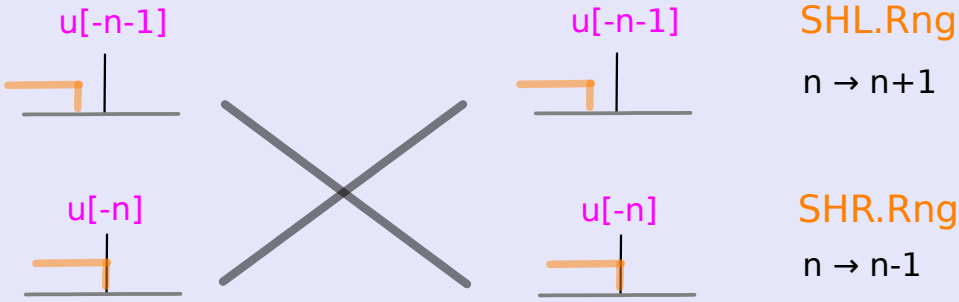
n → n+1



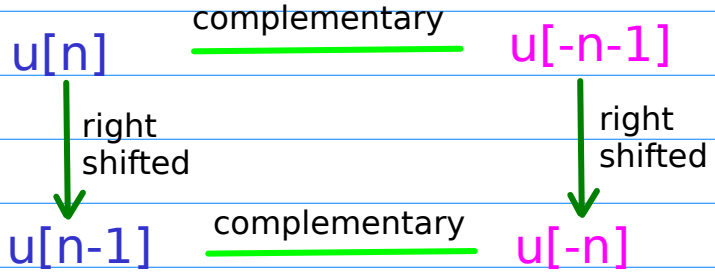
SHR.Rng

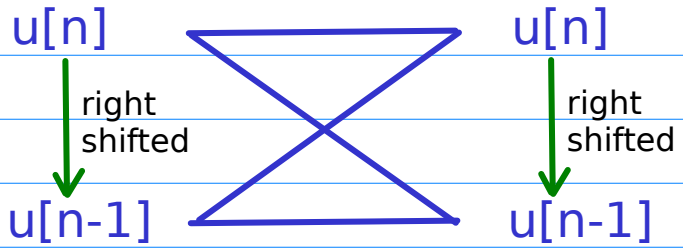
n → n-1

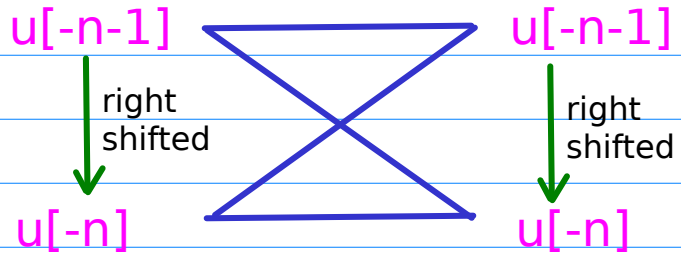




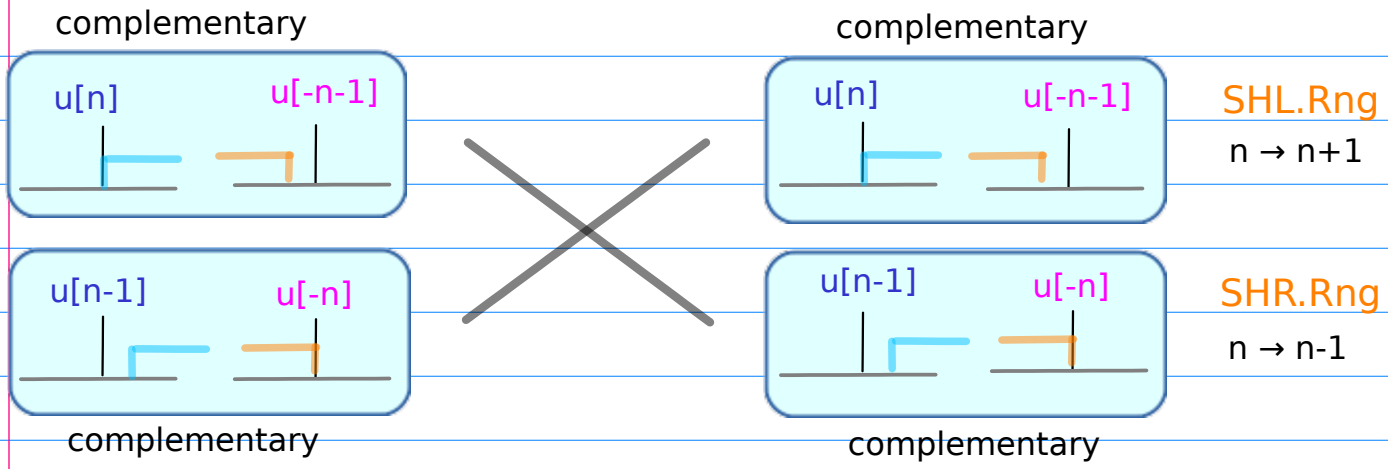
Shifting Property (3) - Layout Orders

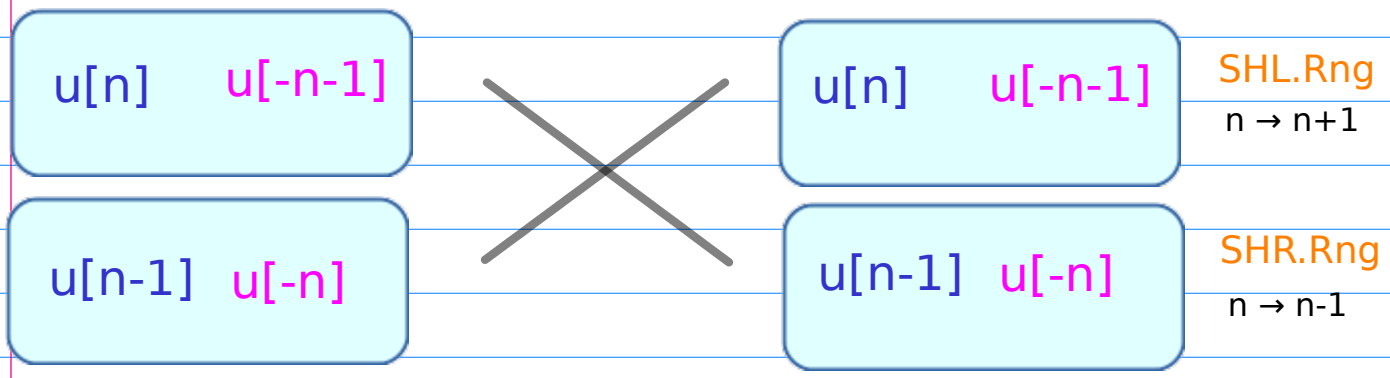


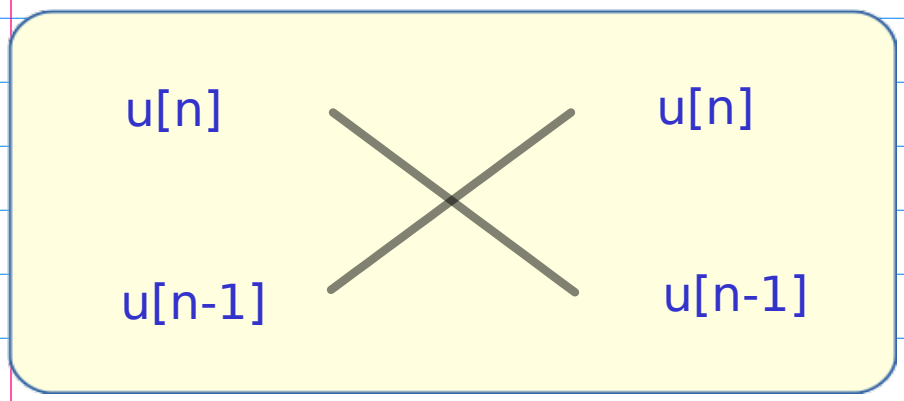


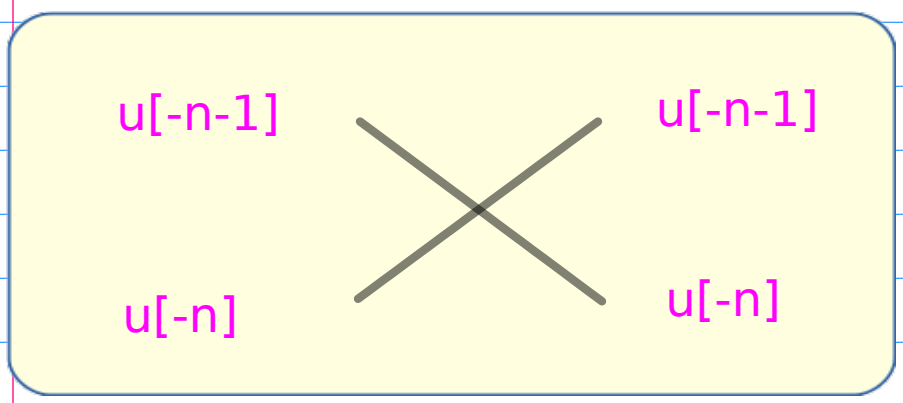


Shifting Property (4) - Complementary Ranges









Shifting Property (5) - Multiplication Factors

(*z) **Right Shifted Range** ***and***

$u[n] \rightarrow u[n-1]$

$u[-n-1] \rightarrow u[-n]$

SHR.Rng

Right Shifted Exponent

$a^{n} \rightarrow a^{n-1}$

$a^{-n} \rightarrow a^{-n+1}$

SHR.Exp

(/z) **Left Shifted Range** ***and***

$u[n-1] \rightarrow u[n]$

$u[-n] \rightarrow u[-n-1]$

SHL.Rng

Left Shifted Exponent

$a^{n} \rightarrow a^{n+1}$

$a^{-n} \rightarrow a^{-n-1}$

SHL.Exp

[*a] **Left Shifted Exponent** ***or***

Right Shifted Exponent

$a^{n} \rightarrow a^{n+1}$ **SHL.Exp**

$a^{-n} \rightarrow a^{-n+1}$ **SHR.Exp**

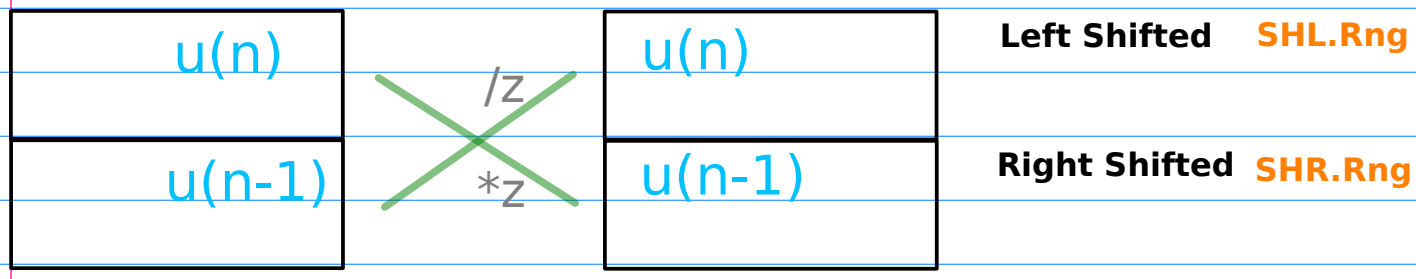
[/a] **Right Shifted Exponent** ***or***

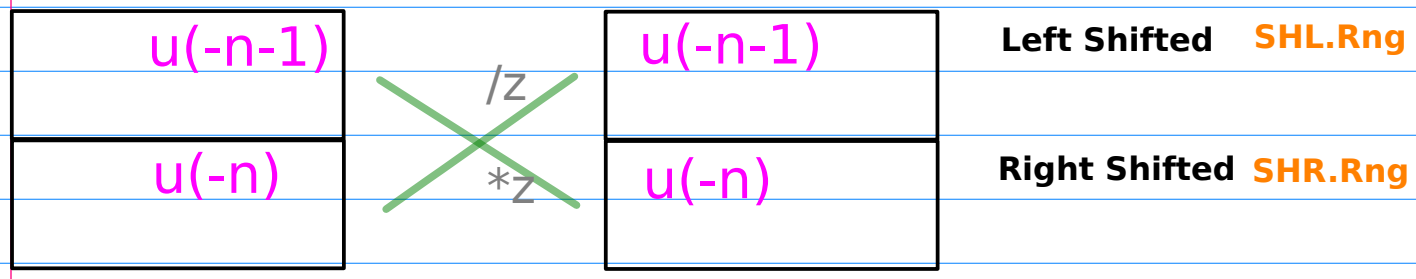
Left Shifted Exponent

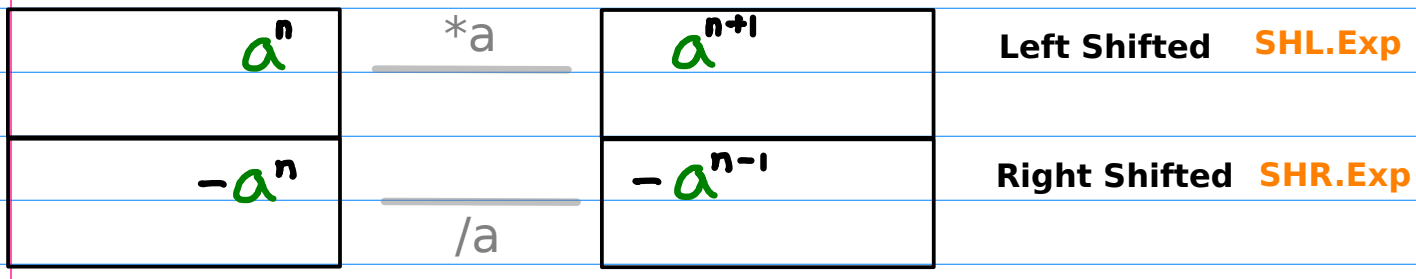
$a^{n} \rightarrow a^{n-1}$ **SHR.Exp**

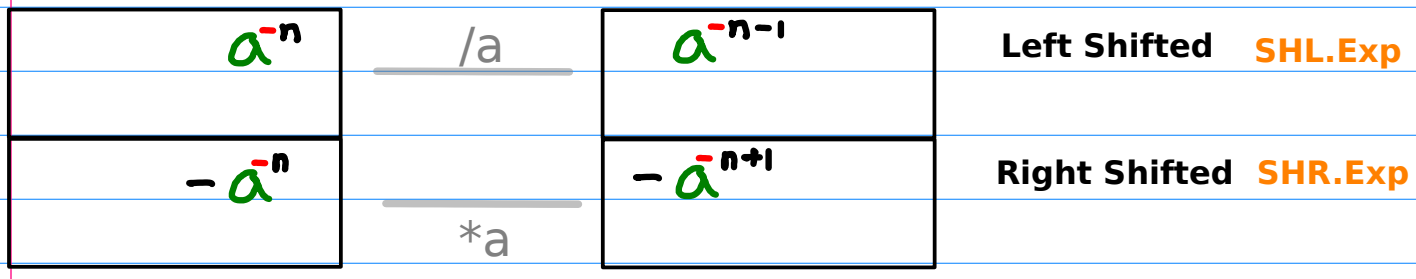
$a^{-n} \rightarrow a^{-n-1}$ **SHL.Exp**

Shifting Property (6) - Shift Combinations

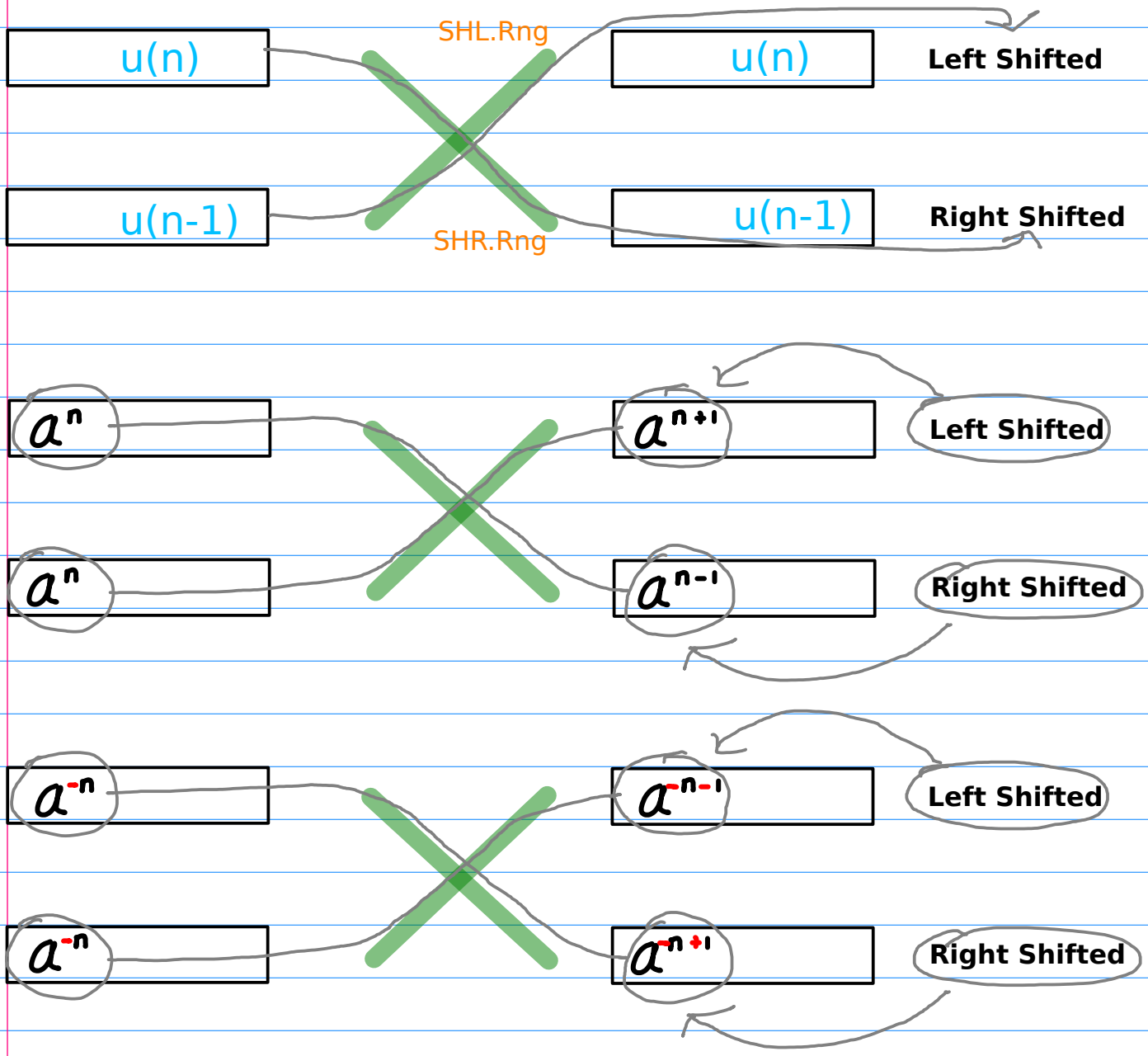




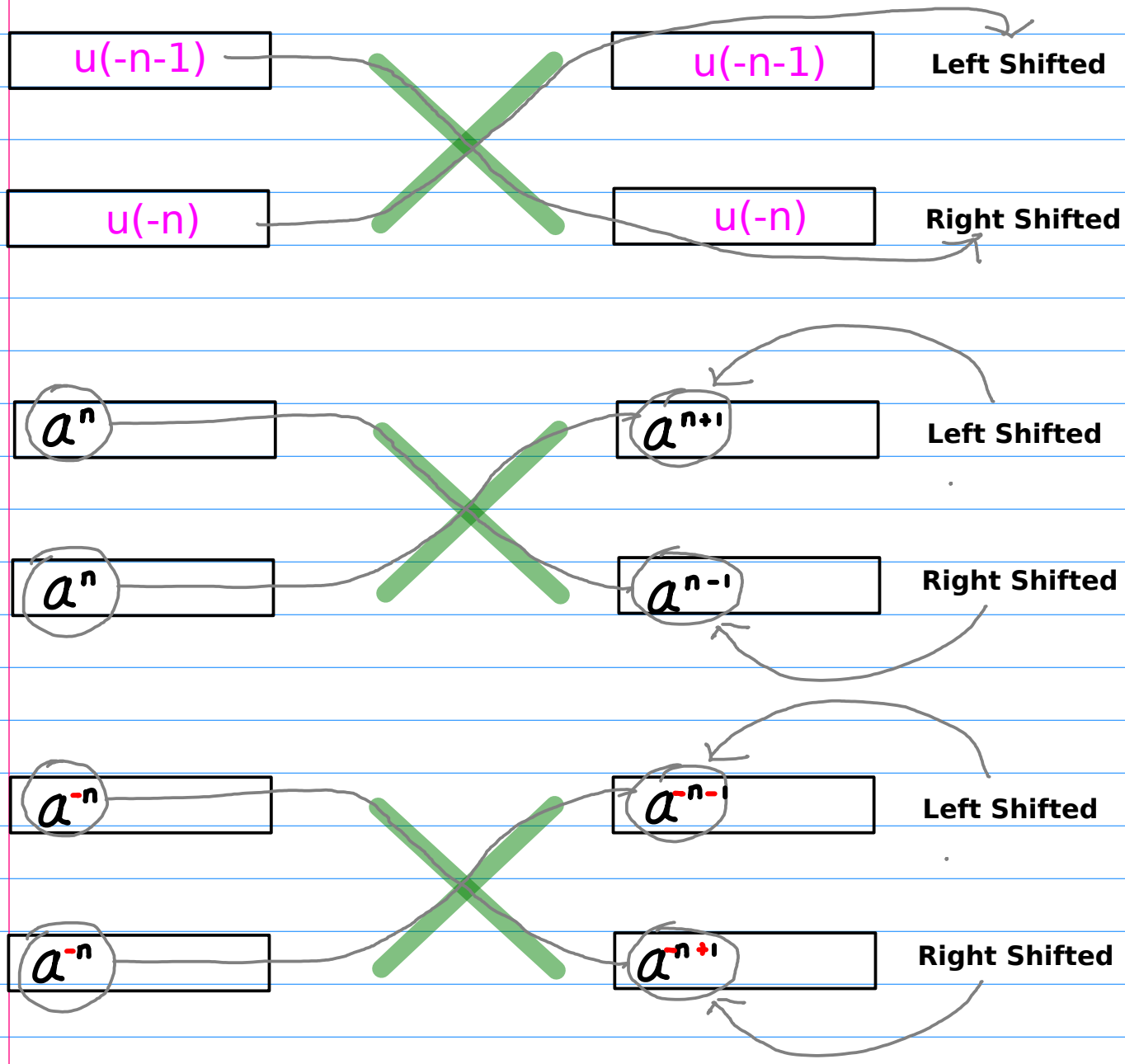


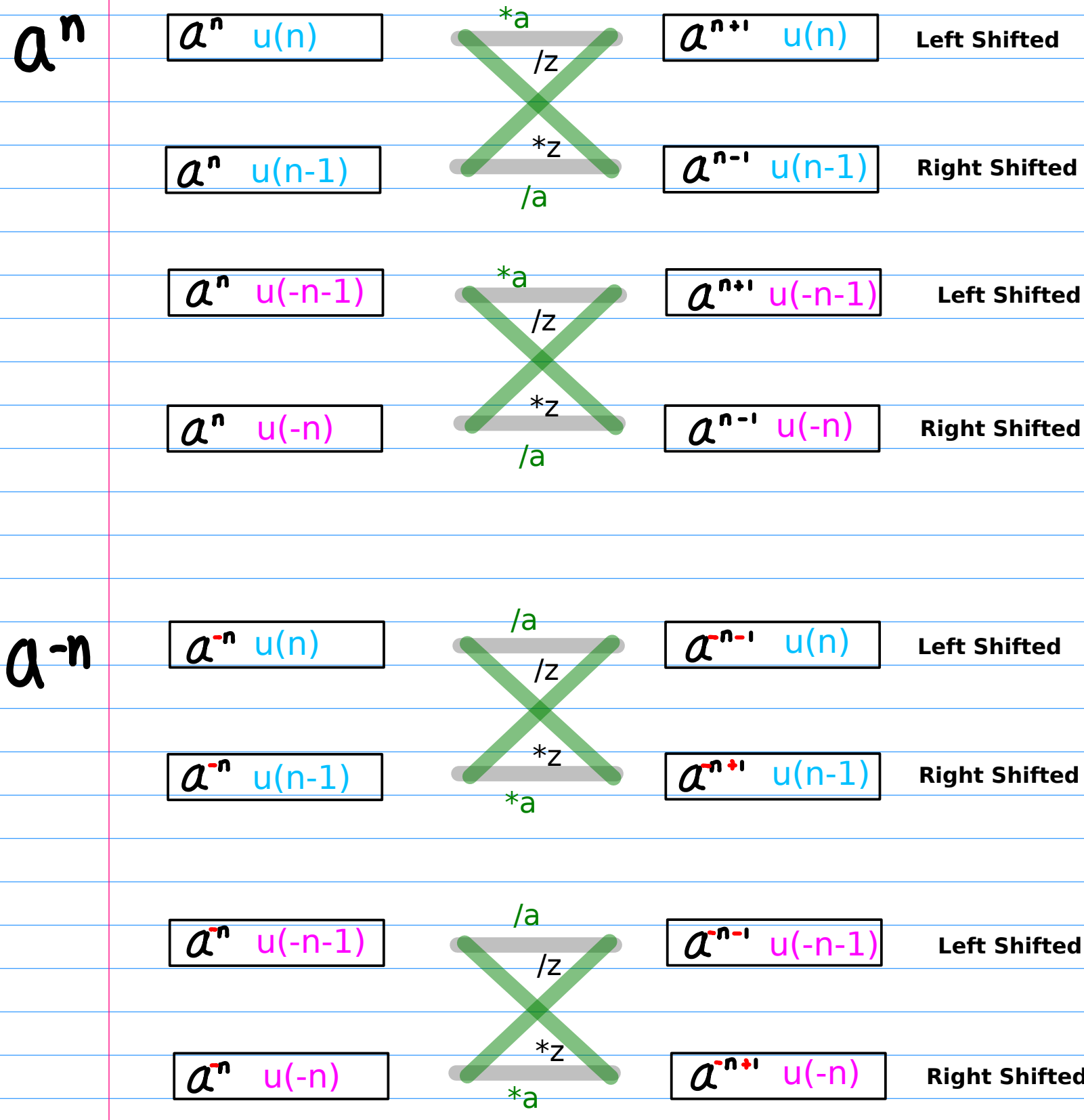


Determining {Left, Right} Shift of a pair of Causal Sequemces

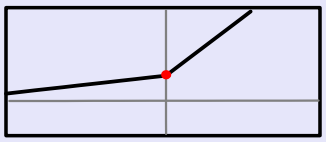


Determining {Left, Right} Shift of a pair of Anti-Causal Sequemces




a^n
a^n u(n)
*a
/z
a^{n+1} u(n)
Left Shifted
a^n u(n-1)
*z
/a
a^{n-1} u(n-1)
Right Shifted
a^n u(-n-1)
*a
/z
a^{n+1} u(-n-1)
Left Shifted
a^n u(-n)
*z
/a
a^{n-1} u(-n)
Right Shifted
a^{-n}
a^{-n} u(n)
/a
/z
a^{-n-1} u(n)
Left Shifted
a^{-n} u(n-1)
*z
*a
a^{-n+1} u(n-1)
Right Shifted
a^{-n} u(-n-1)
/a
/z
a^{-n-1} u(-n-1)
Left Shifted
a^{-n} u(-n)
*z
*a
a^{-n+1} u(-n)
Right Shifted

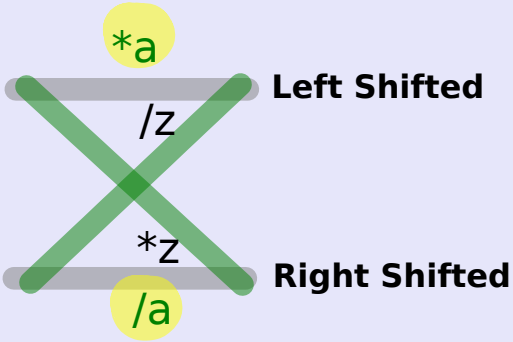
Shifting Combinations (1) Multiplication Factors

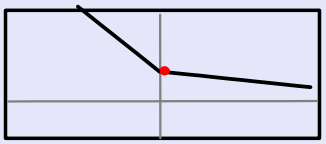


a^n

Causal Sequences

Anti-Causal Sequences

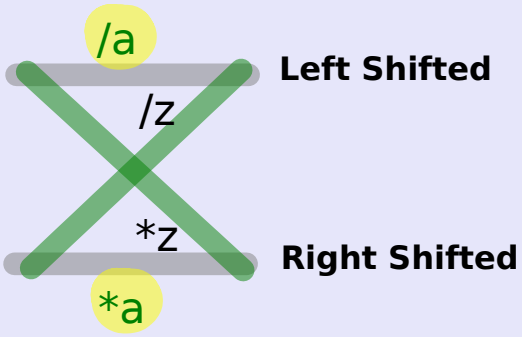


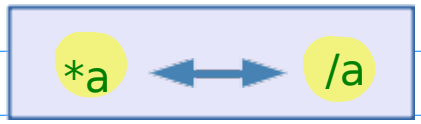


a^{-n}

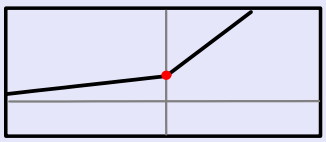
Causal Sequences

Anti-Causal Sequences

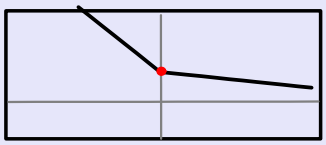




Shifting Combinations (2) - Sequences



a^n



a^{-n}

Causal Sequences

a^n u[n]

a^n u[n-1]

*a

/z

*z

/a

a^{n+1} u[n]

a^{n-1} u[n-1]

Causal Sequences

a^{-n} u[n]

a^{-n} u[n-1]

/a

/z

*z

*a

a^{-n-1} u[n]

a^{-n+1} u[n-1]

Anti-Causal Sequences

$-a^n$ u[-n-1)

$-a^n$ u[-n)

*a

*z

/z

/a

$-a^{n+1}$ u[-n-1]

$-a^{n-1}$ u[-n]

Anti-Causal Sequences

$-a^{-n}$ u[-n-1]

$-a^{-n}$ u[-n]

/a

*z

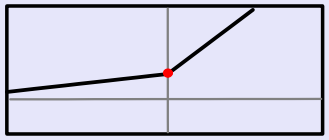
/z

*a

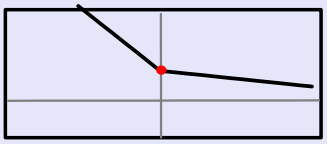
$-a^{-n-1}$ u(-n-1]

$-a^{-n+1}$ u(-n]

Shifting Combinations (3) - Geometric Series



a^n



a^{-n}

Causal Sequences

$$\frac{1}{1-az} \qquad \frac{az}{1-az}$$

*a, /z, *z, /a

$$\frac{a}{1-az} \qquad \frac{z}{1-az}$$

Causal Sequences

$$\frac{1}{1-a^{-1}z} \qquad \frac{a^{-1}z}{1-a^{-1}z}$$

/a, /z, *z, *a

$$\frac{a^{-1}}{1-a^{-1}z} \qquad \frac{z}{1-a^{-1}z}$$

Anti-Causal Sequences

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad -\frac{1}{1-a^{-1}z^{-1}}$$

*a, *z, /z, /a

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}}$$

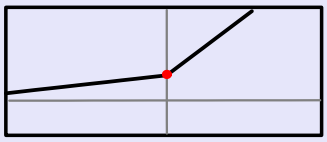
Anti-Causal Sequences

$$-\frac{az^{-1}}{1-az^{-1}} \qquad -\frac{1}{1-az^{-1}}$$

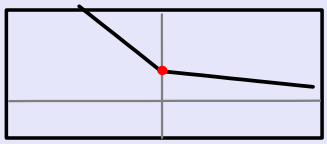
/a, *z, /z, *a

$$-\frac{z^{-1}}{1-az^{-1}} \qquad -\frac{a}{1-az^{-1}}$$

Shifting Combinations (4) - Numerator

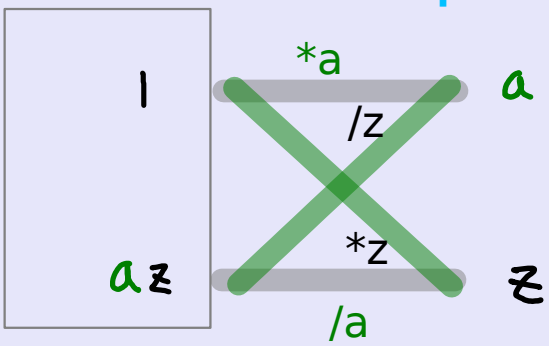


a^n

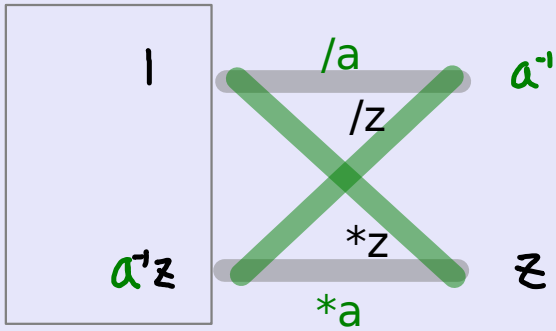


a^{-n}

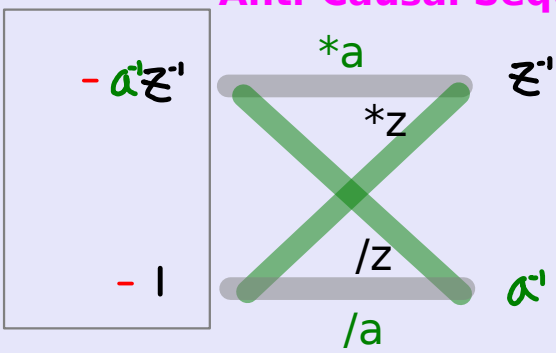
Causal Sequences



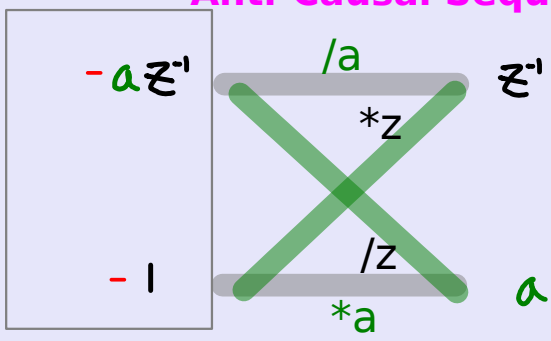
Causal Sequences

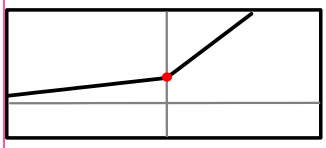


Anti-Causal Sequences



Anti-Causal Sequences

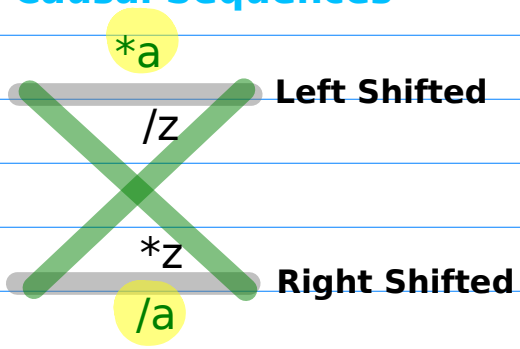




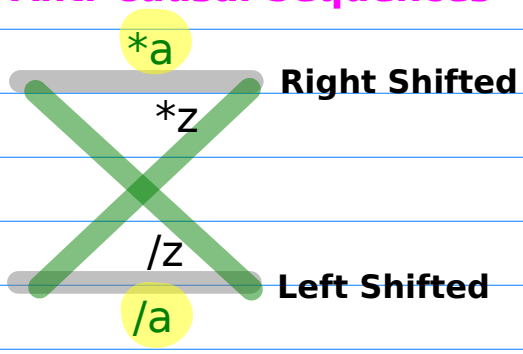
a^n

Geometric Series and Sequences

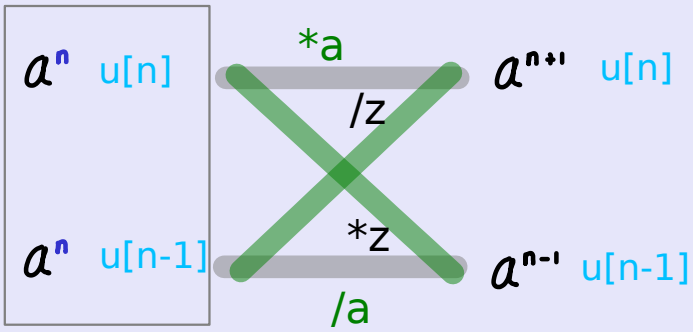
Causal Sequences



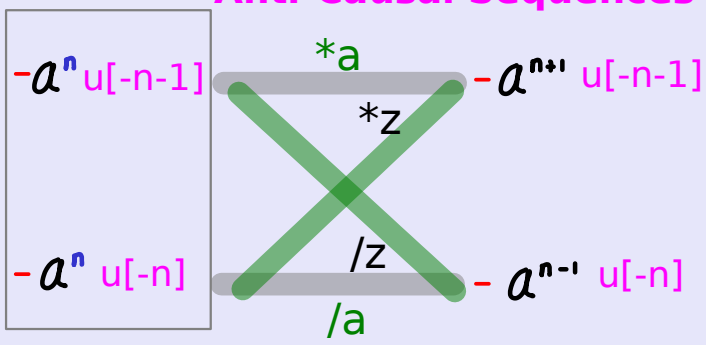
Anti-Causal Sequences



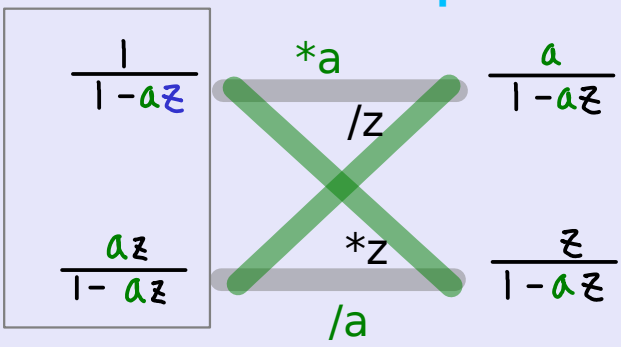
Causal Sequences



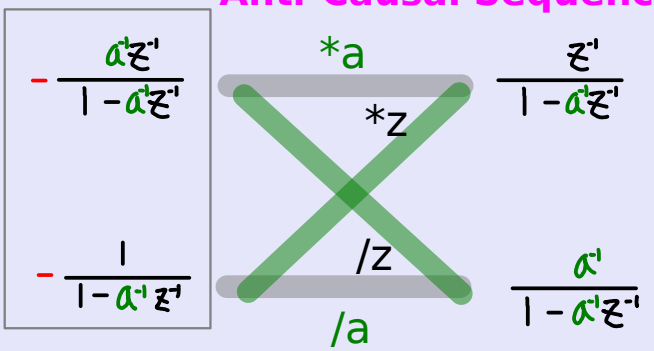
Anti-Causal Sequences

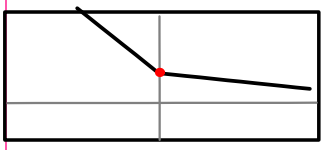


Causal Sequences



Anti-Causal Sequences

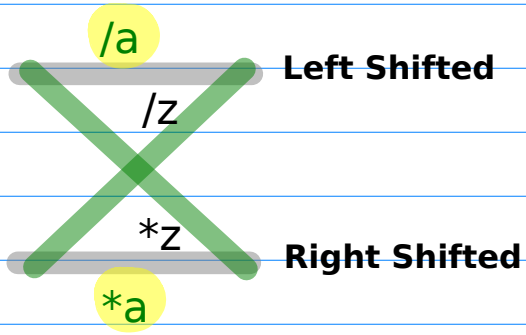




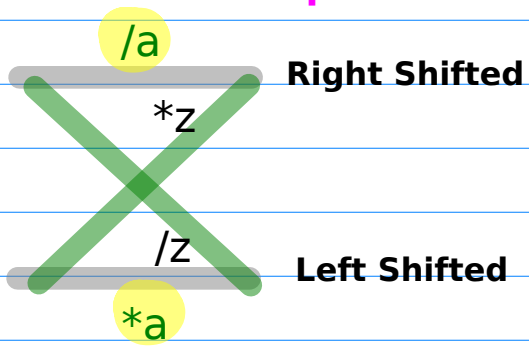
a^{-n}

Geometric Series and Sequences

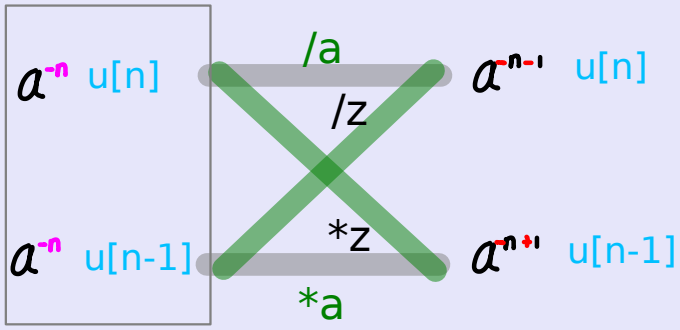
Causal Sequences



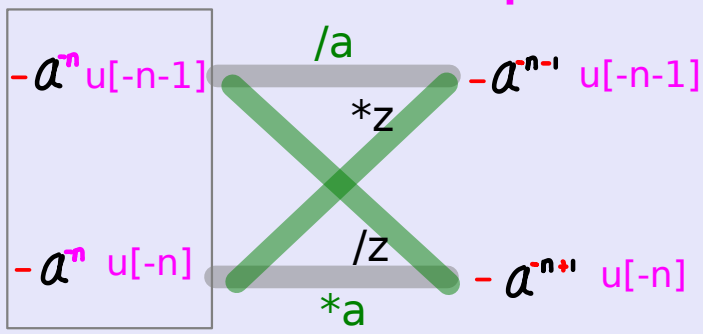
Anti-Causal Sequences



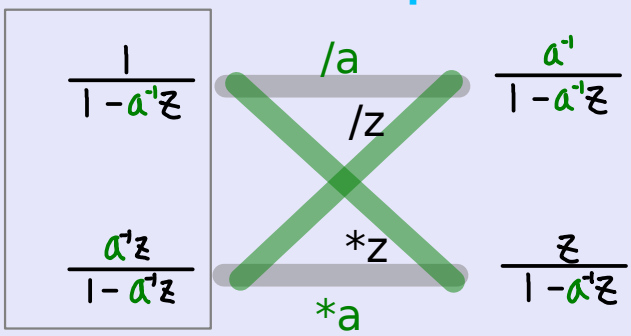
Causal Sequences



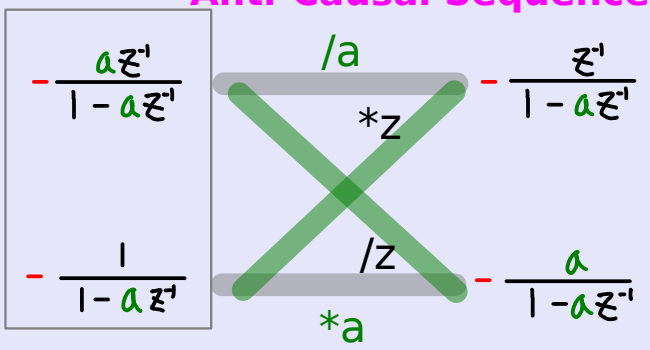
Anti-Causal Sequences

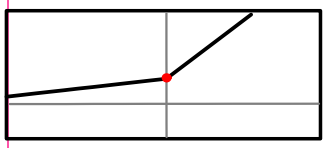


Causal Sequences



Anti-Causal Sequences



a^n 

*a

/a

Shifting a sequence

$\frac{1}{1-az}$ $|z|<a^{-1}$ — $a^n\ u(n)$

*a

/z

$\frac{a}{1-az}$ $|z|<a^{-1}$ — $a^{n+1}\ u(n)$

*z

/a

$\frac{az}{1-az}$ $|z|<a^{-1}$ — $a^n\ u(n-1)$

$\frac{z}{1-az}$ $|z|<a^{-1}$ — $a^{n-1}\ u(n-1)$

$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^n\ u(-n-1)$

*a

/z

$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^{n+1}\ u(-n-1)$

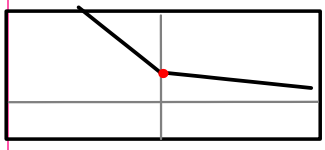
*z

/a

$-\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^n\ u(-n)$

$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^{n-1}\ u(-n)$

Shifting a sequence

a^{-n} 

/a

*a

$\frac{1}{1-a^{-1}z}$ $|z|<a$ — $a^{-n}\,u(n)$

/a

/z

$\frac{a^{-1}}{1-a^{-1}z}$ $|z|<a$ — $a^{-n-1}\,u(n)$

*z

$\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$ — $a^{-n}\,u(n-1)$

*a

$\frac{z}{1-a^{-1}z}$ $|z|<a$ — $a^{-n+1}\,u(n-1)$

$-\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$ — $a^{-n}\,u(-n-1)$

/a

/z

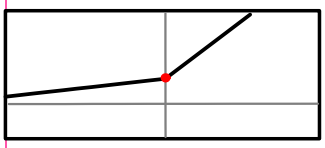
$-\frac{z^{-1}}{1-az^{-1}}$ $|z|>a$ — $a^{-n-1}\,u(-n-1)$

*z

$-\frac{1}{1-az^{-1}}$ $|z|>a$ — $a^{-n}\,u(-n)$

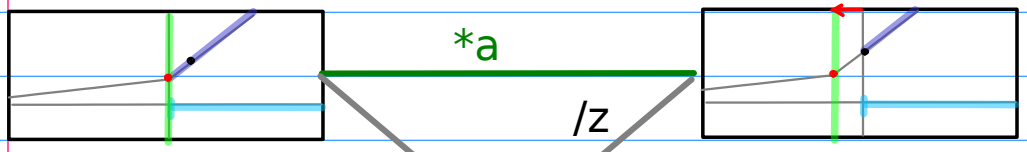
*a

$-\frac{a}{1-az^{-1}}$ $|z|>a$ — $a^{-n+1}\,u(-n)$

a^n 

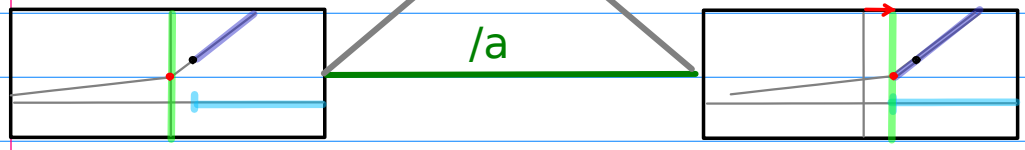
Shifting a sequence

/z $n \leftarrow n+1$ *z $n \leftarrow n-1$



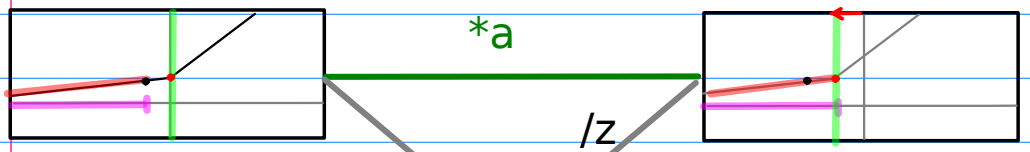
a^n u(n) a^{n+1} u(n)

Shift Left



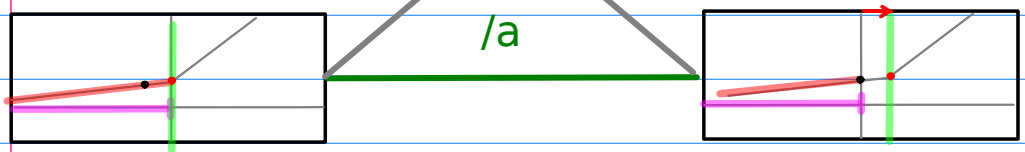
a^n u(n-1) a^{n-1} u(n-1)

Shift Right



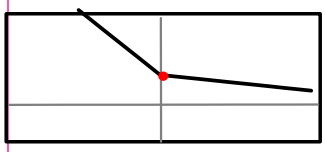
a^n u(-n-1) a^{n+1} u(-n-1)

Shift Left



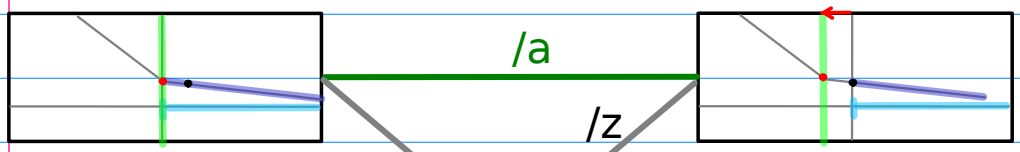
a^n u(-n) a^{n-1} u(-n)

Shift Right

a^{-n} 

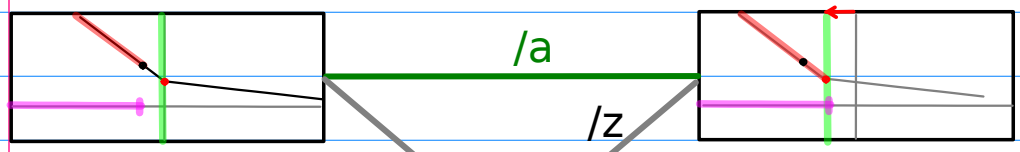
Shifting a sequence

/z $n \leftarrow n+1$ *z $n \leftarrow n-1$



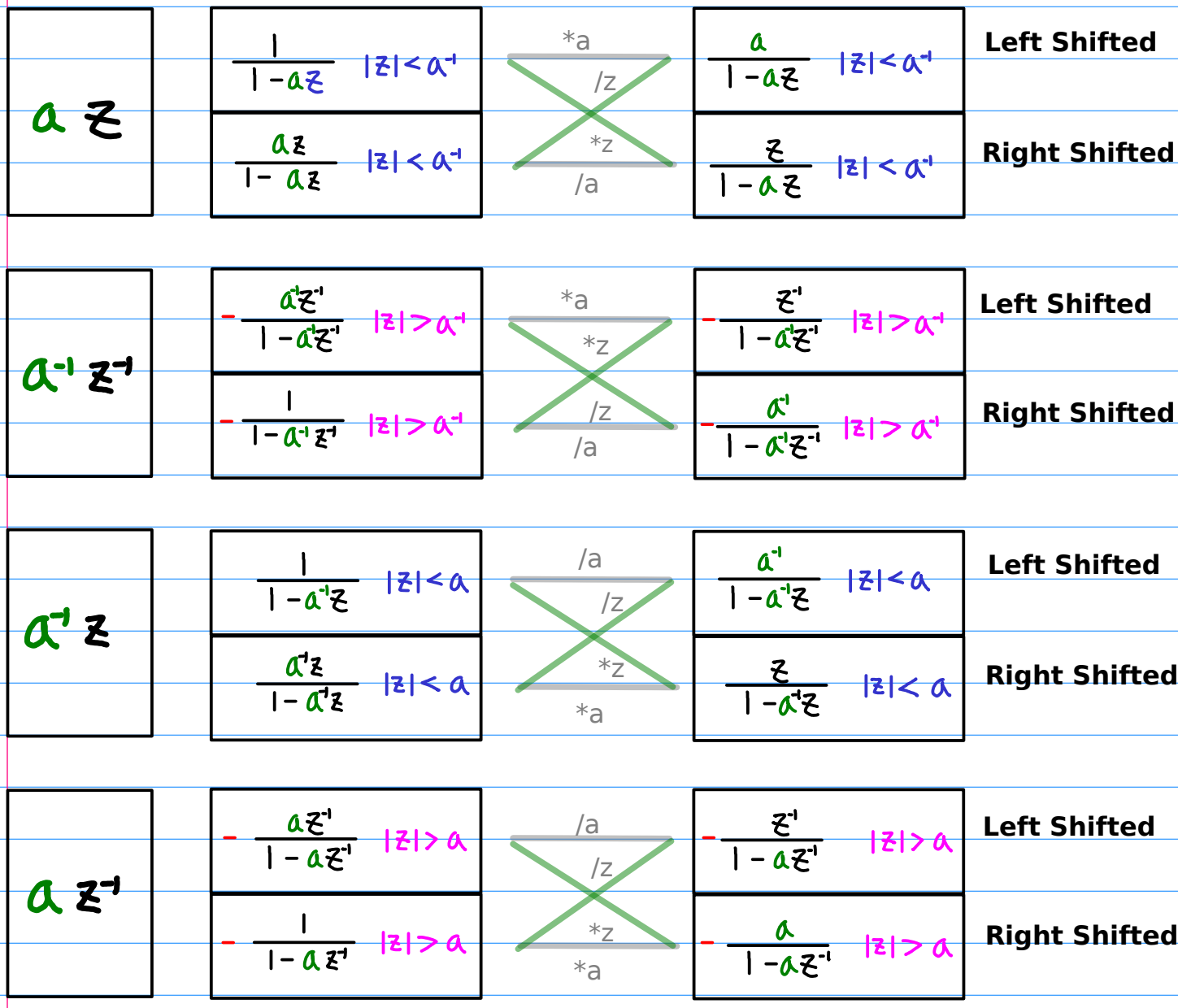
a^{-n} u(n) a^{-n-1} u(n) **Shift Left**

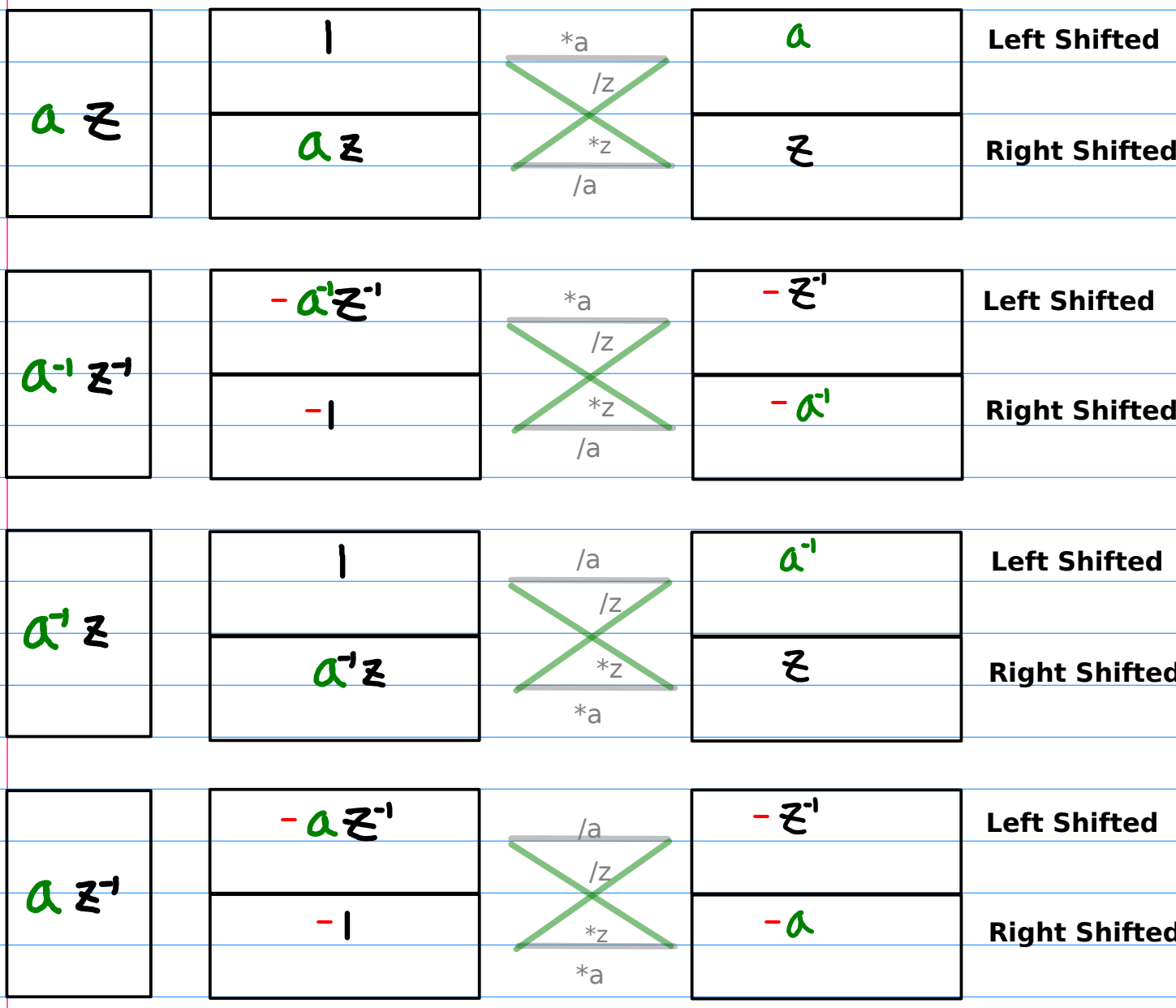
a^{-n} u(n-1) a^{-n+1} u(n-1) **Shift Right**

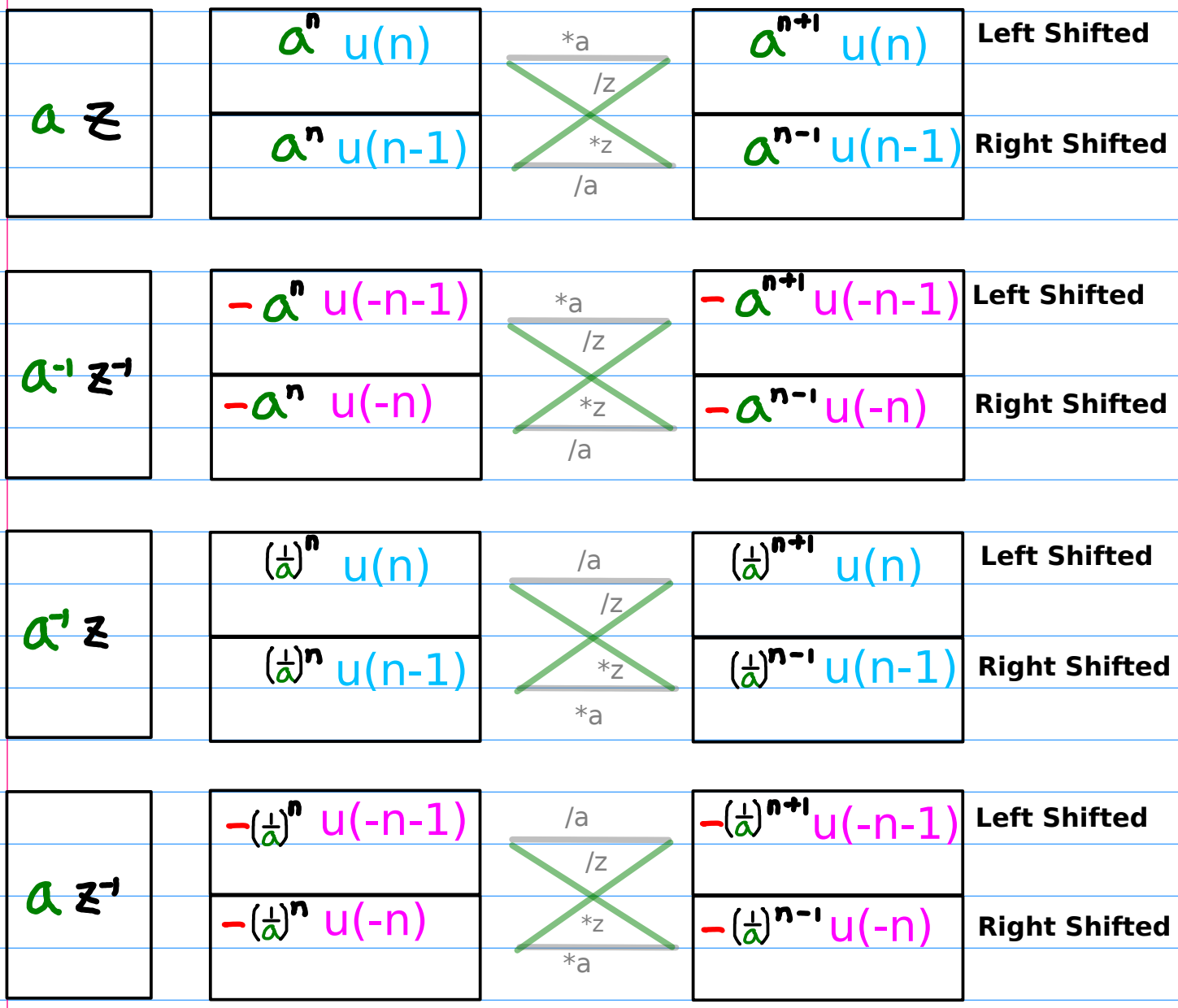


a^{-n} u(-n-1) a^{-n-1} u(-n-1) **Shift Left**

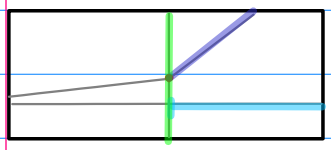
a^{-n} u(-n) a^{-n+1} u(-n) **Shift Right**


a^n
$\frac{1}{1-az}$ $|z|<a^{-1}$
$\frac{az}{1-az}$ $|z|<a^{-1}$
*a
/z
*z
/a
$\frac{a}{1-az}$ $|z|<a^{-1}$
$\frac{z}{1-az}$ $|z|<a^{-1}$
Left Shifted
Right Shifted
a^{-n}
$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$
$-\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$
*a
*z
/z
/a
$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$
$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$
Left Shifted
Right Shifted
a^{-n}
$\frac{1}{1-a^{-1}z}$ $|z|<a$
$\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$
/a
/z
*z
*a
$\frac{a^{-1}}{1-a^{-1}z}$ $|z|<a$
$\frac{z}{1-a^{-1}z}$ $|z|<a$
Left Shifted
Right Shifted
a^{n}
$-\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$
$-\frac{1}{1-az^{-1}}$ $|z|>a$
/a
/z
*z
*a
$-\frac{z^{-1}}{1-az^{-1}}$ $|z|>a$
$-\frac{a}{1-az^{-1}}$ $|z|>a$
Left Shifted
Right Shifted

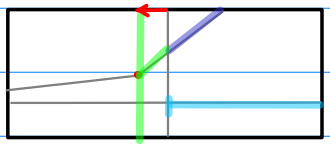

a z
1
a z
*a
/z
*z
/a
a
z
Left Shifted
Right Shifted
a⁻¹ z⁻¹
-a⁻¹z⁻¹
-1
*a
/z
*z
/a
-z⁻¹
-a⁻¹
Left Shifted
Right Shifted
a⁻¹ z
1
a⁻¹z
/a
/z
*z
*a
a⁻¹
z
Left Shifted
Right Shifted
a z⁻¹
-a z⁻¹
-1
/a
/z
*z
*a
-z⁻¹
-a
Left Shifted
Right Shifted

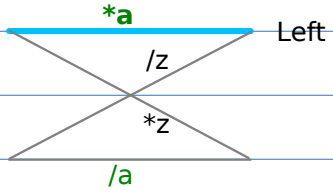

a z
$a^n\ u(n)$
$a^n\ u(n-1)$
*a
/z
*z
/a
$a^{n+1}\ u(n)$
$a^{n-1}\ u(n-1)$
Left Shifted
Right Shifted
$a^{-1}\ z^{-1}$
$-a^n\ u(-n-1)$
$-a^n\ u(-n)$
*a
/z
*z
/a
$-a^{n+1}\ u(-n-1)$
$-a^{n-1}\ u(-n)$
Left Shifted
Right Shifted
$a^{-1}\ z$
$(\frac{1}{a})^n\ u(n)$
$(\frac{1}{a})^n\ u(n-1)$
/a
/z
*z
*a
$(\frac{1}{a})^{n+1}\ u(n)$
$(\frac{1}{a})^{n-1}\ u(n-1)$
Left Shifted
Right Shifted
$a\ z^{-1}$
$-(\frac{1}{a})^n\ u(-n-1)$
$-(\frac{1}{a})^n\ u(-n)$
/a
/z
*z
*a
$-(\frac{1}{a})^{n+1}\ u(-n-1)$
$-(\frac{1}{a})^{n-1}\ u(-n)$
Left Shifted
Right Shifted

Left Shifting Sequences a^n



SHL.Exp

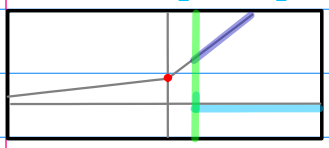




a^n u[n] a^{n+1} u[n]]

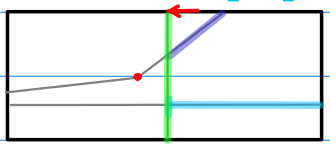
=

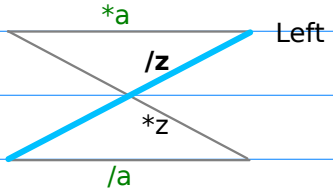
a^n u[n-1] a^{n+1} u[n]

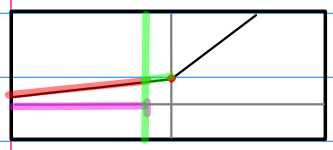


SHL.Exp

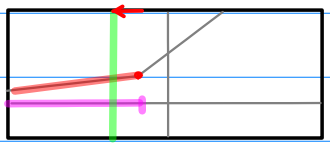
SHL.Rng

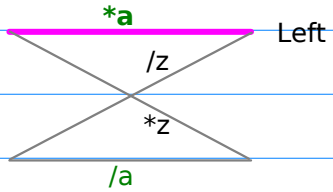






SHL.Exp

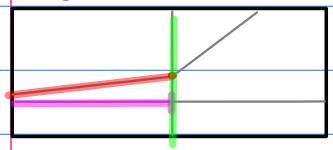




a^n u[-n-1] a^{n+1} u[-n-1]

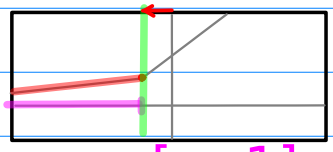
=

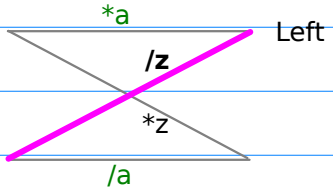
a^n u[-n] a^{n+1}



SHL.Exp

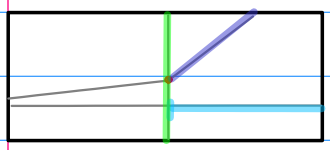
SHL.Rng



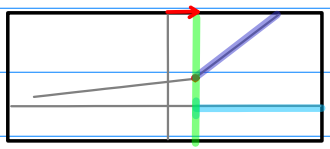


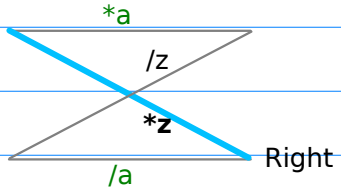
u[-n-1]

Right Shifting Sequences a^n



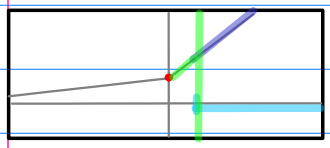
SHR.Exp





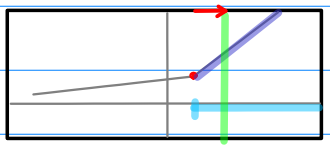
a^n u[n] a^{n-1} u[n-1]

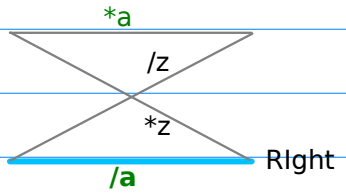
a^n u[n-1] a^{n-1} u[n-1]

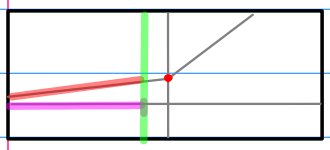


SHR.Exp

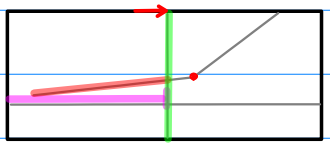
SHR.Rng

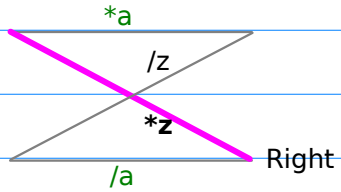






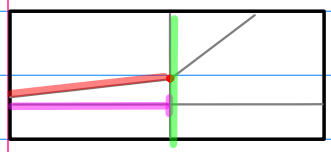
SHR.Exp





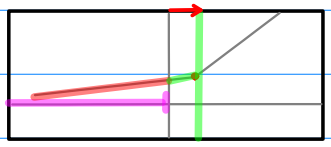
a^n u[-n-1] a^{n-1} u[-n]

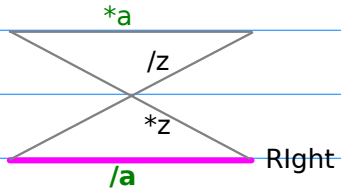
a^n u[-n] a^{n-1} u[-n]



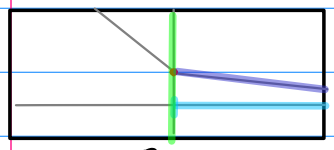
SHR.Exp

SHR.Rng



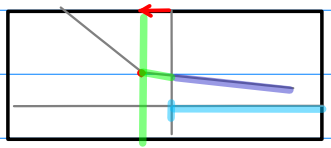


Left Shifting Sequences a^{-n}

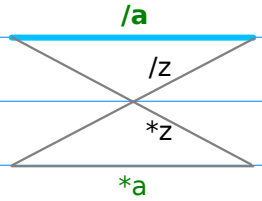


$a^{-n}\,u[n]$

SHL.Exp

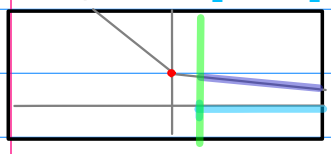


$a^{-n-1}\,u[n]$



$=$

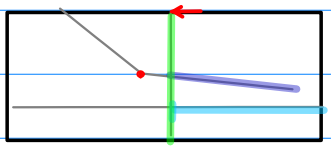
$a^{-n}\,u[n-1]$

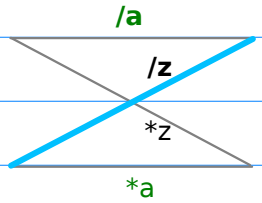


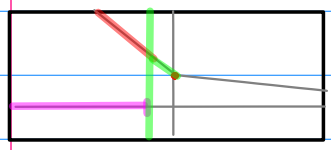
SHL.Exp

SHL.Rng

$a^{-n-1}\,u[n]$

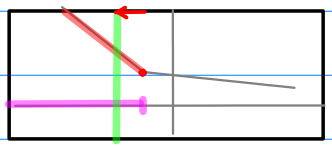




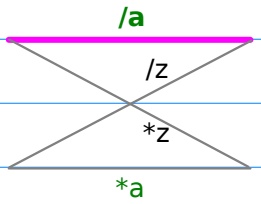


$a^{-n}\,u[-n-1]$

SHL.Exp

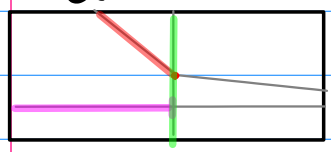


$a^{-n-1}\,u[-n-1]$



$=$

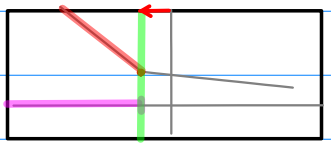
$a^{-n}\,u[-n]$

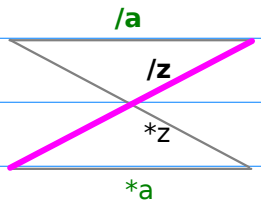


SHL.Exp

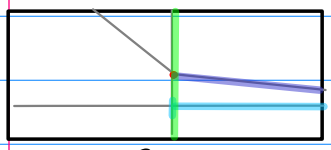
SHL.Rng

$a^{-n-1}\,u[-n-1]$

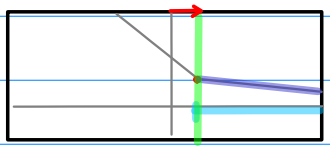


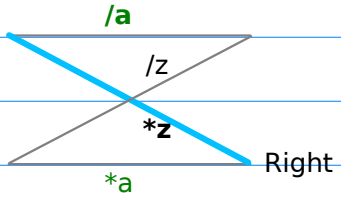


Right Shifting Sequences a^{-n}



SHR.Exp

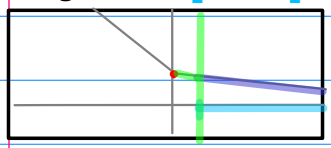




a^{-n} u[n] a^{-n+1} u[n-1]

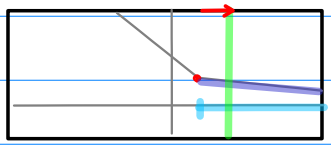
=

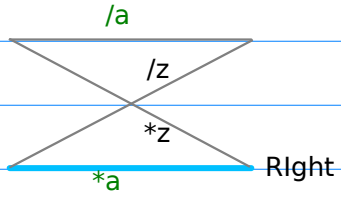
a^{-n} u[n-1] a^{-n+1} u[n-1]

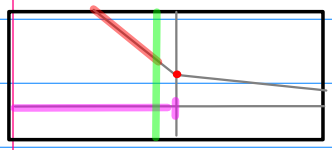


SHR.Exp

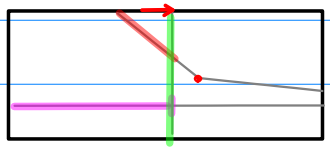
SHR.Rng

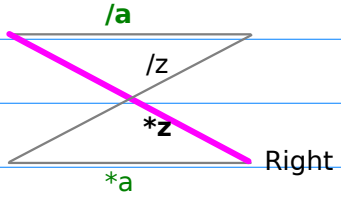






SHR.Exp

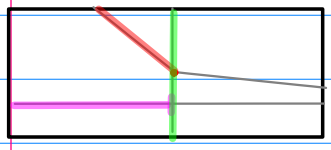




a^{-n} u[-n-1] a^{-n+1} u[-n]

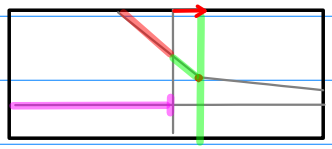
=

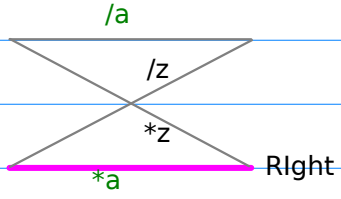
a^{-n} u[-n] a^{-n+1} u[-n]



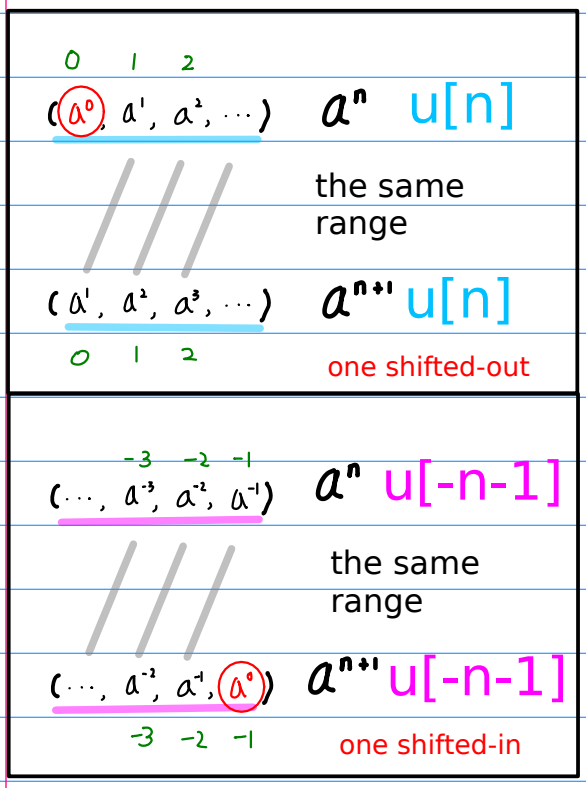
SHR.Exp

SHR.Rng

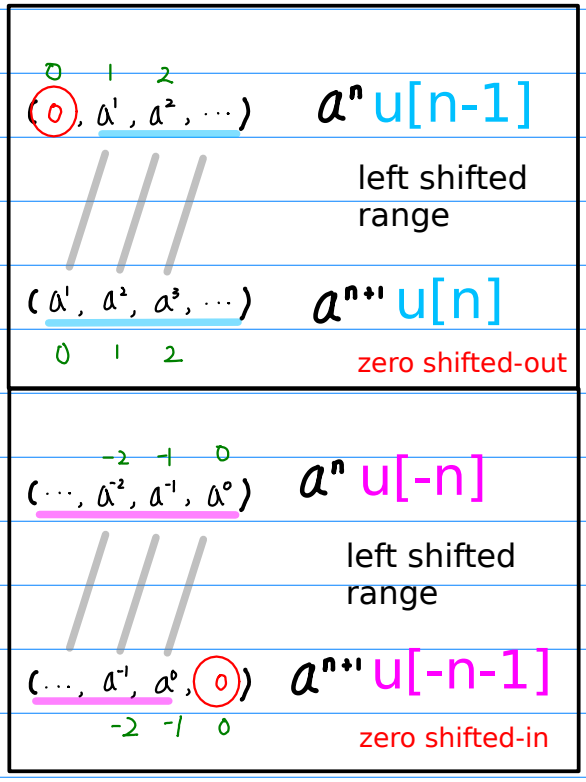




Left Shifting Sequences

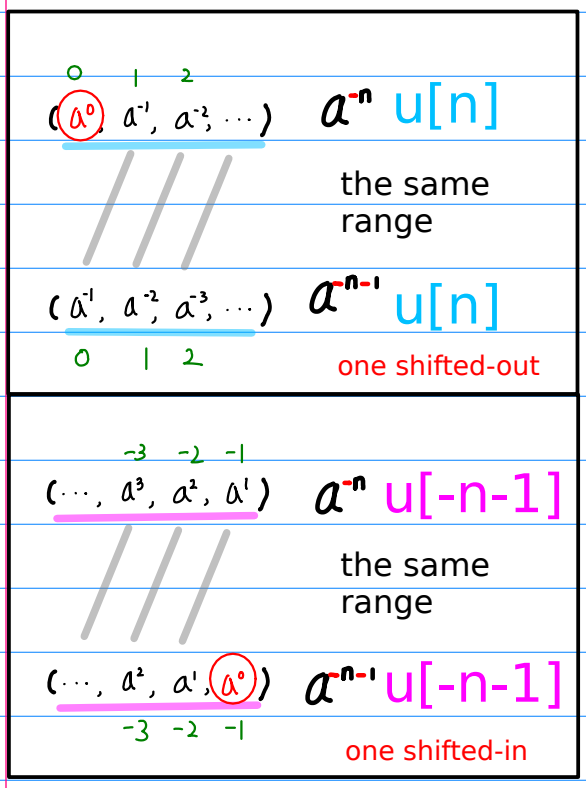


SHL.Exp ... a^n

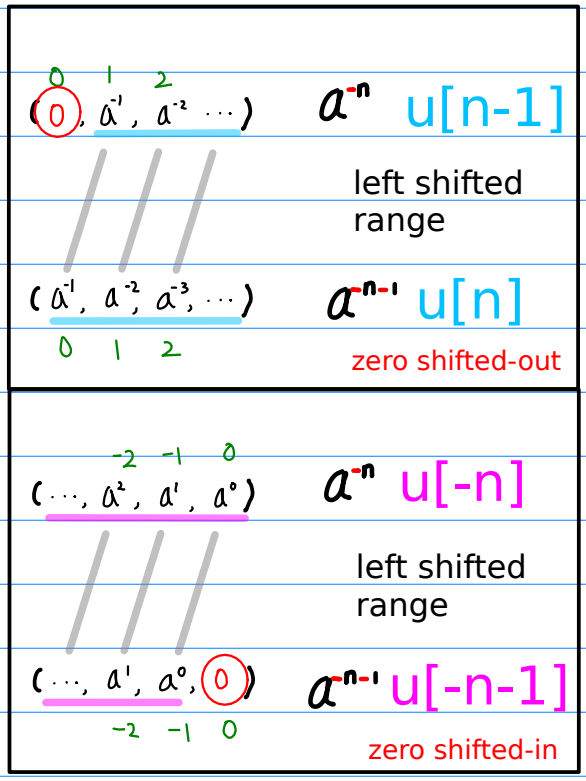


SHL.Rng, SHL.Exp ... a^n

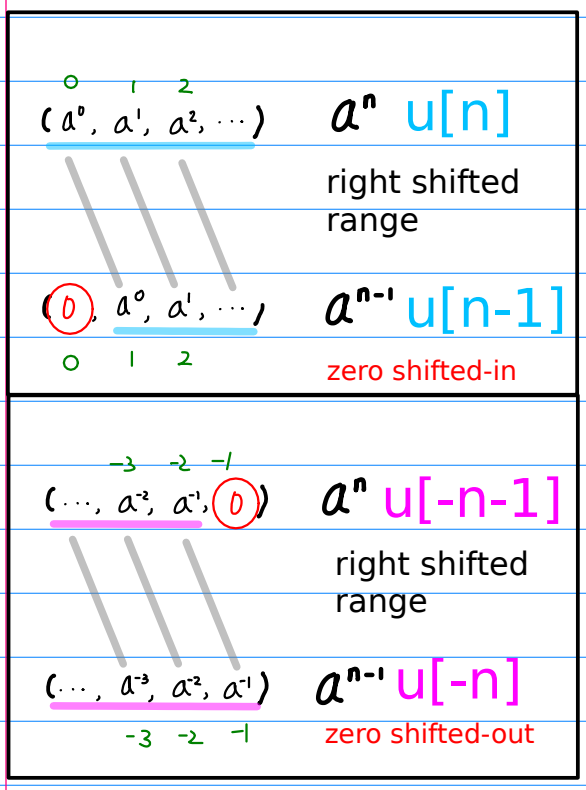
SHL.Exp ... a^-n



SHL.Rng, SHL.Exp ... a^-n



Right Shifting Sequences

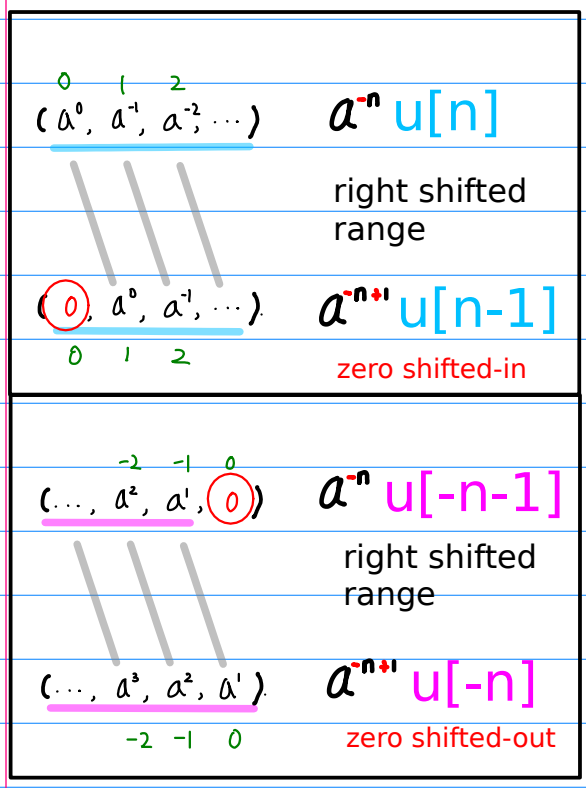


SHR.Exp ... a^n

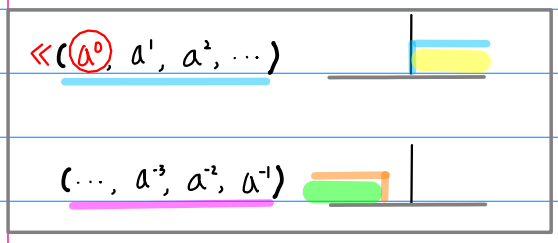
SHR.Rng, SHR.Exp ... a^n

SHR.Exp ... a^-n

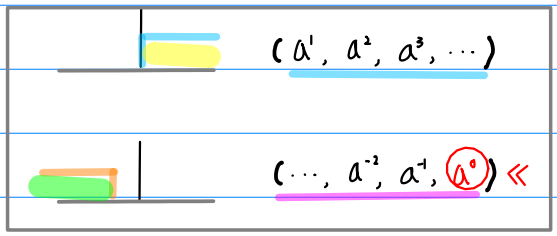
SHR.Rng, SHR.Exp ... a^-n

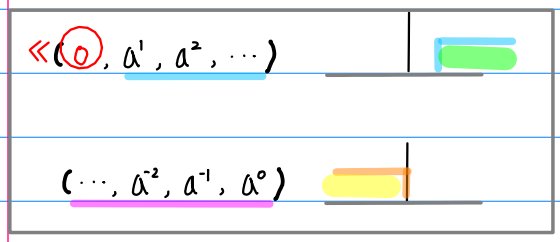


Original Sequence → Shifted Sequence

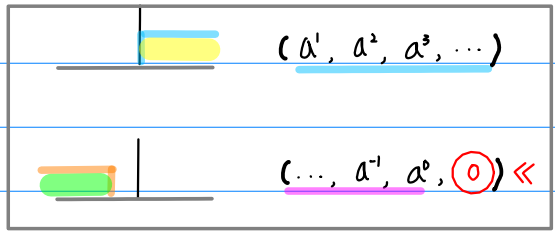


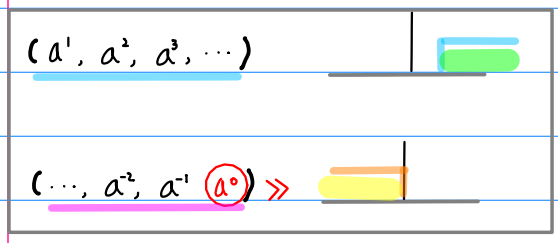
* no shift
* non-zero shift in
* a new value introduced



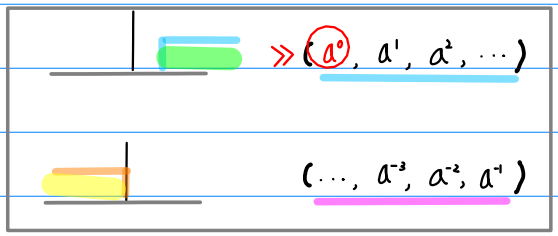


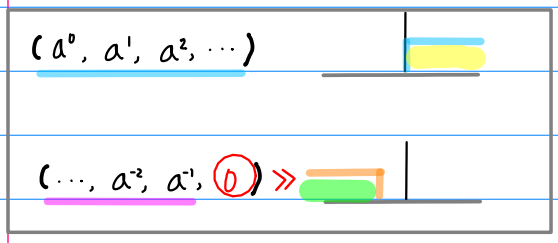
* left shift
* zero shift in
* the same set of values



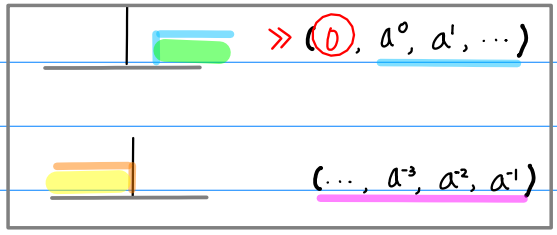


* no shift
* non-zero shift in
* a new value introduced

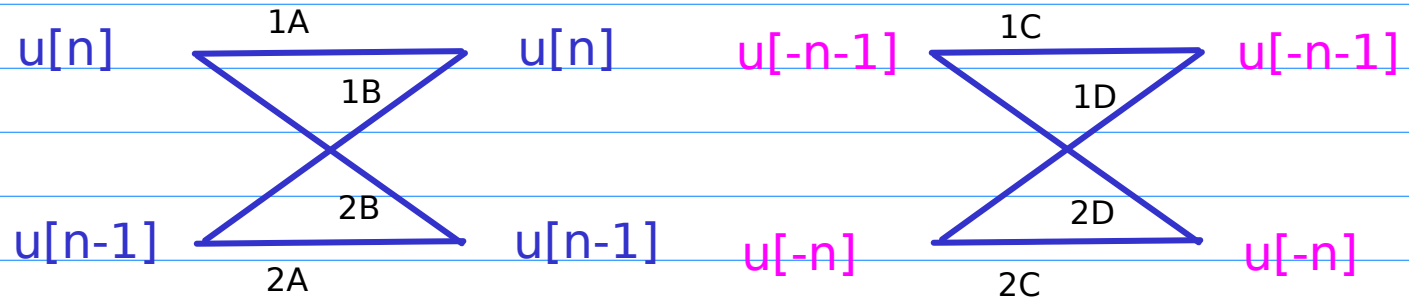


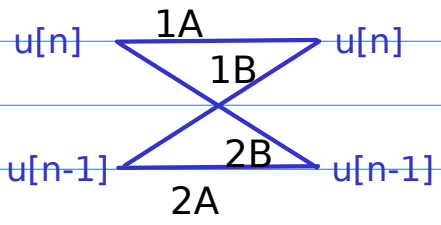


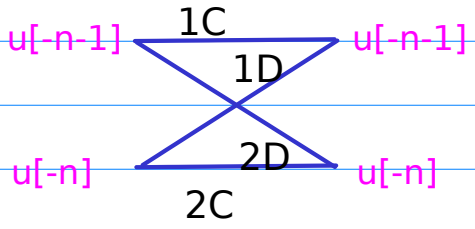
* right shift
* zero shift in
* the same set of values



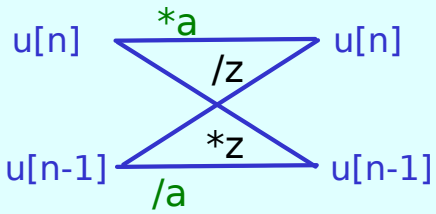
1-A	Left Shift	Causal	u(n)	u(n)	ID.Rng
1-B	Left Shift	Causal	u(n-1)	u(n)	SHL.Rng
1-C	Left Shift	Anti-Causal	u(-n-1)	u(-n-1)	ID.Rng
1-D	Left Shift	Anti-Causal	u(-n)	u(-n-1)	SHL.Rng
2-A	Right Shift	Causal	u(n-1)	u(n-1)	ID.Rng
2-B	Right Shift	Causal	u(n)	u(n-1)	SHR.Rng
2-C	Right Shift	Anti-Causal	u(-n)	u(-n)	ID.Rng
2-D	Right Shft	Anti-Causal	u(-n-1)	u(-n)	SHR.Rng

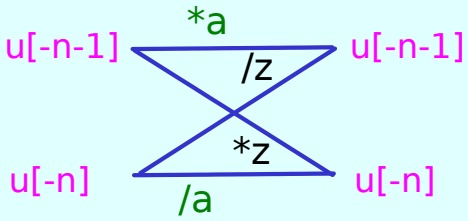



u[n]
1A
u[n]
1B
u[n-1]
2B
u[n-1]
2A

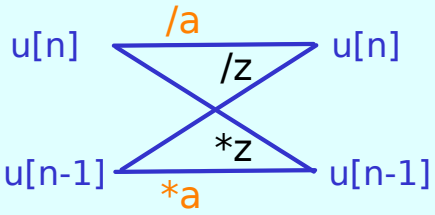

u[-n-1]
1C
u[-n-1]
1D
u[-n]
2D
u[-n]
2C

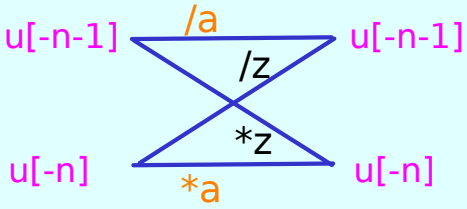
a^n


u[n]
*a
u[n]
/z
u[n-1]
*z
u[n-1]
/a

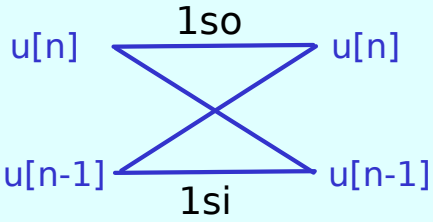

u[-n-1]
*a
u[-n-1]
/z
u[-n]
*z
u[-n]
/a

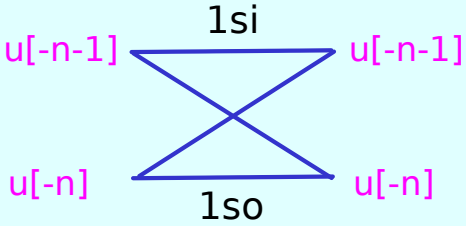
a^{-n}


u[n]
/a
u[n]
/z
u[n-1]
*z
u[n-1]
*a

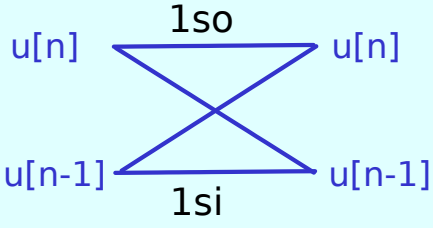

u[-n-1]
/a
u[-n-1]
/z
u[-n]
*z
u[-n]
*a

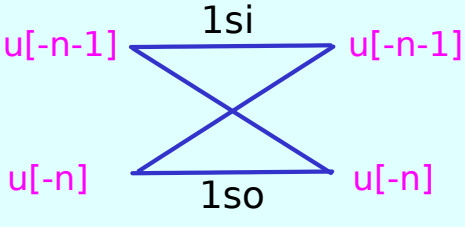
a^n


u[n]
1so
u[n]
u[n-1]
u[n-1]
1si


u[-n-1]
1si
u[-n-1]
u[-n]
u[-n]
1so

a^{-n}

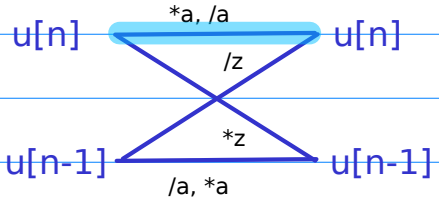

u[n]
1so
u[n]
u[n-1]
u[n-1]
1si


u[-n-1]
1si
u[-n-1]
u[-n]
u[-n]
1so

I-A Left Shfted Sequence $(a,\ a^{-1})$ Causal

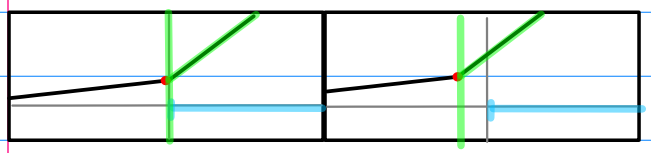
$a^n \cdot u(n) \longrightarrow a^{n+1} \cdot u(n)$ ID.Rng

$a^{-n} \cdot u(n) \longrightarrow a^{-n-1} \cdot u(n)$ ID.Rng



1 shifted out

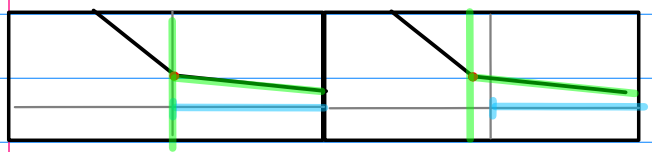
*a



$\frac{1}{1-az}$ $|z|<a^{-1}$

$\frac{a}{1-az}$ $|z|<a^{-1}$

/a



$\frac{1}{1-a^{-1}z}$ $|z|<a$

$\frac{a^{-1}}{1-a^{-1}z}$ $|z|<a$

$\frac{1}{1-az} = a^0z^0 + a^1z^1 + a^2z^2 + \cdots \Rightarrow (a^0, a^1, a^2, \cdots)$ $u(n)$

$|z|<a^{-1}$

$\frac{a}{1-az} = a^1z^0 + a^2z^1 + a^3z^2 + \cdots \Rightarrow (a^1, a^2, a^3, \cdots)$ $u(n)$

$|z|<a^{-1}$

$\frac{1}{1-a^{-1}z} = a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots \Rightarrow (a^0, a^{-1}, a^{-2}, \cdots)$ $u(n)$

$|z|<a^{-1}$

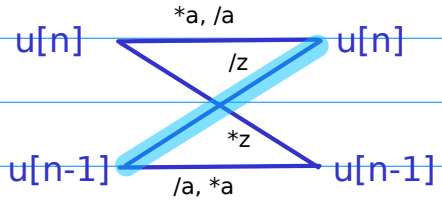
$\frac{a^{-1}}{1-a^{-1}z} = a^{-1}z^0 + a^{-2}z^1 + a^{-3}z^2 + \cdots \Rightarrow (a^{-1}, a^{-2}, a^{-3}, \cdots)$ $u(n)$

$|z|<a^{-1}$

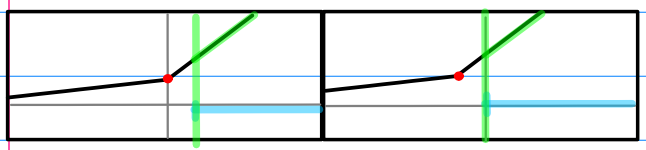
1-B Left Shfted Sequence (z^{-1}, z^{-1}) Causal

$a^n \cdot u(n-1) \longrightarrow a^{n+1} \cdot u(n)$ SHL.Rng

$a^{-n} \cdot u(n-1) \longrightarrow a^{-n-1} \cdot u(n)$ SHL.Rng



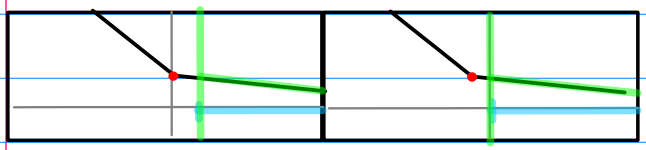
/z



$\frac{az}{1-az}$ $|z| < a^{-1}$

$\frac{a}{1-az}$ $|z| < a^{-1}$

/z



$\frac{a^{-1}z}{1-a^{-1}z}$ $|z| < a$

$\frac{a^{-1}}{1-a^{-1}z}$ $|z| < a$

$\frac{az}{1-az} = a^1z^1 + a^2z^2 + a^3z^3 + \cdots \Rightarrow (0, a^1, a^2, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

$\frac{a}{1-az} = a^1z^0 + a^2z^1 + a^3z^2 + \cdots \Rightarrow (a^1, a^2, a^3, \cdots)$ $u(n)$

$|z| < a^{-1}$

$\frac{a^{-1}z}{1-a^{-1}z} = a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \cdots \Rightarrow (0, a^{-1}, a^{-2} \cdots)$ $u(n-1)$

$|z| < a^{-1}$

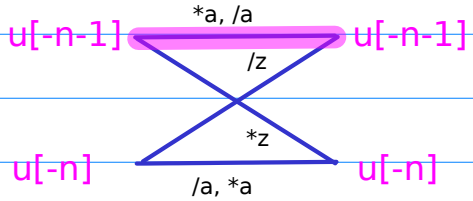
$\frac{a^{-1}}{1-a^{-1}z} = a^{-1}z^0 + a^{-2}z^1 + a^{-3}z^2 + \cdots \Rightarrow (a^{-1}, a^{-2}, a^{-3}, \cdots)$ $u(n)$

$|z| < a^{-1}$

1-C Left Shfted Sequence (a, a^{-1}) Anti-Causal

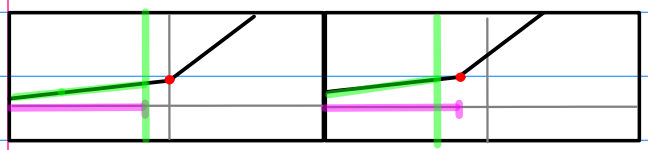
$a^n \cdot u(-n-1) \longrightarrow a^{n+1} \cdot u(-n-1)$ ID.Rng

$a^{-n} \cdot u(-n-1) \longrightarrow a^{-n-1} \cdot u(-n-1)$ ID.Rng



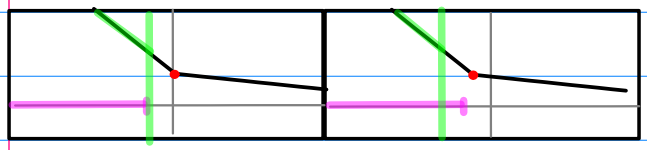
1 shifted in

*a



$\frac{az^{-1}}{1-az^{-1}}$ $|z|>a^{-1}$ $\quad$ $\frac{z^{-1}}{1-az^{-1}}$ $|z|>a^{-1}$

/a



$\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$ $\quad$ $\frac{z^{-1}}{1-az^{-1}}$ $|z|>a$

$\frac{az^{-1}}{1-az^{-1}} = \cdots + a^{-3}z^{-3} + a^{-2}z^{-2} + a^{-1}z^{-1} \Rightarrow (\cdots, a^{-3}, a^{-2}, a^{-1})$ $u(-n-1)$

$|z|>a^{-1}$

$\frac{z^{-1}}{1-az^{-1}} = \cdots + a^{-2}z^{-3} + a^{-1}z^{-2} + a^{0}z^{-1} \Rightarrow (\cdots, a^{-2}, a^{-1}, a^{0})$ $u(-n-1)$

$|z|>a^{-1}$

$\frac{az^{-1}}{1-az^{-1}} = \cdots + a^{3}z^{-3} + a^{2}z^{-2} + a^{1}z^{-1} \Rightarrow (\cdots, a^{3}, a^{2}, a^{1})$ $u(-n-1)$

$|z|>a$

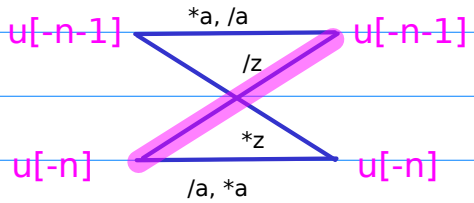
$\frac{z^{-1}}{1-az^{-1}} = \cdots + a^{2}z^{-3} + a^{1}z^{-2} + a^{0}z^{-1} \Rightarrow (\cdots, a^{2}, a^{1}, a^{0})$ $u(-n-1)$

$|z|>a$

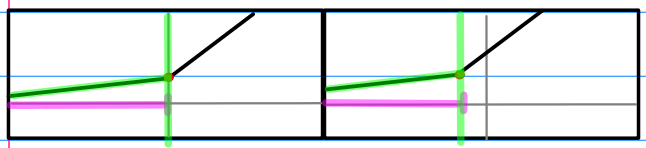
1-D Left Shfted Sequence (z^{-1}, z^{-1}) Anti-Causal

$a^n \cdot u(-n) \longrightarrow a^{n+1} \cdot u(-n-1)$ SHL.Rng

$a^{-n} \cdot u(-n) \longrightarrow a^{-n-1} \cdot u(-n-1)$ SHL.Rng



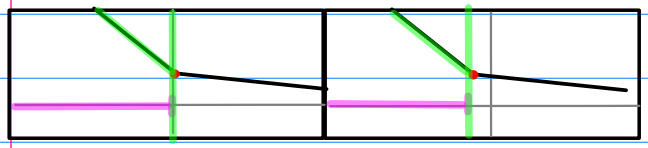
/z



$\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

/z



$\frac{1}{1-az^{-1}}$ $|z|>a^{-1}$

$\frac{z^{-1}}{1-az^{-1}}$ $|z|>a^{-1}$

$\frac{1}{1-a^{-1}z^{-1}} = \cdots + a^{-2}z^{-2} + a^{-1}z^{-1} + a^0 z^0 \Rightarrow (\cdots, a^{-2}, a^{-1}, a^0)$ $u(-n)$

$|z|>a^{-1}$

$\frac{z^{-1}}{1-a^{-1}z^{-1}} = \cdots + a^{-2}z^{-3} + a^{-1}z^{-2} + a^0 z^{-1} \Rightarrow (\cdots, a^{-1}, a^0, 0)$ $u(-n-1)$

$|z|>a^{-1}$

$\frac{1}{1-az^{-1}} = \cdots + a^2 z^{-2} + a^1 z^{-1} + a^0 z^0 \Rightarrow (\cdots, a^2, a^1, a^0)$ $u(-n)$

$|z|>a^{-1}$

$\frac{z^{-1}}{1-az^{-1}} = \cdots + a^2 z^{-3} + a^1 z^{-2} + a^0 z^{-1} \Rightarrow (\cdots, a^1, a^0, 0)$ $u(-n-1)$

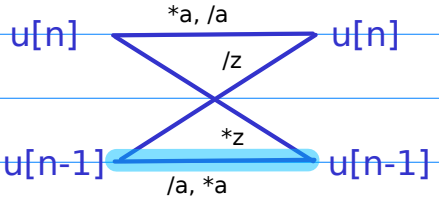
$|z|>a^{-1}$

2-A Right Shfted Sequence (a^{-1}, a)

Causal

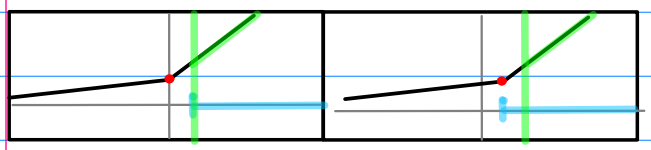
$a^n \cdot u(n-1) \longrightarrow a^{n-1} \cdot u(n-1)$ ID.Rng

$a^{-n} \cdot u(n-1) \longrightarrow a^{-n+1} \cdot u(n-1)$ ID.Rng



1 shifted in

/a



$\frac{az}{1-az}$ $|z| < a^{-1}$

$\frac{z}{1-az}$ $|z| < a^{-1}$

*a

$\frac{a^{-1}z}{1-a^{-1}z}$ $|z| < a$

$\frac{z}{1-a^{-1}z}$ $|z| < a$

$\frac{az}{1-az} = a^1z^1 + a^2z^2 + a^3z^3 + \cdots \Rightarrow (a^1, a^2, a^3, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

$\frac{z}{1-az} = a^0z^1 + a^1z^2 + a^2z^3 + \cdots \Rightarrow (a^0, a^1, a^2, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

$\frac{a^{-1}z}{1-a^{-1}z} = a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \cdots \Rightarrow (a^{-1}, a^{-2}, a^{-3}, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

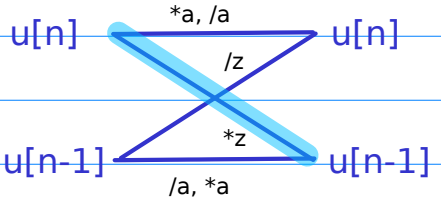
$\frac{z}{1-a^{-1}z} = a^0z^1 + a^{-1}z^2 + a^{-2}z^3 + \cdots \Rightarrow (a^0, a^{-1}, a^{-2}, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

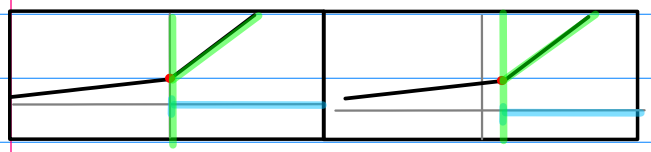
2-B Right Shfted Sequence (Z, z) Causal

$a^n \cdot u(n) \longrightarrow a^{n-1} \cdot u(n-1)$ SHR.Rng

$a^{-n} \cdot u(n) \longrightarrow a^{-n+1} \cdot u(n-1)$ SHR.Rng



*Z



$\frac{1}{1-az}$ $|z| < a^{-1}$

$\frac{z}{1-az}$ $|z| < a^{-1}$

*Z

$\frac{1}{1-a^{-1}z}$ $|z| < a$

$\frac{z}{1-a^{-1}z}$ $|z| < a$

$\frac{1}{1-az} = a^0z^0 + a^1z^1 + a^2z^2 + \cdots \Rightarrow (a^0, a^1, a^2, \cdots)$ $u(n)$

$|z| < a^{-1}$

$\frac{z}{1-az} = a^0z^1 + a^1z^2 + a^2z^3 + \cdots \Rightarrow (0, a^0, a^1, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

$\frac{1}{1-a^{-1}z} = a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots \Rightarrow (a^0, a^{-1}, a^{-2}, \cdots)$ $u(n)$

$|z| < a^{-1}$

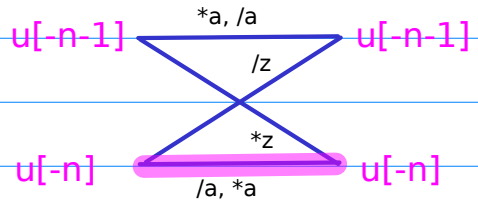
$\frac{z}{1-a^{-1}z} = a^0z^1 + a^{-1}z^2 + a^{-2}z^3 + \cdots \Rightarrow (0, a^0, a^{-1}, \cdots)$ $u(n-1)$

$|z| < a^{-1}$

2-c Right Shfted Sequence (a^{-1}, a) Anti-Causal

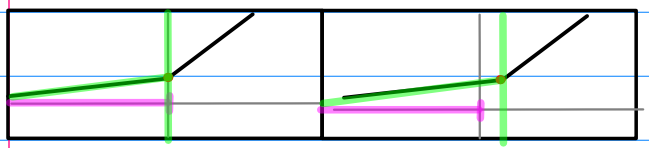
$a^n \cdot u(-n) \longrightarrow a^{n-1} \cdot u(-n)$ ID.Rng

$a^{-n} \cdot u(-n) \longrightarrow a^{-n+1} \cdot u(-n)$ ID.Rng



1 shifted out

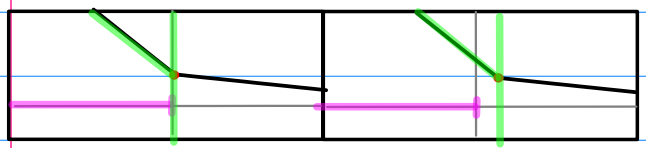
/a



$\frac{1}{1-a^{-1}z^{-1}}$ $|z| > a^{-1}$

$\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z| > a^{-1}$

*a



$\frac{1}{1-az^{-1}}$ $|z| > a$

$\frac{a}{1-az^{-1}}$ $|z| > a$

$\frac{1}{1-a^{-1}z^{-1}} = \cdots + a^{-2}z^{-2} + a^{-1}z^{-1} + a^0 z^0 \Rightarrow (\cdots, a^{-2}, a^{-1}, a^0)$ $u(-n)$

$|z| > a^{-1}$

$\frac{a^{-1}}{1-a^{-1}z^{-1}} = \cdots + a^{-3}z^{-2} + a^{-2}z^{-1} + a^{-1}z^0 \Rightarrow (\cdots, a^{-3}, a^{-2}, a^{-1})$ $u(-n)$

$|z| > a^{-1}$

$\frac{1}{1-az^{-1}} = \cdots + a^2 z^{-2} + a^1 z^{-1} + a^0 z^0 \Rightarrow (\cdots, a^2, a^1, a^0)$ $u(-n)$

$|z| > a$

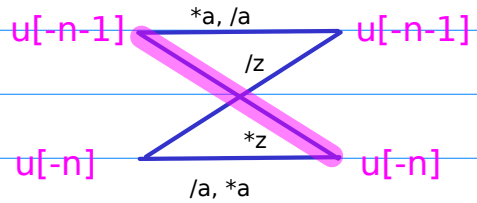
$\frac{a}{1-az^{-1}} = \cdots + a^3 z^{-2} + a^2 z^{-1} + a^1 z^0 \Rightarrow (\cdots, a^3, a^2, a^1)$ $u(-n)$

$|z| > a$

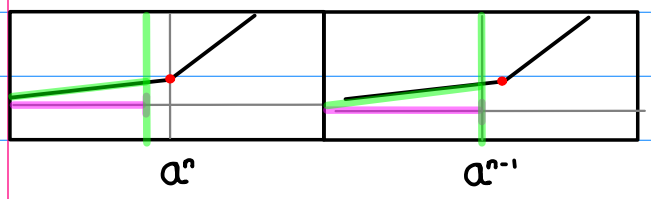
2-D Right Shfted Sequence (z, z) Anti-Causal

$a^n \cdot u(-n-1) \longrightarrow a^{n-1} \cdot u(-n)$ SHR.Rng

$a^{-n} \cdot u(-n-1) \longrightarrow a^{-n+1} \cdot u(-n)$ SHR.Rng

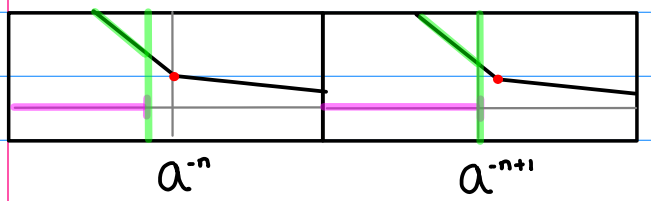


*Z



$\frac{az^{-1}}{1-az^{-1}}$ $|z|>a^{-1}$ $\qquad$ $\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

*Z



$\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$ $\qquad$ $\frac{a}{1-az^{-1}}$ $|z|>a$

$\frac{az^{-1}}{1-az^{-1}} = \cdots + a^3z^{-3} + a^2z^{-2} + a^1z^{-1} \Rightarrow (\cdots, a^2, a^1, 0)$ $u(-n-1)$

$|z|>a^{-1}$

$\frac{a^{-1}}{1-a^{-1}z^{-1}} = \cdots + a^{-3}z^{-2} + a^{-2}z^{-1} + a^{-1}z^{0} \Rightarrow (\cdots, a^{-3}, a^{-2}, a^{-1})$ $u(-n)$

$|z|>a^{-1}$

$\frac{az^{-1}}{1-az^{-1}} = \cdots + a^3z^{-3} + a^2z^{-2} + a^1z^{-1} \Rightarrow (\cdots, a^2, a^1, 0)$ $u(-n-1)$

$|z|>a$

$\frac{a}{1-az^{-1}} = \cdots + a^3z^{-3} + a^2z^{-2} + a^1z^{-1} \Rightarrow (\cdots, a^3, a^2, a^1)$ $u(-n)$

$|z|>a$

Making Shifted Sequences

making left shifted sequences

causal

the same set of slots

left shifted set of samples

anti-causal

the same set of slots

left shifted set of samples

causal

left shifted set of slots

the same set of samples

anti-causal

left shifted set of slots

the same set of samples

making right shifted sequences

causal

the same set of slots

right shifted set of samples

anti-causal

the same set of slots

right shifted set of samples

causal

right shifted set of slots

the same set of samples

anti-causal

right shifted set of slots

the same set of samples

Making Shifted Sequences

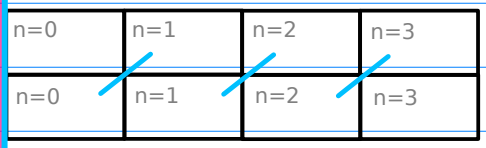
making left shifted sequences

the same set of slots

left shifted set of samples

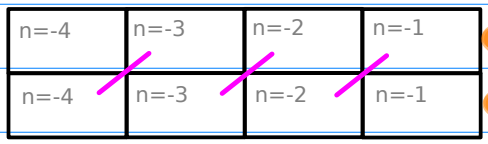
u[n]

u[n]



u[-n-1]

u[-n-1]

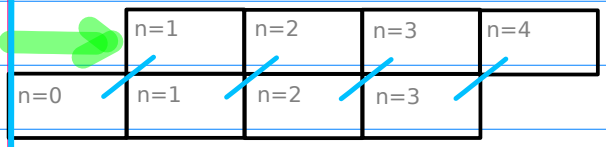


left shifted set of slots

the same set of samples

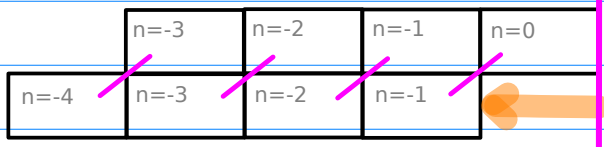
u[n-1]

u[n]



u[-n]

u[-n-1]



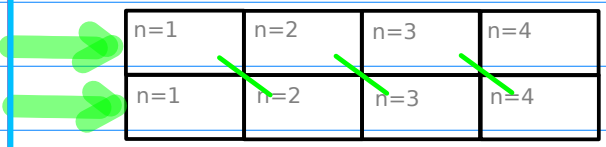
making right shifted sequences

the same set of slots

right shifted set of samples

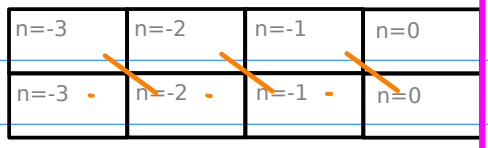
u[n-1]

u[n-1]



u[-n]

u[-n]

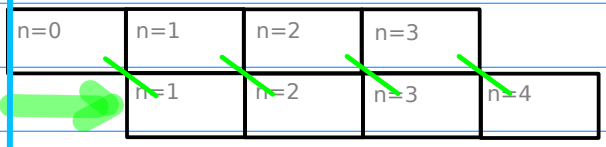


right shifted set of slots

the same set of samples

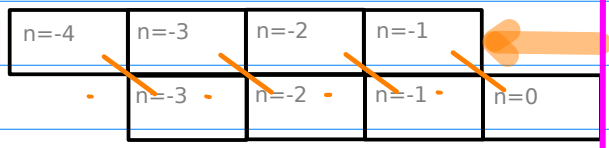
u[n]

u[n-1]



u[-n-1]

u[-n]

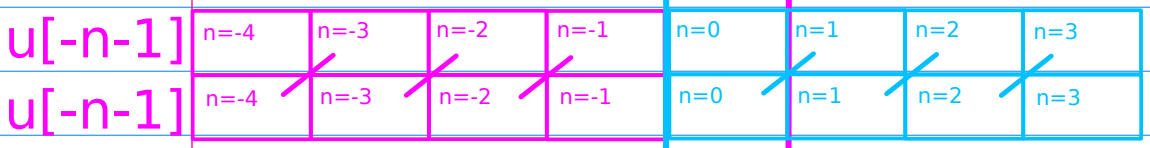


Making Shifted Sequences

making left shifted sequences

the same set of slots

left shifted set of samples

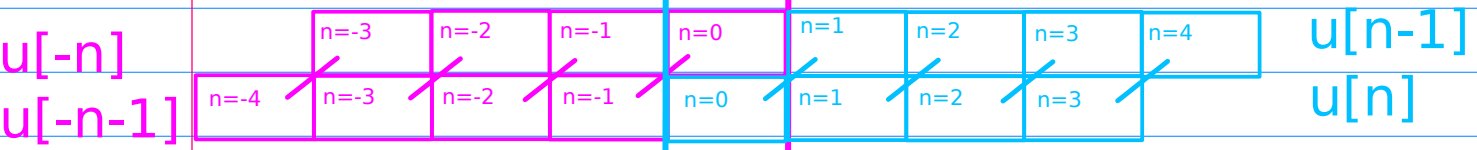


the same set of slots

left shifted set of samples

left shifted set of slots

the same set of samples



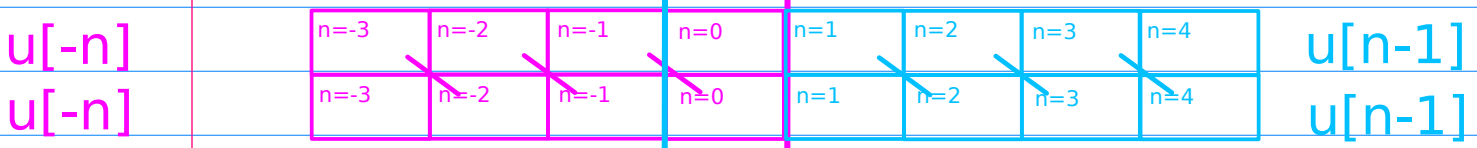
left shifted set of slots

the same set of samples

making right shifted sequences

the same set of slots

right shifted set of samples

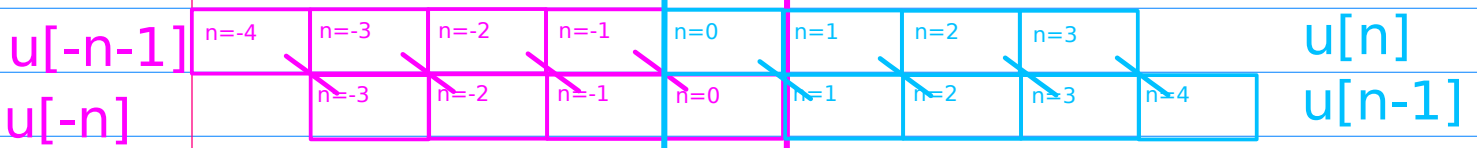


the same set of slots

right shifted set of samples

right shifted set of slots

the same set of samples



right shifted set of slots

the same set of samples

Complementary Ranges

Two Types of Left-Shifted Causal Sequences

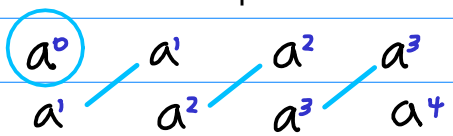
the same fixed slots

u[n]

u[n]

n=0	n=1	n=2	n=3
n=0	n=1	n=2	n=3

left-shift samples



a^n

a^{n+1}

a^0	a^1	a^2	a^3
a^1	a^2	a^3	a^4

left-shifted sequence (I)

the same set of slots

left shifted set of samples

*a

a^n u[n]

a^{n+1} u[n]

SHL.Exp

right-shift pre-slot

u[n-1]

u[n]

	n=1	n=2	n=3	n=4
n=0	n=1	n=2	n=3	

fixed samples

a^1	a^2	a^3	a^4
a^1	a^2	a^3	a^4

a^n

a^{n+1}

	a^1	a^2	a^3	a^4
a^1	a^2	a^3	a^4	

left-shifted sequence (II)

left shifted set of slots

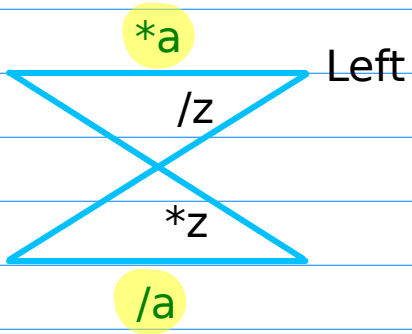
the same set of samples

/z

a^n u[n-1]

a^{n+1} u[n]

SHL.Rng & SHL.Exp



Two Types of Left-Shifted Anti-Causal Sequences

left shift both slots

u[-n-1]
u[-n-1]

n=-4	n=-3	n=-2	n=-1
n=-4	n=-3	n=-2	n=-1

left-shift samples

a^{-4} a^{-3} a^{-2} a^{-1}
a^{-3} a^{-2} a^{-1} a^{0}

a^n
a^{n+1}

a^{-4}	a^{-3}	a^{-2}	a^{-1}
a^{-3}	a^{-2}	a^{-1}	a^{0}

left-shifted sequence (I)

the same set of slots
left shifted set of samples

a^n u[-n-1]
a^{n+1} u[-n-1]

SHL.Exp

*a

left-shift post-slot

u[-n]
u[-n-1]

	n=-3	n=-2	n=-1	n=0
n=-4	n=-3	n=-2	n=-1	

fixed samples

a^{-3}	a^{-2}	a^{-1}	a^{0}
a^{-3}	a^{-2}	a^{-1}	a^{0}

a^n
a^{n+1}

	a^{-3}	a^{-2}	a^{-1}	a^{0}
a^{-3}	a^{-2}	a^{-1}	a^{0}	

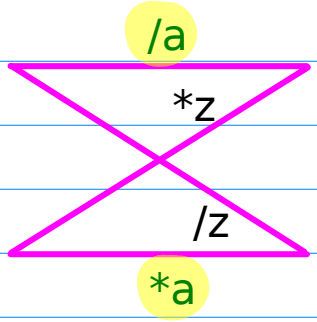
left-shifted sequence (II)

left shifted set of slots
the same set of samples

a^n u[-n]
a^{n+1} u[-n-1]

SHL.Rng & SHL.Exp

/z



Two Types of Right-Shifted Causal Sequences

right shift both slots

u[n-1]
u[n-1]

	n=1	n=2	n=3	n=4
u[n-1]				
u[n-1]				

right shift post-samples

a^1 a^2 a^3 a^4
a^0 a^1 a^2 a^3

a^n
a^{n-1}

a^n	a^1	a^2	a^3	a^4
a^{n-1}	a^0	a^1	a^2	a^3

right-shifted sequence (I)

the same set of slots
right shifted set of samples

a^n u[n-1]
a^{n-1} u[n-1]

SHR.Rng

/a

right shift post-slot

u[n]
u[n-1]

	n=0	n=1	n=2	n=3	n=4
u[n]					
u[n-1]					

fixed samples

a^0	a^1	a^2	a^3	
a^0	a^1	a^2	a^3	

a^n
a^{n-1}

a^0	a^1	a^2	a^3	
	a^0	a^1	a^2	a^3

right-shifted sequence (II)

right shifted set of slots
the same set of samples

a^n u[n]
a^{n-1} u[n-1]

SHR.Rng & SHR.Exp

*z

*a
/z
*z
/a

Right

Two Types of Right-Shifted Anti-Causal Sequences

the same fixed slots

u[-n]

u[-n]

n=-3	n=-2	n=-1	n=0
n=-3	n=-2	n=-1	n=0

right shift post-samples

a^{-3} a^{-2} a^{-1} a^{0}

a^{-4} a^{-3} a^{-2} a^{-1}

a^n

a^{n-1}

a^{-3}	a^{-2}	a^{-1}	a^{0}
a^{-4}	a^{-3}	a^{-2}	a^{-1}

right-shifted sequence (I)

the same set of slots

right shifted set of samples

/a

a^n u[-n]

a^{n-1} u[-n]

SHR.Rng

left shift pre-slot

u[-n-1]

u[-n]

n=-4	n=-3	n=-2	n=-1
	n=-3	n=-2	n=-1	n=0

fixed samples

a^{-4}	a^{-3}	a^{-2}	a^{-1}
a^{-4}	a^{-3}	a^{-2}	a^{-1}

a^n

a^{n-1}

a^{-4}	a^{-3}	a^{-2}	a^{-1}
	a^{-4}	a^{-3}	a^{-2}	a^{-1}

right-shifted sequence (II)

right shifted set of slots

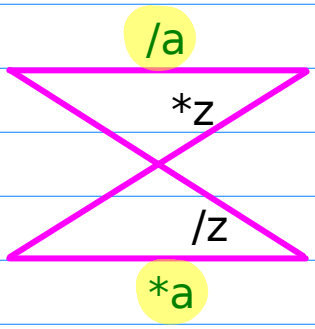
the same set of samples

*z

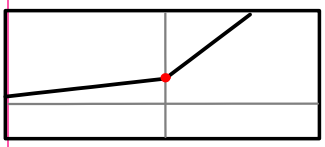
a^n u[-n-1]

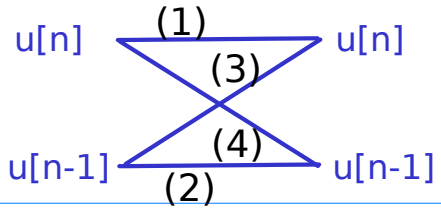
a^{n-1} u[-n]

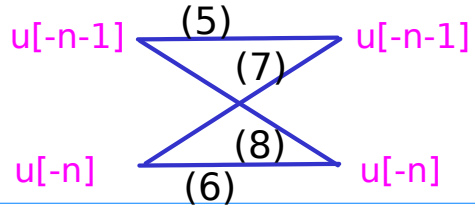
SHR.Rng & SHR.Exp



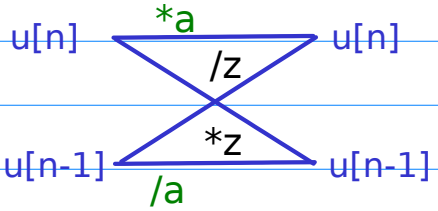
a^n

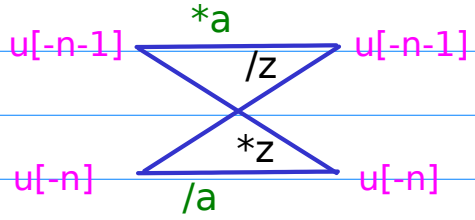




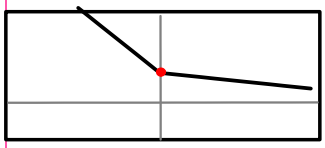


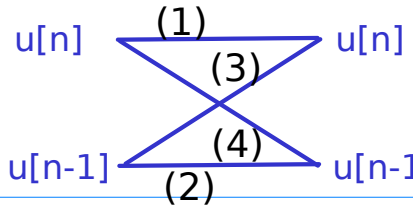
(1)	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n) $(a^0, a^1, a^2, \cdots)$	a^{n+1} u(n) $(a^1, a^2, a^3, \cdots)$
(2)	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n-1) $(a^1, a^2, a^3, \cdots)$	a^{n-1} u(n-1) $(a^0, a^1, a^2, \cdots)$
(3)	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n-1) $(a^1, a^2, a^3, \cdots)$	a^{n+1} u(n) $(a^1, a^2, a^3, \cdots)$
(4)	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n) $(a^0, a^1, a^2, \cdots)$	a^{n-1} u(n-1) $(a^0, a^1, a^2, \cdots)$
(5)	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n-1) $-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-a^{n+1}$ u(-n-1) $-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$
(6)	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n) $-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-a^{n-1}$ u(-n) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$
(7)	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^0})$	$-a^{n+1}$ u(-n-1) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$
(8)	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n-1) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$-a^{n-1}$ u(-n) $-(\cdots, \frac{1}{a^4}, \frac{1}{a^3}, \frac{1}{a^2})$

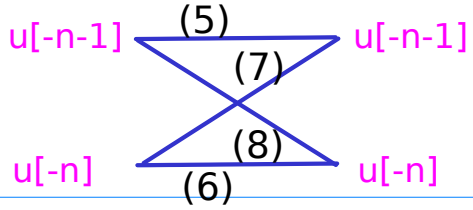




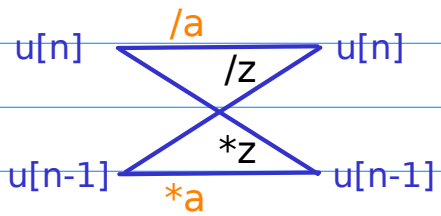
a^{-n}

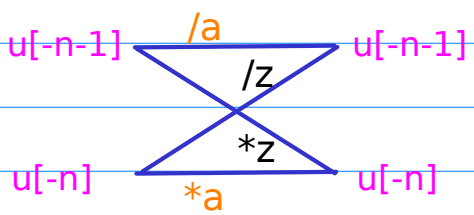




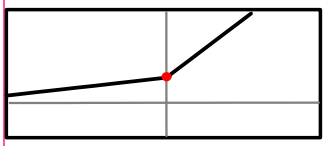


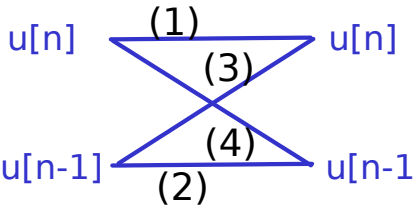
(1)	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n$ u(n) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n+1}$ u(n) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$
(2)	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n$ u(n-1) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n-1}$ u(n-1) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$
(3)	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n$ u(n-1) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n+1}$ u(n) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$
(4)	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n$ u(n) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n-1}$ u(n-1) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$
(5)	$-\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n$ u(-n-1) $-\left(\cdots, a^3, a^2, a^1\right)$	$-\left(\frac{1}{a}\right)^{n+1}$ u(-n-1) $-\left(\cdots, a^2, a^1, a^0\right)$
(6)	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n$ u(-n) $-\left(\cdots, a^3, a^1, a^0\right)$	$-\left(\frac{1}{a}\right)^{n-1}$ u(-n) $-\left(\cdots, a^3, a^2, a^1\right)$
(7)	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n$ u(-n) $-\left(\cdots, a^2, a^1, a^0\right)$	$-\left(\frac{1}{a}\right)^{n+1}$ u(-n-1) $-\left(\cdots, a^2, a^1, a^0\right)$
(8)	$-\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n$ u(-n-1) $-\left(\cdots, a^3, a^2, a^1\right)$	$-\left(\frac{1}{a}\right)^{n-1}$ u(-n) $-\left(\cdots, a^3, a^2, a^1\right)$

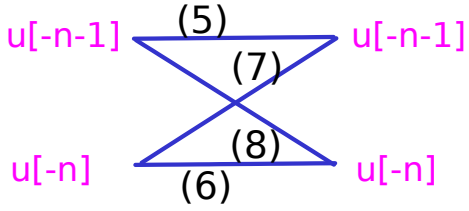




a^n

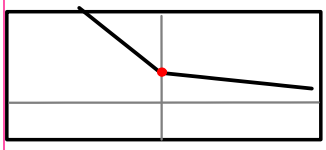


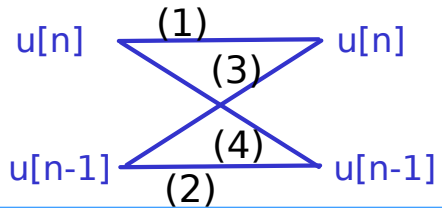


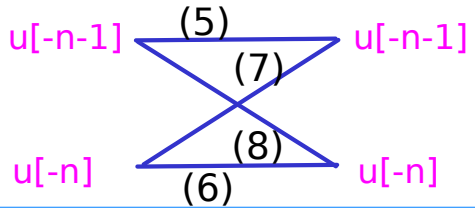


	scale(a)		ID.Rng	
(1)	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n) $(a^0, a^1, a^2, \cdots)$	a^{n+1} u(n) $(a^1, a^2, a^3, \cdots)$
(5)	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n-1) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$-a^{n+1}$ u(-n-1) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$
	scale(1/a)		ID.Rng	
(2)	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n-1) $(a^1, a^2, a^3, \cdots)$	a^{n-1} u(n-1) $(a^0, a^1, a^2, \cdots)$
(6)	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n) $-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-a^{n-1}$ u(-n) $-(\cdots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^1})$
	scale(1/z)		SHL.Rng	
(3)	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n-1) $(a^1, a^2, a^3, \cdots)$	a^{n+1} u(n) $(a^1, a^2, a^3, \cdots)$
(7)	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^1}, \frac{1}{a^0})$	$-a^{n+1}$ u(-n-1) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^1}, \frac{1}{a^0})$
	scale(z)		SHR.Rng	
(4)	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n u(n) $(a^0, a^1, a^2, \cdots)$	a^{n-1} u(n-1) $(a^0, a^1, a^2, \cdots)$
(8)	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ u(-n-1) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$-a^{n-1}$ u(-n) $-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$
	Original Series	Scaled Series	Original Sequence	Shifted Sequence

a^{-n}







scale(1/a)

			ID.Rng	
(1)	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\left(\frac{1}{a}\right)^n$ u(n) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n+1}$ u(n) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$
(5)	$-\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\left(\frac{1}{a}\right)^n$ u(-n-1) $-\left(\cdots, a^3, a^2, a^1\right)$	$-\left(\frac{1}{a}\right)^{n+1}$ u(-n-1) $-\left(\cdots, a^2, a^1, a^0\right)$

scale(a)

			ID.Rng	
(2)	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\left(\frac{1}{a}\right)^n$ u(n-1) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n-1}$ u(n-1) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$
(6)	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\left(\frac{1}{a}\right)^n$ u(-n) $\left(\cdots, a^2, a^1, a^0\right)$	$-\left(\frac{1}{a}\right)^{n-1}$ u(-n) $\left(\cdots, a^3, a^2, a^1\right)$

scale(1/z)

			SHL.Rng	
(3)	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\left(\frac{1}{a}\right)^n$ u(n-1) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$	$\frac{1}{a}^{n+1}$ u(n) $\left(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots\right)$
(7)	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\left(\frac{1}{a}\right)^n$ u(-n) $-\left(\cdots, a^2, a^1, a^0\right)$	$-\left(\frac{1}{a}\right)^{n+1}$ u(-n-1) $-\left(\cdots, a^2, a^1, a^0\right)$

scale(z)

			SHR.Rng	
(4)	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\left(\frac{1}{a}\right)^n$ u(n) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$	$\left(\frac{1}{a}\right)^{n-1}$ u(n-1) $\left(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots\right)$
(8)	$-\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\left(\frac{1}{a}\right)^n$ u(-n-1) $-\left(\cdots, a^3, a^2, a^1\right)$	$-\left(\frac{1}{a}\right)^{n-1}$ u(-n) $-\left(\cdots, a^3, a^2, a^1\right)$

Original Series | Scaled Series | Original Sequence | Shifted Sequence