Linear Equations with Constant Coefficients (2A)

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Homogeneous Linear Equations
with constant coefficients

Types of First Order ODEs

A General Form of First Order Differential Equations

$$\frac{dy}{dx} = g(x, y)$$

$$y' = g(x, y)$$

Separable Equations

$$\frac{dy}{dx} = g_1(x) g_2(y)$$

$$y' = g_1(x) g_2(y)$$

$$y = f(x)$$

Linear Equations

$$a_1(x) \frac{dy}{dx} + a_0(x) y = g(x)$$

$$a_1(x) y' + a_0(x) y = g(x)$$

$$y = f(x)$$

Exact Equations

$$M(x, y) dx + N(x, y) dy = 0$$

$$\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = 0$$

$$z = f(x, y)$$

Second Order ODEs

First Order ***Linear Equations***

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

$$a_1(x)y' + a_0(x)y = g(x)$$

Second Order ***Linear Equations***

$$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x)$$

Second Order ***Linear Equations*** ***with Constant Coefficients***

$$a_2\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_0y = g(x)$$

$$a_2y'' + a_1y' + a_0y = g(x)$$

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = g(x)$$

$$ay'' + by' + cy = g(x)$$

Auxiliary Equation

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

try a solution $y = e^{mx}$

$$a\frac{d^2}{dx^2}\{e^{mx}\} + b\frac{d}{dx}\{e^{mx}\} + c\{e^{mx}\} = 0$$

$$a\{e^{mx}\}'' + b\{e^{mx}\}' + c\{e^{mx}\} = 0$$

$$a\{m^2e^{mx}\} + b\{me^{mx}\} + c\{e^{mx}\} = 0$$

$$a\{m^2e^{mx}\} + b\{me^{mx}\} + c\{e^{mx}\} = 0$$

$$(am^2 + bm + c)\cdot e^{mx} = 0$$

$$(am^2 + bm + c)\cdot e^{mx} = 0$$

auxiliary equation

$$(am^2 + bm + c) = 0$$

$$(am^2 + bm + c) = 0$$

Roots of the Auxiliary Equation

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

auxiliary equation

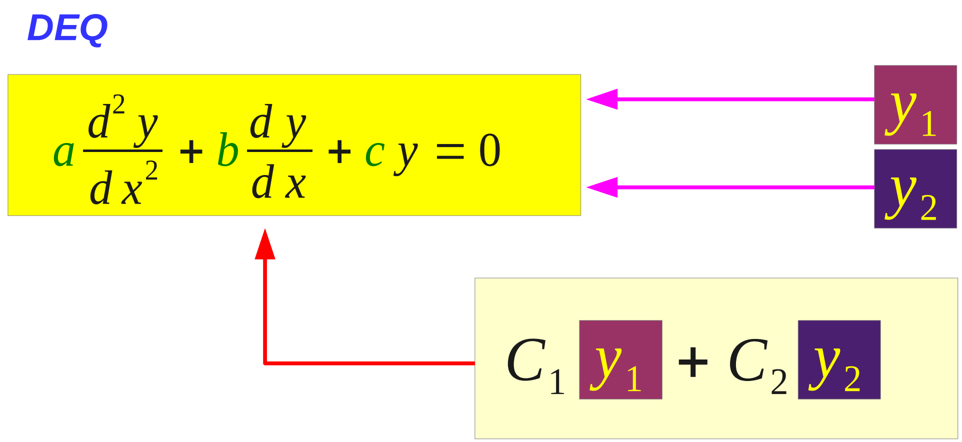
try a solution $y = e^{mx}$ ➡ $(am^2 + bm + c) = 0$

$$m_1 = (-b + \sqrt{b^2 - 4ac})/2a$$

$$m_2 = (-b - \sqrt{b^2 - 4ac})/2a$$

$y_1 = e^{m_1 x}, \quad y_2 = e^{m_2 x}$	**(A)** $b^2 - 4ac > 0$	Real, distinct m_1, m_2
$y_1 = e^{m_1 x} = y_2 = e^{m_2 x}$	**(B)** $b^2 - 4ac = 0$	Real, equal m_1, m_2
$y_1 = e^{m_1 x}, \quad y_2 = e^{m_2 x}$	**(C)** $b^2 - 4ac < 0$	Conjugate complex m_1, m_2

Linear Combination of Solutions



$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

y_1, y_2

$C_1 y_1 + C_2 y_2$

$$a y_1'' + b y_1' + c y_1 = 0$$
$$a y_2'' + b y_2' + c y_2 = 0$$

$$a(y_1''+y_2'') + b(y_1'+y_2') + c(y_1+y_2) = 0$$
$$a(y_1+y_2)'' + b(y_1+y_2)' + c(y_1+y_2) = 0$$

$$y_3 = y_1 + y_2$$
$$y_4 = y_1 - y_2$$
$$y_5 = y_3 + 2y_4$$
$$y_6 = y_3 - 2y_4$$

$$a(C_1y_1''+C_2y_2'') + b(C_1y_1'+C_2y_2') + c(C_1y_1+C_2y_2) = 0$$
$$a(C_1y_1+C_2y_2)'' + b(C_1y_1+C_2y_2)' + c(C_1y_1+C_2y_2) = 0$$

Solutions of 2nd Order ODEs

DEQ

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0 \quad \leftarrow y_1, \; y_2$$

$$C_1 y_1 + C_2 y_2$$

$$\begin{cases} y_1 = e^{m_1 x} \\ y_2 = e^{m_2 x} \end{cases} \quad (D > 0) \qquad \begin{cases} y_1 = e^{m_1 x} \end{cases} \quad (D = 0) \qquad \begin{cases} y_1 = e^{m_1 x} \\ y_2 = e^{m_2 x} \end{cases} \quad (D < 0)$$

$$\begin{cases} y = C_1 e^{m_1 x} + C_2 e^{m_2 x} & (D > 0) \\ y = C_1 e^{m_1 x} \quad ? & (D = 0) \\ y = C_1 e^{m_1 x} + C_2 e^{m_2 x} & (D < 0) \end{cases}$$

auxiliary equation

$$(am^2 + bm + c) = 0$$

$$m_1 = \left(-b + \sqrt{b^2 - 4ac}\right)/2a$$

$$m_2 = \left(-b - \sqrt{b^2 - 4ac}\right)/2a$$

Linear Independent Functions

$$C_1 y_1 + C_2 y_2 = 0 \Rightarrow C_1 = C_2 = 0$$

always zero means all coefficients must be zero

y_1 and y_2 .are linearly independent functions

if not all zero, y_1 can be represented by y_2, and vice versa.

$$C_1 \neq 0 \Rightarrow y_1 = -\frac{C_2}{C_1} y_2 = a\, y_2$$

$$C_2 \neq 0 \Rightarrow y_2 = -\frac{C_1}{C_2} y_1 = b\, y_1$$

y_1 and y_2 .are linearly dependent functions

y_1 simply a constant multiple of y_2, and vice versa.

Linear Independent Functions Example (1)

$$y_1 = e^{2x} \qquad y_2 = 3e^{2x}$$

$$\frac{y_1}{y_2} = \frac{e^{2x}}{3e^{2x}} = \frac{1}{3} = c$$

C_1 C_2

many solutions of C1, C2

$$3 \cdot \{e^{2x}\} - 1 \cdot \{3e^{2x}\} = 0$$

$$-6 \cdot \{e^{2x}\} + 2 \cdot \{3e^{2x}\} = 0$$

$$\cdots \qquad \cdots$$

~~linearly independent~~ functions

$$y_1 = e^{2x} \qquad y_2 = xe^{2x}$$

$$\frac{y_1}{y_2} = \frac{e^{2x}}{xe^{2x}} = \frac{1}{x} = u(x)$$

C_1 C_2

the only solution C1, C2

$$0\{e^{2x}\} + 0 \cdot \{xe^{2x}\} = 0$$

linearly independent functions

Linear Independent Functions Example (2)

linearly dependent functions

$$y_1 = e^{2x} \qquad y_2 = 3e^{2x}$$

$$y = c_1 e^{2x} + c_2 \cdot 3e^{2x}$$

$$y = (c_1 + 3c_2)e^{2x} = C e^{2x}$$

~~general solution~~

linearly independent functions

$$y_1 = e^{2x} \qquad y_2 = x e^{2x}$$

$$y = c_1 e^{2x} + c_2 x e^{2} x$$

general solution

$$y'' - 4y' + 4y = 0$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$y_1 = e^{2x}$$

$$y_1' = 2e^{2x}$$

$$y_1'' = 4e^{2x}$$

$$4y_1 = 4e^{2x}$$

$$-4y_1' = -8e^{2x}$$

$$y_1'' = 4e^{2x}$$

$$0$$

$$y_2 = x e^{2x}$$

$$y_2' = e^{2x} + 2x e^{2x}$$

$$y_2'' = 2e^{2x} + 2e^{2x} + 4x e^{2x}$$

$$4y_2 = 4x e^{2x}$$

$$-4y_2' = -4e^{2x} - 8x e^{2x}$$

$$y_2'' = 4e^{2x} + 4x e^{2x}$$

$$0$$

Fundamental Set of Solutions

Second Order EQ

$$a\frac{d^2 y}{dx^2} + b\frac{dy}{dx} + c\,y = 0$$

y_1

y_2

$$C_1 y_1 + C_2 y_2$$

Functions y_1 and y_2 are either

- linearly independent functions or
- linearly dependent functions

$$\{y_1,\ \ y_2\}$$

Second Order

there can be at most **two** linearly independent functions

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$$

any *n* linearly independent solutions of the homogeneous linear *n-th* order differential equation

Fundamental Set of Solutions

Linear Independent Functions and Wronskian

$$C_1 y_1 + C_2 y_2 = 0 \quad \Rightarrow \quad C_1 = C_2 = 0$$

always zero means all coefficients must be zero

y_1 and y_2 .are linearly independent functions

$$C_1 y_1 + C_2 y_2 = 0$$
$$C_1 y_1' + C_2 y_2' = 0$$

$$\begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If the inverse matrix exists

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0 \quad \Leftrightarrow \quad \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

the only solution: trivial

$$W(y_1, y_2) \neq 0$$

(A) Real Distinct Roots Case

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

try a solution $y = e^{mx}$ → $(am^2 + bm + c) = 0$ auxiliary equation

$$m_1 = \left(-b + \sqrt{b^2 - 4ac}\right)/2a \qquad y_1 = e^{m_1 x}$$

$$m_2 = \left(-b - \sqrt{b^2 - 4ac}\right)/2a \qquad y_2 = e^{m_2 x}$$

$b^2 - 4ac > 0$	Real, distinct m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$
$b^2 - 4ac = 0$	Real, equal m_1, m_2	$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$
$b^2 - 4ac < 0$	Conjugate complex m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$

(B) Repeated Real Roots Case

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

try a solution $y = e^{mx}$ → $(am^2 + bm + c) = 0$ auxiliary equation

$$m_1 = (-b + \sqrt{b^2 - 4ac})/2a$$

$$m_2 = (-b - \sqrt{b^2 - 4ac})/2a$$

→ $b^2 - 4ac = 0$ →

$$m_1 = -b/2a$$

$$m_2 = -b/2a$$

$$e^{m_1 x} = e^{m_2 x} = e^{-\frac{b}{2a}x}$$

$b^2 - 4ac > 0$	Real, distinct m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$
$b^2 - 4ac = 0$	Real, equal m_1, m_2	$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$
$b^2 - 4ac < 0$	Conjugate complex m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$

(C) Complex Roots of the Auxiliary Equation

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

try a solution $y = e^{mx}$ → $(am^2 + bm + c) = 0$ auxiliary equation

$m_1 = (-b + \sqrt{b^2 - 4ac})/2a$ → $m_1 = (-b + \sqrt{4ac - b^2}\, i)/2a$ $\quad y_1 = e^{m_1 x}$

$m_2 = (-b - \sqrt{b^2 - 4ac})/2a$ → $m_2 = (-b - \sqrt{4ac - b^2}\, i)/2a$ $\quad y_2 = e^{m_2 x}$

$b^2 - 4ac > 0$	Real, distinct m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$
$b^2 - 4ac = 0$	Real, equal m_1, m_2	$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$
$b^2 - 4ac < 0$	Conjugate complex m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$

Complex Exponential Conversion

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2 y}{dx^2} + b\frac{dy}{dx} + c\,y = 0$$

$$a\,y'' + b\,y' + c\,y = 0$$

$$m_1 = (-b + \sqrt{b^2 - 4ac})/2a \;\Rightarrow\; m_1 = (-b + \sqrt{4ac - b^2}\; i)/2a = \alpha + i\beta$$

$$m_2 = (-b - \sqrt{b^2 - 4ac})/2a \;\Rightarrow\; m_2 = (-b - \sqrt{4ac - b^2}\; i)/2a = \alpha - i\beta$$

$$y_1 = e^{m_1 x}$$

$$y_2 = e^{m_2 x}$$

$b^2 - 4\text{ac} < 0$ Conjugate complex m_1, m_2

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$$

Pick **two** homogeneous solution

$$y_1 = \{e^{(\alpha + i\beta)x} + e^{(\alpha - i\beta)x}\}/2 = e^{\alpha x}\cos(\beta x) \qquad (C_1 = +1/2, \quad C_2 = +1/2)$$

$$y_2 = \{e^{(\alpha + i\beta)x} - e^{(\alpha - i\beta)x}\}/2i = e^{\alpha x}\sin(\beta x) \qquad (C_1 = +1/2i, \quad C_2 = -1/2i)$$

$$y = C_3 e^{\alpha x}\cos(\beta x) + C_4 e^{\alpha x}\sin(\beta x) = e^{\alpha x}(C_3\cos(\beta x) + C_4\sin(\beta x))$$

Wronskian

Second Order EQ

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

y_1: $e^{(\alpha+i\beta)x}$

y_2: $e^{(\alpha-i\beta)x}$

$$\begin{vmatrix} e^{(\alpha+i\beta)x} & e^{(\alpha-i\beta)x} \\ (e^{(\alpha+i\beta)x})' & (e^{(\alpha-i\beta)x})' \end{vmatrix}$$

$$= \begin{vmatrix} e^{(\alpha+i\beta)x} & e^{(\alpha-i\beta)x} \\ (\alpha+i\beta)e^{(\alpha+i\beta)x} & (\alpha-i\beta)e^{(\alpha-i\beta)x} \end{vmatrix}$$

$$= (\alpha-i\beta)e^{2\alpha x} - (\alpha+i\beta)e^{2\alpha x}$$

$$= (-i2\beta)e^{2\alpha x} \neq 0$$

$$y_3 = \frac{1}{2}y_1 + \frac{1}{2}y_2 \qquad \{e^{(\alpha+i\beta)x} + e^{(\alpha-i\beta)x}\}/2 = e^{\alpha x}\cos(\beta x)$$

$$y_4 = \frac{1}{2i}y_1 - \frac{1}{2i}y_2 \qquad \{e^{(\alpha+i\beta)x} - e^{(\alpha-i\beta)x}\}/2i = e^{\alpha x}\sin(\beta x)$$

$$\begin{vmatrix} e^{\alpha x}\cos(\beta x) & e^{\alpha x}\sin(\beta x) \\ (e^{\alpha x}\cos(\beta x))' & (e^{\alpha x}\sin(\beta x))' \end{vmatrix} = \begin{vmatrix} e^{\alpha x}\cos(\beta x) & e^{\alpha x}\sin(\beta x) \\ (\alpha e^{\alpha x}\cos(\beta x) - \beta e^{\alpha x}\sin(\beta x)) & (\alpha e^{\alpha x}\sin(\beta x) + \beta e^{\alpha x}\cos(\beta x)) \end{vmatrix}$$

$$= e^{\alpha x}\cos(\beta x)(\alpha e^{\alpha x}\sin(\beta x) + \beta e^{\alpha x}\cos(\beta x)) - e^{\alpha x}\sin(\beta x)(\alpha e^{\alpha x}\cos(\beta x) - \beta e^{\alpha x}\sin(\beta x))$$

$$= e^{\alpha x}\cos(\beta x)\beta e^{\alpha x}\cos(\beta x) + e^{\alpha x}\sin(\beta x)\beta e^{\alpha x}\sin(\beta x) = \beta e^{2\alpha x}(\cos^2(\beta x) + \sin^2(\beta x)) = \beta e^{2\alpha x} \neq 0$$

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Wronskian : Linear Independent

Second Order EQ

$$a\frac{d^2 y}{dx^2} + b\frac{dy}{dx} + c\,y = 0$$

y_1: $e^{(\alpha+i\beta)x}$

y_2: $e^{(\alpha-i\beta)x}$

$$\begin{vmatrix} e^{(\alpha+i\beta)x} & e^{(\alpha-i\beta)x} \\ (e^{(\alpha+i\beta)x})' & (e^{(\alpha-i\beta)x})' \end{vmatrix} \neq 0$$

linearly independent ⇒ Fundamental Set of Solutions

$$y_3 = \frac{1}{2}y_1 + \frac{1}{2}y_2 \qquad \{e^{(\alpha+i\beta)x} + e^{(\alpha-i\beta)x}\}/2 = e^{\alpha x}\cos(\beta x)$$

$$y_4 = \frac{1}{2i}y_1 - \frac{1}{2i}y_2 \qquad \{e^{(\alpha+i\beta)x} - e^{(\alpha-i\beta)x}\}/2i = e^{\alpha x}\sin(\beta x)$$

$$\begin{vmatrix} e^{\alpha x}\cos(\beta x) & e^{\alpha x}\sin(\beta x) \\ (e^{\alpha x}\cos(\beta x))' & (e^{\alpha x}\sin(\beta x))' \end{vmatrix} \neq 0$$

linearly independent ⇒ Fundamental Set of Solutions

Fundamental Set Examples (1)

Second Order EQ

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

y_1 $\quad e^{(\alpha+i\beta)x}$

y_2 $\quad e^{(\alpha-i\beta)x}$

$$y_3 = \frac{1}{2}y_1 + \frac{1}{2}y_2$$

$$y_4 = \frac{1}{2i}y_1 - \frac{1}{2i}y_2$$

$$\{e^{(\alpha+i\beta)x} + e^{(\alpha-i\beta)x}\}/2 = e^{\alpha x}\cos(\beta x)$$

$$\{e^{(\alpha+i\beta)x} - e^{(\alpha-i\beta)x}\}/2i = e^{\alpha x}\sin(\beta x)$$

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Fundamental Set Examples (2)

Second Order EQ

$$a\frac{d^2 y}{dx^2} + b\frac{dy}{dx} + c\,y = 0$$

$$y_1 = y_3 + i\, y_4 \qquad e^{(\alpha + i\beta)x}$$

$$y_2 = y_3 - i\, y_4 \qquad e^{(\alpha - i\beta)x}$$

$$y_3 = e^{\alpha x}\cos(\beta x)$$

$$y_4 = e^{\alpha x}\sin(\beta x)$$

$$e^{\alpha x}[\cos(\beta x) + i\sin(\beta x)]$$

$$e^{\alpha x}[\cos(\beta x) - i\sin(\beta x)]$$

General Solution Examples

Second Order EQ

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

linearly independent → Fundamental Set of Solutions

$$\{y_1, \ y_2\} = \{e^{(\alpha+i\beta)x}, \ e^{(\alpha-i\beta)x}\}$$

$$C_1 y_1 + C_2 y_2$$

$$C_1 e^{(\alpha+i\beta)x} + C_2 e^{(\alpha-i\beta)x}$$

→ General Solution

linearly independent → Fundamental Set of Solutions

$$\{y_3, \ y_4\} = \{e^{\alpha x}\cos(\beta x), \ e^{\alpha x}\sin(\beta x)\}$$

$$c_3 y_3 + c_3 y_4$$

$$c_3 e^{\alpha x}\cos(\beta x) + c_4 e^{\alpha x}\sin(\beta x)$$
$$= e^{\alpha x}(c_3\cos(\beta x) + c_4\sin(\beta x))$$

→ General Solution

General Solutions

- Homogeneous Equation

- Non-homogeneous Equation

Homogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

auxiliary equation

$$(am^2 + bm + c) = 0$$

$$m_1 = \left(-b + \sqrt{b^2 - 4ac}\right)/2a$$
$$m_2 = \left(-b - \sqrt{b^2 - 4ac}\right)/2a$$

(A)	$b^2 - 4ac > 0$	Real, distinct m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$
(B)	$b^2 - 4ac = 0$	Real, equal m_1, m_2	$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$
(C)	$b^2 - 4ac < 0$	Conjugate complex m_1, m_2	$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$

General Solution – Nonhomogeneous Equations

Nonhomogeneous Second Order DEs with Constant Coefficients

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = g(x)$$

$$ay'' + by' + cy = g(x)$$

$$y = \underbrace{c_1 y_1 + c_2 y_2}_{\text{complementary function}} + \underbrace{y_p}_{\text{particular solution}}$$

complementary function **particular solution**

The general solution for a nonhomogeneous linear n-th order differential equation

Homogeneous Second Order DEs with Constant Coefficients

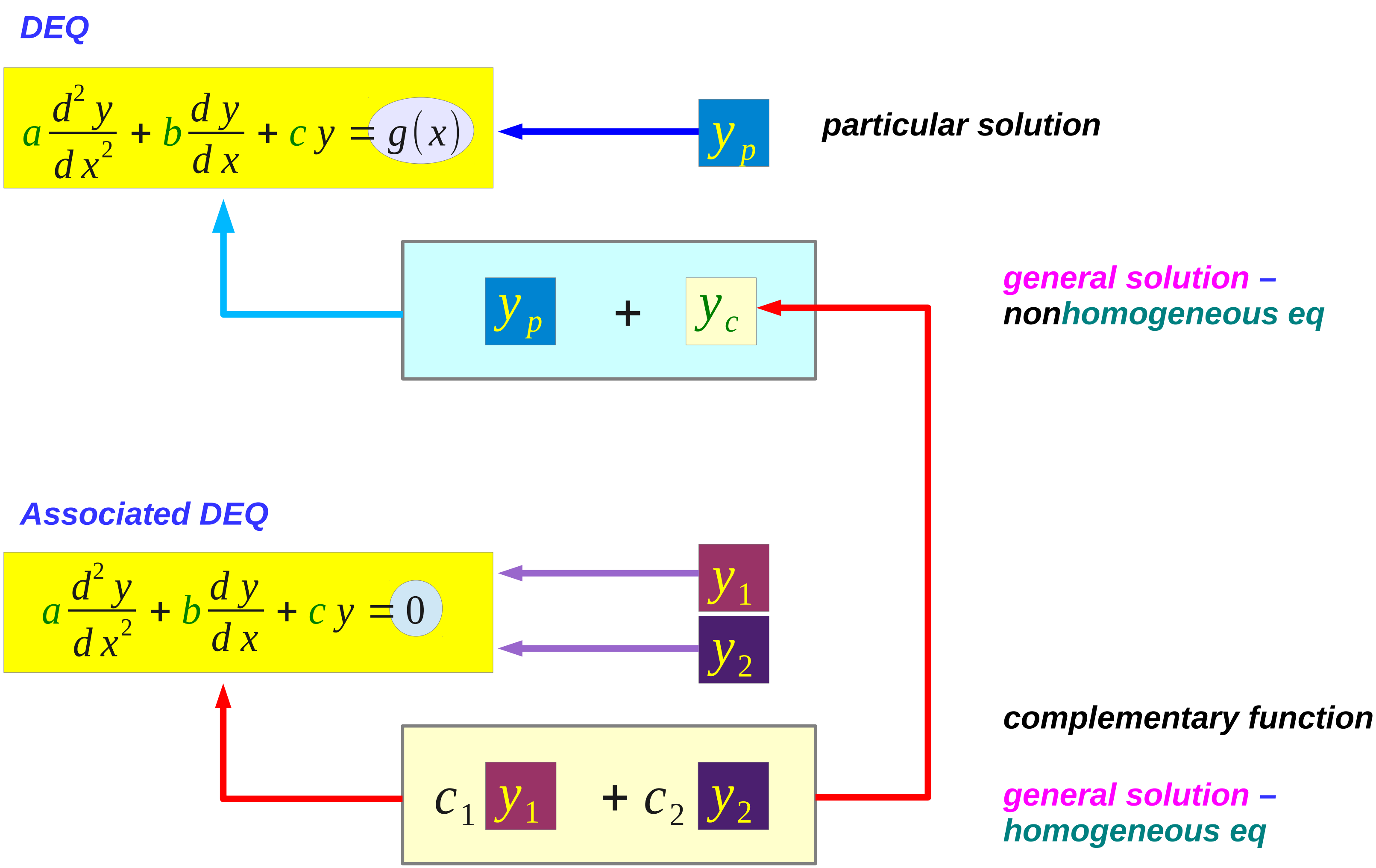
$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

$$ay'' + by' + cy = 0$$

$$y = c_1 y_1 + c_2 y_2$$

The general solution for a homogeneous linear n-th order differential equation

Complementary Function



y_c and y_p

DEQ

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c\,y = g(x) \quad \leftarrow \quad y_p$$

particular solution

$$y_c + y_p$$

general solution – nonhomogeneous eq

$$a\frac{d^2y_c}{dx^2} + b\frac{dy_c}{dx} + c\,y_c \Rightarrow 0$$

many such complementary functions

c_i ***many possible coefficients***

$$a\frac{d^2y_p}{dx^2} + b\frac{dy_p}{dx} + c\,y_p \Rightarrow g(x)$$

only one particular functions

coefficients can be determined

References

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