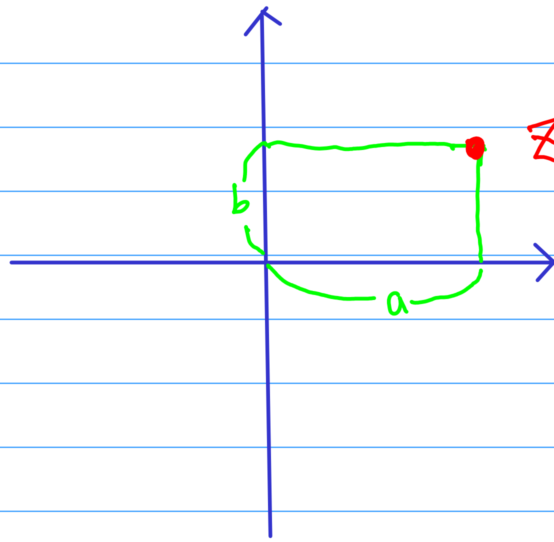
Complex Numbers (H.1)

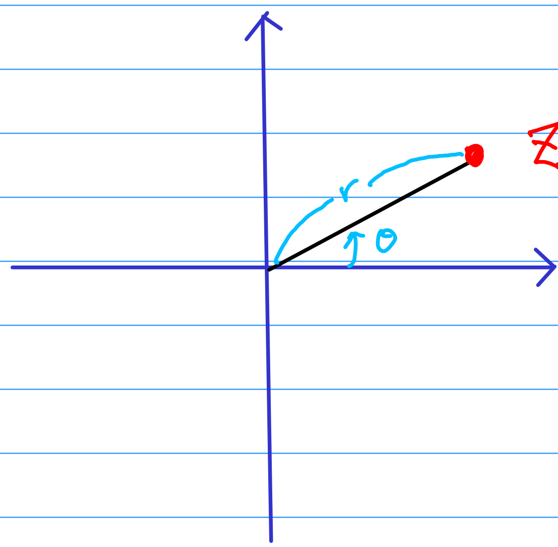
20150112

Rectangular Form

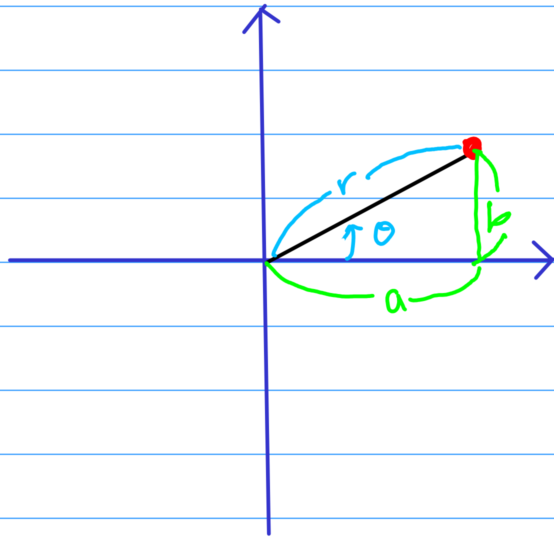


$z = a + bi$

Polar Form



$z = r \cdot e^{j\theta}$

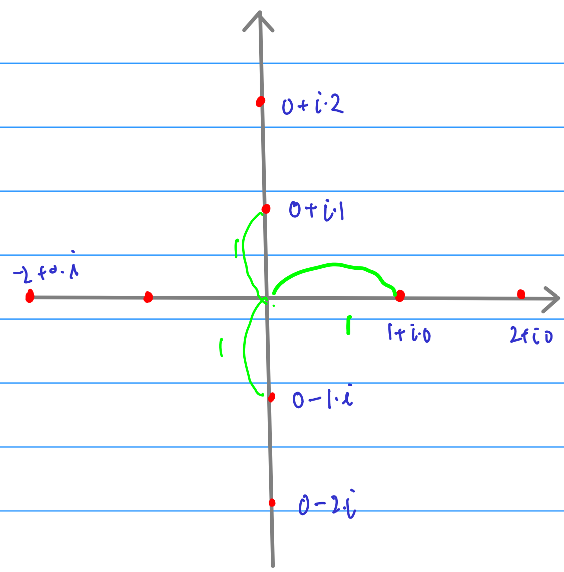


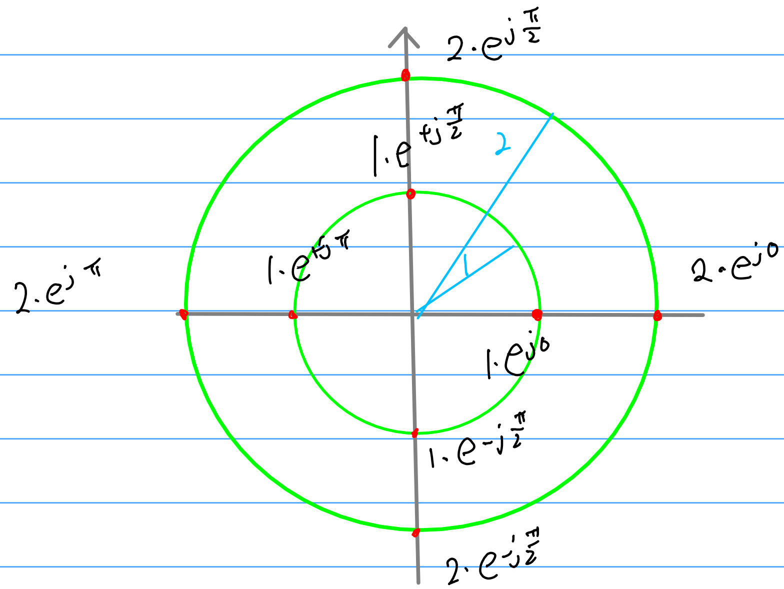
$$\boxed{\begin{aligned} r\cos\theta &\Rightarrow a \\ r\sin\theta &\Rightarrow b \end{aligned}}$$

$$\boxed{\begin{aligned} \sqrt{a^2+b^2} &\Rightarrow r \\ \frac{b}{a} &\Rightarrow \tan\theta \end{aligned}}$$

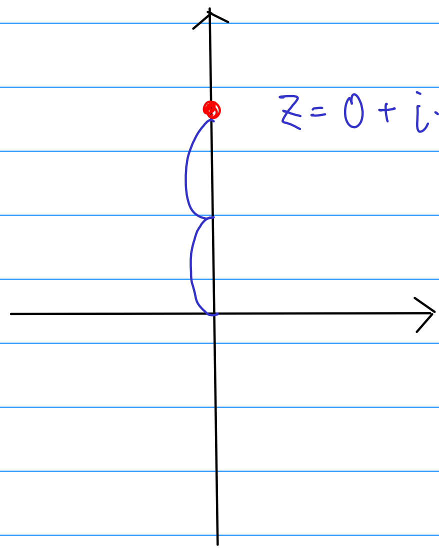
polar $\Rightarrow$ rect

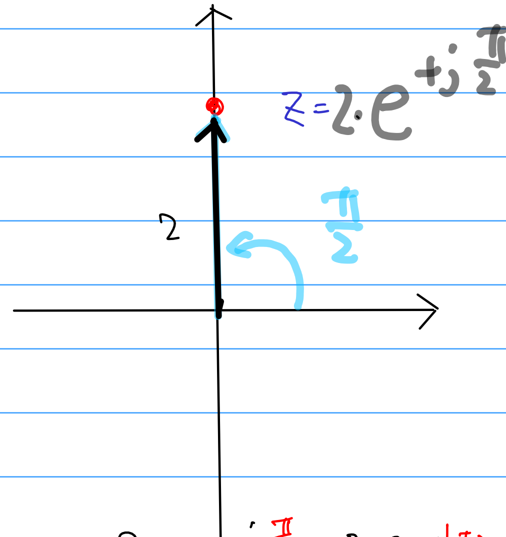
rect $\Rightarrow$ polar


0+i·2
0+i·1
-2+0·i
1+i·0
2+i·0
0-1·i
0-2i


2·e^{jπ/2}
1·e^{+jπ/2}
2
1
1·e^{±jπ}
2·e^{jπ}
2·e^{j0}
1·e^{j0}
1·e^{-jπ/2}
2·e^{-jπ/2}

Converting z into the polar form



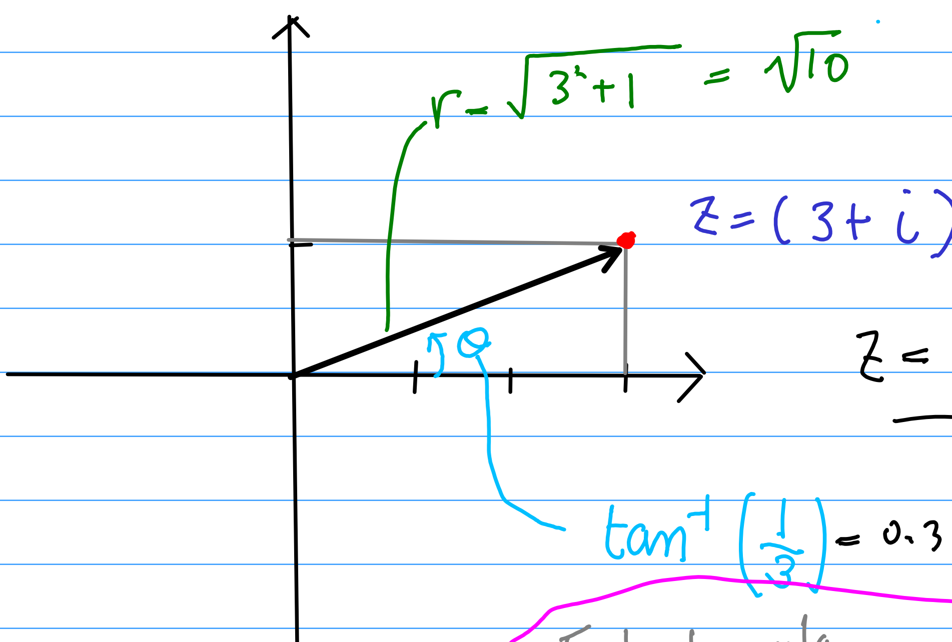


$$2 \cdot e^{j\frac{\pi}{2}} = 2 \cdot \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right)$$

$$= 0 + i2$$

1. connect the origin and a complex number
2. measure the distance $\Rightarrow r$
3. measure the angle $\Rightarrow \theta$

$$\boxed{re^{j\theta}}$$



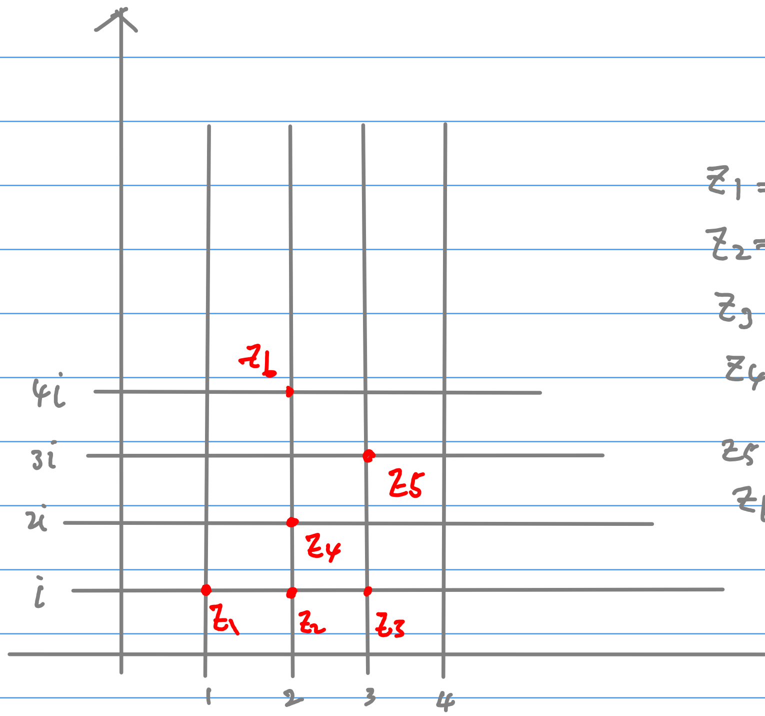
$$z = \sqrt{10} \cdot e^{i\,0.321}$$

$$\tan^{-1}\left(\frac{1}{3}\right) = 0.321 \text{ rad}$$

Euler formula

$$\boxed{3 + i}$$

$$\sqrt{10}\left(\cos(0.321) + i \sin(0.321)\right) = \sqrt{10}\cos(0.321) + i\sqrt{10}\sin(0.321)$$



$z_1 = 1 + i = \sqrt{2}\, e^{+j\frac{\pi}{4}}$

$z_2 = 2 + i$

$z_3 = 3 + i$

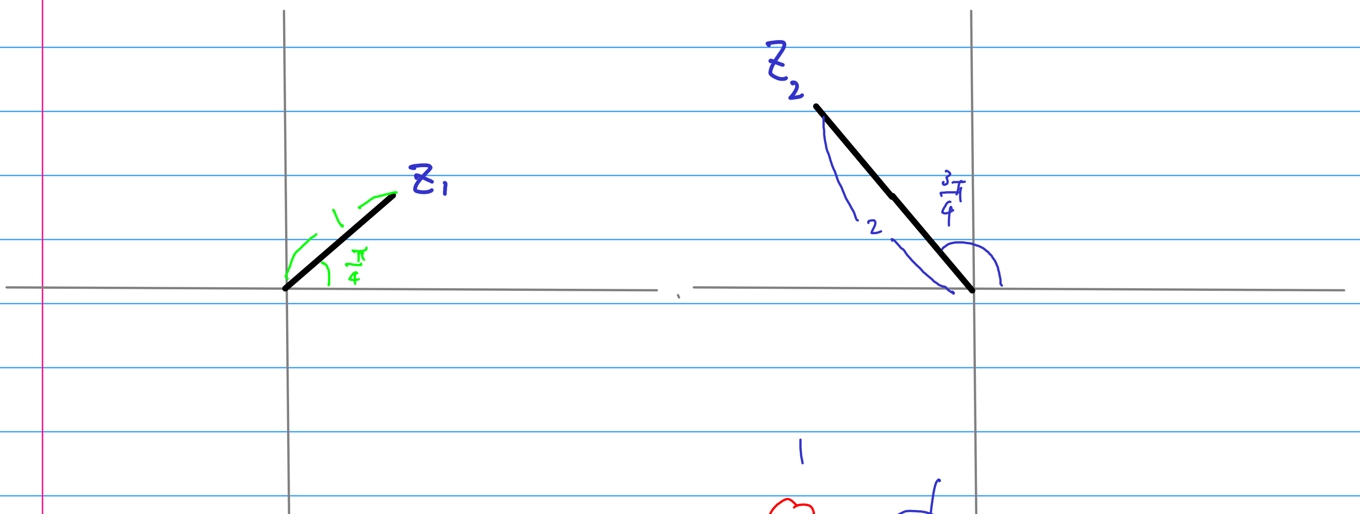
$z_4 = 2 + 2i = 2\sqrt{2}\, e^{+j\frac{\pi}{4}}$

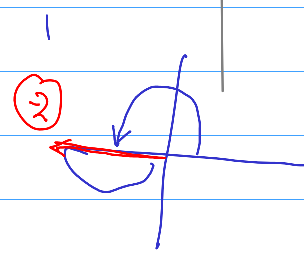
$z_5 = 3 + 3i = 3\sqrt{2}\, e^{+j\frac{\pi}{4}}$

$z_6 = 2 + 4i$

$$z_1 = 1 \cdot e^{j\frac{\pi}{4}} = 1 \cdot \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$z_2 = 2 \cdot e^{+j\frac{3}{4}\pi} = 2\left(\cos\left(\tfrac{3}{4}\pi\right) + i\sin\left(\tfrac{3}{4}\pi\right)\right)$$



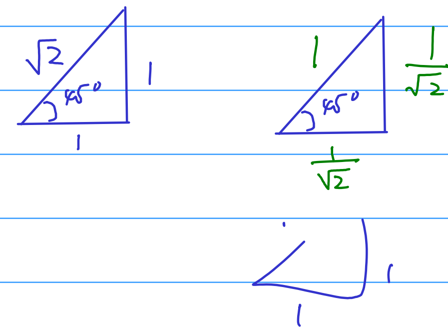


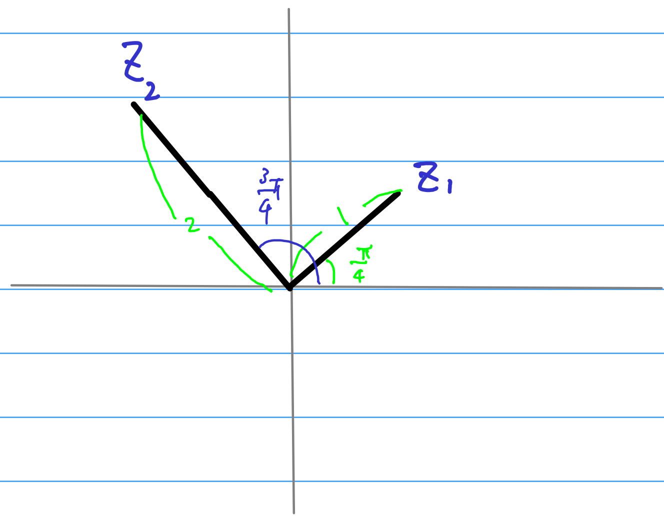
$$z_1 \cdot z_2 = \left(1 \cdot e^{j\frac{\pi}{4}}\right)\left(2 \cdot e^{j\frac{3}{4}\pi}\right)$$

$$= (1 \cdot 2) \cdot \left(e^{j\frac{\pi}{4}} \cdot e^{j\frac{3}{4}\pi}\right)$$

$$= 2 \cdot e^{j\left(\frac{\pi}{4} + \frac{3}{4}\pi\right)} = 2 \cdot e^{j\pi} = -2$$

$$= 2\left(\cos(\pi) + i\sin(\pi)\right)$$





$$z_1 = 1\cdot e^{i\frac{\pi}{4}} = 1\cdot\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$$z_2 = 2\cdot e^{+j\frac{3}{4}\pi} = 2\left(\cos\left(\frac{3}{4}\pi\right) + i\sin\left(\frac{3}{4}\pi\right)\right) = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$$z_1\cdot z_2 = 2\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\cdot 1\cdot\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= 2\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\left(+\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= 2\left(-\left(\frac{\sqrt{2}}{2}\right)^2 + \left(i\frac{\sqrt{2}}{2}\right)^2\right)$$

$$= 2\left(-\frac{2}{4} + (-1)\left(\frac{2}{4}\right)\right)$$

$$= 2\left(-\frac{4}{4}\right) = -2$$

$$(A+B)(A-B) = A^2 - B^2$$

$$z = a+bi$$

$$\bar{z} = a-bi$$

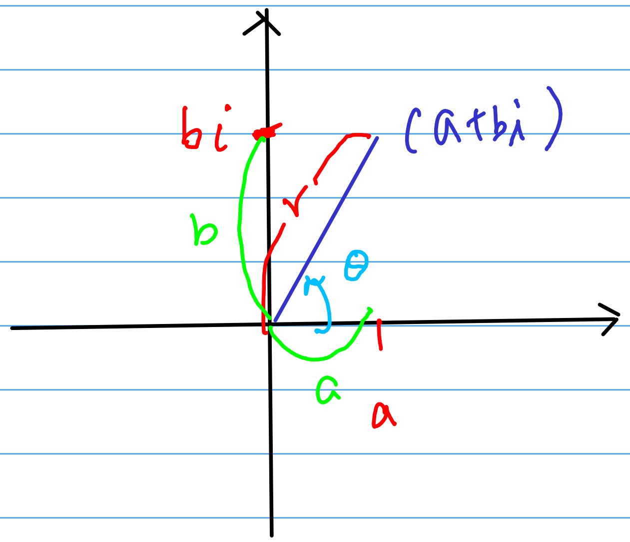
$$z\bar{z} = (a+bi)\cdot(a-bi)$$

$$= \boxed{a}^2 - \boxed{bi}^2$$

$$= a^2 - b^2(-1)$$

$$= a^2+b^2 \Rightarrow |z|^2$$

$$r = \sqrt{a^2+b^2} = abs(z) = |z|$$



$$\tan\theta = \frac{b}{a}$$

$$\theta \sim arg(z)$$

Rectangular

$$Z = a + bi \qquad \bar{z} = z^* = a - bi$$

$$w = c + di \qquad \bar{w} = w^* = c - di$$

$$\frac{1}{z} = (\text{Real part}) + i\,(\text{Imag Part})$$

$$\frac{1}{z} = \frac{1}{a+bi}$$

$$\frac{1}{z} \times \left(\frac{\bar{z}}{\bar{z}}\right) = \frac{\bar{z}}{z\bar{z}} = \frac{a-bi}{a^2+b^2}$$

$$\frac{1}{z} = \left(\frac{a}{a^2+b^2}\right) + i\left(\frac{-b}{a^2+b^2}\right)$$

polar

$$z = a + bi = r_1 e^{+i\theta_1} \qquad \left(r_1 = \sqrt{a^2+b^2} \quad \tan\theta_1 = \frac{b}{a} \right.$$

$$w = c + di = r_2 e^{+i\theta_2} \qquad \left. r_2 = \sqrt{c^2+d^2} \quad \tan\theta_2 = \frac{d}{c} \right)$$

$$\bar{z}_1 = z^* = a - bi = r_1 e^{-i\theta_1}$$

$$\bar{w} = w^* = c - di = r_2 e^{-i\theta_2}$$

$$z \cdot w = r_1 e^{+i\theta_1} \cdot r_2 e^{+i\theta_2} = r_1 r_2 e^{+i\theta_1} e^{+i\theta_2}$$

$$= r_1 r_2 e^{+i(\theta_1+\theta_2)}$$

$$\boxed{\begin{array}{ccc} r_1 & \cdot & r_2 \\ \theta_1 & + & \theta_2 \end{array}} \quad \begin{array}{l} \text{mult} \\ \text{add} \end{array}$$

$$z / w = \frac{r_1 e^{+i\theta_1}}{r_2 e^{+i\theta_2}} = \frac{r_1}{r_2} e^{+i(\theta_1-\theta_2)}$$

$$r_1 = \mathrm{abs}(z) = |z| \qquad r_2 = \mathrm{abs}(w) = |w|$$

$$\theta_1 = \arg(z) \qquad \theta_2 = \arg(w)$$

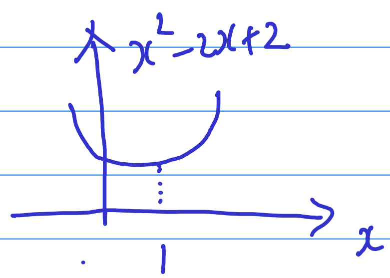
$f(x) = x^3 - x^2 + 2 \Rightarrow$

$f(x) = 0$ 3차 방정식

```
(%i7) f(x) := x^3 - x^2 + 2;
(%o7) f(x):=x^3-x^2+2
(%i10) factor(f(x));
(%o10) (x+1)(x^2-2x+2)
```

$$f(x) = (x+1)(x^2-2x+2) = 0$$

$$\begin{cases} x+1 = 0 \\ x^2-2x+2 \neq 0 \end{cases} \longrightarrow \boxed{x = -1}$$



$$D = b^2 - 4ac = 4 - 4\cdot 1\cdot 2$$
$$= -4 < 0$$

$D < 0$

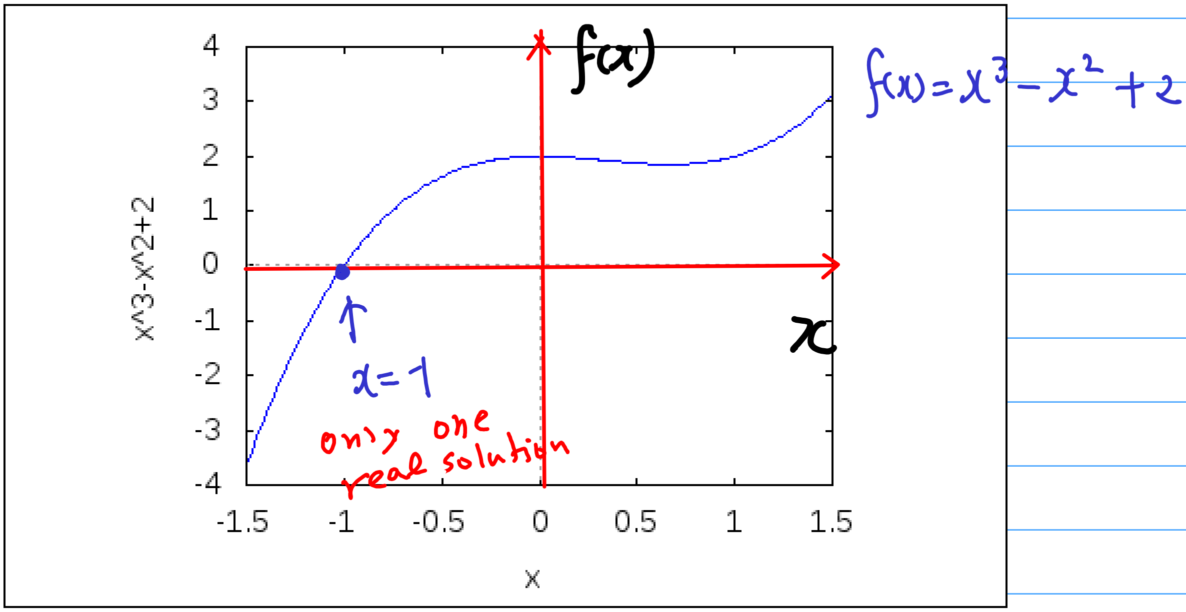
$\boxed{x^2 - 2x + 2 > 0}$

always

$$f(x) = x^3 - x^2 + 2 \qquad x: \text{real}$$

```
(%i14) wxplot2d([f(x)], [x,-1.5,1.5])$
```

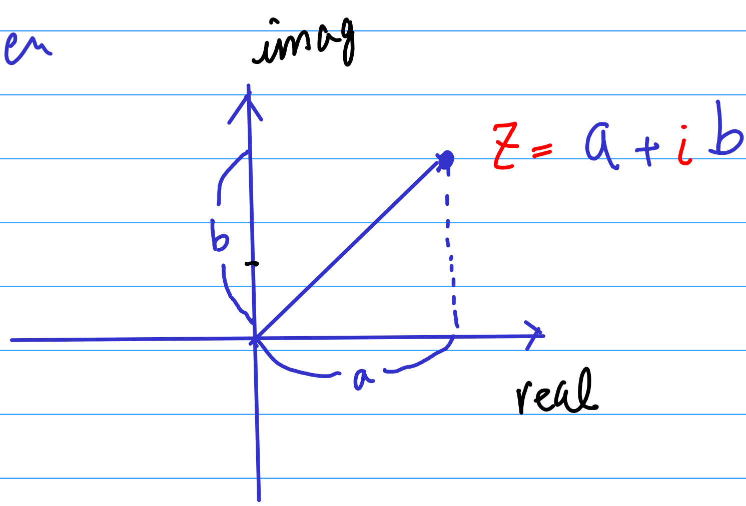
(%t14)



Complex number

i

$i^2 = -1$

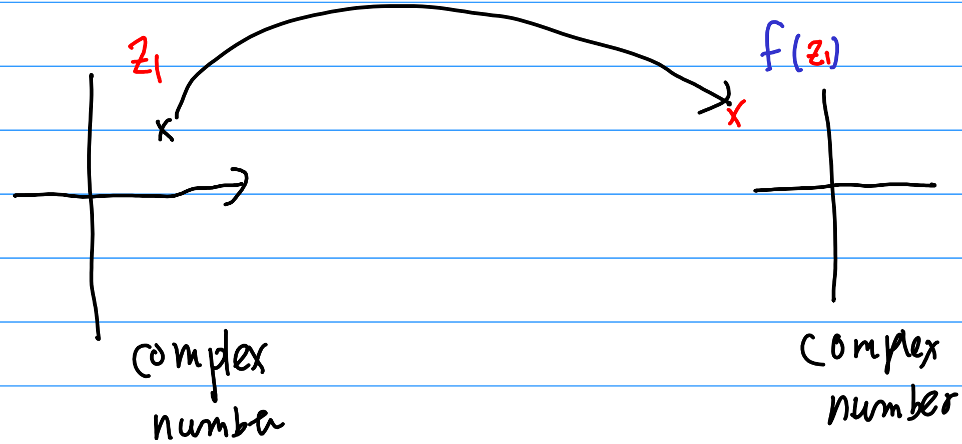


$$f(z) = z^3 - z^2 + 2 \qquad z: \text{complex number}$$

$$f(x) = x^3 - x^2 + 2 = (x+1)(x^2+2x+2)$$

$$f(z) = z^3 - z^2 + 2 = (z+1)(z^2+2z+2)$$

$$\begin{cases} (z+1)=0 & \text{one real sol} \\ \boxed{(z^2+2z+2)=0} & \text{two complex sol's} \end{cases}$$



완전제곱식

$$z^2 - 2z + 2 = 0$$

$$z^2 - 2z + 1 + 1 = 0$$

$$(z-1)^2 + 1 = 0$$

$$(z-1)^2 = -1$$

$$(z-1) = \pm\sqrt{-1} = \pm i$$

$$\begin{cases} \boxed{z = 1 \pm i} \\ \boxed{z = -1} \end{cases}$$

근의 공식

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{+2 \pm \sqrt{4-4\cdot 8}}{2}$$

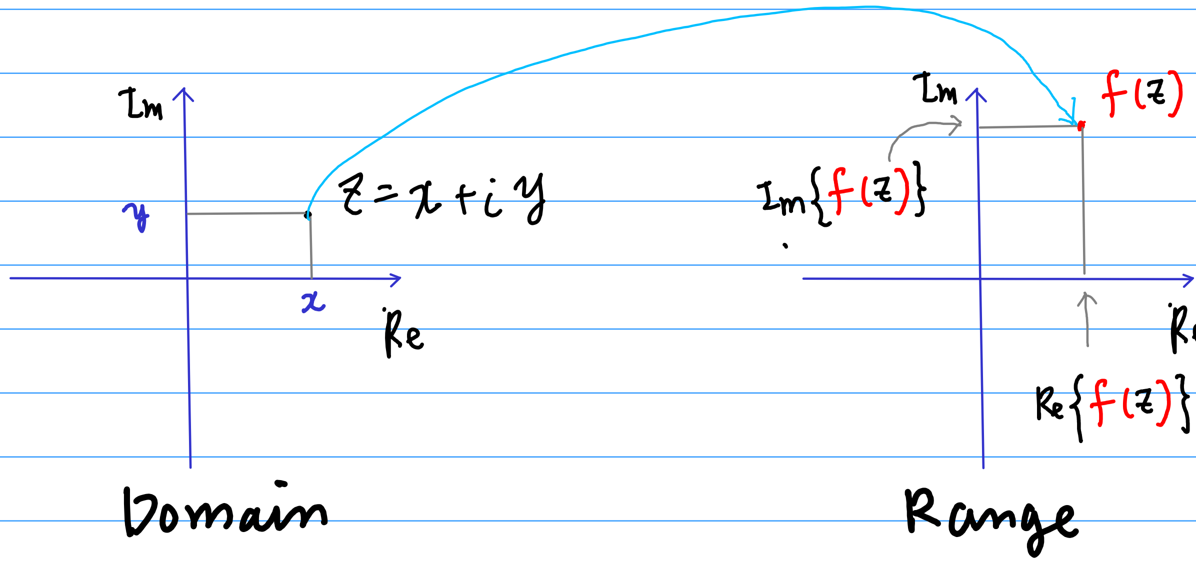
$$= \frac{+2 \pm \sqrt{4(-1)}}{2}$$

$$= \frac{+2 \pm 2\sqrt{-1}}{2}$$

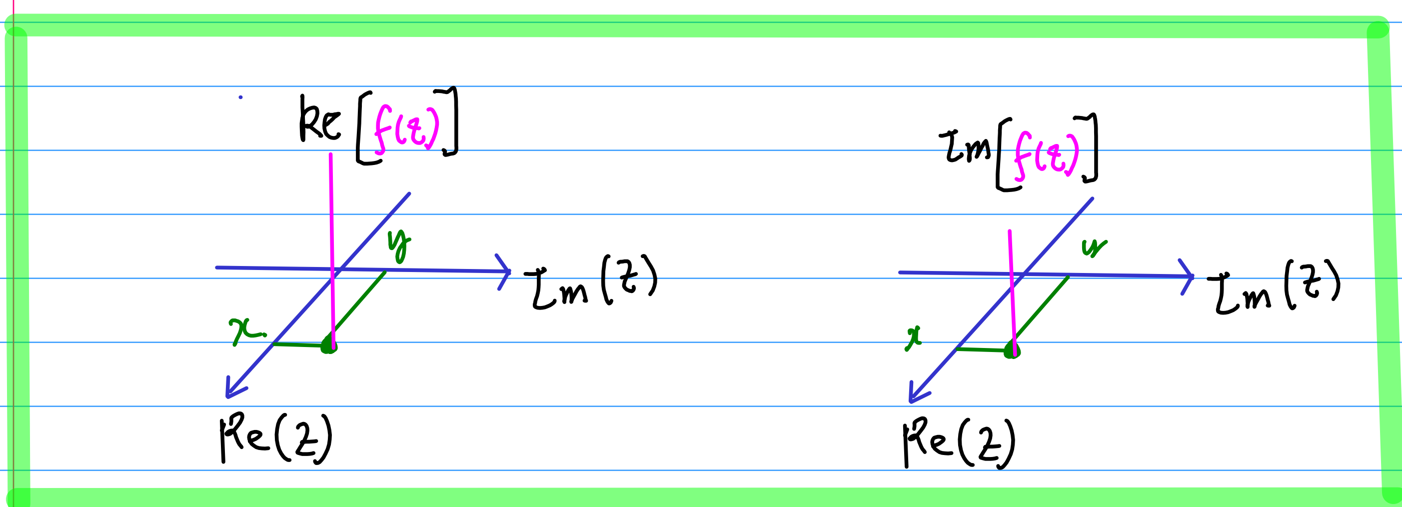
$$= \frac{+2 \pm 2i}{2}$$

$$= \boxed{1 \pm i}$$

Choice 1

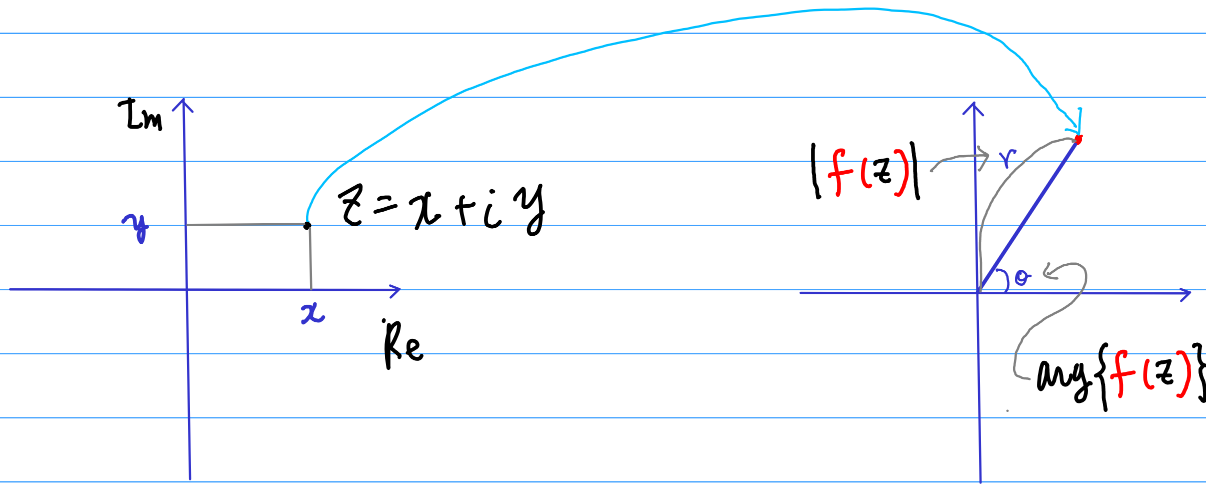


Im
y
z = x + i y
x
Re
Domain
Im
f(z)
Im{f(z)}
Re
Re{f(z)}
Range



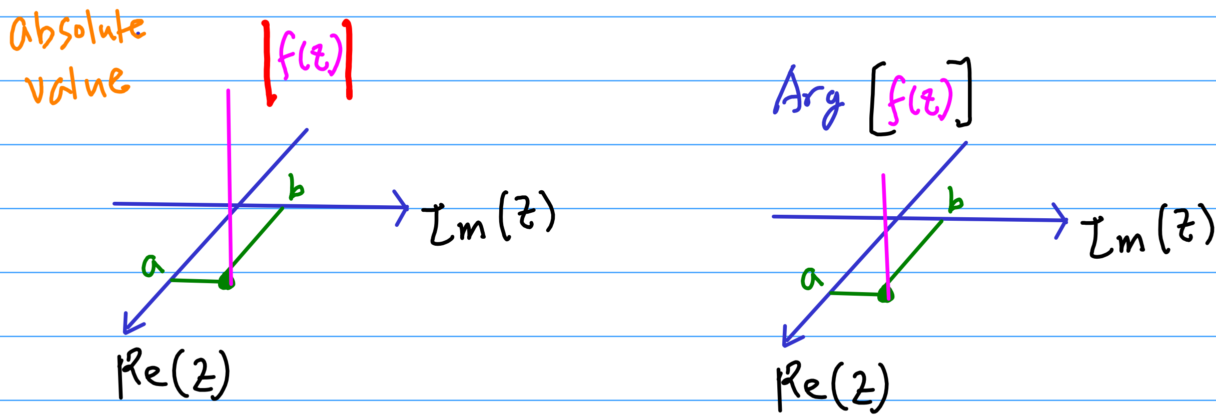
Re[f(z)]
y
Im(z)
x
Re(z)
Im[f(z)]
y
Im(z)
x
Re(z)

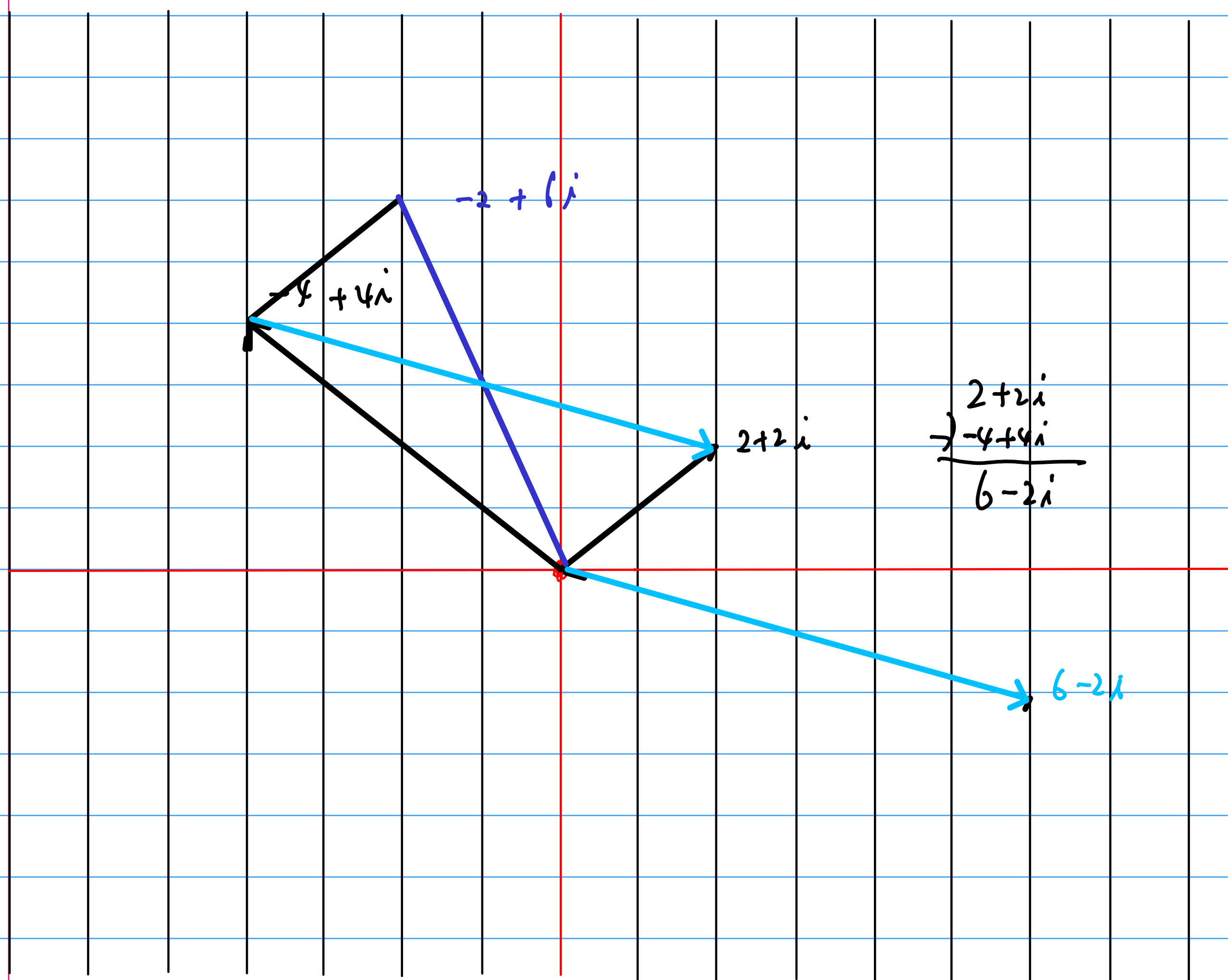
Choice 2



Domain

Range

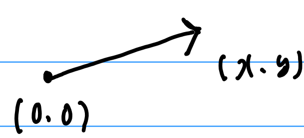




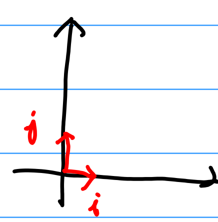
R^2에 어떤 점을 둘 시 $P(x, y)$

① Ordered Pair (x, y) 2개의 real 숫자.

② Vector



standard unit vector



ⓐ component notation $\langle x, y \rangle$

ⓑ $x\vec{i} + y\vec{j}$

③ $x + iy$ Complex number

$$\boxed{z = x + iy}$$

$$f(z) = z^3 - z^2 + 2$$

$$= (x+iy)^3 - (x+iy)^2 + 2$$

$$= (x+iy)^2\left((x+iy) - 1\right) + 2$$

$$= \left(x^2 + 2ixy + (iy)^2\right)(x-1+iy) + 2$$

$$= (x^2 - y^2 + i2xy)(x-1+iy) + 2$$

$$= \left(x^3 - x^2 + 2 - 3xy^2 + y^2\right) + i\left(-y^3 + 3x^2y - 2xy\right)$$

```
(%i23) expand(f(z));
(%o23) -%i y^3-3 x y^2+y^2+3 %i x^2 y-2 %i x y+x^3-x^2+2
```

$$f(z) = f(x+iy)$$

$$= \left(x^3 - x^2 + 2 - 3xy^2 + y^2\right) + i\left(-y^3 + 3x^2y - 2xy\right)$$

$$= u(x,y) + i\,v(x,y)$$

$$= u + i\,v$$

$$f(z) = \boxed{x^3 - x^2 + 2 \quad - 3xy^2 + y^2} + i\,\boxed{\left(-y^3 + 3x^2 y \quad - 2xy\right)}$$

$$= \quad u \quad + i\, v$$

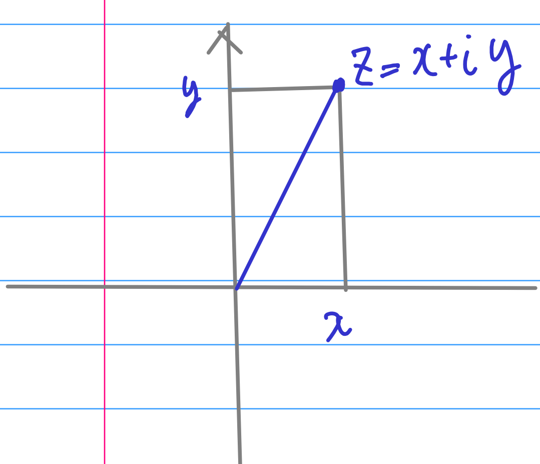
$$= \quad u(x, y) \quad + i \quad v(x, y)$$

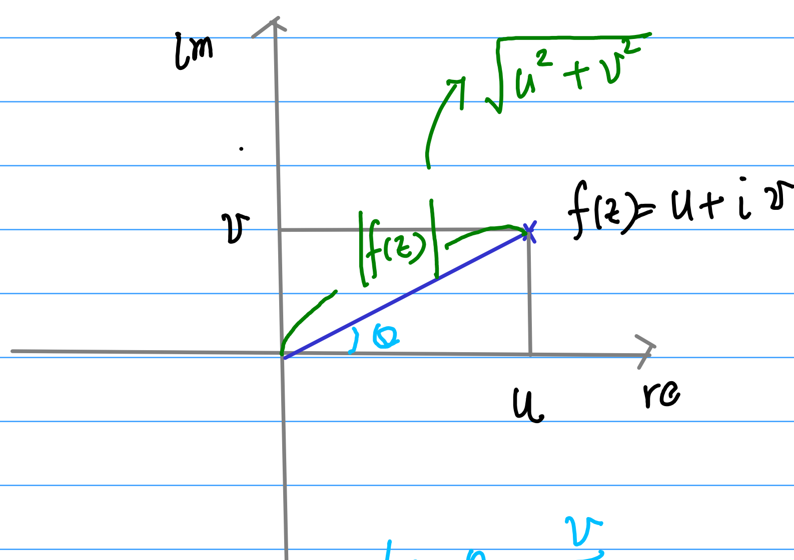
$$\text{real}\,(f(z)) = u$$

$$\text{imag}\,(f(z)) = v$$

$|f(z)|$ $\qquad$ $$\text{abs}\,(f(z)) = \sqrt{u^2 + v^2}$$

$\angle f(z)$ $\qquad$ $$\text{arg}\,(f(z)) = \theta, \quad \tan\theta = \frac{v}{u}$$





$$\tan\theta = \frac{v}{u}$$

$$f(z) = \boxed{x^3 - x^2 + 2 \quad - 3xy^2 + y^2} + i\boxed{\left(-y^3 + 3x^2y \quad - 2xy\right)}$$

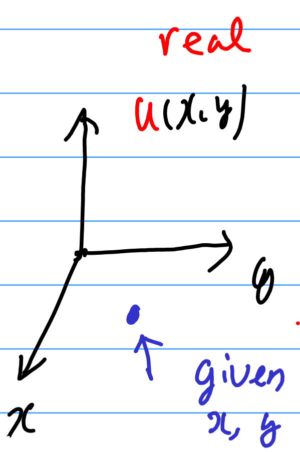
$$= \quad u \quad + i \, v$$

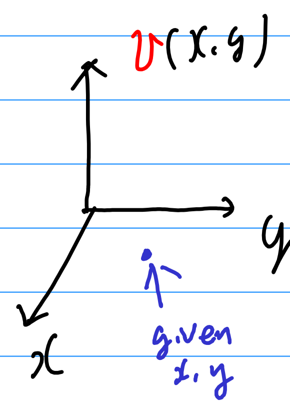
$$= \quad u(x,y) \quad + i \quad v(x,y)$$

$$z = x + iy \qquad\qquad f(z) = u(x,y) + i\,v(x,y)$$

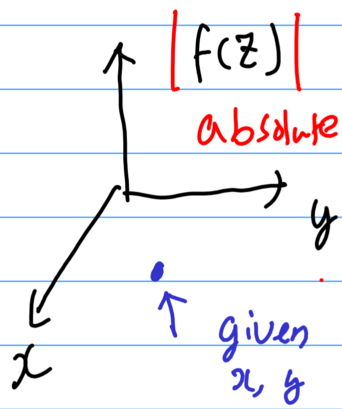
given x, y

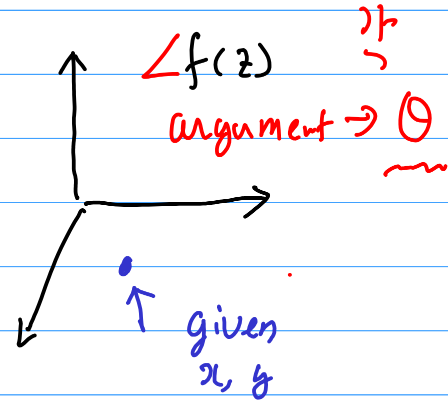
Choice ①



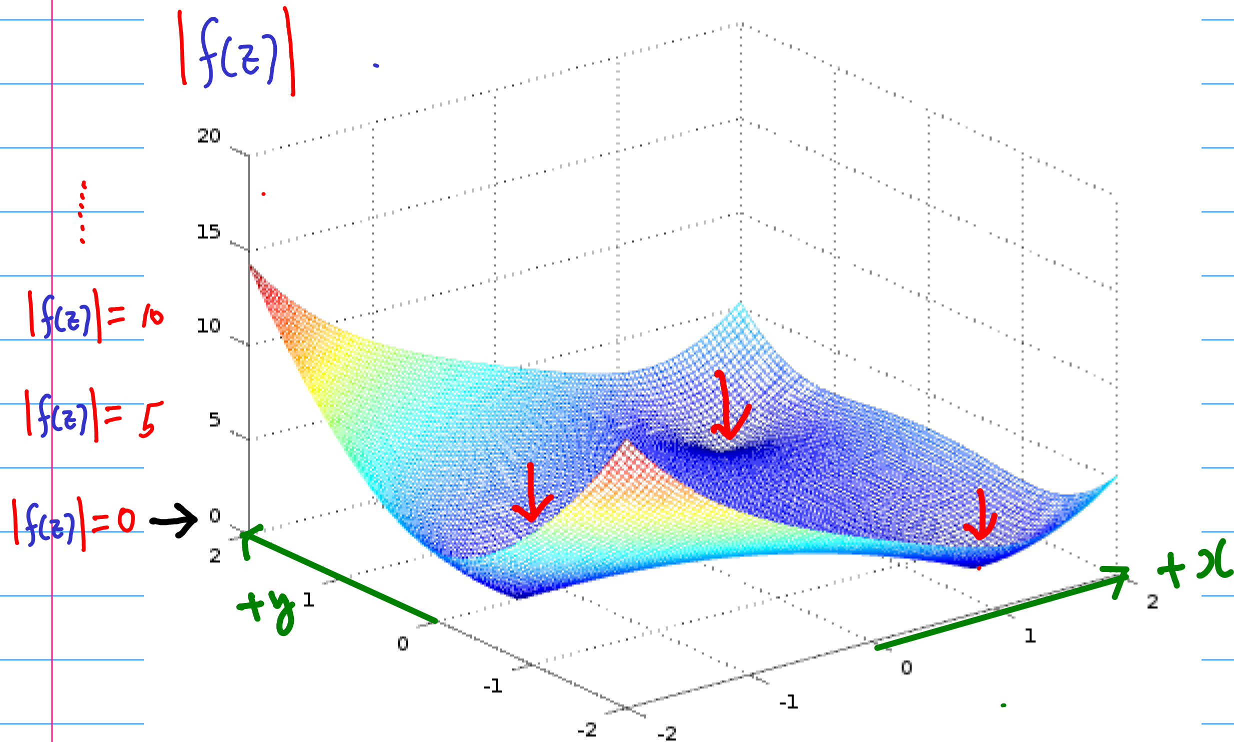


Choice ②

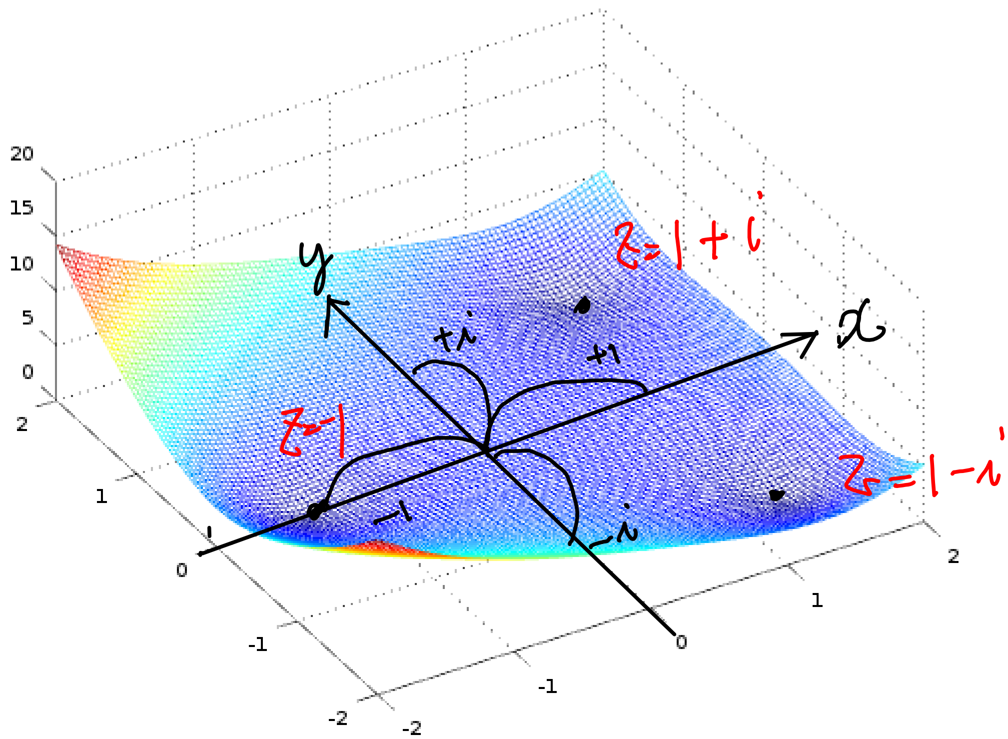


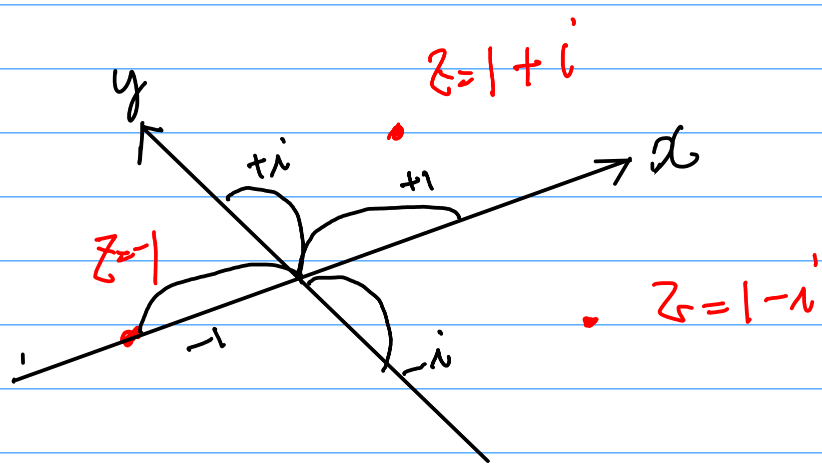


$$f(z) = z^3 - z^2 + 2$$

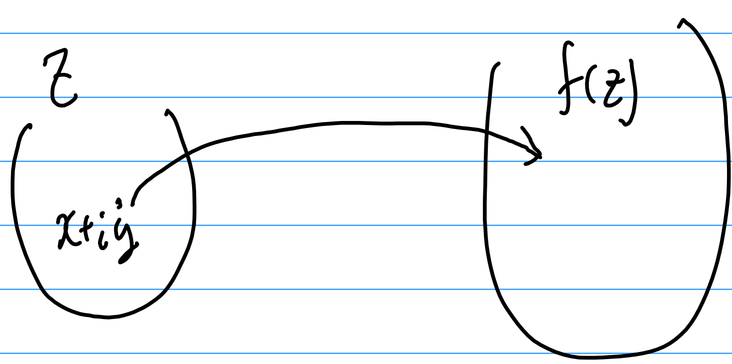


$z = -1,\ 1\pm$

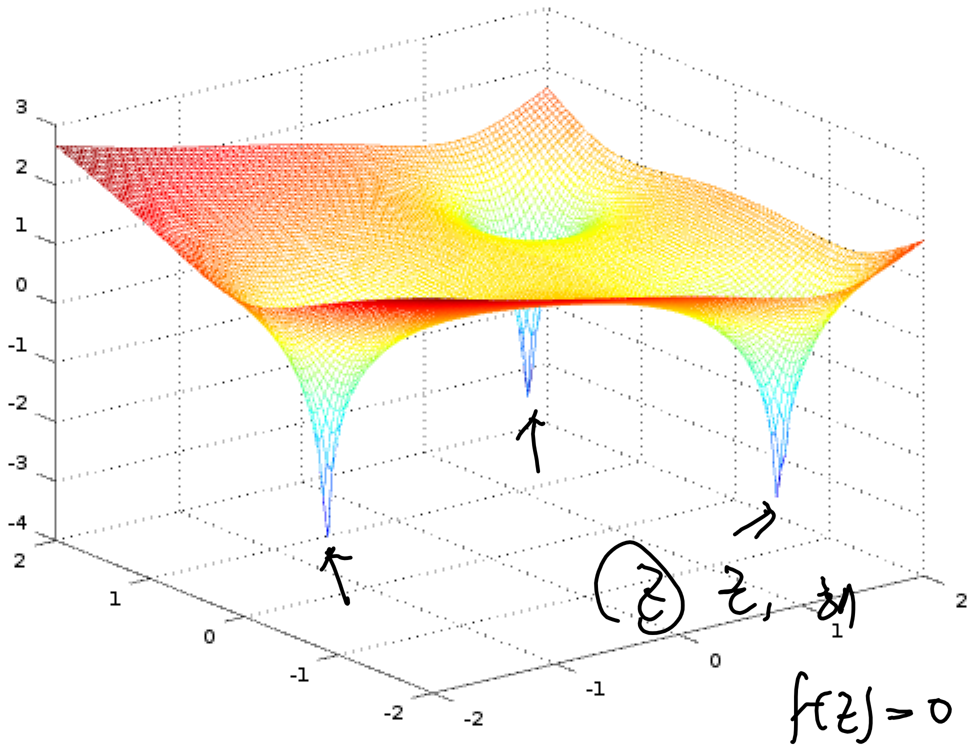




→ Domain



$|f(z)|$



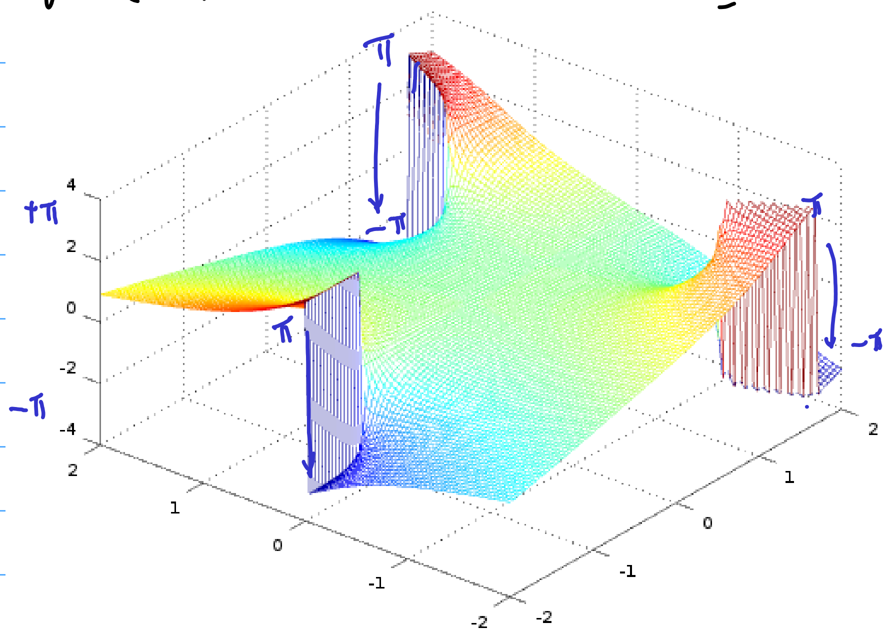
③ z, 해

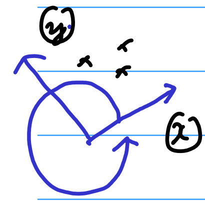
$$f(z) = 0$$

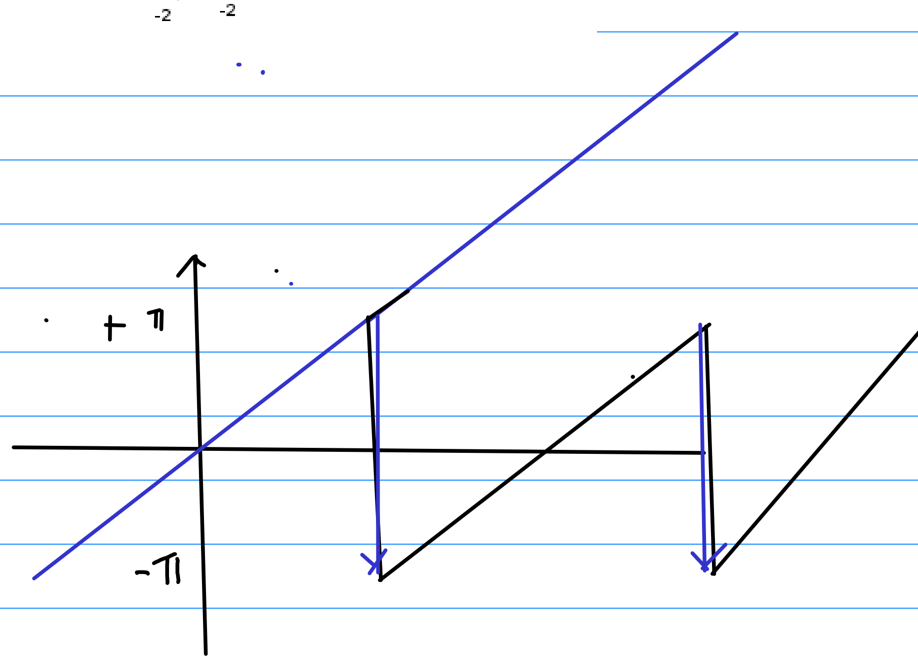
$$|f(z)| = |0| = 0$$

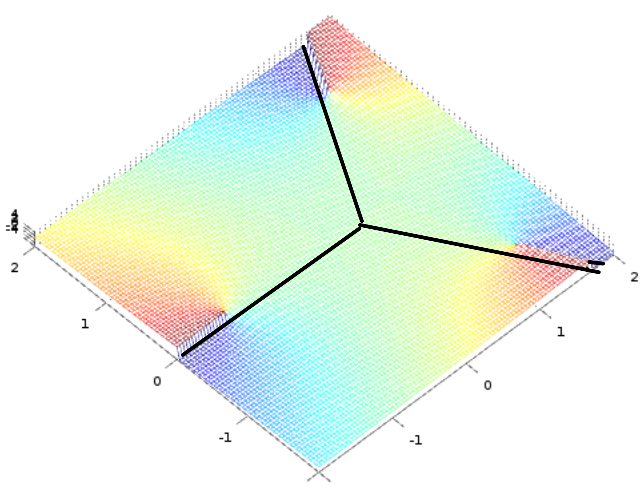
```
octave:35> mesh(xx, yy, abs(f))
octave:36> x = linspace(-2, 2, 101);
octave:37> y = linspace(-2, 2, 101);
octave:38> [xx, yy] = meshgrid(x, y);
octave:39> mesh(xx, yy, abs(f))
octave:40> mesh(xx, yy, log(abs(f)))
```

$\arg\left(f(z)\right) \longrightarrow \theta$







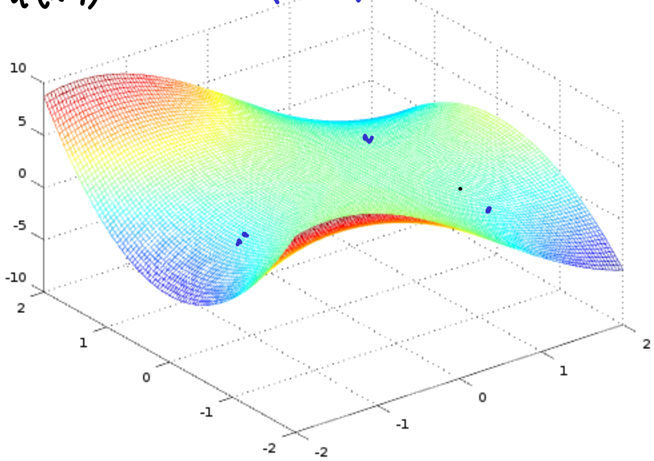


$$f(z) = u + iv$$

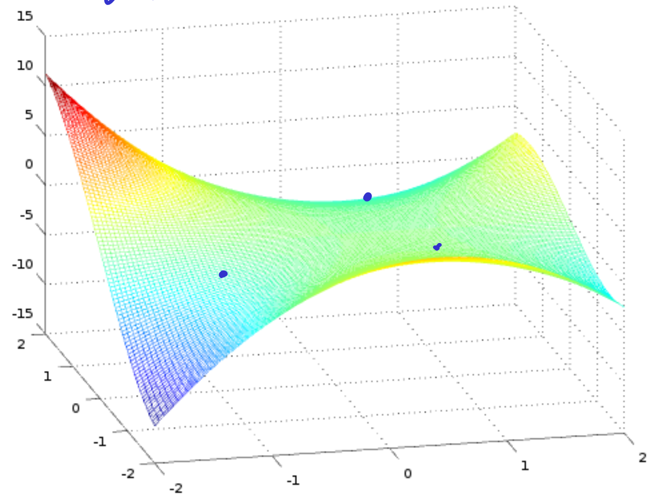
$$= \underline{u(x,y)} + i\,\underline{v(x,y)}$$

$x+iy$

$u(x,y) =$ real$(f(z))$

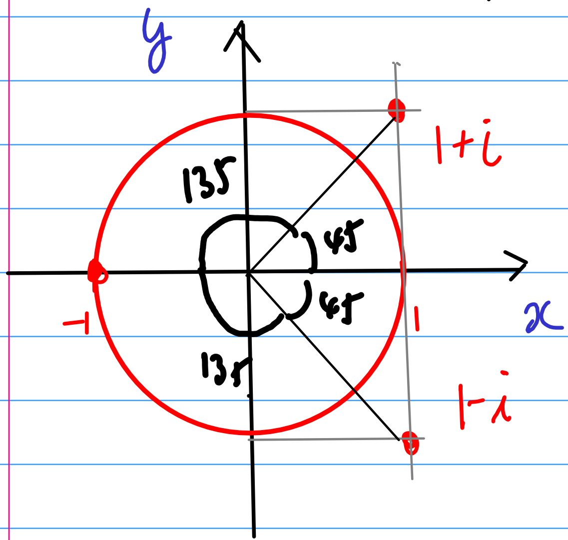


$v(x,y) =$ imag$(f(z))$



$$f(z) = \boxed{x^3 - x^2 + 2 \quad - 3xy^2 + y^2} + i\boxed{\left(-y^3 + 3x^2 y \quad - 2xy\right)}$$

$$z = -1,\ 1 \pm i$$



$f(-1) = 0$

$f(1+i) = 0$

$f(1-i) = 0$

$$f(-1) = \boxed{-1^3 - 1^2 + 2 \quad - 3x0^2 + 0^2} + i\boxed{\left(-0^3 + 3x^2 0 \quad - 2x0\right)} = 0$$

$f(1+i) = 0$

$f(1-i) = 0$

$f(i) = 3 - i$

$f(-1) = 0$

$f(-i) = 1 - i$

```
(%i1) f(z) := z^3 - z^2 + 2;
(%o1) f(z):=z^3-z^2+2
(%i4) ratsimp(f(1+%i));
(%o4) 0
(%i5) ratsimp(f(1-%i));
(%o5) 0
(%i6) ratsimp(f(%i));
(%o6) 3-%i
(%i7) ratsimp(f(-1));
(%o7) 0
(%i8) ratsimp(f(-i));
(%o8) -i^3-i^2+2
```

$-1 + 2$

$1 -$

Powers of Z

$z, z^2, z^3, \ldots$

$x^2 = x^1 \cdot x^1 = x^{1+1}$

$x^3 = \underbrace{x \cdot x \cdot x}_{3} = x^{1+1+1} = x^3$

real num x

complex num z

$z = r e^{+j\theta}$

$z^2 = (r \cdot e^{j\theta}) \cdot (r \cdot e^{j\theta}) = r \cdot r \cdot e^{j\theta} \cdot e^{j\theta} = r^2 \cdot e^{j\theta + j\theta}$

$z^2 = r^2 e^{j2\theta}$

polar form : easy

rectangular form

$z = (a + bi)$

$z^2 = (a+bi)(a+bi) = a^2 - b^2 + 2ab\,i$ ..... difficult to calculate

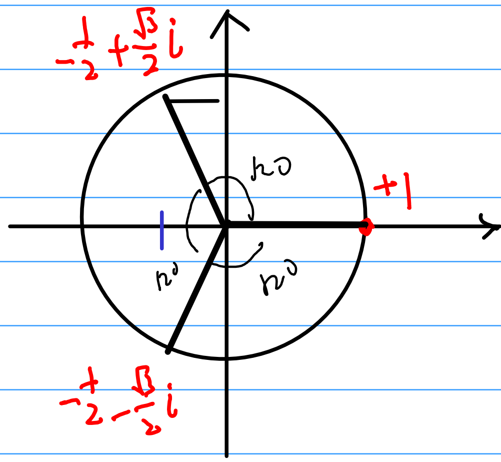
$z^3 = 1$ $\qquad f(z) = z^3 - 1 = 0$.

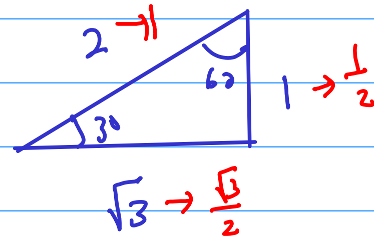
$(z-1)\cdot(z^2 + z + 1) = 0$

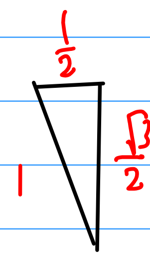
$(z^2 + z + \frac{1}{4}) + \frac{3}{4} = 0$

$(z + \frac{1}{2})^2 = -\frac{3}{4}$ $\qquad (z + \frac{1}{2}) = \pm \frac{\sqrt{3}}{2} i$ $\qquad z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$|z^3| = |1| = 1$







solve

$z^3 = 1 \qquad z = \sqrt[3]{1} \qquad\qquad z^4 = 1 \qquad z = \sqrt[4]{1}$

$z^3 = -1 \qquad z = \sqrt[3]{-1} \qquad\qquad z^4 = -1 \qquad z = \sqrt[4]{-1}$

$z^3 = +i \qquad z = \sqrt[3]{+i} \qquad\qquad z^4 = +i \qquad z = \sqrt[4]{+i}$

$z^3 = -i \qquad z = \sqrt[3]{-i} \qquad\qquad z^4 = -i \qquad z = \sqrt[4]{-i}$

Use the polar form : $\boxed{re^{i\theta}}$

$\Rightarrow$ easier

$$z = r e^{+j\theta}$$

$$z \cdot z = z^2 = r \cdot e^{+j\theta} \cdot r \cdot e^{+j\theta}$$

$$= r^2 \cdot e^{+j\theta + j\theta}$$

$$z^2 = r^2 e^{+j2\theta}.$$

$$z^3 = r^3 e^{+j3\theta}$$

$$(a^x b^y)^m = (a^x)^m \cdot (b^y)^m$$

$$= a^{xm} \cdot b^{ym}$$

$$z^n = \left((r) \cdot (e^{+j\theta})\right)^n$$

$$= (r)^n \cdot (e^{j\theta})^n$$

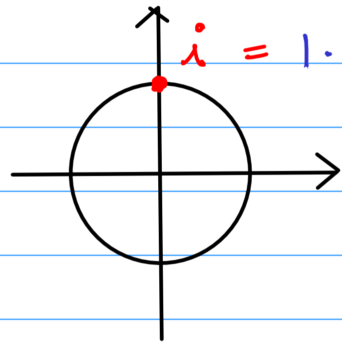
$$= r^n \cdot e^{j(\theta n)}$$

$$= r^n \cdot e^{jn\theta}$$

$z^3 = i \quad z?$

$z = \sqrt[3]{i}$

$= (i)^{\frac{1}{3}}$

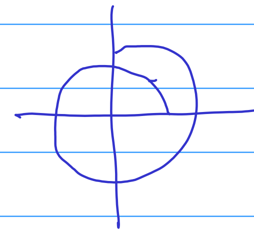


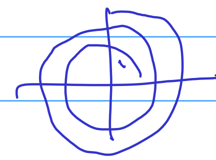
$i = 1\cdot e^{+j\frac{\pi}{2}}$

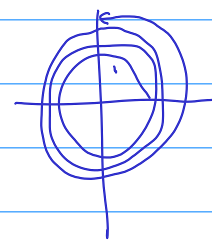
$= 1\cdot e^{j(\frac{\pi}{2}+2\pi)}$

$= 1\cdot e^{j(\frac{\pi}{2}+4\pi)}$

$= 1\cdot e^{j(\frac{\pi}{2}+6\pi)}$



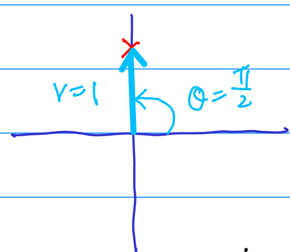




$z^3 = r\, e^{j\theta} = r\cdot e^{j(\theta+2k\pi)}$

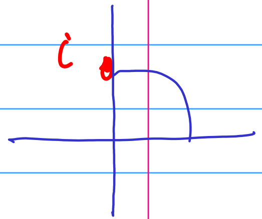
$z = \left(r\, e^{j\theta}\right)^{\frac{1}{3}} = r^{\frac{1}{3}}\, e^{j\frac{(\theta+2k\pi)}{3}}$

$z^3 = i = 1\cdot e^{j\frac{\pi}{2}}$

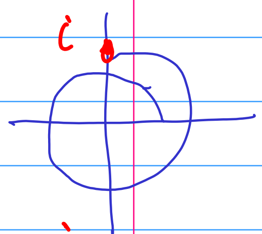


$z = \left(1\cdot e^{j(\frac{\pi}{2}+2k\pi)}\right)^{\frac{1}{3}}$

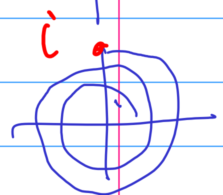
$= \sqrt[3]{1}\;\; e^{j(\frac{\pi}{2}+2k\pi)/3}$



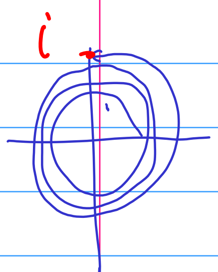
$i = 1\cdot e^{j(\frac{\pi}{2})}$ $\qquad (i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2})/3}}$ $\qquad \frac{\pi}{6}$



$i = 1\cdot e^{j(\frac{\pi}{2}+2\pi)}$ $\qquad (i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2}+2\pi)/3}}$ $\qquad \frac{5}{6}\pi$



$i = 1\cdot e^{j(\frac{\pi}{2}+4\pi)}$ $\qquad (i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2}+4\pi)/3}}$ $\qquad \frac{9}{6}\pi$

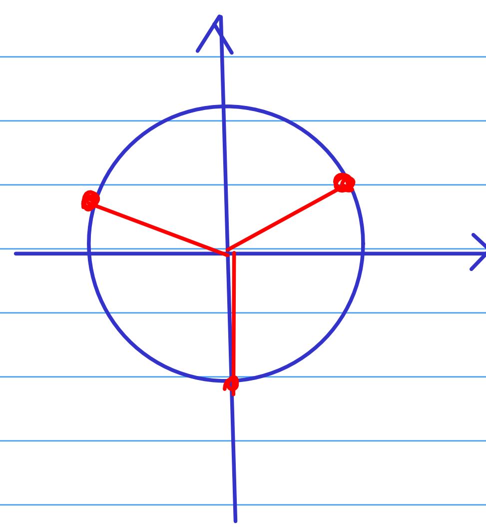


$i = 1\cdot e^{j(\frac{\pi}{2}+6\pi)}$ $\qquad (i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2}+6\pi)/3}}$ $\qquad \frac{13\pi}{6}$

$$(i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2})/3}} = 1\cdot e^{j\frac{\pi}{6}}$$

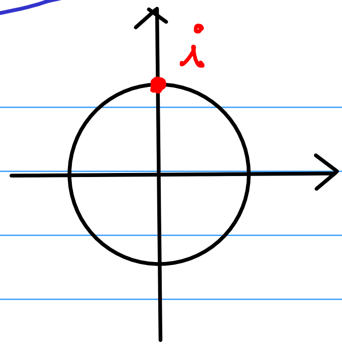
$$(i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2}+2\pi)/3}} = 1\cdot e^{j\frac{5}{6}\pi}$$

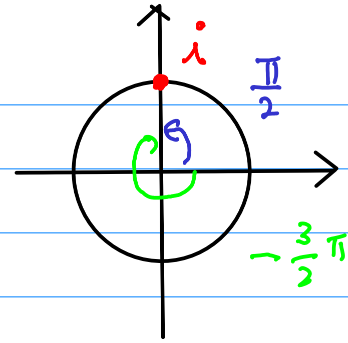
$$(i)^{\frac{1}{3}} = 1^{\frac{1}{3}}\cdot e^{j\underline{(\frac{\pi}{2}+4\pi)/3}} = 1\cdot e^{j\frac{9}{6}\pi}$$



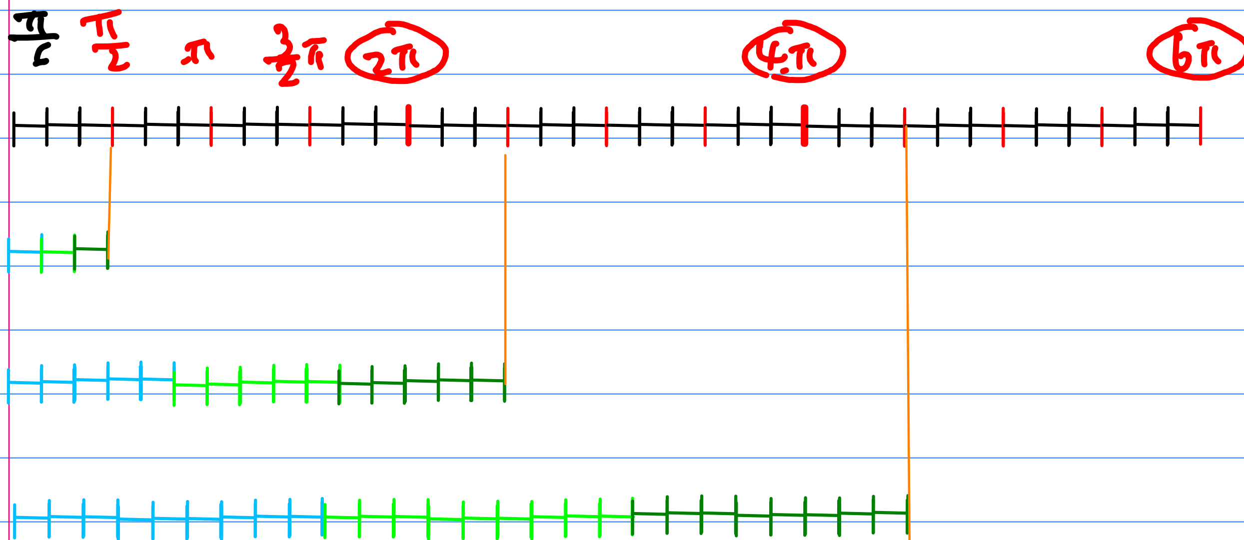
$\boxed{z^3 = i}$

$z = \sqrt[3]{i} = i^{\frac{1}{3}}$





$\arg(i) = \{ \frac{\pi}{2}, \frac{9\pi}{2}, \frac{17\pi}{2}$



$0 \quad 2\pi \quad 4\pi \quad 6\pi \quad 7\pi$

$\frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{\pi}{2}$

$\arg(i) = \{ \frac{\pi}{2} \quad \frac{5\pi}{2} \quad \frac{9\pi}{2} \quad \frac{13\pi}{2} \quad \frac{15}{2}\pi \ldots \}$

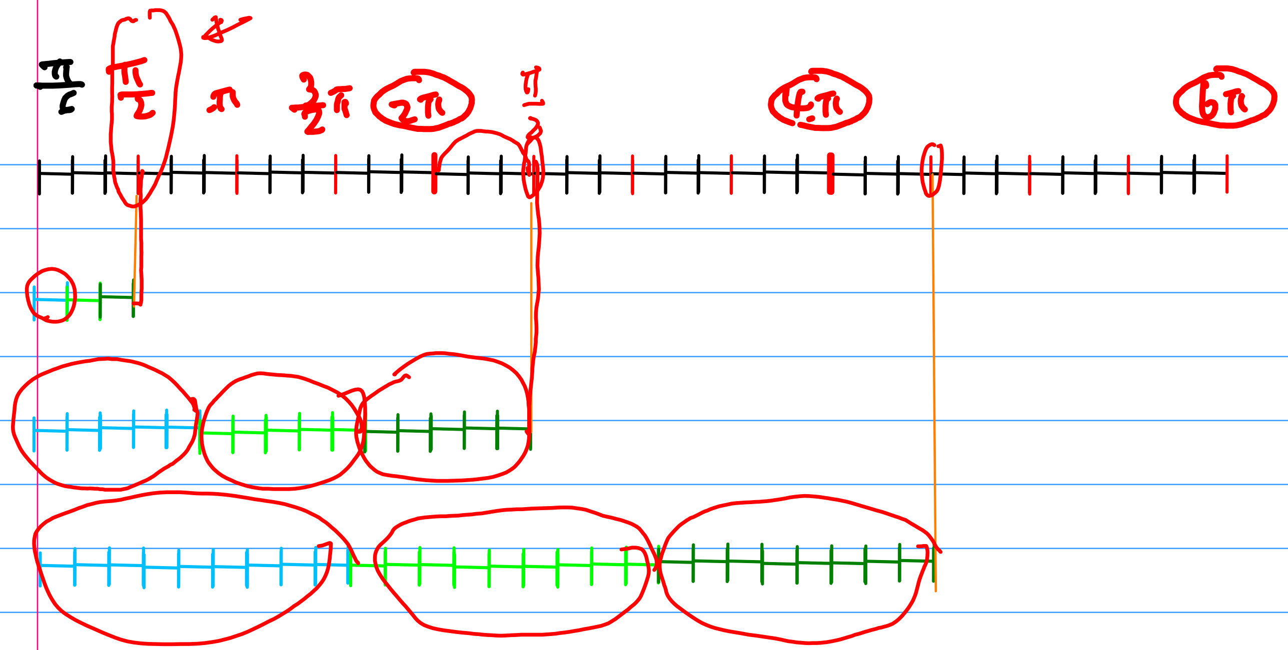
$0 \quad 2\pi \quad 4\pi \quad 6\pi \quad 7\pi$

$-\frac{3}{2}\pi \quad -\frac{3}{2}\pi \quad -\frac{3}{2}\pi \quad -\frac{3}{2}\pi \quad -\frac{3}{2}\pi$

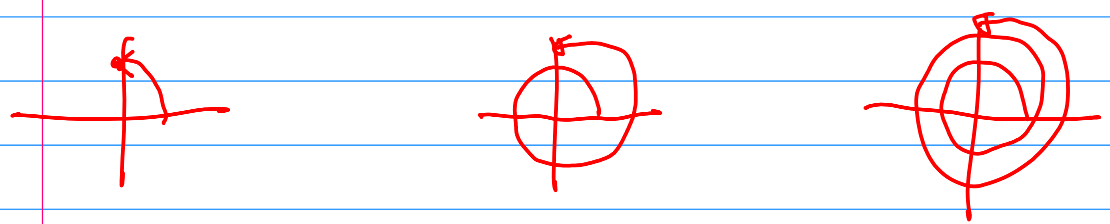
$-\frac{3}{2}\pi \quad \frac{1}{2}\pi \quad \frac{5\pi}{2} \quad \frac{9}{2}\pi$

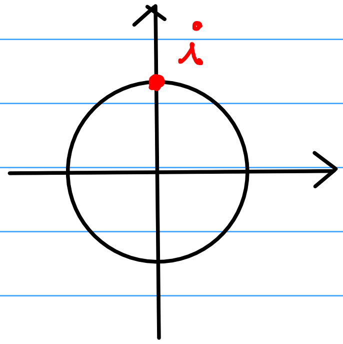
$\arg(i) = \{ \ldots, -\frac{11}{2}\pi, -\frac{7}{2}\pi, -\frac{3}{2}\pi, \frac{\pi}{2}, \frac{5}{2}\pi, \frac{9}{2}\pi \ldots \}$

$\mathrm{Arg}(i) = \frac{\pi}{2}$ $\qquad -\pi \le \mathrm{Arg}(z) < \pi$



π/2
π
3/2 π
2π
π/2
4π
6π





$$i = 1\cdot e^{+j\frac{\pi}{2}} = 1\cdot\left[\cos\left(\frac{\pi}{2}+0\right) + i\sin\left(\frac{\pi}{2}+0\right)\right]$$

$$= 1\cdot e^{j\left(\frac{\pi}{2}+2\pi\right)} = 1\cdot\left[\cos\left(\frac{\pi}{2}+2\pi\right) + i\sin\left(\frac{\pi}{2}+2\pi\right)\right]$$

$$= 1\cdot e^{j\left(\frac{\pi}{2}+4\pi\right)} = 1\cdot\left[\cos\left(\frac{\pi}{2}+4\pi\right) + i\sin\left(\frac{\pi}{2}+4\pi\right)\right]$$

$$= 1\cdot e^{j\left(\frac{\pi}{2}+6\pi\right)} = 1\cdot\left[\cos\left(\frac{\pi}{2}+6\pi\right) + i\sin\left(\frac{\pi}{2}+6\pi\right)\right]$$

$$z^3 = i$$

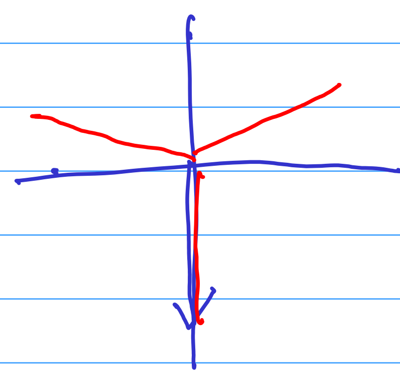
$$(z^3)^{\frac{1}{3}} = (i)^{\frac{1}{3}} = \left(1\cdot e^{+j\frac{\pi}{2}}\right)^{\frac{1}{3}} = 1\cdot e^{j\frac{\pi}{2}\cdot\frac{1}{3}} \qquad \frac{\pi}{6}$$

$$= \left(1\cdot e^{j\left(\frac{\pi}{2}+2\pi\right)}\right)^{\frac{1}{3}} = 1\cdot e^{j\left(\frac{\pi}{2}+2\pi\right)\frac{1}{3}} \qquad \frac{5}{6}\pi$$

$$= \left(1\cdot e^{j\left(\frac{\pi}{2}+4\pi\right)}\right)^{\frac{1}{3}} = 1\cdot e^{j\left(\frac{\pi}{2}+4\pi\right)\frac{1}{3}} \qquad \frac{3}{2}\pi$$

$$= \left(1\cdot e^{j\left(\frac{\pi}{2}+6\pi\right)}\right)^{\frac{1}{3}} = 1\cdot e^{j\left(\frac{\pi}{2}+6\pi\right)\frac{1}{3}}$$

$$\vdots$$



$$z = x + iy = r \cdot e^{j\theta}$$

$$z^2 = z \cdot z = (x+iy)(x+iy) = x^2 - y^2 + 2xy\,i$$

$$= r \cdot e^{j\theta} \cdot r \cdot e^{j\theta} = r^2 \cdot e^{j(2\theta)}$$

adde twice

multiplied twice

$$z^3 = r^3 e^{j(3\theta)}$$

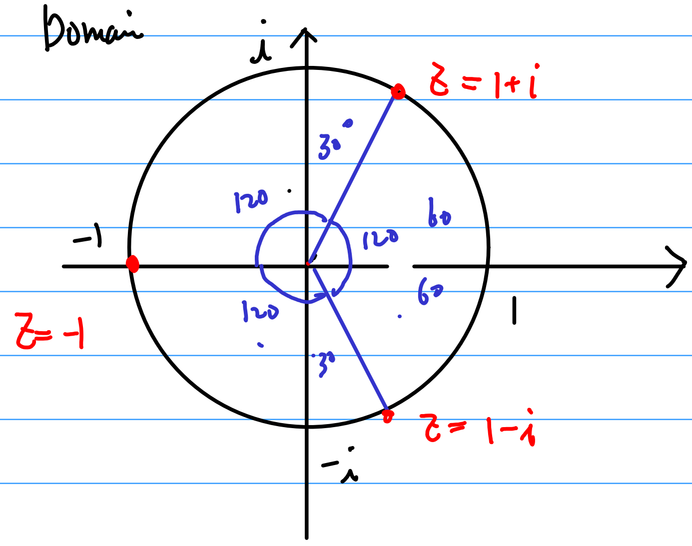
$$\Downarrow$$

$$(z^3)^{\frac{1}{3}} = \left(r^3 e^{j(3\theta)}\right)^{\frac{1}{3}}$$

$$= r^{3 \cdot \frac{1}{3}} \cdot e^{j3\theta \cdot \frac{1}{3}}$$

$$= r e^{j\theta}$$

Domain



$$\boxed{e^{iy} = \cos y + i \sin y}$$

$$z = re^{j\theta}$$

$$z^n = r^n \left(e^{j\theta}\right)^n$$

$$= r^n e^{jn\theta}$$

$$= r^n \left(\cos(\theta) + j \sin(\theta)\right)^n$$

$$= r^n \left(\cos(n\theta) + j \sin(n\theta)\right)$$

$$\boxed{\begin{aligned} &\left(\cos(\theta) + j \sin(\theta)\right)^n \\ &= \left(\cos(n\theta) + j \sin(n\theta)\right) \end{aligned}}$$

$$\boxed{e^{iy} = \cos y + i \sin y}$$

$$z = re^{j\theta}$$

$$z^n = r^n (e^{j\theta})^n$$

$$= r^n e^{jn\theta}$$

$$\sqrt[n]{z} = z^{\frac{1}{n}}$$

$$= \left(re^{j(\theta)}\right)^{\frac{1}{n}}$$ → gives only 1 sol. X

$$= \left(re^{j(\theta + 2\pi k)}\right)^{\frac{1}{n}}$$ → can find all n solutions

$$= r^{\frac{1}{n}} e^{j(\theta + 2\pi k)/n}$$

$$= r^{\frac{1}{n}} \left(\cos\left(\frac{\theta + 2\pi k}{n}\right) + j \sin\left(\frac{\theta + 2\pi k}{n}\right)\right)$$

Complex Roots : $z^4 = 1$

$r = |a|^{\frac{1}{4}}$ $\theta = \frac{1}{4}(-\pi + 2k\pi)$

$z^4 = -1$

$0 \le arg(z) < 2\pi \quad : \; -\pi$

$\theta_0 = -\frac{1}{4}\pi \Rightarrow \theta_0^4 = -\pi$

$2\pi \le arg(z) < 4\pi \quad : \; +\pi$

$\theta_1 = +\frac{1}{4}\pi \Rightarrow \theta_1^4 = -\pi$

$4\pi \le arg(z) < 6\pi \quad : \; +3\pi$

$\theta_2 = +\frac{3}{4}\pi \Rightarrow \theta_2^4 = -\pi$

$6\pi \le arg(z) < 8\pi \quad : \; 5\pi$

$\theta_3 = +\frac{5}{4}\pi \Rightarrow \theta_3^4 = -\pi$