Laurent Series and z-Transform
- Geometric Series
Applications B

20210928 Tue

$f(z)$ causal Laurent $\boxed{z}$

$g(z)$ anti-causal Laurent $\boxed{z^{-1}}$

complementary range relation

$X(z)$ causal z-transform $\boxed{z^{-1}}$

$Y(z)$ anti- causal z-transform $\boxed{z}$

complementary range relation

Laurent Series $f(z)$ $g(z)$

z-transform $X(z)$ $Y(z)$

a_{-3} a_{-2} a_{-1} a_0 a_1 a_2 a_3

z^3 z^2 z^1 z^0 z^{-1} z^{-2} z^{-3}

anti-causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + a_{-3}z^{-3} + a_{-2}z^{-2} + a_{-1}z^{-1} \quad a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots$$

a_{-3} a_{-2} a_{-1} a_0 a_1 a_2 a_3

z^3 z^2 z^1 z^0 z^{-1} z^{-2} z^{-3}

anti-causal z-transform $Y(z)$ causal z-transform $X(z)$

$$\cdots + a_{-3}z^{3} + a_{-2}z^{2} + a_{-1}z^{1} \quad a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots$$

	anti-causal	causal
Laurent	$g(z)$	$f(z)$
z-transform	$Y(z)$	$X(z)$

Laurent Series $f(z^{-1})$ $g(z^{-1})$

a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3
z^3	z^2	z^1	z^0	z^{-1}	z^{-2}	z^{-3}

anti-causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + a_{-3}z^{-3} + a_{-2}z^{-2} + a_{-1}z^{-1} \quad a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots$$

anti-causal Laurent $f(z^{-1})$ causal Laurent $g(z^{-1})$

$$\cdots + a_3 z^{-3} + a_2 z^{-2} + a_1 z^{-1} + a_0 z^0 \quad a_{-1} z^1 + a_{-2} z^2 + a_{-3} z^3 + \cdots$$

anti-causal z-transform $Y(z)$ causal z-transform $X(z)$

$$\cdots + a_{-3} z^3 + a_{-2} z^2 + a_{-1} z^1 \quad a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots$$

anti-causal Laurent	$g(z)$	causal Laurent	$f(z)$
	$\downarrow$		$\downarrow$
causal Laurent	$g(z^{-1})$	anti-causal Laurent	$f(z^{-1})$
	$\|$		$\|$
anti-causal z-transform	$Y(z)$	causal z-transform	$X(z)$

z-transform $X(z^{-1})$ $Y(z^{-1})$

$$\begin{array}{ccccccc} a_{-3} & a_{-2} & a_{-1} & a_0 & a_1 & a_2 & a_3 \\ z^{3} & z^{2} & z^{1} & z^{0} & z^{-1} & z^{-2} & z^{-3} \end{array}$$

anti- causal z-transform $Y(z)$ causal z-transform $X(z)$

$$\cdots + a_{-3}z^{3} + a_{-2}z^{2} + a_{-1}z^{1} \quad a_0 z^{0} + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots$$

anti- causal z-transform $X(z^{-1})$ causal z-transform $Y(z^{-1})$

$$\cdots + a_{3}z^{3} + a_{2}z^{2} + a_{1}z^{1} + a_0 z^{0} \quad a_{-1} z^{-1} + a_{-2} z^{-2} + a_{-3} z^{-3} + \cdots$$

anti- causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + a_{-3}z^{-3} + a_{-2}z^{-2} + a_{-1}z^{-1} \quad a_0 z^{0} + a_1 z^{1} + a_2 z^{2} + a_3 z^{3} + \cdots$$

anti- causal z-transform $Y(z)$ causal z-transform $X(z)$

$\downarrow$ $\downarrow$

causal z-transform $Y(z^{-1})$ anti- causal z-transform $X(z^{-1})$

$\|$ $\|$

anti- causal Laurent $g(z)$ causal Laurent $f(z)$

Composition with the reciprocal function 1/z

causal Laurent	anti- causal Laurent
$f(z)$	$g(z)$
$\parallel$	$\parallel$
$X(z^{-1})$	$Y(z^{-1})$
anti- causal z-transform	causal z-transform

causal z-transform	anti- causal z-transform
$X(z)$	$Y(z)$
$\parallel$	$\parallel$
$f(z^{-1})$	$g(z^{-1})$
anti- causal Laurent	causal Laurent

causal Laurent $f(z)$ = anti- causal z-transform $X(z^{-1})$

different ROC

anti- causal Laurent $g(z)$ = causal z-transform $Y(z^{-1})$

causal z-transform $X(z)$ = anti- causal Laurent $f(z^{-1})$

different ROC

anti- causal z-transform $Y(z)$ = causal Laurent $g(z^{-1})$

anti-causal Laurent $g(z)$

$\downarrow$

causal Laurent $g(z^{-1})$

$\|$

anti-causal z-transform $Y(z)$

causal Laurent $f(z)$

$\downarrow$

anti-causal Laurent $f(z^{-1})$

$\|$

causal z-transform $X(z)$

anti-causal z-transform $Y(z)$

$\downarrow$

causal z-transform $Y(z^{-1})$

$\|$

anti-causal Laurent $g(z)$

causal z-transform $X(z)$

$\downarrow$

anti-causal z-transform $X(z^{-1})$

$\|$

causal Laurent $f(z)$

A unit starting Geometric Series **Origin including**

Laurent Series

z-Transform

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

(1) $+\dfrac{1}{1-\boxed{az}}$ $\boxed{|z|<a^{-1}}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

a^n u(n)	$(n \geq 0)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\boxed{|z|<a}$

$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(3) $+\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\boxed{|z|>a^{-1}}$

$(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

a^n u(-n)	$(n<1)$

(4) $+\dfrac{1}{1-\boxed{az^{-1}}}$ $\boxed{|z|>a}$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$(\frac{1}{a})^n$ u(-n)	$(n<1)$

Geometric Series - a unit start term

z-Transform (n → -n)

(1) $+\dfrac{1}{1-\boxed{az}}$ $\quad$ $\boxed{|z|<a^{-1}}$

$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + \cdots)$

a^{-n} u((-n))	$(-n \geq 0)$
$(\frac{1}{a})^n$ u(-n)	$(n < 1)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\quad$ $\boxed{|z|<a}$

$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

$(\frac{1}{a})^{-n}$ u((-n))	$(-n \geq 0)$
a^n u(-n)	$(n < 1)$

(3) $+\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\quad$ $\boxed{|z|>a^{-1}}$

$(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \cdots)$

a^{-n} u(-(-n))	$(-n < 1)$
$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(4) $+\dfrac{1}{1-\boxed{az^{-1}}}$ $\quad$ $\boxed{|z|>a}$

$(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$(\frac{1}{a})^{-n}$ u(-(-n))	$(-n < 1)$
a^n u(n)	$(n \geq 0)$

Geometric Series

Laurent Series vs. z-Transform (n → -n)

(1)

$+\frac{1}{1-\boxed{az}}$ $\quad$ $|z|<a^{-1}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + \cdots)$

Laurent	a^n u(n)	$(n \geq 0)$
z-Trans	$(\frac{1}{a})^n$ u(-n)	$(n < 1)$

(2)

$+\frac{1}{1-\boxed{a^{-1}z}}$ $\quad$ $|z|<a$

$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

Laurent	$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$
z-Trans	a^n u(-n)	$(n < 1)$

(3)

$+\frac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\quad$ $|z|>a^{-1}$

$(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \cdots)$

Laurent	a^n u(-n)	$(n < 1)$
z-Trans	$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(4)

$+\frac{1}{1-\boxed{az^{-1}}}$ $\quad$ $|z|>a$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

Laurent	$(\frac{1}{a})^n$ u(-n)	$(n < 1)$
z-Trans	a^n u(n)	$(n \geq 0)$

A CR starting Geometric Series **origin excluding**

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Laurent Series

z-Transform

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

(5) $\boxed{+\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}}$ $\boxed{|z|>a^{-1}}$

$-(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$-a^{n}$ u(-n-1)	$(n<0)$

(6) $\boxed{+\dfrac{az^{-1}}{1-az^{-1}}}$ $\boxed{|z|>a}$

$-(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$-\left(\left(\frac{1}{a}\right)^{-1}z^{-1}+\left(\frac{1}{a}\right)^{-2}z^{-2}+\left(\frac{1}{a}\right)^{-3}z^{-3}+\cdots\right)$

$-\left(\frac{1}{a}\right)^{n}$ u(-n-1)	$(n<0)$

(7) $\boxed{+\dfrac{az}{1-az}}$ $\boxed{|z|<a^{-1}}$

$(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

a^{n} u(n-1)	$(n\geq 1)$

(8) $\boxed{+\dfrac{a^{-1}z}{1-a^{-1}z}}$ $\boxed{|z|<a}$

$(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$\left(\left(\frac{1}{a}\right)^{1}z^{1}+\left(\frac{1}{a}\right)^{2}z^{2}+\left(\frac{1}{a}\right)^{3}z^{3}+\cdots\right)$

$\left(\frac{1}{a}\right)^{n}$ u(n-1)	$(n\geq 1)$

Geometric Series - a non-unit start term

z-Transform (n → -n)

(5) $\boxed{+\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}}$ $\boxed{|z| > a^{-1}}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{1}z^{-1} + (\frac{1}{a})^{2}z^{-2} + (\frac{1}{a})^{3}z^{-3} + \cdots)$

$-a^{-n}$ u(-(-n)-1)	$(-n < 0)$
$-(\frac{1}{a})^{n}$ u(n-1)	$(n \geq 1)$

(6) $\boxed{+\dfrac{az^{-1}}{1-az^{-1}}}$ $\boxed{|z| > a}$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

$-(\frac{1}{a})^{-n}$ u(-(-n)-1)	$(-n < 0)$
$-a^{n}$ u(n-1)	$(n \geq 1)$

(7) $\boxed{+\dfrac{az}{1-az}}$ $\boxed{|z| < a^{-1}}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$((\frac{1}{a})^{-1}z^{1} + (\frac{1}{a})^{-2}z^{2} + (\frac{1}{a})^{-3}z^{3} + \cdots)$

a^{-n} u((-n)-1)	$(-n \geq 1)$
$(\frac{1}{a})^{n}$ u(-n-1)	$(n < 0)$

(8) $\boxed{+\dfrac{a^{-1}z}{1-a^{-1}z}}$ $\boxed{|z| < a}$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

$(\frac{1}{a})^{-n}$ u((-n)-1)	$(-n \geq 1)$
a^{n} u(-n-1)	$(n < 0)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform (n → -n)

(5) $+\dfrac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}$ $\quad |z| > a^{-1}$

$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^{1}z^{-1} + \left(\frac{1}{a}\right)^{2}z^{-2} + \left(\frac{1}{a}\right)^{3}z^{-3} + \cdots\right)$

Laurent	$-a^n$	$u(-n-1)$	$(n < 0)$
z-Trans	$-\left(\frac{1}{a}\right)^n$	$u(n-1)$	$(n \geq 1)$

(6) $+\dfrac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}$ $\quad |z| > a$

$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^{-1}z^{-1} + \left(\frac{1}{a}\right)^{-2}z^{-2} + \left(\frac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

Laurent	$-\left(\frac{1}{a}\right)^n$	$u(-n-1)$	$(n < 0)$
z-Trans	$-a^n$	$u(n-1)$	$(n \geq 1)$

(7) $+\dfrac{\boxed{az}}{1-\boxed{az}}$ $\quad |z| < a^{-1}$

$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^{-1}z^{1} + \left(\frac{1}{a}\right)^{-2}z^{2} + \left(\frac{1}{a}\right)^{-3}z^{3} + \cdots\right)$

Laurent	a^n	$u(n-1)$	$(n \geq 1)$
z-Trans	$\left(\frac{1}{a}\right)^n$	$u(-n-1)$	$(n < 0)$

(8) $+\dfrac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}$ $\quad |z| < a$

$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^{1}z^{1} + \left(\frac{1}{a}\right)^{2}z^{2} + \left(\frac{1}{a}\right)^{3}z^{3} + \cdots\right)$

Laurent	$\left(\frac{1}{a}\right)^n$	$u(n-1)$	$(n \geq 1)$
z-Trans	a^n	$u(-n-1)$	$(n < 0)$

4 cases of geometric series **Simple Pole Form**
- 2 representations for each case

using z		using $1/z$	
simple pole p	simple pole $1/p$	simple pole $1/p$	simple pole p
(A) $\boxed{\dfrac{1}{z - p}}$	(B) $\boxed{\dfrac{1}{z - p^{-1}}}$	(C) $\boxed{\dfrac{1}{z^{-1} - p}}$	(D) $\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$
/p $\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}}$	*p $\boxed{-\dfrac{p}{1 - p z}}$	/p $\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$	*p $\boxed{-\dfrac{p}{1 - p z^{-1}}}$
/z $\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}}$	/z $\boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$	*z $\boxed{\dfrac{z}{1 - p z}}$	*z $\boxed{\dfrac{z}{1 - p^{-1} z}}$

p^{-1} $\qquad\qquad$ p^{-1}

z^{-1}

/p $\boxed{-p^{-n-1}\ u(n)}$	*p $\boxed{-p^{n+1}\ u(n)}$	/p $\boxed{-p^{n-1}\ u(-n)}$	*p $\boxed{-p^{-n+1}\ u(-n)}$
/z $\boxed{p^{-n-1}\ u(-n-1)}$	/z $\boxed{p^{n+1}\ u(-n-1)}$	*z $\boxed{p^{n-1}\ u(n-1)}$	*z $\boxed{p^{-n+1}\ u(n-1)}$

4 cases of geometric series **Simple Pole Form**

- 2 representations for each case

using p		using 1/p	
simple pole p	simple pole 1/p	simple pole 1/p	simple pole p
(A) $\frac{1}{z - p}$	(C) $\frac{1}{z^{-1} - p}$	(B) $\frac{1}{z - p^{-1}}$	(D) $\frac{1}{z^{-1} - p^{-1}}$
/p $-\frac{p^{-1}}{1 - p^{-1} z}$	/p $-\frac{p^{-1}}{1 - p^{-1} z^{-1}}$	*p $-\frac{p}{1 - p z}$	*p $-\frac{p}{1 - p z^{-1}}$
/z $\frac{z^{-1}}{1 - p z^{-1}}$	*z $\frac{z}{1 - p z}$	/z $\frac{z^{-1}}{1 - p^{-1} z^{-1}}$	*z $\frac{z}{1 - p^{-1} z}$

z^{-1} (A ↔ C) z^{-1} (B ↔ D)

p^{-1}

/p $-p^{-n-1}$ u(n)	/p $-p^{n-1}$ u(-n)	*p $-p^{n+1}$ u(n)	*p $-p^{-n+1}$ u(-n)
/z p^{-n-1} u(-n-1)	*z p^{n-1} u(n-1)	/z p^{n+1} u(-n-1)	*z p^{-n+1} u(n-1)

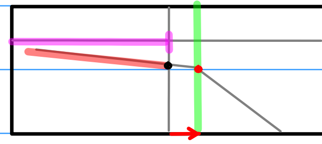
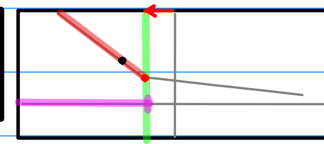
4 cases of geometric series **Simple Pole Form**
- 2 representations for each case

	simple pole p				simple pole $1/p$			
	(A)	$\frac{1}{z-p}$	(D)	$\frac{1}{z^{-1}-p^{-1}}$	(B)	$\frac{1}{z-p^{-1}}$	(C)	$\frac{1}{z^{-1}-p}$
	/p	$-\frac{p^{-1}}{1-p^{-1}z}$	*p	$-\frac{p}{1-pz^{-1}}$	*p	$-\frac{p}{1-pz}$	/p	$-\frac{p^{-1}}{1-p^{-1}z^{-1}}$
	/z	$\frac{z^{-1}}{1-pz^{-1}}$	*z	$\frac{z}{1-p^{-1}z}$	/z	$\frac{z^{-1}}{1-p^{-1}z^{-1}}$	*z	$\frac{z}{1-pz}$

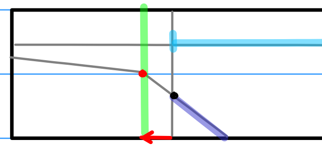
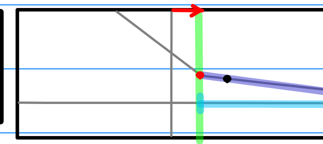
z^{-1} (A – D) $\quad$ z^{-1} (B – C)

p^{-1} (D – B)

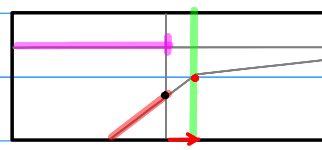
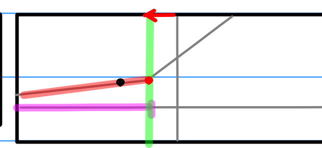
/p	$-p^{-n-1}$ u(n)	*p	$-p^{-n+1}$ u(-n)	*p	$-p^{n+1}$ u(n)	/p	$-p^{n-1}$ u(-n)
/z	p^{-n-1} u(-n-1)	*z	p^{-n+1} u(n-1)	/z	p^{n+1} u(-n-1)	*z	p^{n-1} u(n-1)

(A) $\boxed{\dfrac{1}{z-p}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$  $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ 

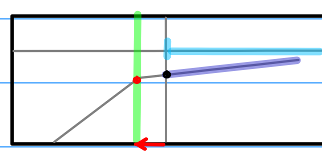
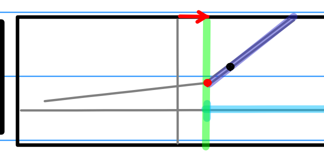
$-p^{-n-1}\ u(n)$ $p^{-n-1}\ u(-n-1)$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$  $\boxed{\dfrac{z}{1-p^{-1}z}}$ 

$-p^{-n+1}\ u(-n)$ $p^{-n+1}\ u(n-1)$

(B) $\boxed{\dfrac{1}{z-p^{-1}}}$ $\boxed{-\dfrac{p}{1-pz}}$  $\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ 

$-p^{n+1}\ u(n)$ $p^{n+1}\ u(-n-1)$

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$  $\boxed{\dfrac{z}{1-pz}}$ 

$-p^{n-1}\ u(-n)$ $p^{n-1}\ u(n-1)$

Assumption $p > 0$

$f(z)$, $g(z)$ are positive series
$X(z)$, $Y(z)$ are positive series

$f(z)$ causal Laurent $\boxed{z}$
$g(z)$ anti-causal Laurent $\boxed{z^{-1}}$

complementary range relation

$X(z)$ causal z-transform $\boxed{z^{-1}}$
$Y(z)$ anti- causal z-transform $\boxed{z}$

complementary range relation

Laurent Series

$g(z)$ | $f(z)$

(A) (D)
(B) (C)

(A) $\boxed{\dfrac{1}{z-p}}$ $g(z)$: $p^{-n-1}u(-n-1)$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ $-f(z)$: $-p^{-n-1}u(n)$

$$\cdots + p^2 z^{-3} + p^1 z^{-2} + p^0 z^{-1} \quad p^{-1} z^0 + p^{-2} z^1 + p^{-3} z^2 + p^{-4} z^3 + \cdots$$

$Y(z)$ $X(z)$

$$\cdots + p^2 z^{3} + p^1 z^{2} + p^0 z^{1} \quad p^{-1} z^0 + p^{-2} z^{-1} + p^{-3} z^{-2} + p^{-4} z^{-3} + \cdots$$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$ $-g(z)$: $-p^{-n+1}u(-n)$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\boxed{\dfrac{z}{1-p^{-1}z}}$ $f(z)$: $p^{-n+1}u(n-1)$

$$\cdots + p^4 z^{-3} + p^3 z^{-2} + p^2 z^{-1} + p^1 z^0 \quad p^0 z^1 + p^{-1} z^2 + p^{-2} z^3 + \cdots$$

$Y(z)$ $X(z)$

$$\cdots + p^4 z^{3} + p^3 z^{2} + p^2 z^{1} + p^1 z^0 \quad p^0 z^{-1} + p^{-1} z^{-2} + p^{-2} z^{-3} + \cdots$$

(B) $\boxed{\dfrac{1}{z-p^{-1}}}$ $g(z)$: $p^{n+1}u(-n-1)$ $\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{-\dfrac{p}{1-pz}}$ $-f(z)$: $-p^{n+1}u(n)$

$$\cdots + p^{-2} z^{-3} + p^{-1} z^{-2} + p^0 z^{-1} \quad p^1 z^0 + p^2 z^1 + p^3 z^2 + p^4 z^3 + \cdots$$

$Y(z)$ $X(z)$

$$\cdots + p^{-2} z^{3} + p^{-1} z^{2} + p^0 z^{1} \quad p^1 z^0 + p^2 z^{-1} + p^3 z^{-2} + p^4 z^{-3} + \cdots$$

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$ $-g(z)$: $-p^{n-1}u(-n)$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\dfrac{z}{1-pz}}$ $f(z)$: $p^{n-1}u(n-1)$

$$\cdots + p^{-4} z^{-3} + p^{-3} z^{-2} + p^{-2} z^{-1} + p^{-1} z^0 \quad p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots$$

$-Y(z)$ $X(z)$

$$\cdots + p^{-4} z^{3} + p^{-3} z^{2} + p^{-2} z^{1} + p^{-1} z^0 \quad p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots$$

$$\boxed{\begin{matrix} g(z) & f(z) \\ \| & \| \\ Y(z^{-1}) & X(z^{-1}) \end{matrix}} \qquad \boxed{\begin{matrix} X(z) & Y(z) \\ \| & \| \\ f(z^{-1}) & g(z^{-1}) \end{matrix}}$$

(A) (D)
(B) (C)

(A) $\boxed{\dfrac{1}{z - p}}$

$g(z)\ \boxed{\dfrac{z^{-1}}{1 - p z^{-1}}}$ $\qquad -f(z)\ \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}}$

p^{-n-1} u(-n-1) $\qquad -p^{-n-1}$ u(n)

$Y(z)\ \boxed{\dfrac{z}{1 - p z}}$ $\qquad -X(z)\ \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$

(D) $\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$

$-g(z)\ \boxed{-\dfrac{p}{1 - p z^{-1}}}$ $\qquad f(z)\ \boxed{\dfrac{z}{1 - p^{-1} z}}$

$-p^{-n+1}$ u(-n) $\qquad p^{-n+1}$ u(n-1)

$-Y(z)\ \boxed{-\dfrac{p}{1 - p z}}$ $\qquad X(z)\ \boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$

$g(z)\ \boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$ $\qquad -f(z)\ \boxed{-\dfrac{p}{1 - p z}}$

p^{n+1} u(-n-1) $\qquad -p^{n+1}$ u(n)

$Y(z)\ \boxed{\dfrac{z}{1 - p^{-1} z}}$ $\qquad -X(z)\ \boxed{-\dfrac{p}{1 - p z^{-1}}}$

(C) $\boxed{\dfrac{1}{z^{-1} - p}}$

$-g(z)\ \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$ $\qquad f(z)\ \boxed{\dfrac{z}{1 - p z}}$

$-p^{n-1}$ u(-n) $\qquad p^{n-1}$ u(n-1)

$-Y(z)\ \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}}$ $\qquad X(z)\ \boxed{\dfrac{z^{-1}}{1 - p z^{-1}}}$

Laurent Series $g(z)$ $f(z)$

z Transform $Y(z)$ $X(z)$

(1)

(A) (D)
(B) (C)

(A) $\boxed{\dfrac{1}{z-p}}$ anti- causal Laurent $g(z)$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ $-f(z)$ causal Laurent

$$\cdots + p^2 z^{-3} + p^1 z^{-2} + p^0 z^{-1} \quad p^{-1} z^0 + p^{-2} z^1 + p^{-3} z^2 + p^{-4} z^3 + \cdots$$

p^{-n-1} u(-n-1) p^{-n-1} u(n)

$$\cdots + p^2 z^3 + p^1 z^2 + p^0 z^1 \quad p^{-1} z^0 + p^{-2} z^{-1} + p^{-3} z^{-2} + p^{-4} z^{-3} + \cdots$$

anti- causal z-transform $Y(z)$ $\boxed{\dfrac{z}{1-pz}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $-X(z)$ causal z-transform

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$ anti- causal Laurent $-g(z)$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\boxed{\dfrac{z}{1-p^{-1}z}}$ $f(z)$ causal Laurent

$$\cdots + p^4 z^{-3} + p^3 z^{-2} + p^2 z^{-1} + p^1 z^0 \quad p^0 z^1 + p^{-1} z^2 + p^{-2} z^3 + \cdots$$

p^{-n+1} u(-n) p^{-n+1} u(n-1)

$$\cdots + p^4 z^3 + p^3 z^2 + p^2 z^1 + p^1 z^0 \quad p^0 z^{-1} + p^{-1} z^{-2} + p^{-2} z^{-3} + \cdots$$

anti- causal z-transform $-Y(z)$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ $X(z)$ causal z-transform

Laurent Series $g(z)$ $f(z)$

z Transform $Y(z)$ $X(z)$

(2)

(A) (D)
(B) (C)

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$

anti- causal Laurent $g(z)$ $\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{-\dfrac{p}{1-pz}}$ $-f(z)$ causal Laurent

$$\cdots + p^{-2}z^{-3} + p^{-1}z^{-2} + p^{0}z^{-1} \quad p^{1}z^{0} + p^{2}z^{1} + p^{3}z^{2} + p^{4}z^{3} + \cdots$$

p^{n+1} u(-n-1) $\qquad$ p^{n+1} u(n)

$$\cdots + p^{-2}z^{3} + p^{-1}z^{2} + p^{0}z^{1} \quad p^{1}z^{0} + p^{2}z^{-1} + p^{3}z^{-2} + p^{4}z^{-3} + \cdots$$

anti- causal z-transform $Y(z)$ $\boxed{\dfrac{z}{1-p^{-1}z}}$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $-X(z)$ causal z-transform

(C) $\boxed{\dfrac{1}{z^{-1} - p}}$

anti- causal Laurent $-g(z)$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\dfrac{z}{1-pz}}$ $f(z)$ causal Laurent

$$\cdots + p^{-4}z^{-3} + p^{-3}z^{-2} + p^{-2}z^{-1} + p^{-1}z^{0} \quad p^{0}z^{1} + p^{1}z^{2} + p^{2}z^{3} + \cdots$$

p^{n-1} u(-n) $\qquad$ p^{n-1} u(n-1)

$$\cdots + p^{-4}z^{3} + p^{-3}z^{2} + p^{-2}z^{1} + p^{-1}z^{0} \quad p^{0}z^{-1} + p^{1}z^{-2} + p^{2}z^{-3} + \cdots$$

anti- causal z-transform $-Y(z)$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $X(z)$ causal z-transform

Laurent Series
z Transform

$g(z)$ $f(z)$ / $X(z^{-1})$ $Y(z^{-1})$

(3)

(A) (D)
(B) (C)

(A) $\boxed{\dfrac{1}{z-p}}$

$g(z)$: $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\quad$ $-f(z)$: $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$

p^{-n-1} u(-n-1) $\quad$ $-p^{-n-1}$ u(n)

anti-causal Laurent $g(z)$ $\quad$ causal Laurent $f(z)$

$$\cdots + p^{2}z^{-3} + p^{1}z^{-2} + p^{0}z^{-1} \quad p^{-1}z^{0} + p^{-2}z^{1} + p^{-3}z^{2} + p^{-4}z^{3} + \cdots$$

anti-causal z-transform $X(z^{-1})$ $\quad$ causal z-transform $Y(z^{-1})$

$$\cdots + p^{-4}z^{3} + p^{-3}z^{2} + p^{-2}z^{1} + p^{-1}z^{0} \quad p^{0}z^{-1} + p^{1}z^{-2} + p^{2}z^{-3} + \cdots$$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$

$-g(z)$: $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\quad$ $f(z)$: $\boxed{\dfrac{z}{1-p^{-1}z}}$

$-p^{-n+1}$ u(-n) $\quad$ p^{-n+1} u(n-1)

anti-causal Laurent $g(z)$ $\quad$ causal Laurent $f(z)$

$$\cdots + p^{4}z^{-3} + p^{3}z^{-2} + p^{2}z^{-1} + p^{1}z^{0} \quad p^{0}z^{1} + p^{-1}z^{2} + p^{-2}z^{3} + \cdots$$

anti-causal z-transform $X(z^{-1})$ $\quad$ causal z-transform $Y(z^{-1})$

$$\cdots + p^{-2}z^{3} + p^{-1}z^{2} + p^{0}z^{1} \quad p^{1}z^{0} + p^{2}z^{-1} + p^{3}z^{-2} + p^{4}z^{-3} + \cdots$$

when the pole is p

2 formulas **Simple Pole Form**

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p^{-1}}}$$

2 representations each **Shifted Geometric Series Form**

(A) $\boxed{\dfrac{1}{z-p}}$

$/p$: $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \triangleq -f(z) = -X(z^{-1})$

causal Laurent; anti- causal z-transform

|| different ROC ||

$/z$: $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \triangleq g(z) = Y(z^{-1})$

anti- causal Laurent; causal z-transform

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$

$*z$: $\boxed{\dfrac{z}{1-p^{-1}z}} \triangleq f(z) = X(z^{-1})$

causal Laurent; anti- causal z-transform

|| different ROC ||

$*p$: $\boxed{-\dfrac{p}{1-pz^{-1}}} \triangleq -g(z) = -Y(z^{-1})$

anti- causal Laurent; causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

when the pole is 1/p

2 formulas

Simple Pole Form

$$\boxed{\frac{1}{z - p^{-1}}} \qquad \boxed{\frac{1}{z^{-1} - p}}$$

2 representations each

Shifted Geometric Series Form

(B) $\boxed{\frac{1}{z - p^{-1}}}$

*p

$$\boxed{-\frac{p}{1 - pz}} \triangleq -f(z) = -X(z^{-1})$$

causal Laurent

anti- causal z-transform

|| different ROC ||

/z

$$\boxed{\frac{z^{-1}}{1 - p^{-1}z^{-1}}} \triangleq g(z) = Y(z^{-1})$$

anti- causal Laurent

causal z-transform

(C) $\boxed{\frac{1}{z^{-1} - p}}$

/p

$$\boxed{\frac{z}{1 - pz}} \triangleq f(z) = X(z^{-1})$$

causal Laurent

anti- causal z-transform

|| different ROC ||

*z

$$\boxed{-\frac{p^{-1}}{1 - p^{-1}z^{-1}}} \triangleq -g(z) = -Y(z^{-1})$$

anti- causal Laurent

causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

(A) $\dfrac{1}{z - p}$

/p → $-\dfrac{p^{-1}}{1 - p^{-1}z}$

$-(p^{-1}z^{0} + p^{-2}z^{1} + p^{-3}z^{2} + \cdots)$ $-f(z)$ causal Laurent

$-((\frac{1}{p})^{1}z^{0} + (\frac{1}{p})^{2}z^{1} + (\frac{1}{p})^{3}z^{2} + \cdots)$ $-X(z^{-1})$ anti- causal z-transform

/z → $\dfrac{z^{-1}}{1 - pz^{-1}}$

$(p^{0}z^{-1} + p^{1}z^{-2} + p^{2}z^{-3} + \cdots)$ $g(z)$ anti- causal Laurent

$((\frac{1}{p})^{0}z^{-1} + (\frac{1}{p})^{-1}z^{-2} + (\frac{1}{p})^{-2}z^{-3} + \cdots)$ $Y(z^{-1})$ causal z-transform

(D) $\dfrac{1}{z^{-1} - p^{-1}}$

*z → $\dfrac{z}{1 - p^{-1}z}$

$(p^{0}z^{1} + p^{-1}z^{2} + p^{-2}z^{3} + \cdots)$ $f(z)$ causal Laurent

$((\frac{1}{p})^{0}z^{1} + (\frac{1}{p})^{1}z^{2} + (\frac{1}{p})^{2}z^{3} + \cdots)$ $X(z^{-1})$ anti- causal z-transform

*p → $-\dfrac{p}{1 - pz^{-1}}$

$-(p^{1}z^{0} + p^{2}z^{-1} + p^{3}z^{-2} + \cdots)$ $-g(z)$ anti- causal Laurent

$-((\frac{1}{p})^{-1}z^{0} + (\frac{1}{p})^{-2}z^{-1} + (\frac{1}{p})^{-3}z^{-2} + \cdots)$ $-Y(z^{-1})$ causal z-transform

(B) $\dfrac{1}{z - p^{-1}}$

*p → $-\dfrac{p}{1 - pz}$

$-(p^{1}z^{0} + p^{2}z^{1} + p^{3}z^{2} + \cdots)$ $-f(z)$ causal Laurent

$-((\frac{1}{p})^{-1}z^{0} + (\frac{1}{p})^{-2}z^{1} + (\frac{1}{p})^{-3}z^{2} + \cdots)$ $-X(z^{-1})$ anti- causal z-transform

/z → $\dfrac{z^{-1}}{1 - p^{-1}z^{-1}}$

$(p^{0}z^{-1} + p^{-1}z^{-2} + p^{-2}z^{-3} + \cdots)$ $g(z)$ anti- causal Laurent

$((\frac{1}{p})^{0}z^{-1} + (\frac{1}{p})^{1}z^{-2} + (\frac{1}{p})^{2}z^{-3} + \cdots)$ $Y(z^{-1})$ causal z-transform

(C) $\dfrac{1}{z^{-1} - p}$

/p → $\dfrac{z}{1 - pz}$

$(p^{0}z^{1} + p^{1}z^{2} + p^{2}z^{3} + \cdots)$ $f(z)$ causal Laurent

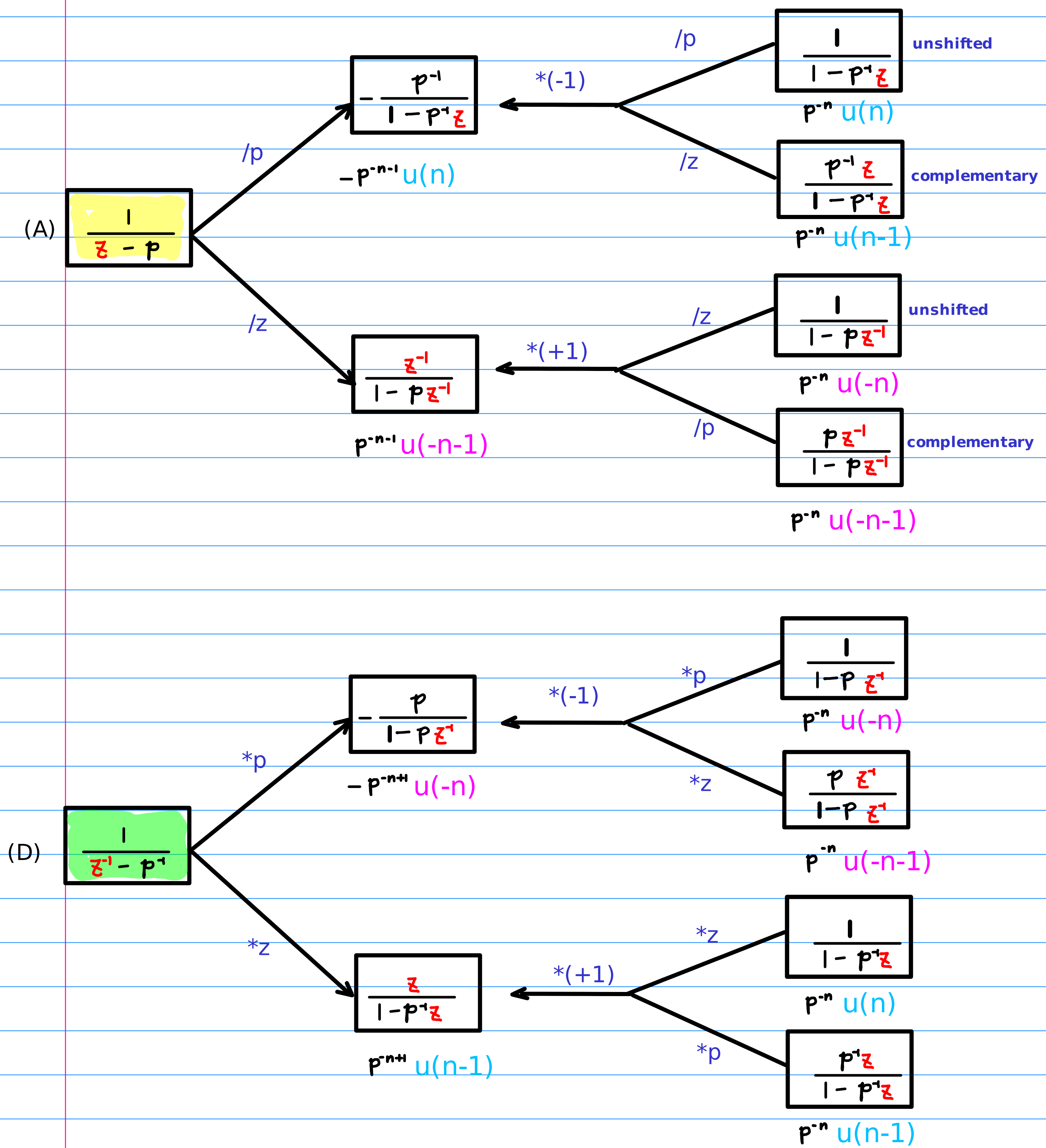
$((\frac{1}{p})^{0}z^{1} + (\frac{1}{p})^{-1}z^{2} + (\frac{1}{p})^{-2}z^{3} + \cdots)$ $X(z^{-1})$ anti- causal z-transform

*z → $-\dfrac{p}{1 - pz^{-1}}$

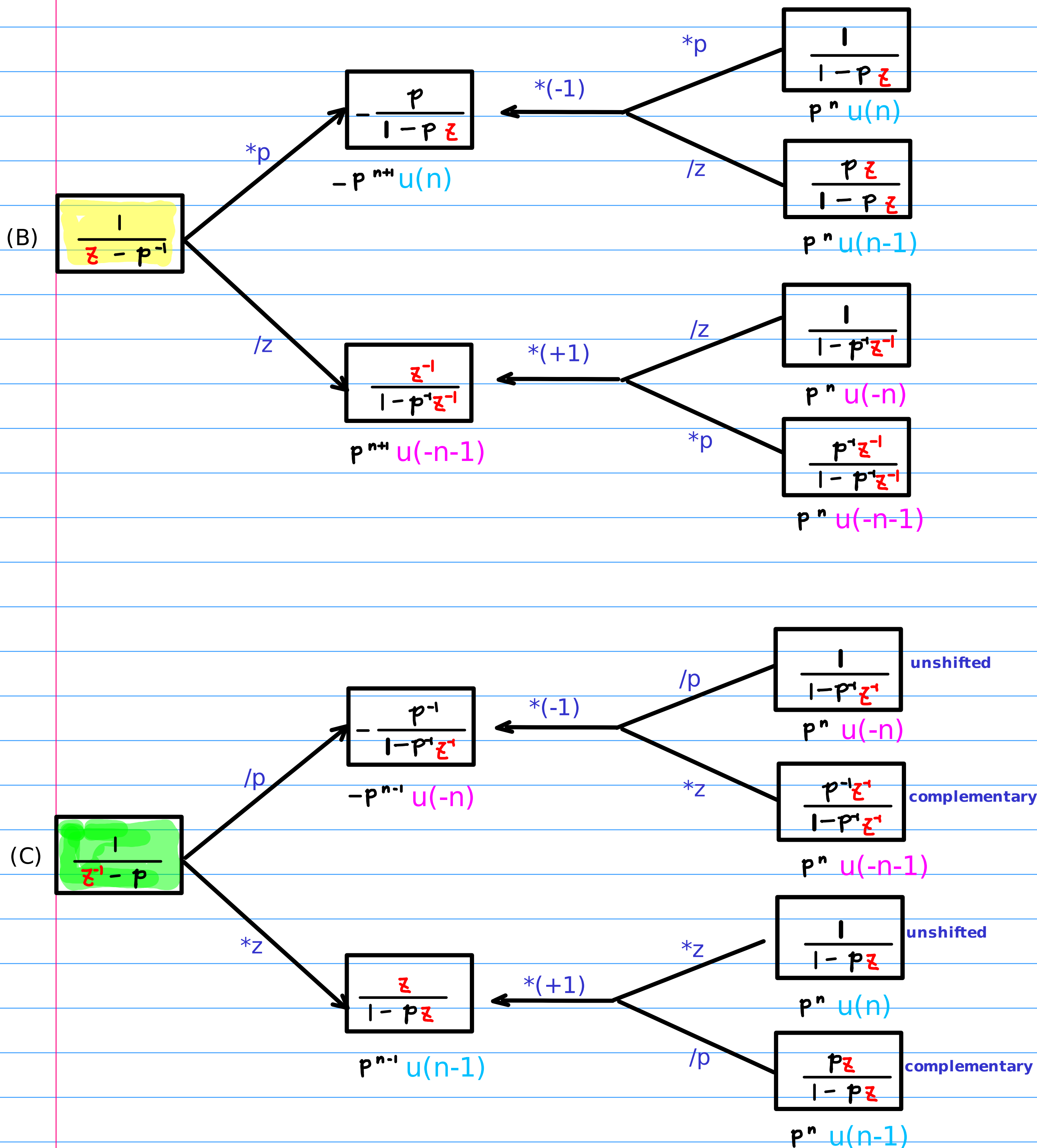
$-(p^{-1}z^{0} + p^{-2}z^{-1} + p^{-3}z^{-2} + \cdots)$ $-g(z)$ anti- causal Laurent

$-((\frac{1}{p})^{1}z^{0} + (\frac{1}{p})^{2}z^{-1} + (\frac{1}{p})^{3}z^{-2} + \cdots)$ $-Y(z^{-1})$ causal z-transform

Shifted Geometric Series (1) p



Shifted Geometric Series (2) 1/p



Laurent Series

z Transform

$g(z)$ $f(z)$

$X(z^{-1})$ $Y(z^{-1})$

(4)

(A) (D)
(B) (C)

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$

$g(z)$: $\boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$ $f(z)$: $\boxed{- \dfrac{p}{1 - p z}}$

p^{n+1} u(-n-1) p^{n+1} u(n)

anti-causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + p^{-2} z^{-3} + p^{-1} z^{-2} + p^{0} z^{-1} \quad p^{1} z^{0} + p^{2} z^{1} + p^{3} z^{2} + p^{4} z^{3} + \cdots$$

anti-causal z-transform $X(z^{-1})$ causal z-transform $Y(z^{-1})$

$$\cdots + p^{4} z^{3} + p^{3} z^{2} + p^{2} z^{1} + p^{1} z^{0} \quad p^{0} z^{-1} + p^{-1} z^{-2} + p^{-2} z^{-3} + \cdots$$

(C) $\boxed{\dfrac{1}{z^{-1} - p}}$

$X(z)$: $\boxed{- \dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$ $Y(z)$: $\boxed{\dfrac{z}{1 - p z}}$

p^{n-1} u(-n) p^{n-1} u(n-1)

anti-causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + p^{-4} z^{-3} + p^{-3} z^{-2} + p^{-2} z^{-1} + p^{-1} z^{0} \quad p^{0} z^{1} + p^{1} z^{2} + p^{2} z^{3} + \cdots$$

anti-causal z-transform $X(z^{-1})$ causal z-transform $Y(z^{-1})$

$$\cdots + p^{2} z^{3} + p^{1} z^{2} + p^{0} z^{1} \quad p^{-1} z^{0} + p^{-2} z^{-1} + p^{-3} z^{-2} + p^{-4} z^{-3} + \cdots$$

(1) $h_1(a,z) = f(z) = \dfrac{1}{1-az}$ (1') $h_1'(a,z) = f_2(z) = \dfrac{a}{1-az}$

(2) $h_2(a,z) = g(z) = \dfrac{1}{1-a^{-1}z}$ (2') $h_2'(a,z) = g_2(z) = \dfrac{a^{-1}}{1-a^{-1}z}$

(3) $h_3(a,z) = \bar{f}_1(z) = \dfrac{1}{1-a^{-1}z^{-1}}$ (3') $h_3'(a,z) = \bar{f}_3(z) = \dfrac{a^{-1}}{1-a^{-1}z^{-1}}$

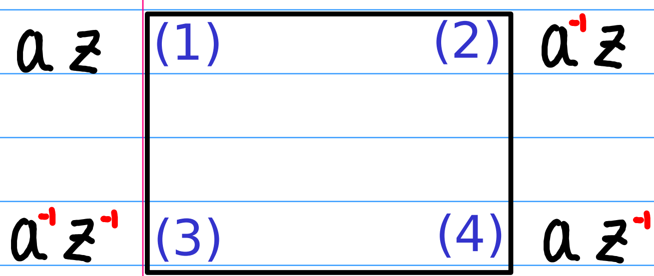
(4) $h_4(a,z) = \bar{g}_1(z) = \dfrac{1}{1-az^{-1}}$ (4') $h_4'(a,z) = \bar{g}_3(z) = \dfrac{a^{-1}}{1-a^{-1}z^{-1}}$

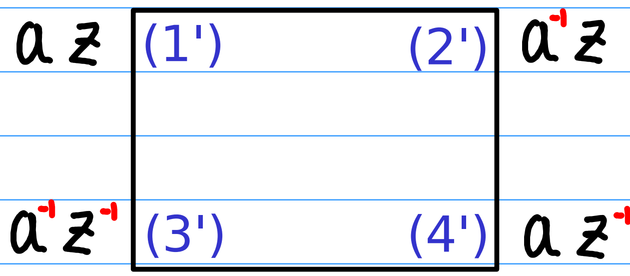
(5) $h_5(a,z) = \bar{f}(z) = \dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ (5') $h_5'(a,z) = \bar{f}_2(z) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$

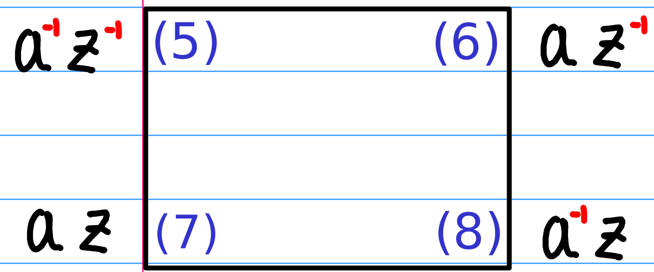
(6) $h_6(a,z) = \bar{g}(z) = \dfrac{az^{-1}}{1-az^{-1}}$ (6') $h_6'(a,z) = \bar{g}_2(z) = \dfrac{z^{-1}}{1-az^{-1}}$

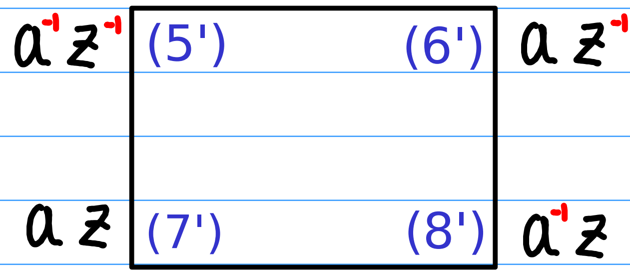
(7) $h_7(a,z) = f_1(z) = \dfrac{az}{1-az}$ (7') $h_7'(a,z) = f_3(z) = \dfrac{z}{1-az}$

(8) $h_8(a,z) = g_1(z) = \dfrac{a^{-1}z}{1-a^{-1}z}$ (8') $h_8'(a,z) = g_3(z) = \dfrac{z}{1-a^{-1}z}$


$a z$
(1)
(2)
$a^{-1} z$
$a^{-1} z^{-1}$
(3)
(4)
$a z^{-1}$


$a z$
(1')
(2')
$a^{-1} z$
$a^{-1} z^{-1}$
(3')
(4')
$a z^{-1}$

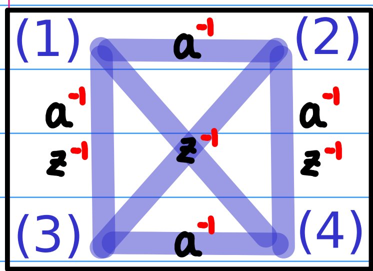

$a^{-1} z^{-1}$
(5)
(6)
$a z^{-1}$
$a z$
(7)
(8)
$a^{-1} z$

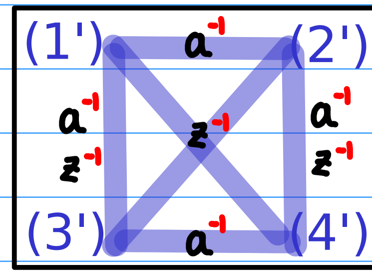

$a^{-1} z^{-1}$
(5')
(6')
$a z^{-1}$
$a z$
(7')
(8')
$a^{-1} z$

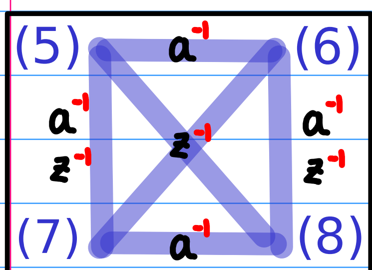
$z < a^{-1}$	(1)	(2)	$z < a$	$z < a^{-1}$	(1')	(2')	$z < a$
$z > a^{-1}$	(3)	(4)	$z > a$	$z > a^{-1}$	(3')	(4')	$z > a$

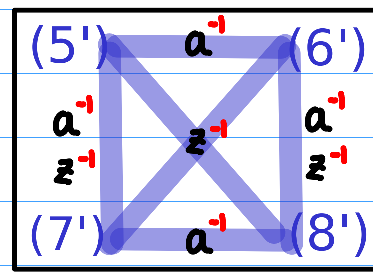
$z > a^{-1}$	(5)	(6)	$z > a$	$z > a^{-1}$	(5')	(6')	$z > a$
$z < a^{-1}$	(7)	(8)	$z < a$	$z < a^{-1}$	(7')	(8')	$z < a$

Substitute with a Multiplicative Inverse









Inv(a) Symm(base)

a^{n} (1) (2) a^{-n} | a^{n+1} (1') (2') a^{-n-1}

(a^{-1}, z) | (a^{-1}, z)

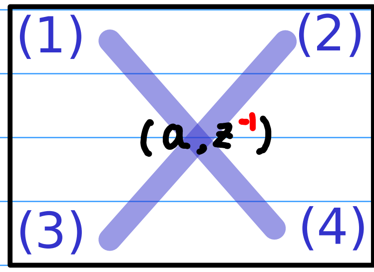
a^{n} (3) (4) a^{-n} | a^{n-1} (3') (4') a^{-n+1}

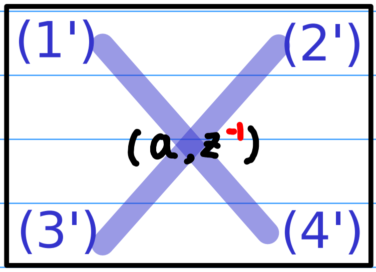
a^{n} (5) (6) a^{-n} | a^{n+1} (5') (6') a^{-n-1}

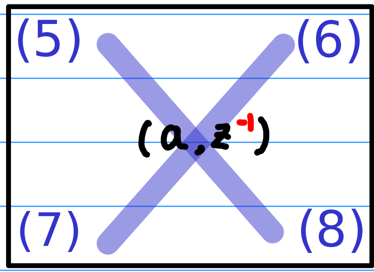
(a^{-1}, z) | (a^{-1}, z)

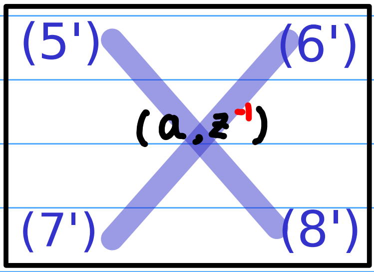
a^{n} (7) (8) a^{-n} | a^{n-1} (7') (8') a^{-n+1}

Inv(z) Symm(base, range)

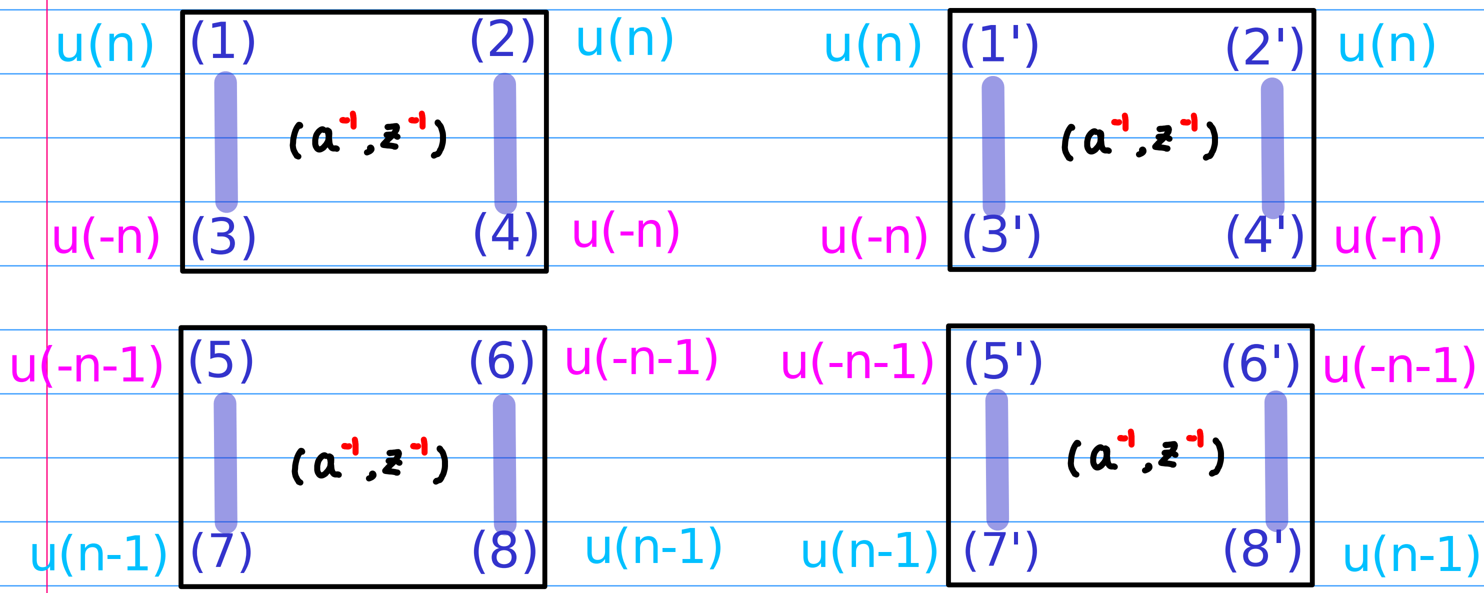








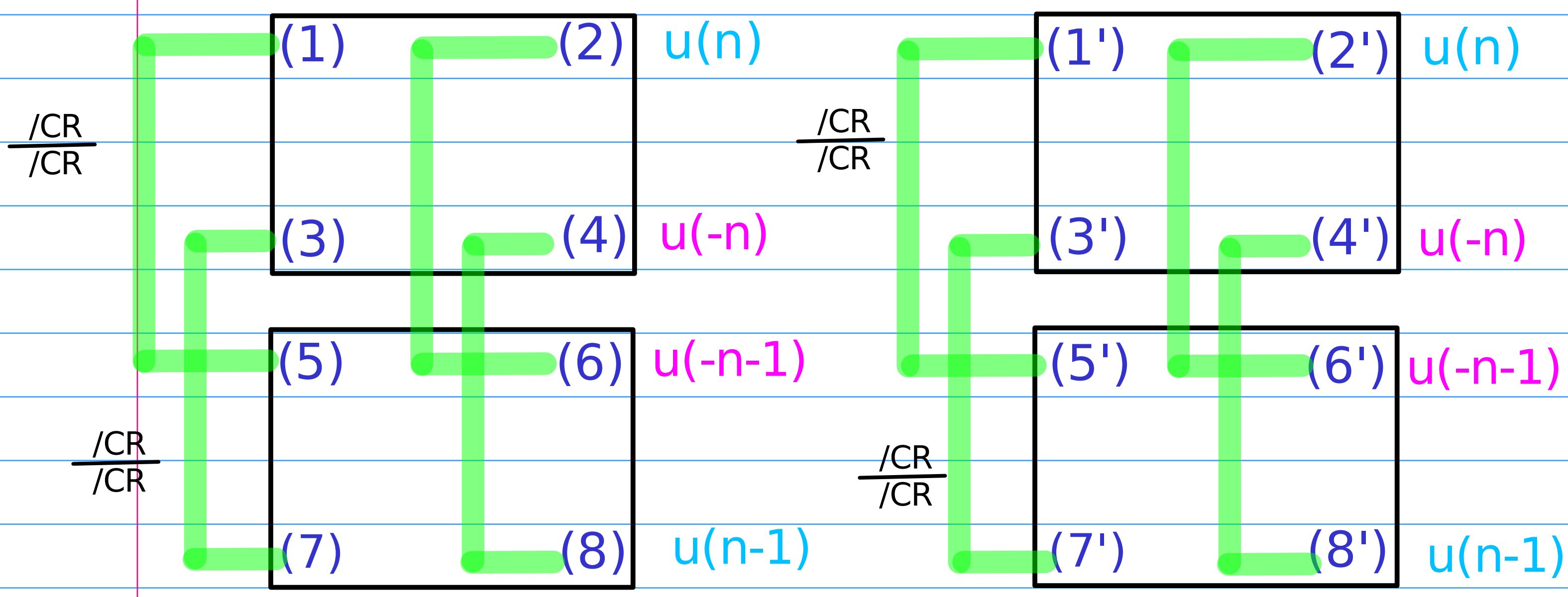
Inv(a,z) Symm(range)



$z < a^{-1}$ u(n)	(1)	(2)	$z < a$ u(n)	$z < a^{-1}$ u(n)	(1')	(2')	$z < a$ u(n)
$z > a^{-1}$ u(-n)	(3)	(4)	$z > a$ u(-n)	$z > a^{-1}$ u(-n)	(3')	(4')	$z > a$ u(-n)

$z > a^{-1}$ u(-n-1)	(5)	(6)	$z > a$ u(-n-1)	$z > a^{-1}$ u(-n-1)	(5')	(6')	$z > a$ u(-n-1)
$z < a^{-1}$ u(n-1)	(7)	(8)	$z < a$ u(n-1)	$z < a^{-1}$ u(n-1)	(7')	(8')	$z < a$ u(n-1)

$\frac{/CR}{/CR}$ Comp(range)



Unshifted Geometric Series

Expressions

$f(z)$ $f_1(z)$ $g(z)$ $g_1(z)$
$\bar{f}(z)$ $\bar{f}_1(z)$ $\bar{g}(z)$ $\bar{g}_1(z)$

(1)

$f(z) = \dfrac{1}{1 - a z}$	$\|z\| < a^{-1}$
$a^n \; u(n)$	$(n \geq 0)$

(2)

$g(z) = \dfrac{1}{1 - a^{-1} z}$	$\|z\| < a$
$\left(\frac{1}{a}\right)^n \; u(n)$	$(n \geq 0)$

(3)

$\bar{f}_1(z) = \dfrac{1}{1 - a^{-1} z^{-1}}$	$\|z\| > a^{-1}$
$a^n \; u(-n)$	$(n < 1)$

(4)

$\bar{g}_1(z) = \dfrac{1}{1 - a z^{-1}}$	$\|z\| > a$
$\left(\frac{1}{a}\right)^n \; u(-n)$	$(n < 1)$

(5)

$\bar{f}(z) = \dfrac{a^{-1} z^{-1}}{1 - a^{-1} z^{-1}}$	$\|z\| > a^{-1}$
$a^n \; u(-n-1)$	$(n < 0)$

(6)

$\bar{g}(z) = \dfrac{a z^{-1}}{1 - a z^{-1}}$	$\|z\| > a$
$\left(\frac{1}{a}\right)^n \; u(-n-1)$	$(n < 0)$

(7)

$f_1(z) = \dfrac{a z}{1 - a z}$	$\|z\| < a^{-1}$
$a^n \; u(n-1)$	$(n \geq 1)$

(8)

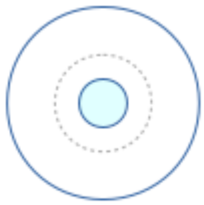
$g_1(z) = \dfrac{a^{-1} z}{1 - a^{-1} z}$	$\|z\| < a$
$\left(\frac{1}{a}\right)^n \; u(n-1)$	$(n \geq 1)$

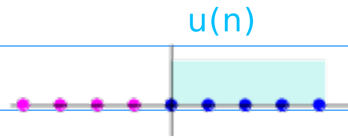
Unshifted Geometric Series

Expressions

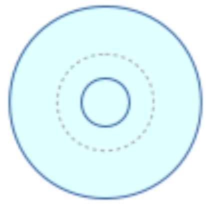
(1)

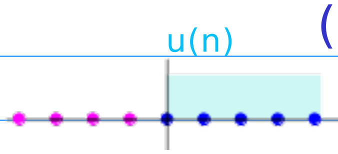
$|z|<a^{-1}$ $(n \geq 0)$





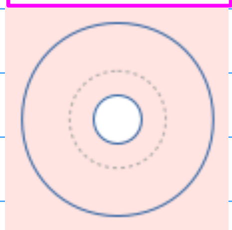
$|z|<a$ $(n \geq 0)$ (2)

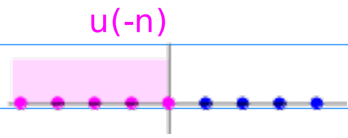




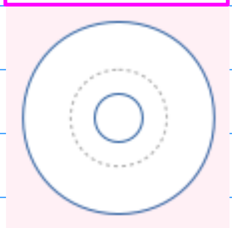
(3)

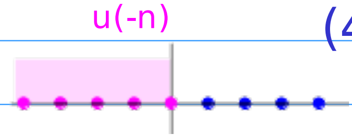
$|z|>a^{-1}$ $(n<1)$





$|z|>a$ $(n<1)$ (4)

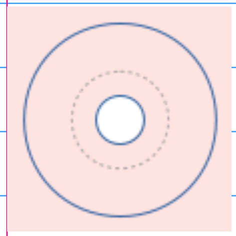


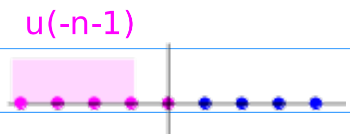


u(n-1)

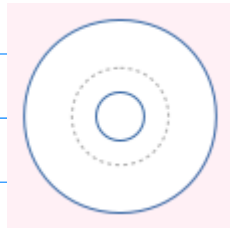
(5)

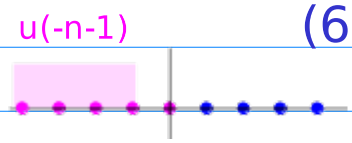
$|z|>a^{-1}$ $(n<0)$





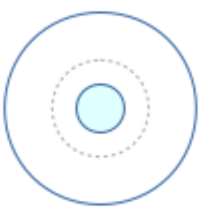
$|z|>a$ $(n<0)$ (6)

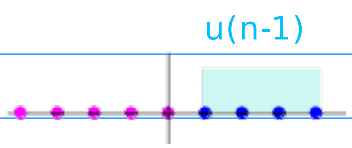




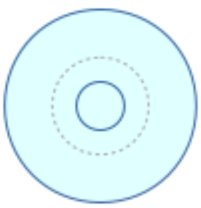
(7)

$|z|<a^{-1}$ $(n \geq 1)$





$|z|<a$ $(n \geq 1)$



u(n-1) (8)

Inv(a) Symm(base)

Inv(a,z) Symm(range)

(1)

$f(z) = \frac{1}{1-az}$	$\vert z\vert < a^{-1}$
a^n u(n)	$(n \geq 0)$

inv(a)

inv(a)

symm(base)

(2)

$g(z) = \frac{1}{1-a^{-1}z}$	$\vert z\vert < a$
$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

inv(a,z) rng(-n) symm(range)

inv(a,z) rng(-n) symm(rang

(3)

$\bar{f}_1(z) = \frac{1}{1-a^{-1}z^{-1}}$	$\vert z\vert > a^{-1}$
a^n u(-n)	$(n < 1)$

inv(a)

inv(a)

symm(base)

(4)

$\bar{g}_1(z) = \frac{1}{1-az^{-1}}$	$\vert z\vert > a$
$(\frac{1}{a})^n$ u(-n)	$(n < 1)$

(5)

$\bar{f}(z) = \frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\vert z\vert > a^{-1}$
a^n u(-n-1)	$(n < 0)$

inv(a)

inv(a)

symm(base)

(6)

$\bar{g}(z) = \frac{az^{-1}}{1-az^{-1}}$	$\vert z\vert > a$
$(\frac{1}{a})^n$ u(-n-1)	$(n < 0)$

inv(a,z) rng(-n) symm(range)

inv(a,z) rng(-n) symm(rang

(7)

$f_1(z) = \frac{az}{1-az}$	$\vert z\vert < a^{-1}$
a^n u(n-1)	$(n \geq 1)$

inv(a)

inv(a)

symm(base)

(8)

$g_1(z) = \frac{a^{-1}z}{1-a^{-1}z}$	$\vert z\vert < a$
$(\frac{1}{a})^n$ u(n-1)	$(n \geq 1)$

$$\frac{/CR}{/CR}$$ Comp(range)

(1)

$f(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^n u(n)	$(n \geq 0)$

(2)

$g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

$\frac{/cr}{/cr}$ dual rng(-n) -1 comp(range)

$\frac{/cr}{/cr}$ dual rng(-n) -1 comp(range)

(3)

$\bar{f}_1(z) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^n u(-n)	$(n < 1)$

(4)

$\bar{g}_1(z) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$(\frac{1}{a})^n$ u(-n)	$(n < 1)$

$\frac{/cr}{/cr}$ dual rng(-n) -1 comp(range)

$\frac{/cr}{/cr}$ dual rng(-n) -1 comp(range

(5)

$\bar{f}(z) = \dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^n u(-n-1)	$(n < 0)$

(6)

$\bar{g}(z) = \dfrac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$(\frac{1}{a})^n$ u(-n-1)	$(n < 0)$

(7)

$f_1(z) = \dfrac{az}{1-az}$	$\lvert z\rvert < a^{-1}$
a^n u(n-1)	$(n \geq 1)$

(8)

$g_1(z) = \dfrac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$(\frac{1}{a})^n$ u(n-1)	$(n \geq 1)$

Inv(z) Symm(base, range)

(1)

$f(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^n u(n)	$(n \geq 0)$

(2)

$g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

inv(z) inv(a) rng(-n) symm(base, range)

(3)

$\bar{f}_1(z) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^n u(-n)	$(n < 1)$

(4)

$\bar{g}_1(z) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$(\frac{1}{a})^n$ u(-n)	$(n < 1)$

(5)

$\bar{f}(z) = \dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^n u(-n-1)	$(n < 0)$

(6)

$\bar{g}(z) = \dfrac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$(\frac{1}{a})^n$ u(-n-1)	$(n < 0)$

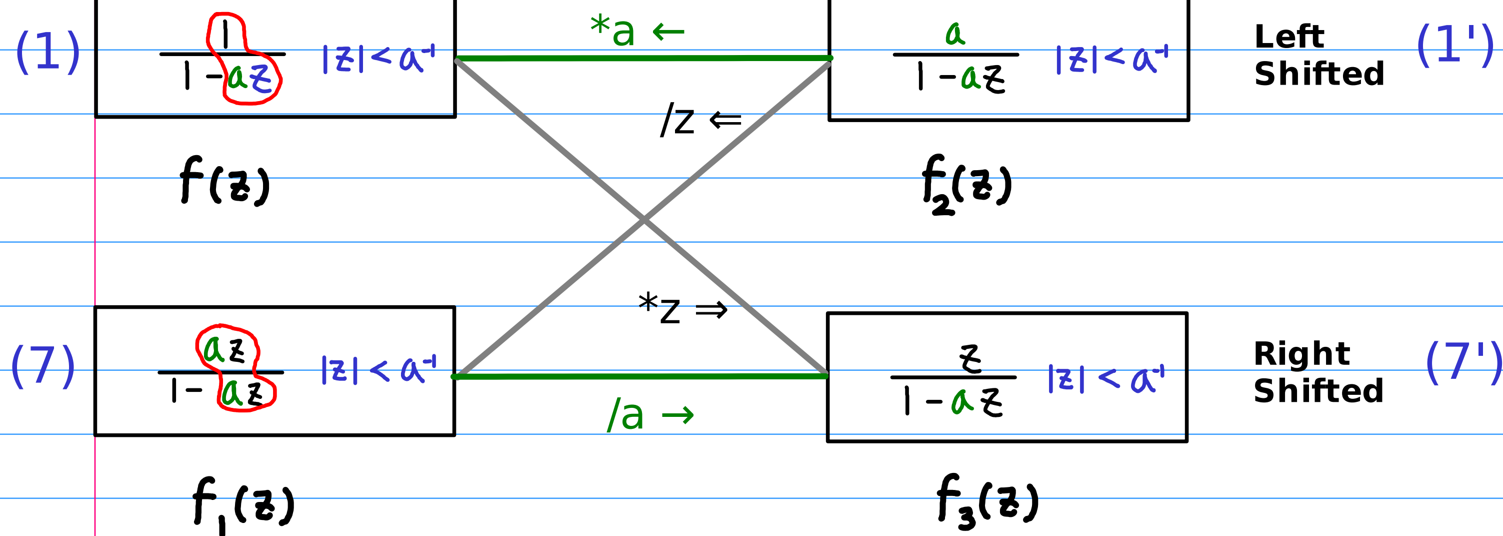
inv(z) inv(a) rng(-n) symm(base, range

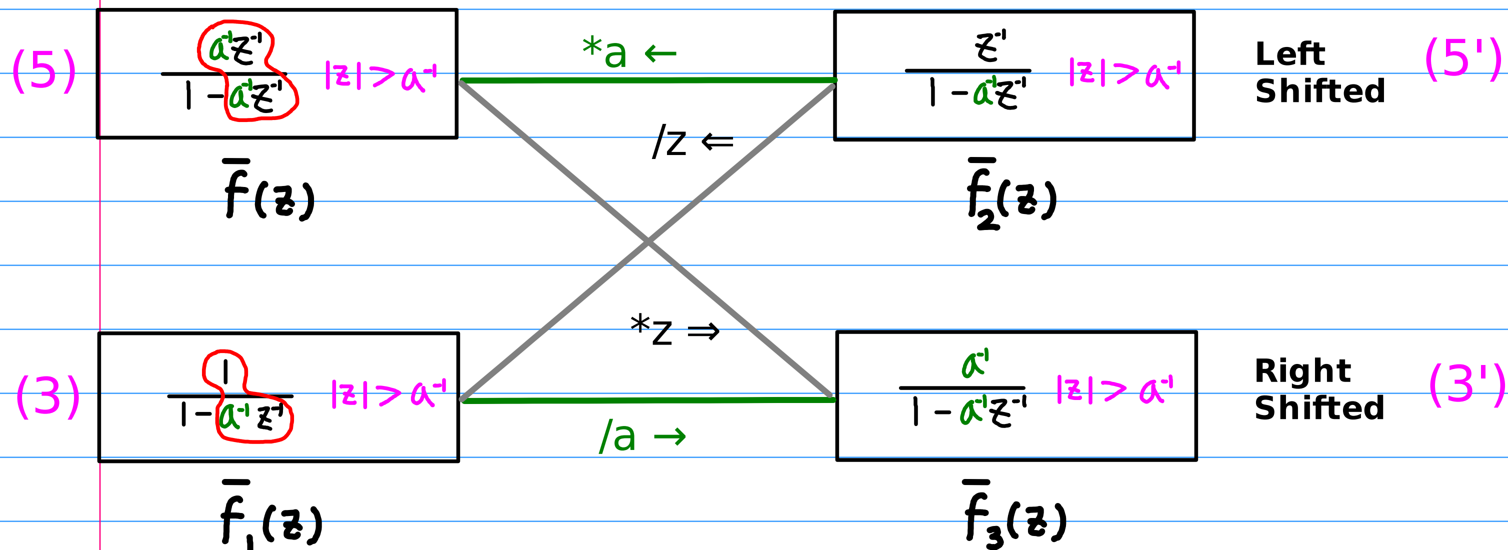
(7)

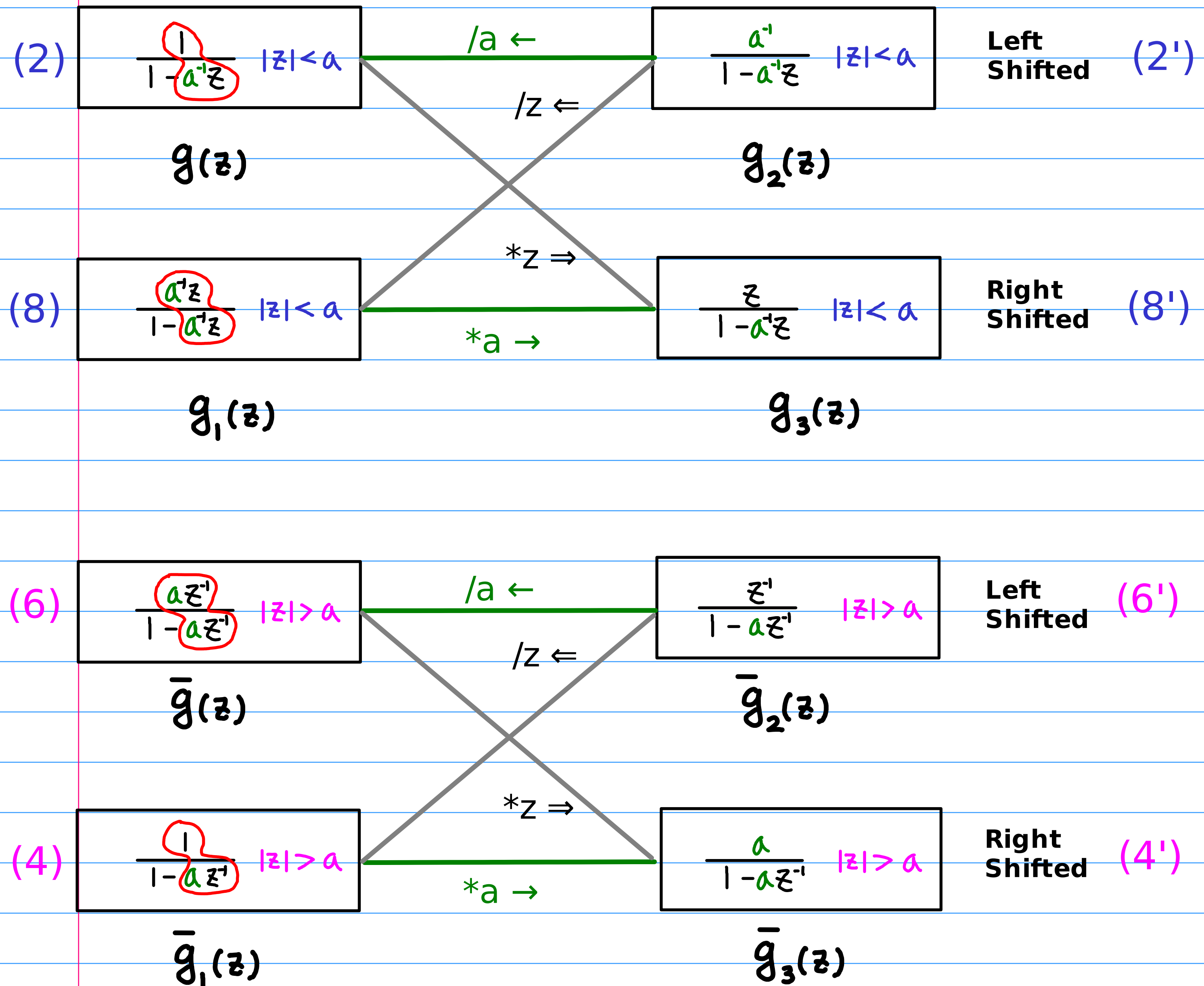
$f_1(z) = \dfrac{az}{1-az}$	$\lvert z\rvert < a^{-1}$
a^n u(n-1)	$(n \geq 1)$

(8)

$g_1(z) = \dfrac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$(\frac{1}{a})^n$ u(n-1)	$(n \geq 1)$


(1)
1 / (1 - az) |z| < a^-1
f(z)
*a ←
/z ⇐
*z ⇒
/a →
a / (1 - az) |z| < a^-1
f_2(z)
Left Shifted
(1')
(7)
az / (1 - az) |z| < a^-1
f_1(z)
z / (1 - az) |z| < a^-1
f_3(z)
Right Shifted
(7')


(5)
a^-1 z^-1 / (1 - a^-1 z^-1) |z| > a^-1
f̄(z)
*a ←
/z ⇐
*z ⇒
/a →
z^-1 / (1 - a^-1 z^-1) |z| > a^-1
f̄_2(z)
Left Shifted
(5')
(3)
1 / (1 - a^-1 z^-1) |z| > a^-1
f̄_1(z)
a^-1 / (1 - a^-1 z^-1) |z| > a^-1
f̄_3(z)
Right Shifted
(3')



(2)
$\frac{1}{1-a^{-1}z}$ $|z|<a$
$g(z)$
/a ←
/z ⇐
$\frac{a^{-1}}{1-a^{-1}z}$ $|z|<a$
$g_2(z)$
Left Shifted
(2')
(8)
$\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$
$g_1(z)$
*z ⇒
*a →
$\frac{z}{1-a^{-1}z}$ $|z|<a$
$g_3(z)$
Right Shifted
(8')
(6)
$\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$
$\bar{g}(z)$
/a ←
/z ⇐
$\frac{z^{-1}}{1-az^{-1}}$ $|z|>a$
$\bar{g}_2(z)$
Left Shifted
(6')
(4)
$\frac{1}{1-az^{-1}}$ $|z|>a$
$\bar{g}_1(z)$
*z ⇒
*a →
$\frac{a}{1-az^{-1}}$ $|z|>a$
$\bar{g}_3(z)$
Right Shifted
(4')

Shifted Geometric Series

Expressions

$f_2(z)$ $f_3(z)$ $g_2(z)$ $g_3(z)$

$\bar{f}_2(z)$ $\bar{f}_3(z)$ $\bar{g}_2(z)$ $\bar{g}_3(z)$

(1')

$f_2(z) = \dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$a^{n+1}\,u(n)$	$(n \geq 0)$

(2')

$g_2(z) = \dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n+1} u(n)$	$(n \geq 0)$

(3')

$\bar{f}_3(z) = \dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a^{n-1}\,u(-n)$	$(n < 1)$

(4')

$\bar{g}_3(z) = \dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n-1} u(-n)$	$(n < 1)$

(5')

$\bar{f}_2(z) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a^{n+1}\,u(-n-1)$	$(n < 0)$

(6')

$\bar{g}_2(z) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n+1} u(-n-1)$	$(n < 0)$

(7')

$f_3(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a^{n-1}\,u(n-1)$	$(n \geq 1)$

(8')

$g_3(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n-1} u(n-1)$	$(n \geq 1)$

Shifted Geometric Series

Expressions

$f_2(z)$ $f_3(z)$ $g_2(z)$ $g_3(z)$

$\bar{f}_2(z)$ $\bar{f}_3(z)$ $\bar{g}_2(z)$ $\bar{g}_3(z)$

(1')

$f_2(z) = \dfrac{a}{1 - az}$	$\lvert z\rvert < a^{-1}$
$a^{n+1}\,u(n)$	$(n \geq 0)$

(2')

$g_2(z) = \dfrac{a^{-1}}{1 - a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n+1} u(n)$	$(n \geq 0)$

(5')

$\bar{f}_2(z) = \dfrac{z^{-1}}{1 - a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a^{n+1}\,u(-n-1)$	$(n < 0)$

(6')

$\bar{g}_2(z) = \dfrac{z^{-1}}{1 - az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n+1} u(-n-1)$	$(n < 0)$

(3')

$\bar{f}_3(z) = \dfrac{a^{-1}}{1 - a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a^{n-1}\,u(-n)$	$(n < 1)$

(4')

$\bar{g}_3(z) = \dfrac{a}{1 - az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n-1} u(-n)$	$(n < 1)$

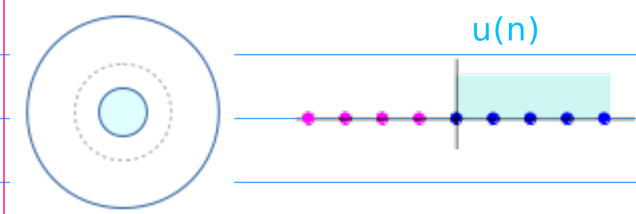
(7')

$f_3(z) = \dfrac{z}{1 - az}$	$\lvert z\rvert < a^{-1}$
$a^{n-1}\,u(n-1)$	$(n \geq 1)$

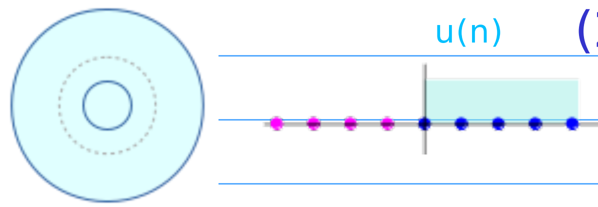
(8')

$g_3(z) = \dfrac{z}{1 - a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n-1} u(n-1)$	$(n \geq 1)$

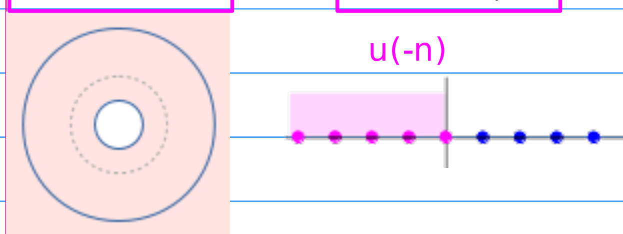
(1') $|z|<a^{-1}$ $(n \geq 0)$



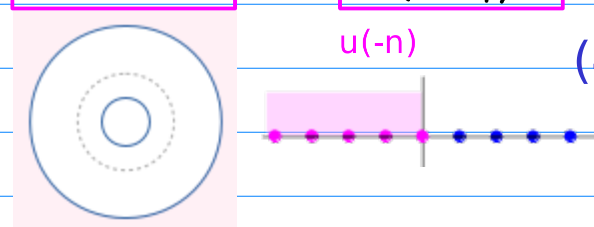
(2') $|z|<a$ $(n \geq 0)$



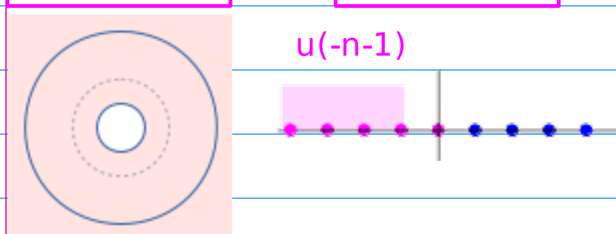
(3') $|z|>a^{-1}$ $(n<1)$



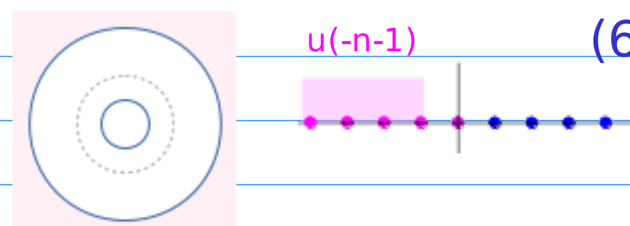
(4') $|z|>a$ $(n<1)$



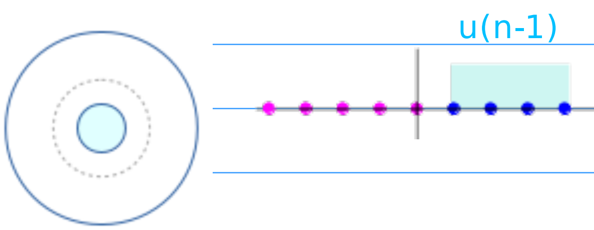
(5') $|z|>a^{-1}$ $(n<0)$



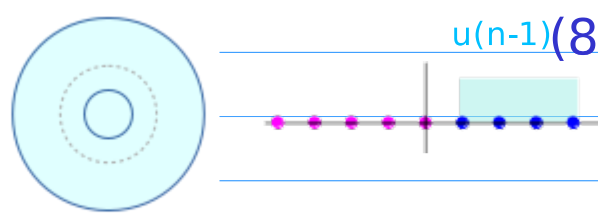
(6') $|z|>a$ $(n<0)$



(7') $|z|<a^{-1}$ $(n \geq 1)$



(8') $|z|<a$ $(n \geq 1)$



(1')

$f_2(z) = \dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
a^{n+1} u(n)	$(n \geq 0)$

inv(a)

inv(a)

symm(base)

(2')

$g_2(z) = \dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n+1}$ u(n)	$(n \geq 0)$

inv(a,z)

inv(a,z)

(3')

$\bar{f}_3(z) = \dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^{n-1} u(-n)	$(n < 1)$

inv(a)

inv(a)

symm(base)

(4')

$\bar{g}_3(z) = \dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n-1}$ u(-n)	$(n < 1)$

(5')

$\bar{f}_2(z) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^{n+1} u(-n-1)	$(n < 0)$

inv(a)

inv(a)

symm(base)

(6')

$\bar{g}_2(z) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n+1}$ u(-n-1)	$(n < 0)$

inv(a,z)

inv(a,z)

(7')

$f_3(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
a^{n-1} u(n-1)	$(n \geq 1)$

inv(a)

inv(a)

symm(base)

(8')

$g_3(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n-1}$ u(n-1)	$(n \geq 1)$

(1')

$f_2(z) = \frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$a^{n+1}\ u(n)$	$(n \geq 0)$

$\frac{/cr}{/cr}$ dual rng(-n) -1 comp(range)

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(1')

$\displaystyle f_2(z) = \frac{a}{1 - az}$	$\lvert z \rvert < a^{-1}$
$a^{n+1}\,u(n)$	$(n \geq 0)$

(2')

$\displaystyle g_2(z) = \frac{a^{-1}}{1 - a^{-1}z}$	$\lvert z \rvert < a$
$\left(\frac{1}{a}\right)^{n+1} u(n)$	$(n \geq 0)$

inv(z)

(3')

$\displaystyle \bar{f}_3(z) = \frac{a^{-1}}{1 - a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$
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