Laurent Series and z-Transform
- Geometric Series
Double Pole Properties B

20190227 Wed

2 formulas of z

① $$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\dfrac{-1}{(z-1)(z-2)}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$$\boxed{+\frac{1}{z-1} - \frac{1}{z-2}} \qquad \boxed{-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<1}$$

causal $f(z)$

anti-causal $X(z)$

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>1}$$

anti-causal $f(z)$

causal $X(z)$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $f(z)$

causal $X(z)$

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

causal $f(z)$

anti-causal $X(z)$

$$① \quad \frac{-1}{(z-1)(z-2)} \qquad\qquad ② \quad \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$+\frac{1}{z-1} - \frac{1}{z-2} \qquad\qquad -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$$\cdot 1 \downarrow \quad \cdot \tfrac{1}{2} \downarrow \qquad\qquad \cdot \tfrac{1}{z} \downarrow \quad \cdot \tfrac{1}{z} \downarrow$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad \boxed{|z|<1} \qquad\qquad -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad \boxed{|z|>1}$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{|z|>2} \qquad\qquad +\frac{z}{1-z} - \frac{z}{1-2z} \quad \boxed{|z|<0.5}$$

$$\cdot \tfrac{1}{z} \uparrow \quad \cdot \tfrac{1}{z} \uparrow \qquad\qquad \cdot \tfrac{1}{1} \uparrow \quad \cdot 2 \uparrow$$

$$+\frac{1}{z-1} - \frac{1}{z-2} \qquad\qquad -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

① $\dfrac{-1}{(z-1)(z-2)}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$|z|<1 \qquad |0.5z|<1$

$|z|<1 \qquad |z|<2$

$|z^{-1}|<1 \qquad |0.5z^{-1}|<1$

$1<|z| \qquad 0.5<|z|$

$\boxed{-\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z} \;\Big|\; |z|<1}$

$\boxed{-\dfrac{1}{1-z^{-1}}+\dfrac{0.5}{1-0.5z^{-1}} \;\Big|\; |z|>1}$

$\boxed{+\dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}} \;\Big|\; |z|>2}$

$\boxed{+\dfrac{z}{1-z}-\dfrac{z}{1-2z} \;\Big|\; |z|<0.5}$

$|z^{-1}|<1 \qquad |2z^{-1}|<1$

$1<|z| \qquad 2<|z|$

$|z|<1 \qquad |2z|<1$

$|z|<1 \qquad |z|<0.5$

$$\begin{array}{ccc|ccc}
-\dfrac{1}{1-z} & +\dfrac{0.5}{1-0.5z} & |z|<1 & -\dfrac{1}{1-z^{-1}} & +\dfrac{0.5}{1-0.5z^{-1}} & |z|>1 \\
\Big\downarrow \cdot\dfrac{1}{z} & \Big\downarrow \cdot\dfrac{2}{z} & & \Big\downarrow \cdot z & \Big\downarrow \cdot 2z & \\
+\dfrac{z^{-1}}{1-z^{-1}} & -\dfrac{z^{-1}}{1-2z^{-1}} & |z|>2 & +\dfrac{z}{1-z} & -\dfrac{z}{1-2z} & |z|<0.5
\end{array}$$

$$\begin{array}{ccc|ccc}
-\dfrac{1}{1-z} & +\dfrac{0.5}{1-0.5z} & |z|<1 & -\dfrac{1}{1-z^{-1}} & +\dfrac{0.5}{1-0.5z^{-1}} & |z|>1 \\
\Big\uparrow \cdot z & \Big\uparrow \cdot\dfrac{z}{2} & & \Big\uparrow \cdot\dfrac{1}{z} & \Big\uparrow \cdot\dfrac{1}{2z} & \\
+\dfrac{z^{-1}}{1-z^{-1}} & -\dfrac{z^{-1}}{1-2z^{-1}} & |z|>2 & +\dfrac{z}{1-z} & -\dfrac{z}{1-2z} & |z|<0.5
\end{array}$$

Causal sequence a_n & x_n

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \boxed{|z|<1}$$

causal $f_1(z) =$

$$-\left[1+1^2z^1+1^3z^2+\cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \boxed{|z|>1}$$

causal $Y_1(z) =$

$$-\left[1^1z^0+1^2z^{-1}+1^3z^{-2}+\cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \boxed{|z|>2}$$

causal $X_2(z)$

$$+\left[\left(\tfrac{1}{1}\right)^0z^{-1}+\left(\tfrac{1}{1}\right)^1z^{-2}+\left(\tfrac{1}{1}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{1}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

1 2 3

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \boxed{|z|<0.5}$$

causal $g_2(z)$

$$+\left[\left(\tfrac{1}{1}\right)^0z^1+\left(\tfrac{1}{1}\right)^1z^2+\left(\tfrac{1}{1}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{1}\right)^{n-1}$$
$$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$

1 2 3

Anti-causal sequence a_n & x_n

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$$

anti-causal $X_1(z)$

$$\boxed{\begin{array}{c} 2=\left(\frac{1}{2}\right)^{-1} \\ \left(\frac{1}{2}\right)=2^{-1} \end{array}}$$

$$-\left[\left(\frac{1}{1}\right)^{-1}+\left(\frac{1}{1}\right)^{-2}z^{1}+\left(\frac{1}{1}\right)^{-3}z^{2}+\cdots\right] \quad -\left(\frac{1}{1}\right)^{n-1}$$

$$+\left[2^{-1}+2^{-2}z^{1}+2^{-3}z^{2}+\cdots\right] \quad +2^{n-1}$$

0 -1 -2

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$$

anti-causal $g_1(z)$

$$-\left[\left(\frac{1}{1}\right)^{-1}z^{0}+\left(\frac{1}{1}\right)^{-2}z^{-1}+\left(\frac{1}{1}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\frac{1}{1}\right)^{n-1}$$

$$+\left[2^{-1}z^{0}+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$

0 -1 -2

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$\boxed{\begin{array}{c} 2=\left(\frac{1}{2}\right)^{-1} \\ \left(\frac{1}{2}\right)=2^{-1} \end{array}}$$

$$+\left[1^{0}z^{-1}+1^{-1}z^{-2}+1^{-2}z^{-3}+\cdots\right] \quad +1^{n+1}$$

$$-\left[\left(\frac{1}{2}\right)^{0}z^{-1}+\left(\frac{1}{2}\right)^{-1}z^{-2}+\left(\frac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

anti-causal $Y_2(z)$

$$+\left[1^{0}z^{1}+1^{-1}z^{2}+1^{-2}z^{3}+\cdots\right] \quad +1^{n+1}$$

$$-\left[\left(\frac{1}{2}\right)^{0}z^{1}+\left(\frac{1}{2}\right)^{-1}z^{2}+\left(\frac{1}{2}\right)^{-2}z^{3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \quad \boxed{|z|<1}$$

causal $f_1(z) =$

$$-[1 + 1^2 z^1 + 1^3 z^2 + \cdots] \qquad -1^{n+1}$$

$$+[(\tfrac{1}{2}) + (\tfrac{1}{2})^2 z^1 + (\tfrac{1}{2})^3 z^2 + \cdots] \qquad +(\tfrac{1}{2})^{n+1}$$

0 1 2

anti-causal $X_1(z)$

$$-[(\tfrac{1}{1})^{-1} + (\tfrac{1}{1})^{-2} z^1 + (\tfrac{1}{1})^{-3} z^2 + \cdots] \qquad -(\tfrac{1}{1})^{n-1}$$

$$+[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots] \qquad +2^{n-1}$$

0 -1 -2

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad \boxed{|z|>1}$$

anti-causal $g_1(z)$

$$-[(\tfrac{1}{1})^{-1} z^0 + (\tfrac{1}{1})^{-2} z^{-1} + (\tfrac{1}{1})^{-3} z^{-2} + \cdots] \qquad -(\tfrac{1}{1})^{n-1}$$

$$+[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots] \qquad +2^{n-1}$$

0 -1 -2

causal $Y_1(z) =$

$$-[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots] \qquad -1^{n+1}$$

$$+[(\tfrac{1}{2})^1 z^0 + (\tfrac{1}{2})^2 z^{-1} + (\tfrac{1}{2})^3 z^{-2} + \cdots] \qquad +(\tfrac{1}{2})^{n+1}$$

0 1 2

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$+[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots] \qquad +1^{n+1}$$

$$-[(\tfrac{1}{2})^0 z^{-1} + (\tfrac{1}{2})^{-1} z^{-2} + (\tfrac{1}{2})^{-2} z^{-3} + \cdots] \qquad -(\tfrac{1}{2})^{n+1}$$

-1 -2 -3

causal $X_2(z)$

$$+[(\tfrac{1}{1})^0 z^{-1} + (\tfrac{1}{1})^1 z^{-2} + (\tfrac{1}{1})^2 z^{-3} + \cdots] \qquad +(\tfrac{1}{1})^{n-1}$$

$$-[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots] \qquad -2^{n-1}$$

1 2 3

$$+\frac{z}{1-z}-\frac{z}{1-2z} \quad \boxed{|z|<0.5}$$

causal $g_2(z)$

$$+[(\tfrac{1}{1})^0 z^1 + (\tfrac{1}{1})^1 z^2 + (\tfrac{1}{1})^2 z^3 + \cdots] \qquad +(\tfrac{1}{1})^{n-1}$$

$$-[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots] \qquad -2^{n-1}$$

1 2 3

anti-causal $Y_2(z)$

$$+[1^0 z^1 + 1^{-1} z^2 + 1^{-2} z^3 + \cdots] \qquad +1^{n+1}$$

$$-[(\tfrac{1}{2})^0 z^1 + (\tfrac{1}{2})^{-1} z^2 + (\tfrac{1}{2})^{-2} z^3 + \cdots] \qquad -(\tfrac{1}{2})^{n+1}$$

-1 -2 -3

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<1}$$

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>1}$$

$$f(z) = -\left[1 + 1^2 z + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{x_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$f(z) = +\left[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^1 + \left(\tfrac{1}{1}\right)^1 z^2 + \left(\tfrac{1}{1}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{1}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^1 z^{-2} + \left(\tfrac{1}{1}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{1}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[1^0 z^1 + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{x_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

1-A		2-A	
1-B		2-B	

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

$$\frac{-1}{(z-1)(z-2)} = +\frac{1}{z-1} - \frac{1}{z-2}$$

① - Ⓐ $\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z| < 1}$ $\boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n \geq 0)}$

$$-\big(\underset{n=0}{1} + \underset{n=1}{1^2 z} + \underset{n=2}{1^3 z^2} + \cdots\big) + \big(\underset{n=0}{\left(\tfrac{1}{2}\right)} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\big)$$

① - Ⓑ $\boxed{\frac{z^{-1}}{1-z^{-1}} - \frac{1}{1-2z^{-1}}}$ $\boxed{|z| > 2}$ $\boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n < 0)}$

$$\left(z^{-1} + \left(\tfrac{1}{1}\right)^1 z^{-2} + \left(\tfrac{1}{1}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\big(\underset{n=-1}{1^0 z^{-1}} + \underset{n=-2}{1^{-1} z^{-2}} + \underset{n=-3}{1^{-2} z^{-3}} + \cdots\big) - \big(\underset{n=-1}{\left(\tfrac{1}{2}\right)^0 z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-1} z^{-2}} + \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-2} z^{-3}} + \cdots\big)$$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

② - Ⓐ $\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z| > 1}$ $\boxed{-\left(\frac{1}{1}\right)^{n-1} + 2^{n-1}}$ $\boxed{(n < 1)}$

$$-\left(1 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-\big(\underset{n=0}{\left(\tfrac{1}{1}\right)^{-1}} + \underset{n=-1}{\left(\tfrac{1}{1}\right)^{-2} z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{1}\right)^{-3} z^{-2}} + \cdots\big) + \big(\underset{n=0}{2^{-1}} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots\big)$$

② - Ⓑ $\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}$ $\boxed{|z| < 0.5}$ $\boxed{+1^{n-1} - 2^{n-1}}$ $\boxed{(n \geq 1)}$

$$+\big(\underset{n=1}{z} + \underset{n=2}{1z^2} + \underset{n=3}{1^2 z^3} + \cdots\big) - \big(\underset{n=1}{z} + \underset{n=2}{2z^2} + \underset{n=3}{2^2 z^3} + \cdots\big)$$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

$$\frac{-1}{(z-1)(z-2)} = +\frac{1}{z-1} - \frac{1}{z-2}$$

① - Ⓐ $\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}$ $\quad \boxed{|z| < 1}$ $\quad \boxed{-\left(\frac{1}{1}\right)^{n-1} + 2^{n-1}}$ $\quad \boxed{(n < 1)}$

$$-\left(1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

① - Ⓑ $\boxed{\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\quad \boxed{|z| > 2}$ $\quad \boxed{+\left(\frac{1}{1}\right)^{n-1} - 2^{n-1}}$ $\quad \boxed{(n \geq 1)}$

$$\left(\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^1 z^{-2} + \left(\tfrac{1}{1}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = -\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

② - Ⓐ $\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\quad \boxed{|z| > 1}$ $\quad \boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad \boxed{(n \geq 0)}$

$$-\left(1 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

② - Ⓑ $\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}$ $\quad \boxed{|z| < 0.5}$ $\quad \boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\quad \boxed{(n < 0)}$

$$+\left(z + \left(\tfrac{1}{1}\right) z^2 + \left(\tfrac{1}{1}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$$+\left(1^0 z + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$f(z) \Longleftrightarrow a_n$

$X(z) \Longleftrightarrow x_n$

①-Ⓐ

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<1}$$

$$a_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

②-Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>1}$$

$$a_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

$$x_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

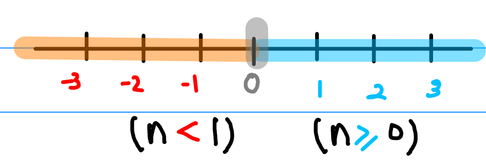
$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

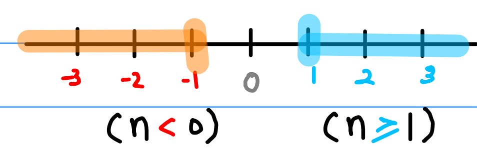
$$a_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$$

$$x_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = a_{-n}$$

$$a_n = x_{-n}$$





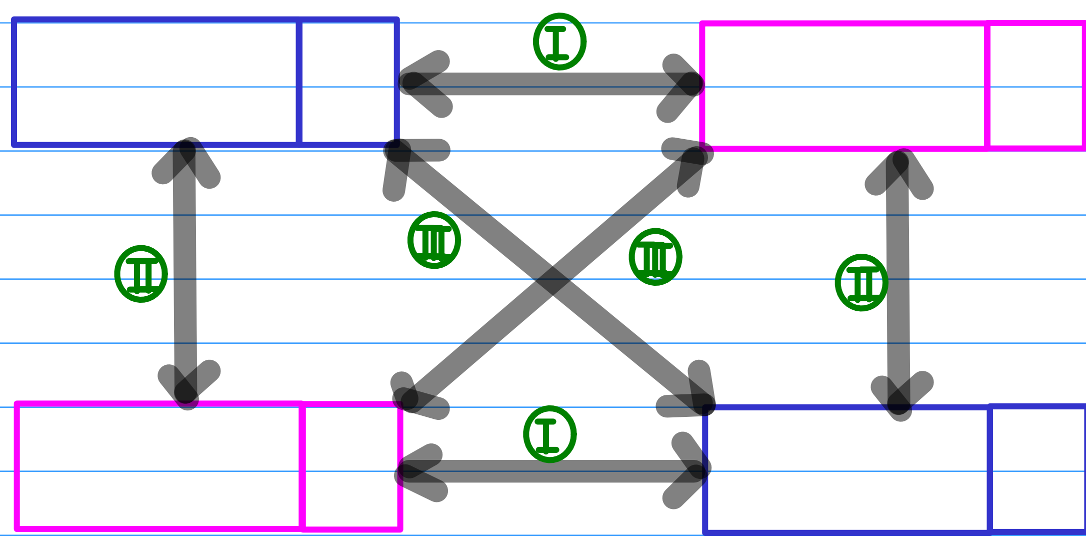
$(n \geq 0) \Longleftrightarrow (n<1)$

$(n \geq 1) \Longleftrightarrow (n<0)$

$$\text{I} \quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$\text{II} \quad (z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$\text{III} \quad (z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$



$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$\textcircled{I}$ $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

①-Ⓐ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \boxed{|z|<1} \quad \xleftrightarrow{z^{-1},\ R^{-1}}$$

$$a_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0) \quad \xleftrightarrow{a_{-n},\ -N}$$

$$x_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n<1) \quad \xleftrightarrow{a_{-n},\ -N}$$

②-Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \boxed{|z|>1}$$

$$a_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n<1)$$

$$x_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \boxed{|z|>2} \quad \xleftrightarrow{z^{-1},\ R^{-1}}$$

$$a_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0) \quad \xleftrightarrow{a_{-n},\ -N}$$

$$x_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1) \quad \xleftrightarrow{a_{-n},\ -N}$$

②-Ⓑ

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \boxed{|z|<0.5}$$

$$a_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$x_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

II $\quad (z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$

① – Ⓐ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<1}$$

$$\parallel \qquad \Updownarrow R^{-1}$$

① – Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

② – Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>1}$$

$$\parallel \qquad \Updownarrow R^{-1}$$

② – Ⓑ

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

① – Ⓐ

$$a_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

① – Ⓑ

$$a_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

② – Ⓐ

$$a_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

② – Ⓑ

$$a_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

① – Ⓐ

$$x_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

① – Ⓑ

$$x_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② – Ⓐ

$$x_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

② – Ⓑ

$$x_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

III ①

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

① - A

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \; \boxed{|z| < 1}$$

$$a_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

② - A

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z| > 1}$$

$$a_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

① - B

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}} \; \boxed{|z| > 2}$$

$$a_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

② - B

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \; \boxed{|z| < 0.5}$$

$$a_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

① - A

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \; \boxed{|z| < 1}$$

$$x_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② - A

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z| > 1}$$

$$x_n = \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

① - B

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}} \; \boxed{|z| > 2}$$

$$x_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

② - B

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \; \boxed{|z| < 0.5}$$

$$x_n = \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

III ②

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \to (z, R^{-1}) \qquad (a_{-n}, -N) \to (-a_n, N^c)$$

①–Ⓐ ②–Ⓑ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<1} \Longleftrightarrow \boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{-1^{n+1} + (\tfrac{1}{2})^{n+1}} \quad (n \geq 0) \quad \xleftrightarrow{-a_{-n},\ (-N)^c} \quad a_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

①–Ⓑ ②–Ⓐ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}}\ \boxed{|z|>2} \Longleftrightarrow \boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$$

$$a_n = \boxed{+1^{n+1} - (\tfrac{1}{2})^{n+1}} \quad (n < 0) \quad \xleftrightarrow{-a_{-n},\ (-N)^c} \quad a_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

①–Ⓐ ②–Ⓑ

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<1} \Longleftrightarrow \boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$x_n = \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1) \quad \xleftrightarrow{-a_{-n},\ (-N)^c} \quad x_n = \boxed{-1^{n+1} + (\tfrac{1}{2})^{n+1}} \quad (n \geq 0)$$

①–Ⓑ ②–Ⓐ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{1}}{1-2z^{-1}}}\ \boxed{|z|>2} \Longleftrightarrow \boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$$

$$x_n = \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1) \quad \xleftrightarrow{-a_{-n},\ (-N)^c} \quad x_n = \boxed{+1^{n+1} - (\tfrac{1}{2})^{n+1}} \quad (n < 0)$$

IV $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

①-Ⓐ

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$$

$$a_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

a_{-n} $\quad -N$

$$x_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

z^{-1}

$-n$

$-n$

②-Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$$

$$a_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

a_{-n} $\quad -N$

$$x_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

I

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

a_{-n} $\quad -N$

$$x_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$$

z^{-1}

$-n$

$-n$

②-Ⓑ

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$$

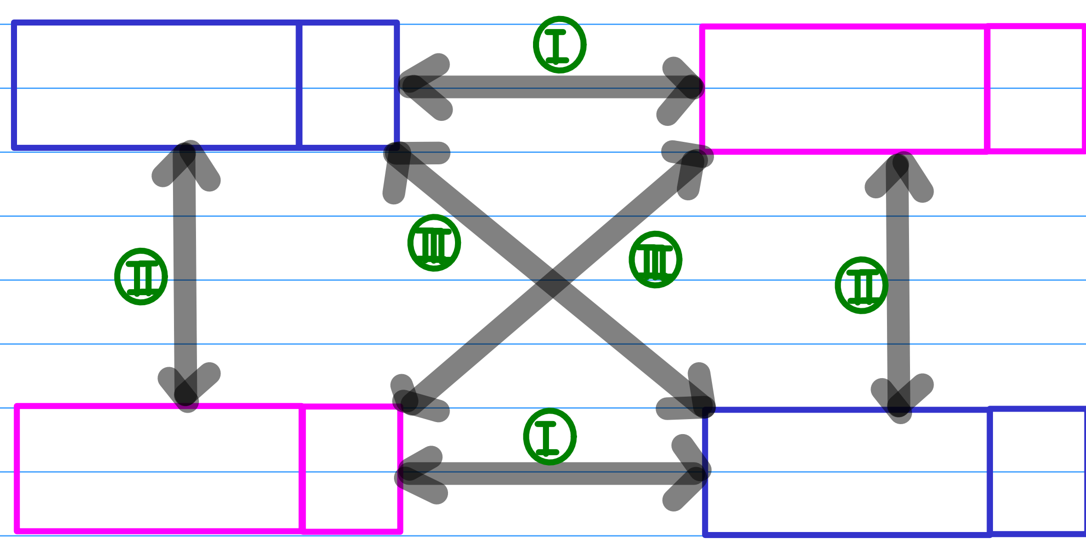
a_{-n} $\quad -N$

$$x_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

I $\quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

II $\quad (z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

III $\quad (z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

Some notations

$R1(z) \quad R1(z^{-1})$

$R2(z) \quad R2(z^{-1})$

R^{-1}

$-N, \quad N^{c}$

ROC's of interests

R1(z)
R2(z)

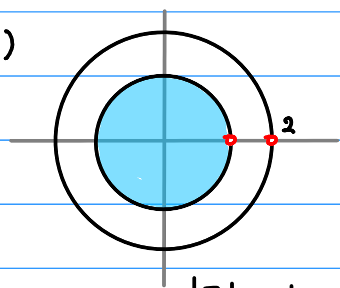
R1(z^{-1})
R2(z^{-1})

① $\dfrac{-1}{(z-1)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$$\boxed{+\frac{1}{z-1}-\frac{1}{z-2}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{-\frac{z}{(z-1)}+\frac{0.5z}{(z-0.5)}}$$

$$p_1 = 1 \qquad\qquad\qquad p_1^{-1} = 1$$
$$p_2 = 2 \quad \xleftrightarrow{z^{-1}} \quad p_2^{-1} = 0.5$$

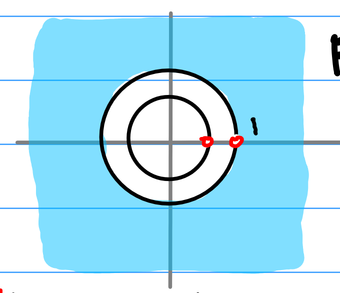
R1(z)



$|z|<1$

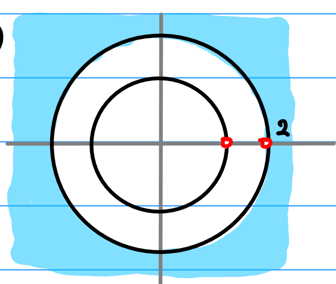
$\xleftrightarrow{z^{-1}}$

R1(z^{-1})



$|z^{-1}|<1 \qquad |z|>1$

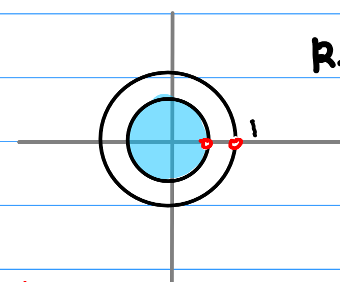
R2(z)



$|z|>2$

$\xleftrightarrow{z^{-1}}$

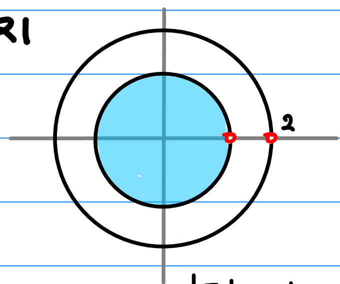
R2(z^{-1})



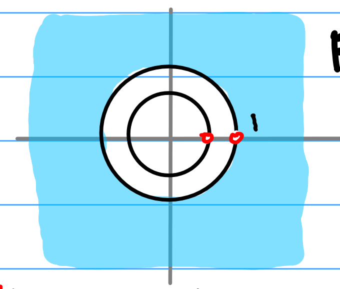
$|z^{-1}|>2 \qquad |z|<0.5$

R^{-1}

$R1(z) \triangleq R1$



$\xleftrightarrow{z^{-1}}$

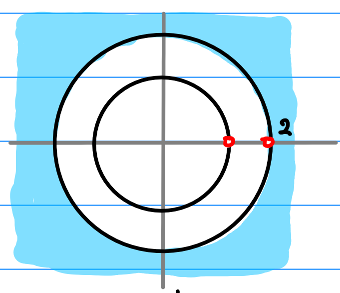


$R1(z^{-1}) \triangleq$

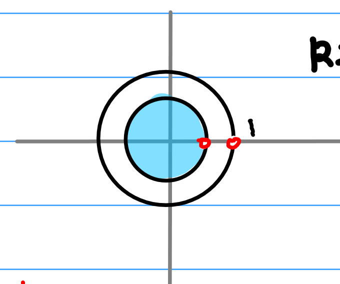
$|z| < 1$

$|z^{-1}| < 1$ $\quad$ $|z| > 1$

$R2(z)$



$\xleftrightarrow{z^{-1}}$



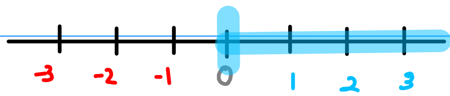
$R2(z^{-1}) \triangleq |$

$|z| > 2$

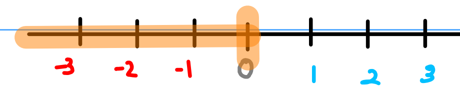
$|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

$-N \quad N^c$

$-N$

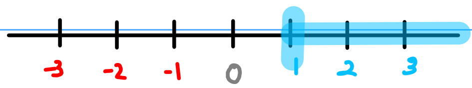


$(n \geq 0)$

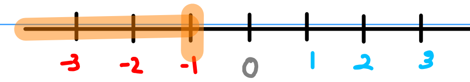


$(n < 1)$

$-N$

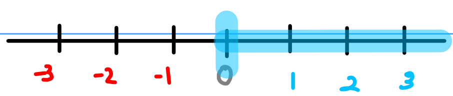


$(n \geq 1)$

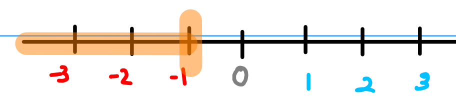


$(n < 0)$

N^c

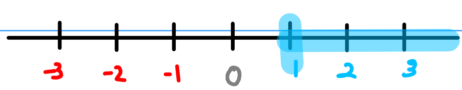


$(n \geq 0)$

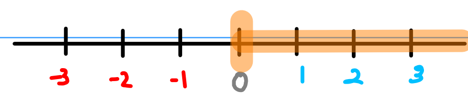


$(n < 0)$

N^c



$(n \geq 1)$



$(n < 1)$

$$(z, R) \Leftrightarrow (a_n, N)$$

$f(z)$	$ROC(z)$	$\Leftrightarrow$	a_n	$RNG(n)$
	$\lvert z \rvert < p$			$n \geq 0$

Ⓘ $$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$f(z^{-1})$	$ROC(z^{-1})$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z \rvert > \frac{1}{p}$			$n < 1$

Ⓘ Ⓘ $$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$f(z)$	$ROC(z^{-1})$	$\Leftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z \rvert > \frac{1}{p}$			$n < 0$

Ⓘ Ⓘ Ⓘ $$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

$f(z^{-1})$	$ROC(z)$	$\Leftrightarrow$	$-a_{-n}$	« $RNG(n)$ »
	$\lvert z \rvert < p$			$n \geq 1$

Ⓘ + Ⓘ Ⓘ

Ⓘ Ⓥ $$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$X(z)$	$ROC(z)$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z \rvert < p$			$n < 1$

$$\text{III} = \text{I} + \text{II}$$

III	$f(z^{-1})$	$ROC(z)$ $\lvert z\rvert < p$	$\Longleftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$ $n \geq 1$	I + II
	$f(z)$	$ROC(z)$ $\lvert z\rvert < p$	$\Longleftrightarrow$	a_n	$RNG(n)$ $n \geq 0$	
I	$f(z^{-1})$	$ROC(z^{-1})$ $\lvert z\rvert > \frac{1}{p}$	$\Longleftrightarrow$	a_{-n}	$RNG(-n)$ $n < 1$	
II	$f(z)$	$ROC(z^{-1})$ $\lvert z\rvert > \frac{1}{p}$	$\Longleftrightarrow$	$-a_n$	$\overline{RNG(n)}$ $n < 0$	
	$f(z^{-1})$	$ROC(z)$ $\lvert z\rvert < p$	$\Longleftrightarrow$	$-a_{-n}$	$\overline{RNG(-n)}$ $n \geq 1$	

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare Ⓘ with Ⓘⓥ

ROC(z)	f(z)	$\Leftrightarrow$	a_n	RNG(n)
$\lvert z \rvert < p$				$n \geq 0$

I $\quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

$f(z^{-1})$	ROC(z^{-1})	$\Leftrightarrow$	a_{-n}	RNG($-n$)
	$\lvert z \rvert > \frac{1}{p}$			$n < 1$

IV $\quad (a_n, N) \Leftrightarrow (x_{-n}, -N)$

$(x_n, N) \Leftrightarrow (a_{-n}, -N)$

$X(z)$	ROC(z)	$\Leftrightarrow$	a_{-n}	RNG($-n$)
	$\lvert z \rvert < p$			$n < 1$

$-n$ $\quad$ $-n$

symmetrical

Ⅰ $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad |z|<1$$

$$f(z) = -\left[1 + 1^2 z + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)}$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \qquad (n < 1)}$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z) = +\left[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)}$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \qquad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^1 + \left(\tfrac{1}{1}\right)^1 z^2 + \left(\tfrac{1}{1}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{1}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)}$$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$\sum_{n=0}^{\infty} a^{n+1} z^n \qquad |z| < a$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^n \qquad |z| > a^{-1}$$

$$\left(\tfrac{1}{a}\right)^{n-1} \qquad n < 1$$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad |z| > a^{-1}$$

$$-a^{n+1} \qquad n < 0$$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^n \qquad |z| < a$$

$$-\left(\tfrac{1}{a}\right)^{n-1} \qquad n \geq 1$$

II $(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \quad |z|<1}$$

$$f(z) = -[1 + 1^1 z + 1^3 z^2 + \cdots] + [(\tfrac{1}{2}) + (\tfrac{1}{2})^2 z + (\tfrac{1}{2})^3 z^2 + \cdots]$$

$$\boxed{a_n = -1^{n+1} + (\tfrac{1}{2})^{n+1} \quad (n \geq 0)}$$

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>1}$$

$$f(z) = -[(\tfrac{1}{1})^{-1} z^0 + (\tfrac{1}{1})^{-2} z^{-1} + (\tfrac{1}{1})^{-3} z^{-2} + \cdots] + [2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$$

$$\boxed{a_n = -(\tfrac{1}{1})^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2}$$

$$f(z) = +[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots] - [(\tfrac{1}{2})^0 z^{-1} + (\tfrac{1}{2})^{-1} z^{-2} + (\tfrac{1}{2})^{-2} z^{-3} + \cdots]$$

$$\boxed{a_n = +1^{n+1} - (\tfrac{1}{2})^{n+1} \quad (n < 0)}$$

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z} \quad |z|<0.5}$$

$$f(z) = +[(\tfrac{1}{1})^0 z^1 + (\tfrac{1}{1})^1 z^2 + (\tfrac{1}{1})^2 z^3 + \cdots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$$

$$\boxed{a_n = +(\tfrac{1}{1})^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$\sum_{n=0}^{\infty} a^{n+1} z^n \qquad |z| < a$

$a^{n+1} \qquad n \geq 0$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$\sum_{n=0}^{-\infty} a^{-n+1} z^n \qquad |z| > a^{-1}$

$(\frac{1}{a})^{n-1} \qquad n < 1$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$-\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad |z| > a^{-1}$

$-a^{n+1} \qquad n < 0$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$-\sum_{n=1}^{\infty} a^{-n+1} z^n \qquad |z| < a$

$-(\frac{1}{a})^{n-1} \qquad n \geq 1$

$$\text{III}\quad (z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z|<1$$

$$f(z) = -[1 + 1^2 z + 1^3 z^2 + \cdots] + [(\tfrac{1}{2}) + (\tfrac{1}{2})^2 z + (\tfrac{1}{2})^3 z^2 + \cdots]$$

$$a_n = -1^{n+1} + (\tfrac{1}{2})^{n+1} \quad (n \geq 0)$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>1$$

$$f(z) = -[(\tfrac{1}{1})^{-1} z^0 + (\tfrac{1}{1})^{-2} z^{-1} + (\tfrac{1}{1})^{-3} z^{-2} + \cdots] + [2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$$

$$a_n = -(\tfrac{1}{1})^{n-1} + 2^{n-1} \quad (n < 1)$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$f(z) = +[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots] - [(\tfrac{1}{2})^0 z^{-1} + (\tfrac{1}{2})^{-1} z^{-2} + (\tfrac{1}{2})^{-2} z^{-3} + \cdots]$$

$$a_n = +1^{n+1} - (\tfrac{1}{2})^{n+1} \quad (n < 0)$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$f(z) = +[(\tfrac{1}{1})^0 z^1 + (\tfrac{1}{1})^1 z^2 + (\tfrac{1}{1})^2 z^3 + \cdots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$$

$$a_n = +(\tfrac{1}{1})^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$\sum_{n=0}^{\infty} a^{n+1} z^n \qquad |z| < a$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^n \qquad |z| > a^{-1}$$

$$(\tfrac{1}{a})^{n-1} \qquad n < 1$$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad |z| > a^{-1}$$

$$-a^{n+1} \qquad n < 0$$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^n \qquad |z| < a$$

$$-(\tfrac{1}{a})^{n-1} \qquad n \geq 1$$

IV $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad |z|<1$$

$$f(z) = -\left[1 + 1^2 z + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{1}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{x_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z) = +\left[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \qquad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^1 + \left(\tfrac{1}{1}\right)^1 z^2 + \left(\tfrac{1}{1}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{1}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{x_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

IV $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$\sum_{n=0}^{\infty} a^{n+1} z^n \qquad \boxed{|z| < a}$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^n \qquad \boxed{|z| > a^{-1}}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad n < 1$$

$$X(z^{-1}) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad n < 1$$

$$X(z) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad \boxed{|z| > a^{-1}}$$

$$-a^{n+1} \qquad n < 0$$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^n \qquad \boxed{|z| < a}$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad n \geq 1$$

$$X(z) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad n \geq 1$$

$$X(z^{-1}) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$-a^{n+1} \qquad n < 0$$

$\boxed{f(z)}$ $\boxed{f(z^{-1})}$

$\boxed{f(z^{-1})}$ $\boxed{f(z)}$

$\boxed{a_n}$ $\boxed{a_{-n}}$

$\boxed{-a_n}$ $\boxed{-a_{-n}}$

$$f(z) = \frac{a}{1 - az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$\sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad \boxed{|z| < a}$

a^{n+1} $\quad \boxed{n \geq 0}$

$$f(z^{-1}) = \frac{a}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$\sum_{n=0}^{-\infty} a^{-n+1} z^n$ $\quad \boxed{|z| > a^{-1}}$

$\left(\frac{1}{a}\right)^{n-1}$ $\quad \boxed{n < 1}$

$$f(z^{-1}) = \frac{-z^{-1}}{1 - a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$-\sum_{n=-1}^{-\infty} a^{n+1} z^n$ $\quad \boxed{|z| > a^{-1}}$

$-a^{n+1}$ $\quad \boxed{n < 0}$

$$f(z) = \frac{-z}{1 - a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$-\sum_{n=1}^{\infty} a^{-n+1} z^n$ $\quad \boxed{|z| < a}$

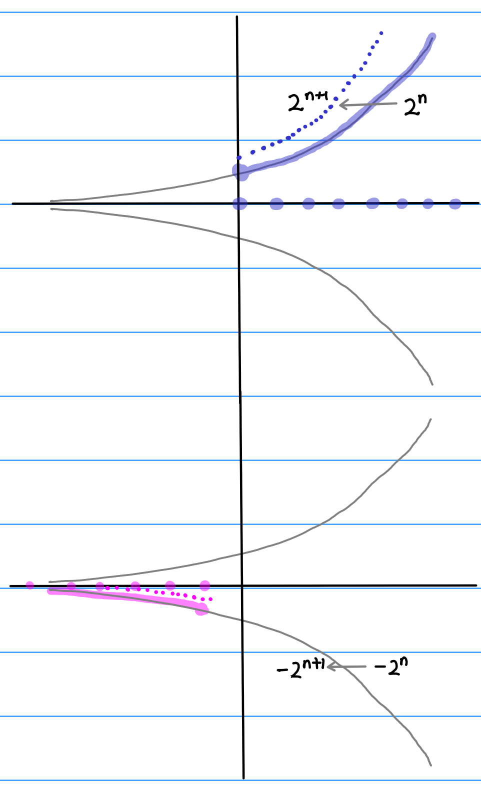
$-\left(\frac{1}{a}\right)^{n-1}$ $\quad \boxed{n \geq 1}$

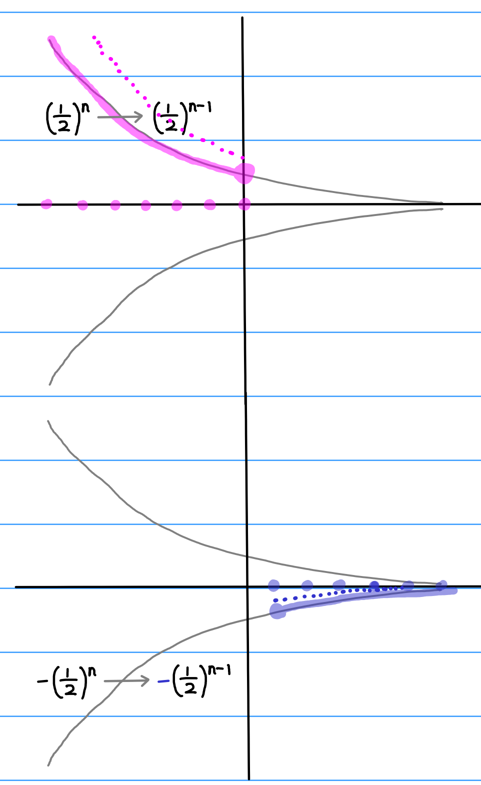
$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$

$-z^{-1} - a^{-1} z^{-2} - a^{-2} z^{-3} - a^{-3} z^{-4} - \cdots$

$a + a^2 z^{-1} + a^3 z^{-2} + a^4 z^{-3} + \cdots$

$-z^1 - a^{-1} z^2 - a^{-2} z^3 - a^{-3} z^4 - \cdots$





$X(z^{-1})$	$X(z)$	x_n	x_{-n}
$X(z)$	$X(z^{-1})$	$-x_n$	$-x_{-n}$

$$X(z^{-1}) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad \boxed{n < 1}$$

$$X(z) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$a^{n+1} \qquad \boxed{n \geq 0}$$

$$X(z) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad \boxed{n \geq 1}$$

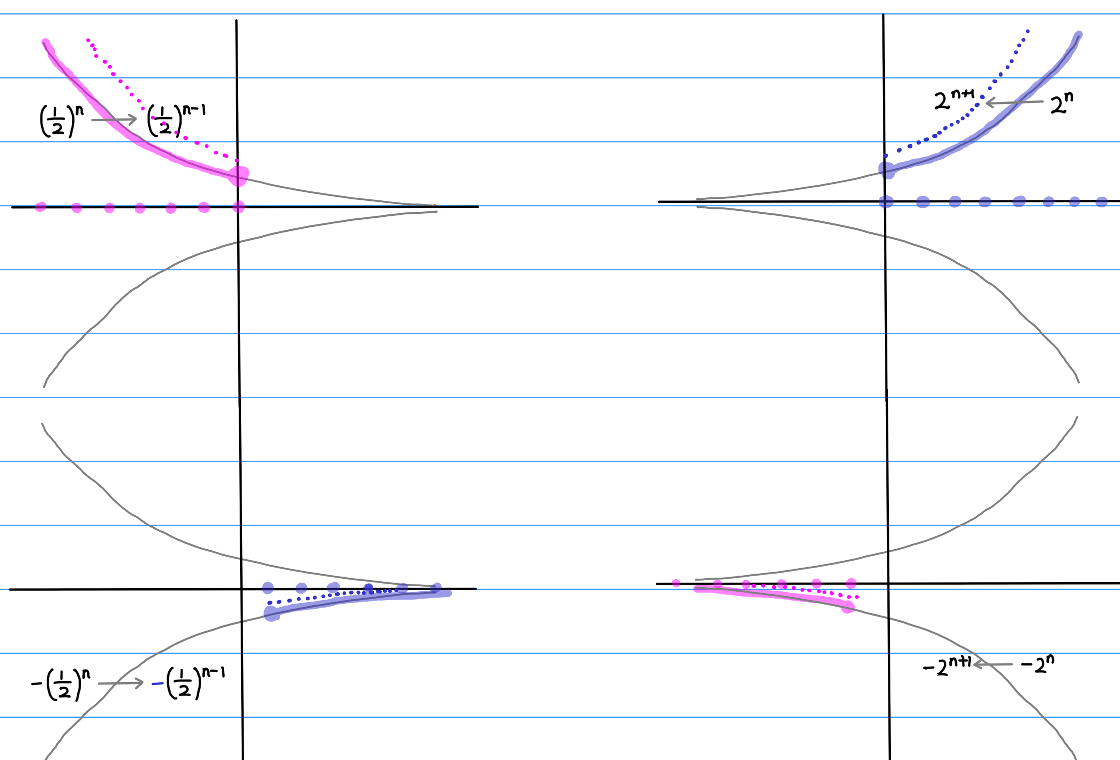
$$X(z^{-1}) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$-a^{n+1} \qquad \boxed{n < 0}$$

$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$$
$$-z^{-1} - a^{-1} z^{-2} - a^{-2} z^{-3} - a^{-3} z^{-4} - \cdots$$

$$a + a^2 z^{-1} + a^3 z^{-2} + a^4 z^{-3} + \cdots$$
$$-z^{1} - a^{-1} z^{2} - a^{-2} z^{3} - a^{-3} z^{4} - \cdots$$



$f(z)$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \qquad |z|<1$$

$f(z^{-1})$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$g(z^{-1})$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$g(z)$

$$+\frac{z}{1-z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$f(z)$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \qquad |z|<1$$

$f(z^{-1})$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>1$$

$g(z^{-1})$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$g(z)$

$$+\frac{z}{1-z}-\frac{z}{1-2z} \qquad |z|<0.5$$