z-Trans-6:
Comparison with the Laurent Series
: Geometric Series View Point

20180207 ~~20170207~~

from Complex Analysis in plain view
Laurent Series and z-Transform
- Geometric Series Examples B 20180207

Causal signal a_n $n \geq 0$

anti-causal signal a_n $n < 0$

Laurent Series $f(z)$

z - Transform $X(z)$

① Causal $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$

$a_n : \left(\frac{1}{2}\right)^0, \quad \left(\frac{1}{2}\right)^1, \quad \left(\frac{1}{2}\right)^2, \quad \cdots \qquad (n \geq 0)$

n = 0 n = 1 n = 2

$$f(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \cdots = \frac{1}{1-\frac{z}{2}} = \frac{2}{2-z}$$

$$\frac{|z|}{2} < 1 \qquad |z| < 2$$

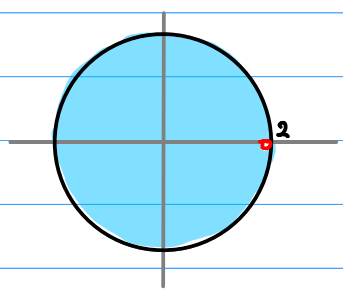
$$X(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots = \frac{1}{1-\frac{1}{2z}} = \frac{z}{z-0.5}$$

$$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$$

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-\frac{z}{2}} \qquad |z| < 2$$

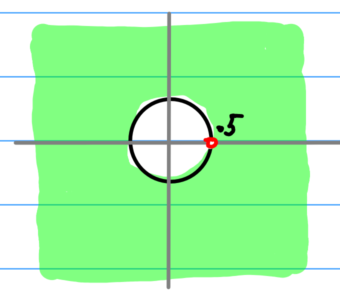
$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

$$= \frac{z}{z-0.5}$$



② Causal $(n \geq 0)$ $a_n = (2)^n$

a_n : $(2)^0$, $(2)^1$, $(2)^2$, $\cdots$ $(n \geq 0)$

n = 0 n = 1 n = 2

$$f(z) = (2)^0 z^0 + (2)^1 z^1 + (2)^2 z^2 + \cdots = \frac{1}{1-2z} = \frac{0.5}{0.5-z}$$

$2|z| < 1$ $|z| < 0.5$

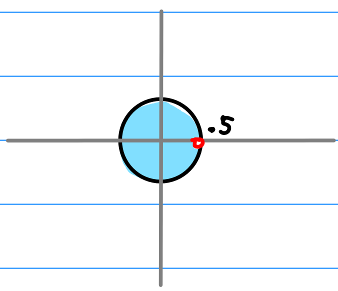
$$X(z) = (2)^0 z^0 + (2)^1 z^{-1} + (2)^2 z^{-2} + \cdots = \frac{1}{1-\frac{2}{z}} = \frac{z}{z-2}$$

$\frac{2}{|z|} < 1$ $|z| > 2$

$a_n = (2)^n$ $(n \geq 0)$

$$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$$

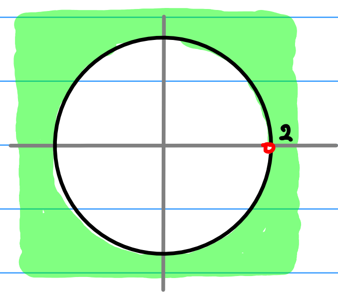
$$= \frac{0.5}{0.5-z}$$



$a_n = (2)^n$ $(n \geq 0)$

$$X(z) = \frac{1}{1-\frac{2}{z}} \quad |z| > 2$$

$$= \frac{z}{z-2}$$



③ Anti-causal $(n<0)$ $a_n = \left(\frac{1}{2}\right)^n$

$$a_n : \quad \underset{n=-1}{\left(\tfrac{1}{2}\right)^{-1}}, \quad \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-2}}, \quad \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-3}}, \quad \cdots \qquad (n<0)$$

$$f(z) = (2)^1 z^{-1} + (2)^2 z^{-2} + (2)^3 z^{-3} + \cdots = \frac{\frac{2}{z}}{1-\frac{2}{z}} = \frac{2}{z-2}$$

$$\frac{2}{|z|} < 1 \qquad |z| > 2$$

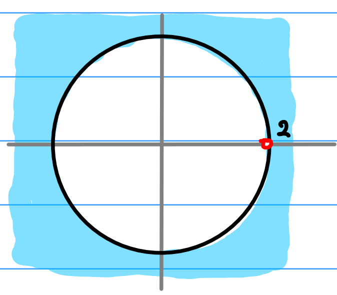
$$X(z) = (2)^1 z^1 + (2)^2 z^2 + (2)^3 z^3 + \cdots = \frac{2z}{1-2z} = \frac{z}{0.5-z}$$

$$2|z| < 1 \qquad |z| < 0.5$$

$$a_n = \left(\frac{1}{2}\right)^n \qquad (n<0)$$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$$

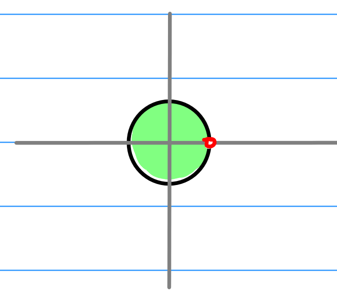
$$= -\frac{2}{2-z}$$



$$a_n = \left(\frac{1}{2}\right)^n \qquad (n<0)$$

$$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$$

$$= -\frac{z}{z-0.5}$$



④ Anti-causal $(n<0)$ $a_n = (2)^n$

$a_n : \quad (2)^{-1}, \quad (2)^{-2}, \quad (2)^{-3}, \quad \cdots \qquad (n<0)$

$\qquad n=-1 \qquad n=-2 \qquad n=-3$

$$f(z) = \left(\tfrac{1}{2}\right)^1 z^{-1} + \left(\tfrac{1}{2}\right)^2 z^{-2} + \left(\tfrac{1}{2}\right)^3 z^{-3} + \cdots = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} = \frac{0.5}{z-0.5}$$

$$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$$

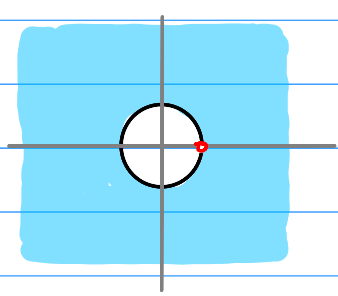
$$X(z) = \left(\tfrac{1}{2}\right)^1 z^{1} + \left(\tfrac{1}{2}\right)^2 z^{2} + \left(\tfrac{1}{2}\right)^3 z^{3} + \cdots = \frac{\frac{z}{2}}{1-\frac{z}{2}} = \frac{z}{2-z}$$

$$\frac{|z|}{2} < 1 \qquad |z| < 2$$

$a_n = (2)^n \quad (n<0)$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

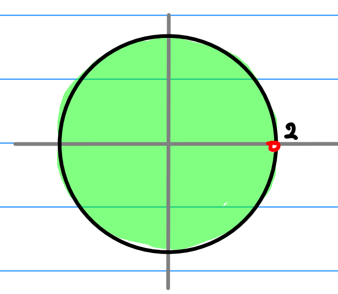
$$= -\frac{0.5}{0.5-z}$$



$a_n = (2)^n \quad (n<0)$

$$X(z) = \frac{\frac{z}{2}}{1-\frac{z}{2}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$

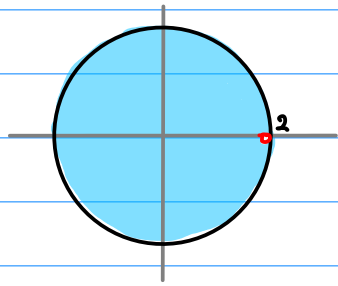


Causal $(n \geq 0)$ $\left(\frac{1}{2}\right)^n, \ (2)^n$

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$

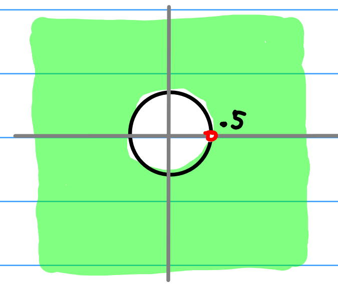
$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$



$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$X(z) = \frac{1}{1-\frac{1}{2z}} \quad |z| > 0.5$

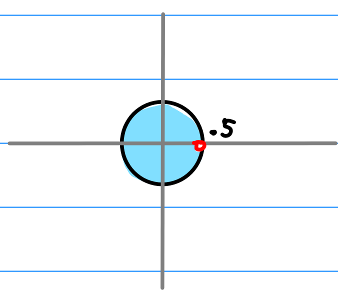
$= \frac{z}{z-0.5}$



$a_n = (2)^n \quad (n \geq 0)$

$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$

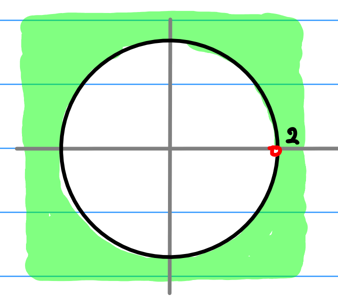
$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$



$a_n = (2)^n \quad (n \geq 0)$

$X(z) = \frac{1}{1-\frac{2}{z}} \quad |z| > 2$

$= \frac{z}{z-2}$



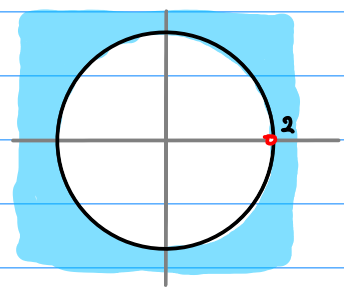
$f(z)$ $X(z)$

Anti-causal $(n<0)$ $\left(\frac{1}{2}\right)^n$, $(2)^n$

$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$$

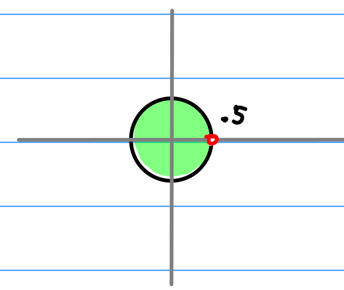
$$= -\frac{2}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$

$$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$$

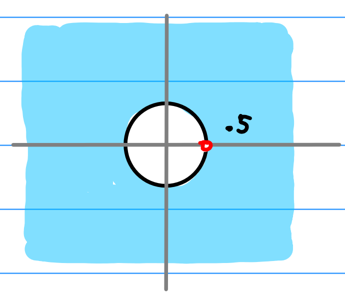
$$= -\frac{z}{z-0.5}$$



$a_n = (2)^n \quad (n<0)$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

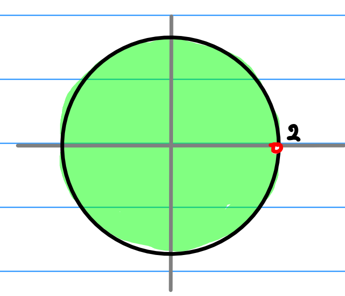
$$= -\frac{0.5}{0.5-z}$$



$a_n = (2)^n \quad (n<0)$

$$X(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$



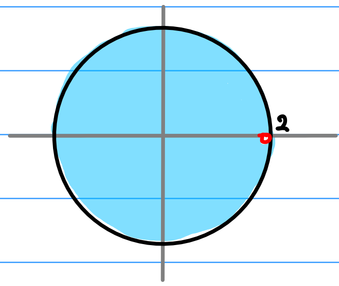
$f(z)$ $\qquad$ $X(z)$

Causal $f(z)$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-\frac{z}{2}} \qquad |z| < 2$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$



$$a_n = (2)^n \quad (n \geq 0)$$

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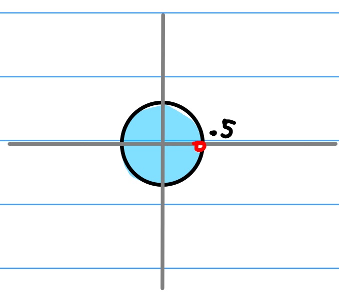
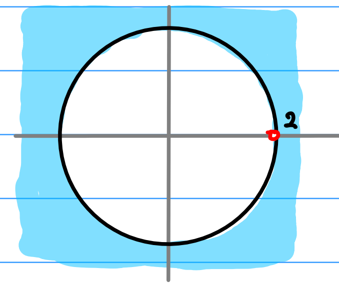


Anti-causal $f(z)$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= -\frac{2}{2-z}$$



$$a_n = (2)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

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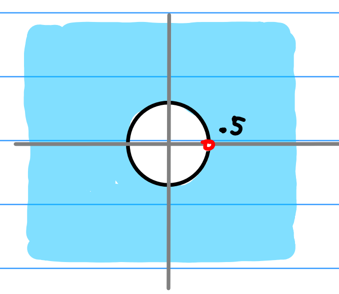
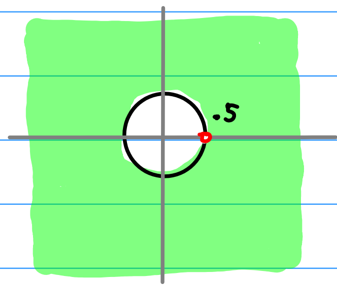


Causal $X(z)$

$$a_n = \left(\tfrac{1}{2}\right)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

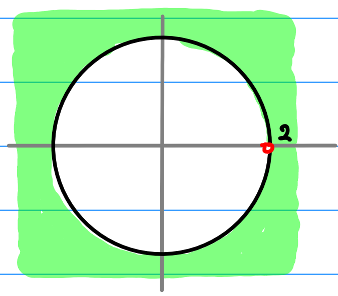
$$= \frac{z}{z-0.5}$$



$$a_n = (2)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= \frac{z}{z-2}$$

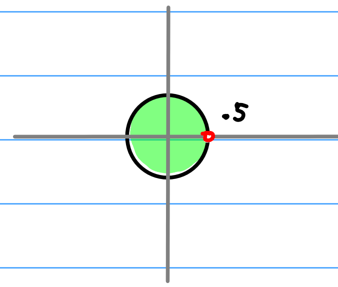


Anti-causal $X(z)$

$$a_n = \left(\tfrac{1}{2}\right)^n \quad (n < 0)$$

$$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$$

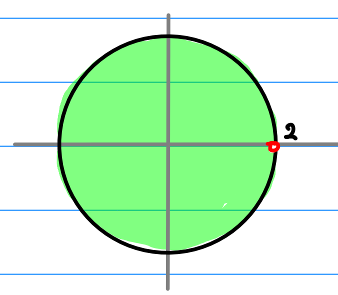
$$= -\frac{z}{z-0.5}$$



$$a_n = (2)^n \quad (n < 0)$$

$$X(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$



Causal b^n — Anti-causal b^n

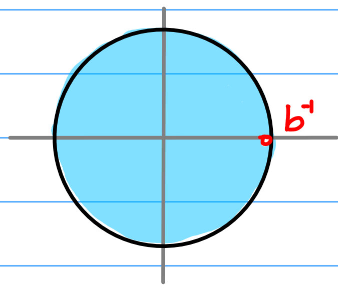
$f(z)$

Causal

$$a_n = (b)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1 - b z} \qquad |z| < b^{-1}$$

$$= \frac{b^{-1}}{b^{-1} - z}$$

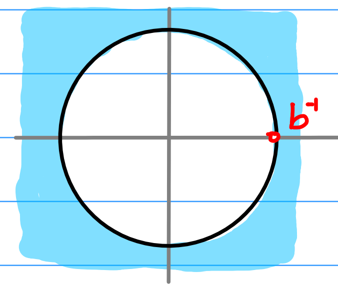


Anti-causal

$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1} z^{-1}}{1 - b^{-1} z^{-1}} \qquad |z| > b^{-1}$$

$$= -\frac{b^{-1}}{b^{-1} - z}$$



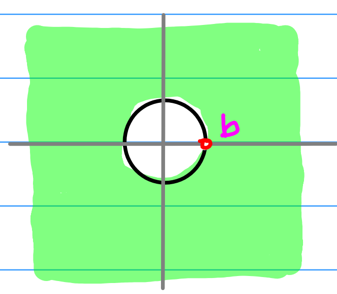
$X(z)$

Causal

$$a_n = (b)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - b z^{-1}} \qquad |z| > b$$

$$= \frac{z}{z - b}$$

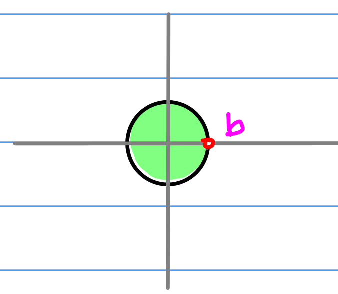


Anti-causal

$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1} z}{1 - b^{-1} z} \qquad |z| < b$$

$$= -\frac{z}{z - b}$$



2 formulas of z

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$

$$\Big\updownarrow z^{-1}$$

② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$

case Ⅰ $f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

case Ⅱ $f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$ $X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅲ $f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅳ $f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$ $X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{\;z^{-1}\;} \quad \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\,\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5}-\frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1}-\frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z}-\frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2}+\frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2}+\frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$\frac{3}{2}\cdot\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

1 – A

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$|z| > 2$ $\quad f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n < 0)$

1 – B

$|z| < 0.5$ $\quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$|z| > 2$ $\quad X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n > 0)$

$$\frac{3}{2}\cdot\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

2 – A

$|z| < 0.5$ $\quad f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\quad \boxed{-2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1}}$ $\quad (n > 0)$

$|z| > 2$ $\quad f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\quad \boxed{+2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}}$ $\quad (n \leq 0)$

2 – B

$|z| < 0.5$ $\quad X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}}$ $\quad (n < 0)$

$|z| > 2$ $\quad X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\quad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}}$ $\quad (n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
Ⓐ $f(z)$	$\lvert z\rvert < \frac{1}{2}$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n > 0)$
	$\lvert z\rvert > 2$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$
Ⓑ $X(z)$	$\lvert z\rvert < \frac{1}{2}$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n < 0)$
	$\lvert z\rvert > 2$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n > 0)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n > 0)$
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n > 0)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	causal $(n \geq 0)$	causal $(n > 0)$
$\lvert z\rvert > 2$	$f(z)$	anticausal $(n < 0)$	anticausal $(n \leq 0)$
$\lvert z\rvert < \frac{1}{2}$	$X(z)$	anticausal $(n \leq 0)$	anticausal $(n < 0)$
$\lvert z\rvert > 2$	$X(z)$	causal $(n > 0)$	causal $(n \geq 0)$

		① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	causal $(n \geq 0)$	causal $(n > 0)$
$\lvert z\rvert < \frac{1}{2}$	$X(z)$	anticausal $(n \leq 0)$	anticausal $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	anticausal $(n < 0)$	anticausal $(n \leq 0)$
$\lvert z\rvert > 2$	$X(z)$	causal $(n > 0)$	causal $(n \geq 0)$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad -\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$\boxed{|z|<0.5}$ $|2z|<1$ $|0.5z|<1$ $\qquad$ $\boxed{|z|<0.5}$ $|2z|<1$ $|0.5z|<1$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$\boxed{|z|>2}$ $|0.5z^{-1}|<1$ $|2z^{-1}|<1$ $\qquad$ $\boxed{|z|>2}$ $|0.5z^{-1}|<1$ $|2z^{-1}|<1$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad -\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$\boxed{|z|<0.5}$ f(z) causal $(n \geq 0)$ $\qquad$ $\boxed{|z|<0.5}$ f(z) causal $(n > 0)$

X(z) anticausal $(n \leq 0)$ $\qquad$ X(z) anticausal $(n < 0)$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$\boxed{|z|>2}$ f(z) anticausal $(n < 0)$ $\qquad$ $\boxed{|z|>2}$ f(z) anticausal $(n \leq 0)$

X(z) causal $(n > 0)$ $\qquad$ X(z) causal $(n \geq 0)$

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

①

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^1+\left(\tfrac{1}{2}\right)^{-3}z^2+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$
$$+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right] \quad +2^{n-1}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

②

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^{-1}z^2+\left(\tfrac{1}{2}\right)^{-2}z^3+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$
$$+\left[2^0z^1+2^{-1}z^2+2^{-2}z^3+\cdots\right] \quad +2^{n+1}$$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$
$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^0+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

①

$|z| < 0.5$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2 + 2^2 z^1 + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n \leq 0)$$

$|z| > 2$

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n > 0)$$

②

$|z| < 0.5$

$$-\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right]$$

$$a_n = -2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1} \qquad (n > 0)$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] + \left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right]$$

$$a_n = -\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1} \qquad (n < 0)$$

$|z| > 2$

$$\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right]$$

$$a_n = +2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1} \qquad (n \leq 0)$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] - \left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right]$$

$$a_n = +\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1} \qquad (n \geq 0)$$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
		$\updownarrow$ -1	$\updownarrow$ -1
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$			
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$		$\updownarrow$ -1	$\updownarrow$ -1
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n>0)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n \geq 0)$

$2^{-n+1} = \left(\frac{1}{2}\right)^{n} \cdot 2 = \left(\frac{1}{2}\right)^{n-1}$ $\qquad$ $\left(\frac{1}{2}\right)^{-n-1} = 2^{n} \cdot 2 = 2^{n+1}$

$\left(\frac{1}{2}\right)^{-n+1} = 2^{n} \cdot \frac{1}{2} = 2^{n-1}$ $\qquad$ $2^{-n-1} = \left(\frac{1}{2}\right)^{n} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^{n+1}$

z^{-1}

		① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n > 0)$
		$-n$	$-n$
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$

z^{-1}

z^{-1}

		① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$			
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n < 0)$
$\lvert z\rvert > 2$		$-n$	$-n$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n > 0)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

z^{-1}

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
		$-n,$	-1
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n\leq 0)$
		$-n,$	-1

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$		$-n,$	-1
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n\leq 0)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$		$-n,$	-1
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n>0)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n\geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-n$
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n \leq 0)$	
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-n$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n>0)$	

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-n$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
	$X(z)$		$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$	$f(z)$	$-n$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$
	$X(z)$		$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅰ $(n \geq 0)$	case Ⅱ $(n > 0)$
	$X(z)$	case Ⅱ $(n \leq 0)$	case Ⅰ $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	case Ⅰ $(n < 0)$	case Ⅱ $(n \leq 0)$
	$X(z)$	case Ⅱ $(n > 0)$	case Ⅰ $(n \geq 0)$

case Ⅰ $f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

case Ⅱ $f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$ $X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅲ $f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅳ $f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$ $X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅰ $(n \geq 0)$	$-n$
	$X(z)$		$-n$ case Ⅰ $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	case Ⅰ $(n < 0)$	$-n$
	$X(z)$		$-n$ case Ⅰ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-n$ $-n$	case Ⅱ $(n > 0)$
	$X(z)$	case Ⅱ $(n \leq 0)$	
$\lvert z\rvert > 2$	$f(z)$	$-n$ $-n$	case Ⅱ $(n \leq 0)$
	$X(z)$	case Ⅱ $(n > 0)$	

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅲ	case Ⅳ
	$X(z)$	case Ⅲ	case Ⅳ
$\lvert z\rvert > 2$	$f(z)$	case Ⅲ	case Ⅳ
	$X(z)$	case Ⅲ	case Ⅳ

case Ⅰ $\quad f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

case Ⅱ $\quad f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \qquad X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅲ $\quad f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅳ $\quad f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \qquad X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

		① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z \rvert < \frac{1}{2}$	$f(z)$	case Ⅲ	$-n$
	$X(z)$	case Ⅲ	
$\lvert z \rvert > 2$	$f(z)$	case Ⅲ	$-n$
	$X(z)$	case Ⅲ	

		① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z \rvert < \frac{1}{2}$	$f(z)$	$-n$	case Ⅳ
	$X(z)$		case Ⅳ
$\lvert z \rvert > 2$	$f(z)$	$-n$	case Ⅳ
	$X(z)$		case Ⅳ

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$\frac{a}{1-az} \qquad \frac{z^{-1}}{a^{-1}z^{-1}-1}$

$$-\left(2 + 2^2 z + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$|z| > 2$ $\quad f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n < 0)$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\quad \boxed{-2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1}}$ $\quad (n > 0)$

$$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$|z| > 2$ $\quad f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\quad \boxed{+2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}}$ $\quad (n \leq 0)$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$$\left(2^{-1} + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5 \qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2 \qquad X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n > 0)$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5 \qquad X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}} \quad (n < 0)$

$$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right) + \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 2 \qquad X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}} \quad (n \geq 0)$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5 \quad f(z) \qquad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \leq 0)$

$|z| > 2 \quad f(z) \qquad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n > 0)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a \quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1}$

$= \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \xrightarrow{-n}$

$n \geq 0 \quad n > 0 \quad n \leq 0 \quad n < 0$

$|z| > a \quad f(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$

$n \leq 0 \quad n < 0 \quad n \geq 0 \quad n > 0$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az} \qquad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \qquad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\qquad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\qquad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\qquad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n > 0 \quad n \leq 0 \quad n < 0$

$\|$ $\qquad \|$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$$

ROC $\qquad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n \leq 0 \quad n < 0 \quad n \geq 0 \quad n > 0$

$$= \left(\frac{1}{a}\right)^{n-1}$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\qquad a^{n+1}$ $\qquad n \geq 0 \quad n > 0 \quad n \leq 0 \quad n < 0$

$z^{-1} \qquad z^{-1} \qquad -n$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n}$$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\qquad \left(\frac{1}{a}\right)^{-n+1}$ $\qquad n \leq 0 \quad n < 0 \quad n \geq 0 \quad n > 0$

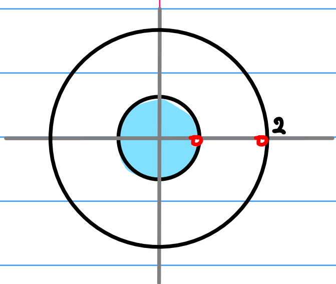
$= a^{n-1}$

Causal $\quad f(z) \qquad X(z)$

$\qquad |z| < 0.5 \qquad |z| > 2$

$$\frac{3}{2}\ \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 2 \qquad |z| < 0.5$



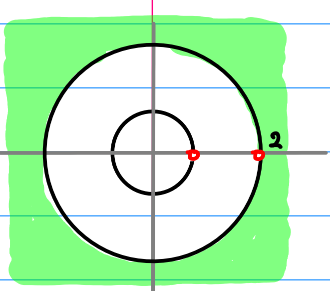
$$f(z) = (-2)\frac{0.5}{0.5-z} + (0.5)\frac{2}{2-z} \qquad (|z| < 0.5)$$

$$a_n = (-2)\ 2^n + (0.5)\ \left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\ \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| > 2$$

$|z| > 2 \qquad |z| > 0.5$



$$X(z) = 0.5\ \frac{z}{z-0.5} - 2\ \frac{z}{z-2} \qquad (|z| > 2)$$

$$a_n = (0.5)\ \left(\tfrac{1}{2}\right)^n - 2 \cdot 2^n \qquad (n \geq 0)$$

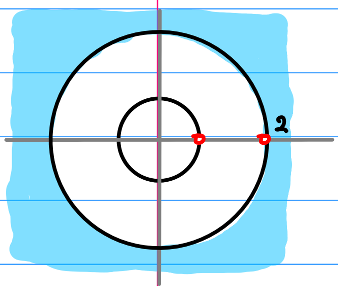
$$\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$$

Anti-causal $\quad f(z) \quad X(z)$

$|z| > 2 \quad |z| < 0.5$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| > 2 \qquad |z| > 0.5$



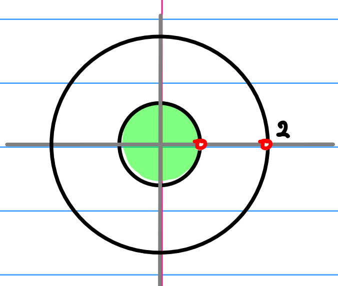
$$f(z) = (-2)\,\frac{-0.5}{0.5-z} + (0.5)\,\frac{-2}{2-z} \qquad (|z| > 0.5)$$

$$a_n = (+2)\;2^n - (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n < 0)$$

$$+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| < 2$$

$|z| < 2 \qquad |z| < 0.5$



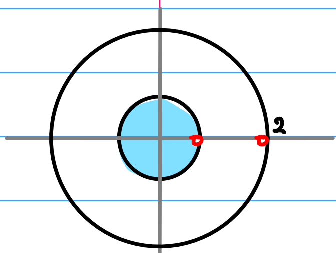
$$X(z) = 0.5\,\frac{-z}{z-0.5} - 2\,\frac{-z}{z-2} \qquad (|z| < 2)$$

$$a_n = -(0.5)\left(\tfrac{1}{2}\right)^n + 2 \cdot 2^n \qquad (n < 0)$$

$$-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}$$

①-Ⓐ $\frac{3}{2}\dfrac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

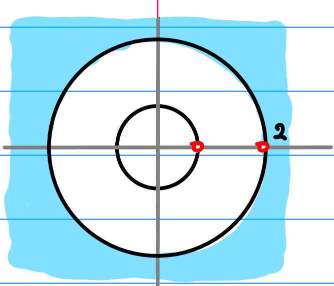


$|z| < 0.5$

$$\begin{aligned} f(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} (z)^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} (z)^n \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}$$

$(n \geq 0)$ $\quad a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



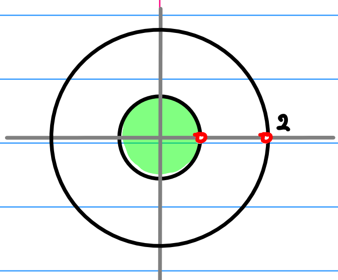
$|z| > 2$

$$\begin{aligned} f(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \qquad \neq \\ &= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n} \left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^{n} \left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \\ &= \sum_{n=-1}^{-\infty} (2)^{n+1} z^{n} - \sum_{n=-1}^{-\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{n} \end{aligned}$$

$(n < 0)$ $\quad a_n = 2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$

① - Ⓑ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



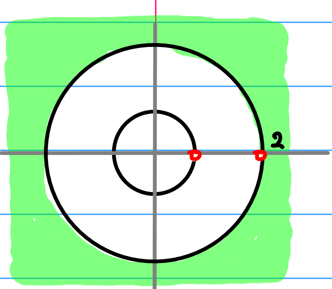
$|z| < 0.5$

$$\boxed{\begin{aligned} X(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty}\left(\tfrac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty}(2)^{n-1} z^{-n}$$

$(n \leq 0)$ $a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



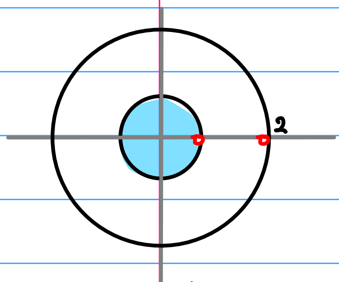
$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n}\left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^{n}\left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}}$$ $\neq$

$(n > 0)$ $a_n = \left(\tfrac{1}{2}\right)^{n-1} - (2)^{n-1}$

② - Ⓐ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

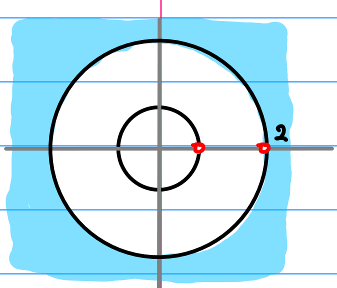


$|z| < 0.5$

$$\boxed{\begin{aligned} f(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^n \end{aligned}} \neq$$

$(n > 0)$ $a_n = -2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

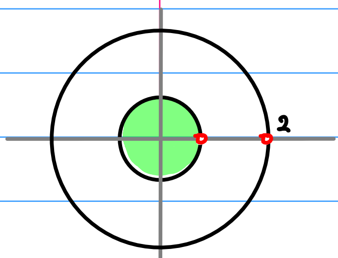
$$\boxed{\begin{aligned} f(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\frac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$$= \sum_{n=0}^{-\infty}(2)^{n-1}z^n - \sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1}z^n$$

$(n \leq 0)$ $a_n = 2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$

② - Ⓑ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



$|z| < 0.5$

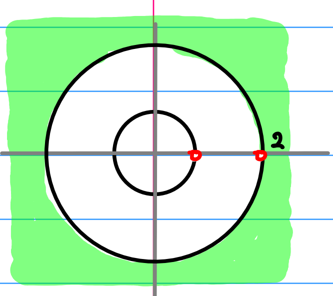
$$\boxed{\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n \end{aligned}}$$

$\neq$

$$= -\sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} + \sum_{n=-1}^{-\infty}(2)^{n+1}z^{-n}$$

$(n < 0)$ $a_n = -\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$(n \geq 0)$ $a_n = \left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$