Vector Calculus (H.1)
Multiple Integrals

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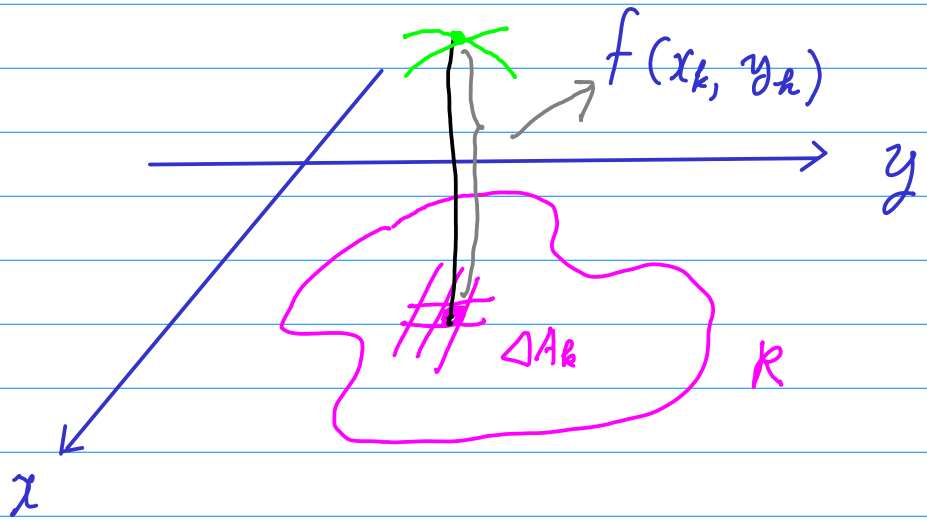
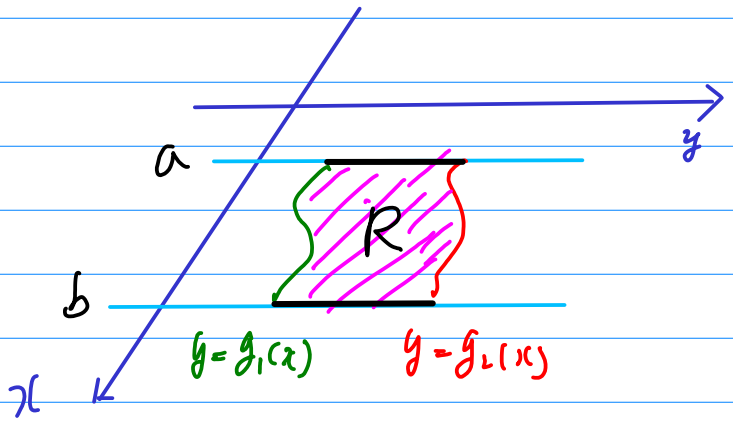
9.10 Double Integral

$$\iint_R f(x,y)\,dA = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(x_k, y_k)\,\Delta A_k$$

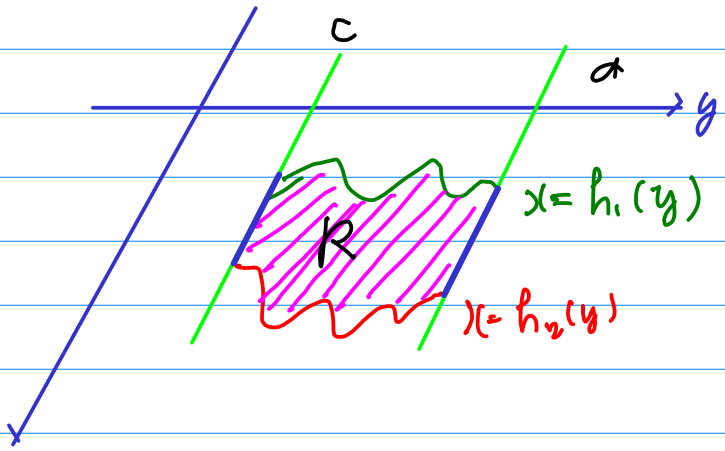


$$\iint_R 1 \cdot dA = \text{area} \qquad A = \iint_R dA$$

Fubini's Theorem

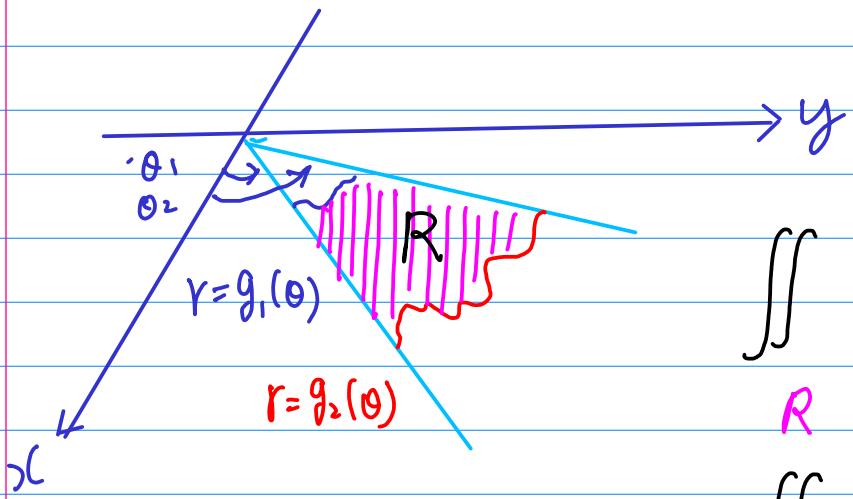


$$\iint_R f(x,y)\,dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right] dx$$



$$\iint_R f(x,y)\,dA = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \right] dy$$

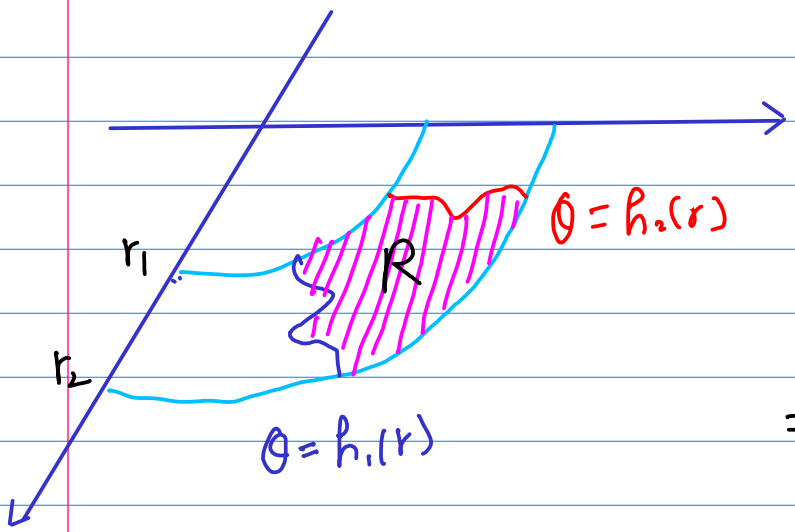
9.11 Double Integral in Polar Coord.



$$\iint_R f(x,y)\,dA$$

$$\iint_R f(r,\theta)\,dA$$

$$= \int_{\theta_1}^{\theta_2} \left[\int_{g_1(\theta)}^{g_2(\theta)} f(r,\theta)\, r\, dr \right] d\theta$$



$$= \int_{r_1}^{r_2} \left[\int_{h_1(r)}^{h_2(r)} f(r,\theta)\, r\, d\theta \right] dr.$$

Change of Variables

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

($x = r\cos\theta$, $y = r\sin\theta$)

r ← Jacobian

x, y $\quad \boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}} \quad r, \theta$

$$\iint_R f(x,y)\,dA$$

$$\int_C f(x,y)\,ds$$