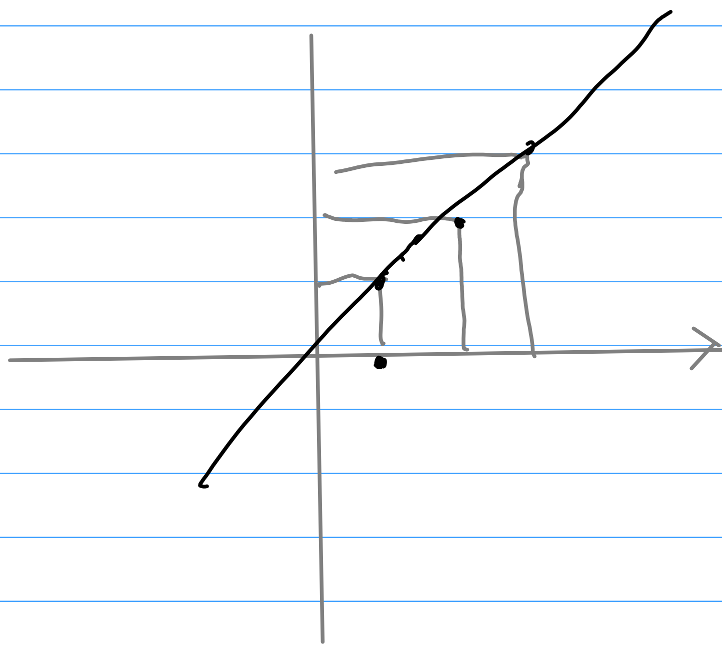
Complex Functions (H.1)

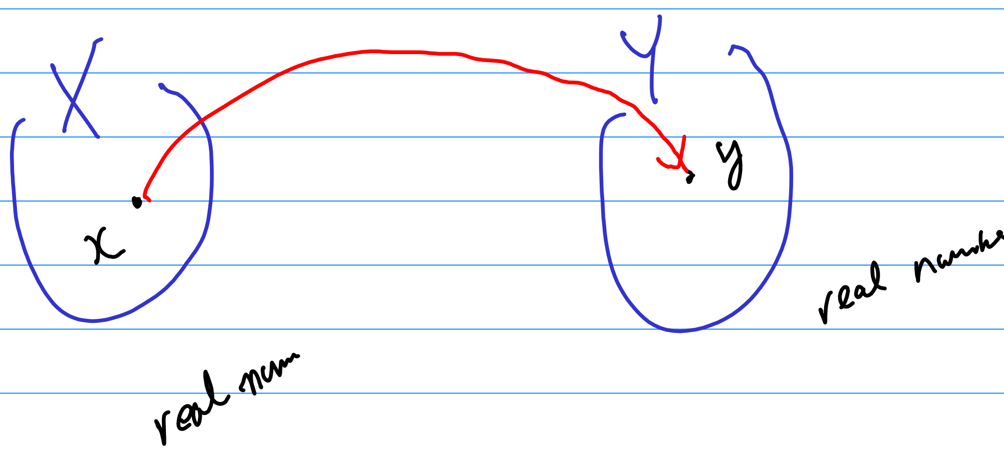
20160112

$y = f(x)$ real function

$\boxed{y = x}$

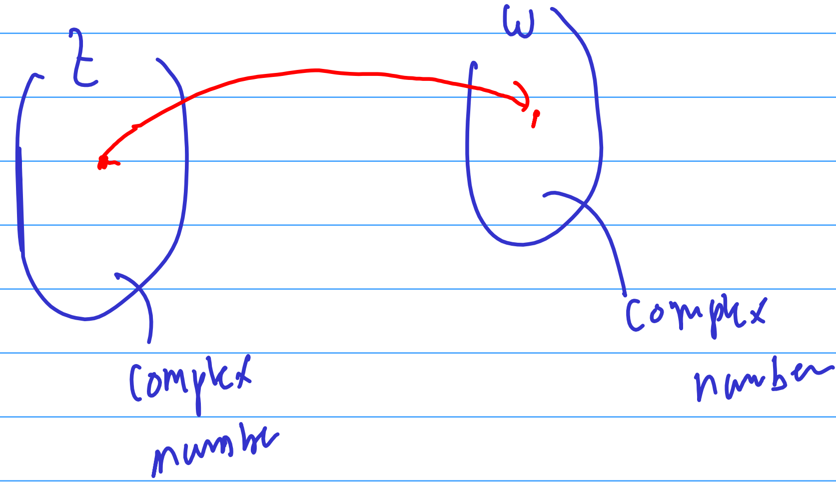
x	-2	-1	0	1	2
y	–	-1	0	1	2

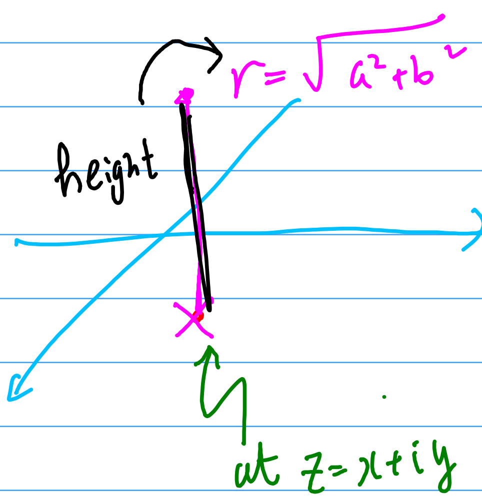




Complex function

$w = f(z)$

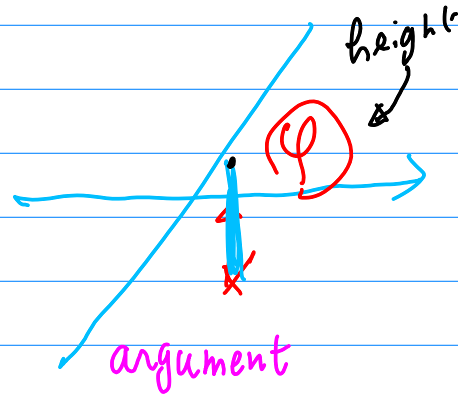




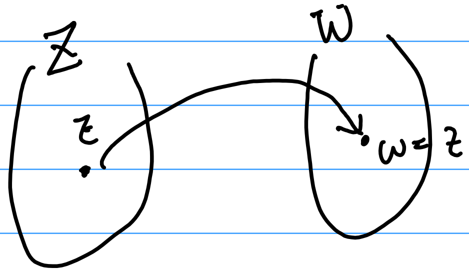
magnitude

absolute

$|z|$

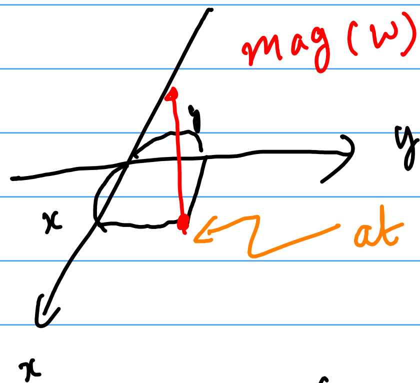


$W = Z$ $(y = x)$

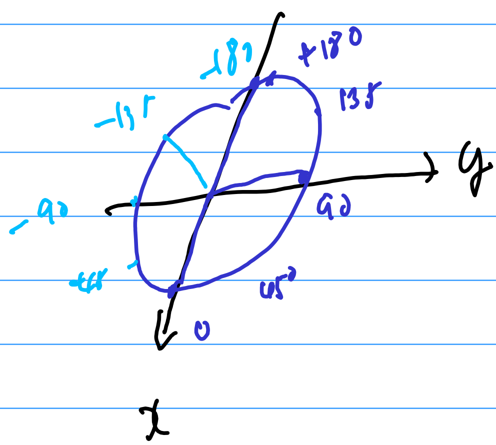


$z = x + iy$

$mag(w) = mag(x) = \sqrt{x^2 + y^2}$ as a height



at $(x + iy)$ .... (x, y)



$-5 \le x \le 5$

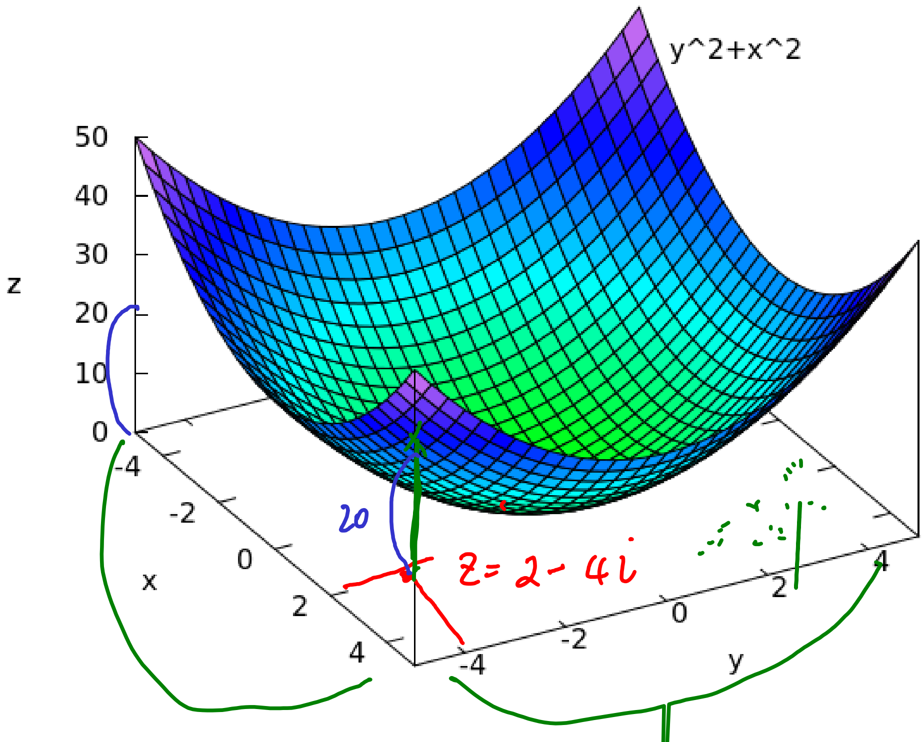
$-5 \le y \le 5$

$z = x + iy \rightarrow w = z^2$

$= re^{i\theta}$

$= x^2 - y^2 + 2xy\,i$

$= r^2 e^{i2\theta}$

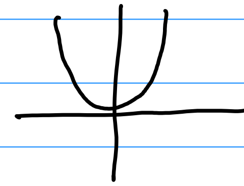


$|z| = |2-4i| = \sqrt{(2-4i)(2+4i)} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

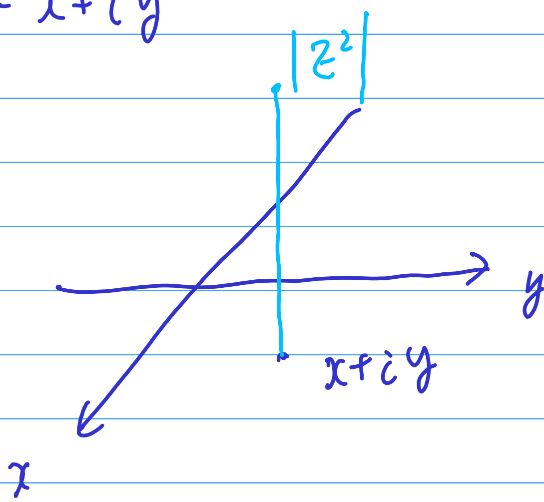
$|w| = |z^2| = |z|^2 = 20$ as a height

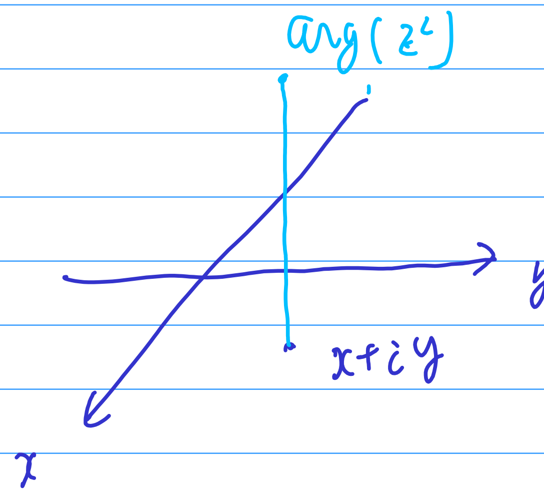
$w = z^2$

$y = x^2$



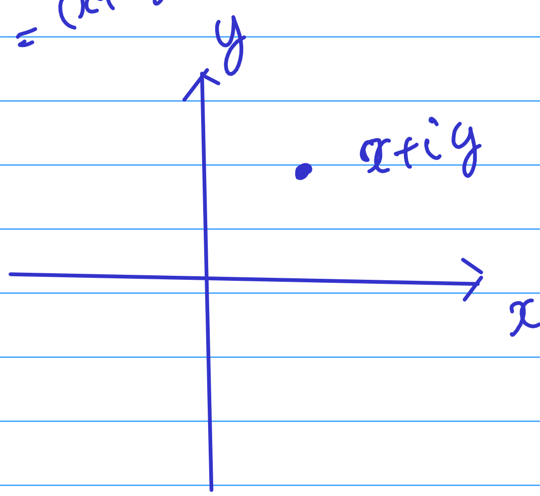
$z = x + iy$





$z = x + iy$

$w = z^2 = (x+iy)^2$

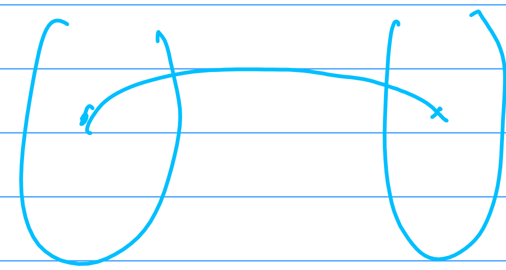


복소수 함수 complex function

$w = f(z)$

w는 z의 함수

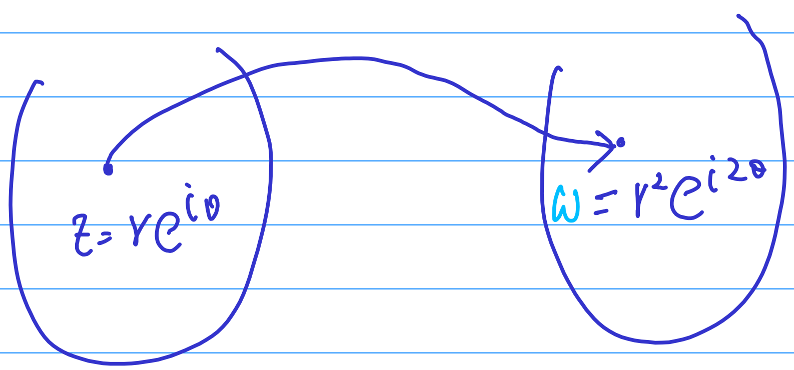
↑ 복소수 ↑ 복소수



$z = x + j y$

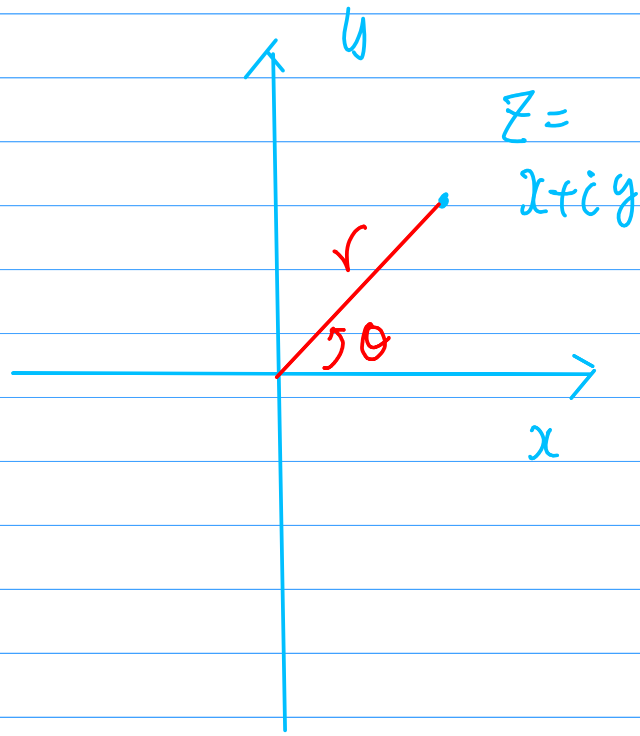
$w = f(z) = z^2 = (x+iy)^2 = (x^2-y^2) + i(2xy)$

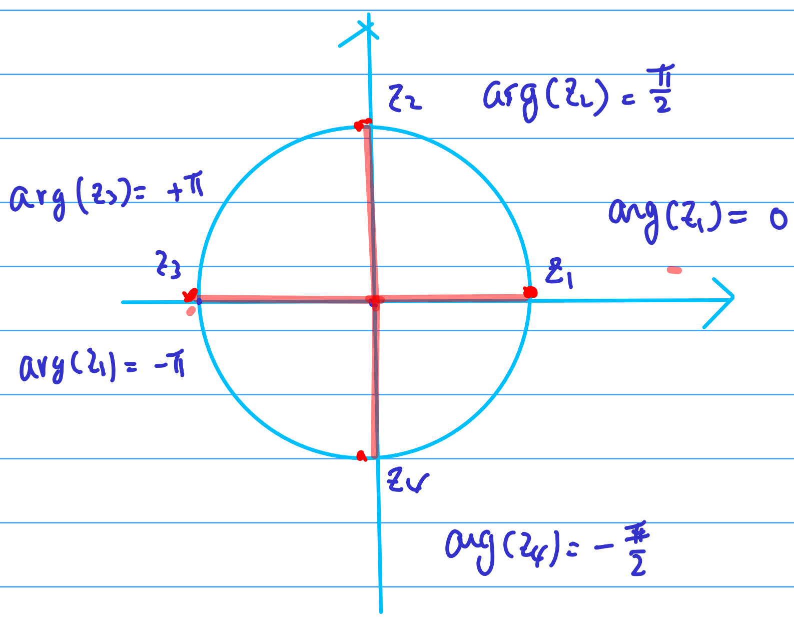
$= (re^{i\theta})^2 = r^2 e^{i2\theta}$



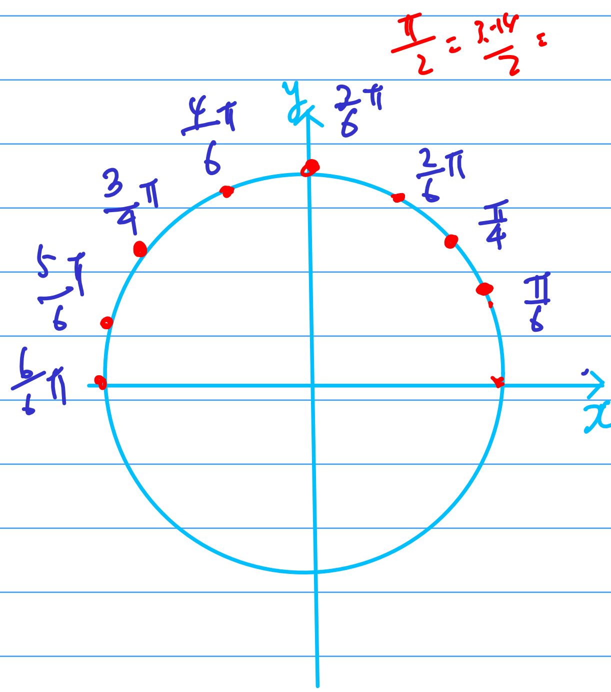
abs(w) , arg(w)

arg(z)

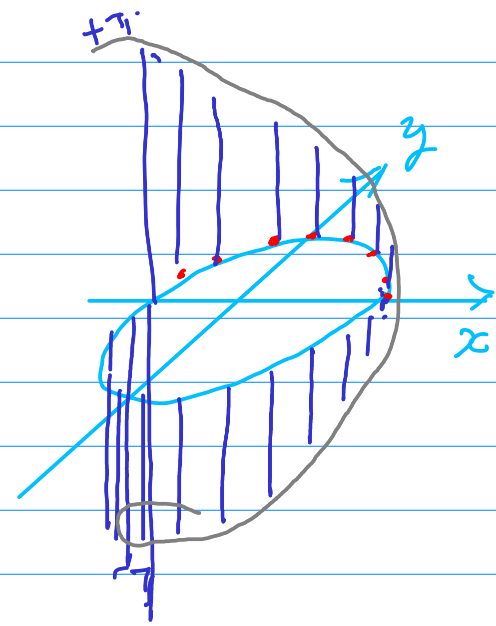




$W = Z$

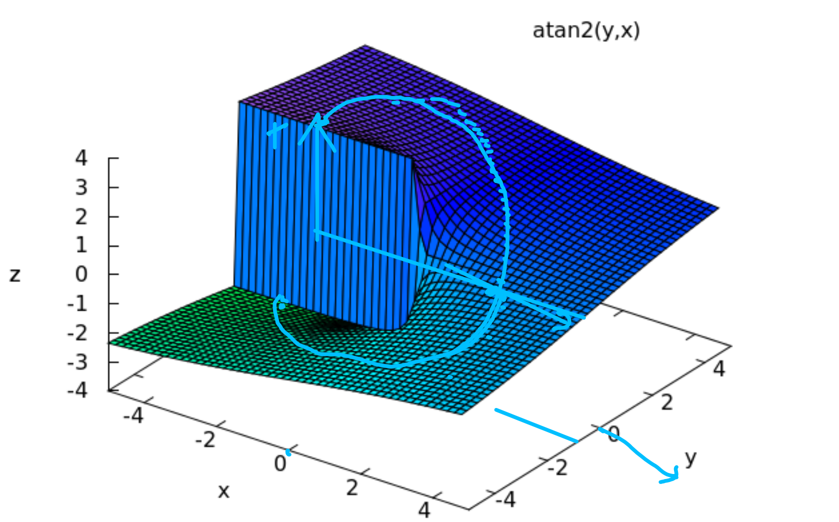


$\frac{\pi}{2} = \frac{3.14}{2} =$
y
x
$\frac{\pi}{6}$
$\frac{\pi}{4}$
$\frac{2}{6}\pi$
$\frac{3}{6}\pi$
$\frac{4}{6}\pi$
$\frac{3}{4}\pi$
$\frac{5}{6}\pi$
$\frac{6}{6}\pi$

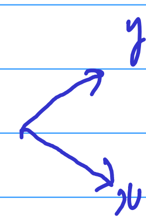


y
x

$\arg(w) = \arg(z)$

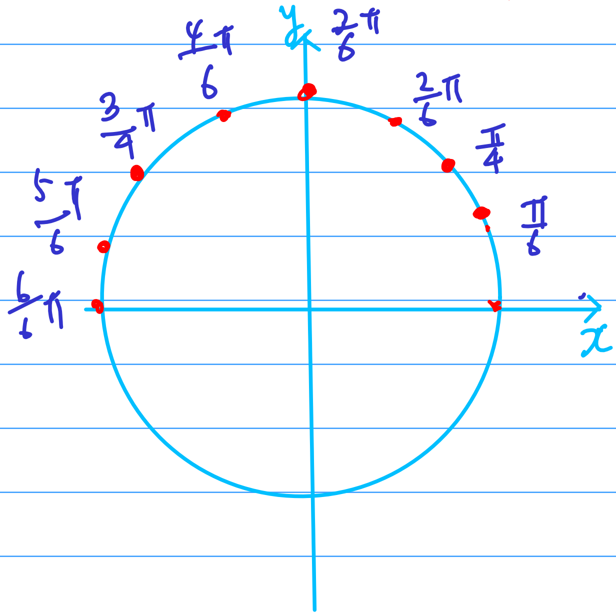


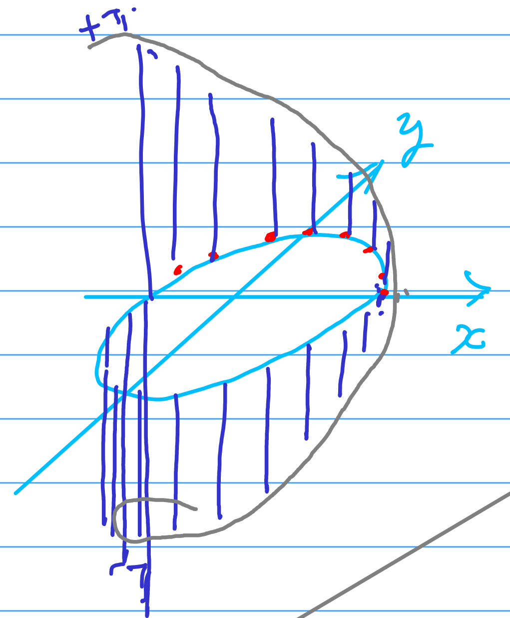
atan2(y,x)
z
x
y
4
3
2
1
0
-1
-2
-3
-4

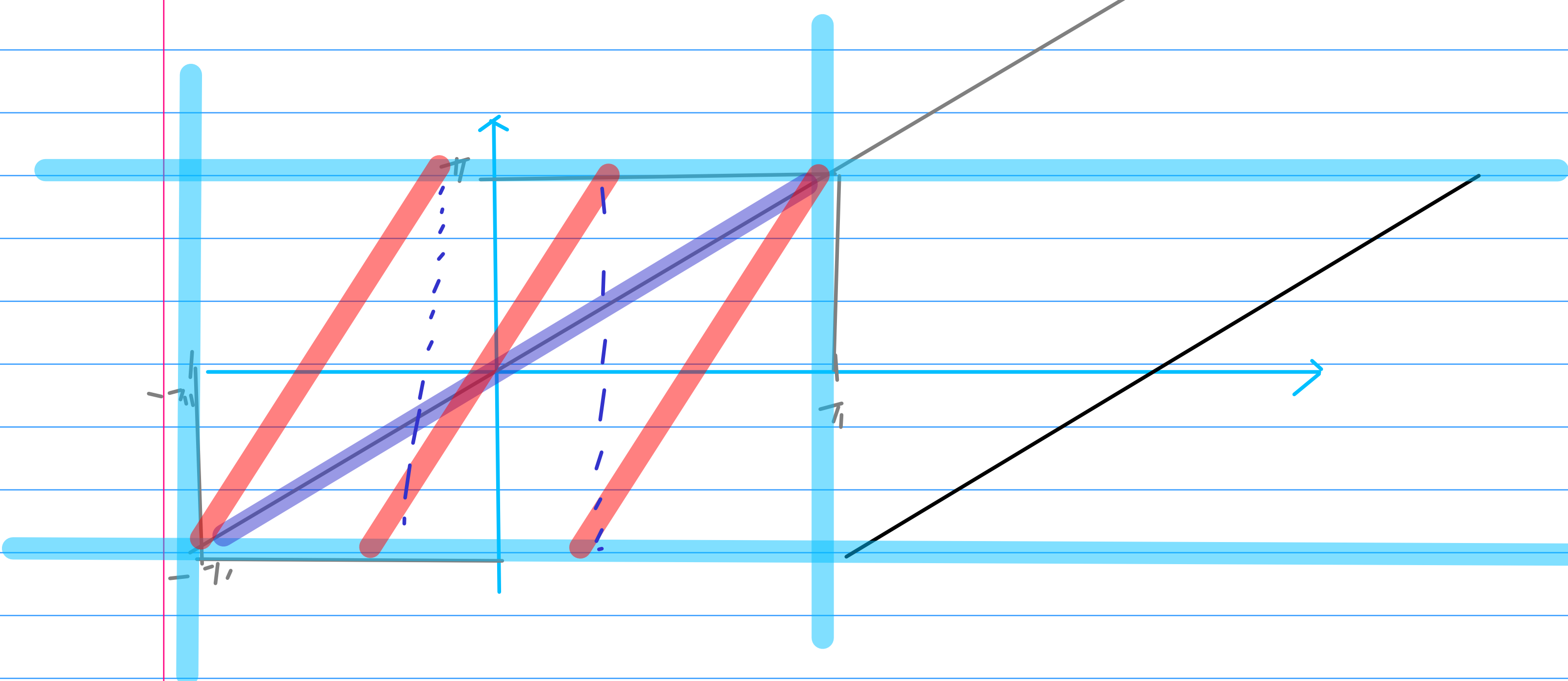


y
x

$\frac{\pi}{2} = \frac{3.14}{2} =$

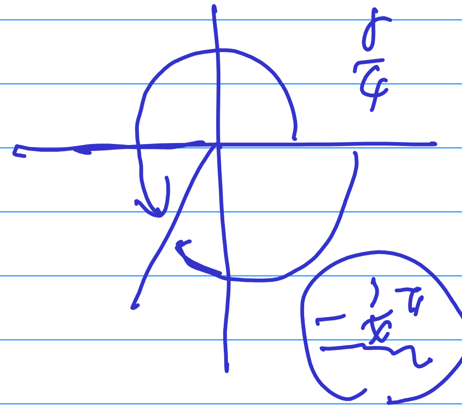






arg(z)

$$\frac{5}{4}\pi \to \frac{4}{4}\pi + \frac{1}{4}\pi$$



$$-\pi < \mathrm{Arg}(z) \le \pi$$

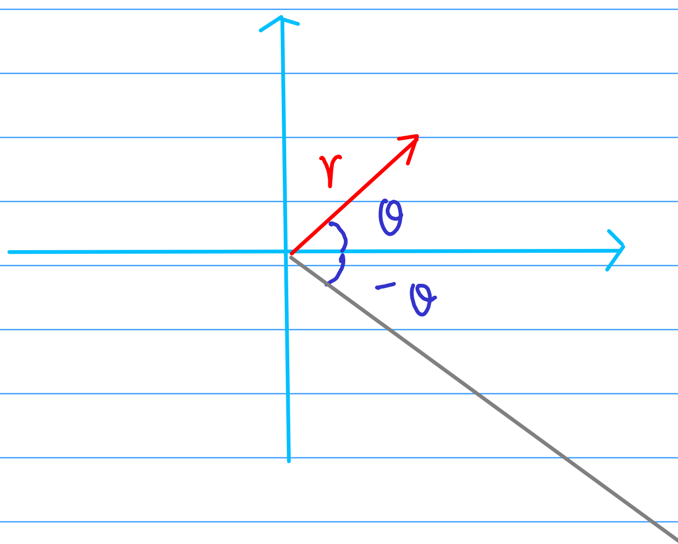
↑ principal argument

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$|e^{i\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$z = re^{j\theta}$$

$$\frac{1}{z} = \frac{1}{re^{j\theta}} = \frac{1}{r}e^{-j\theta}$$

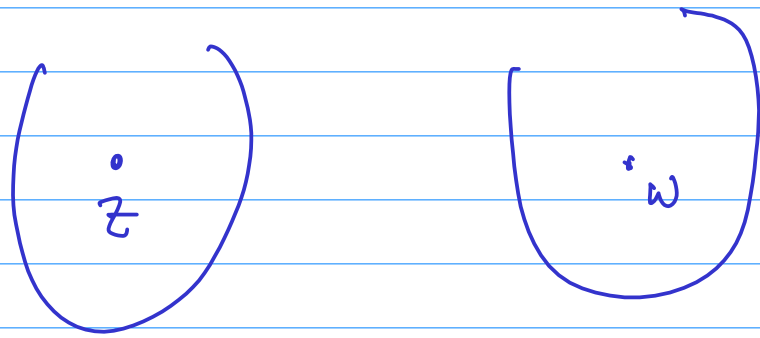


$r < 1$ $\quad \frac{1}{r} > 1$

$r > 1$ $\quad \frac{1}{r} < 1$

Complex Function.

$w = f(z)$



$z = x + iy$

실수 허수

$w = u + iv$

실수 허수

$w = f(z)$

$u(x, y)$ $v(x, y)$

$$w = f(z) = z^2$$

$$w = u + iv$$

$$= (x + iy)(x + iy)$$

$$= (x^2 - y^2) + (2xy)i$$

$$u(x,y) \qquad v(x,y)$$

$$= re^{j\theta}$$

$$r = \sqrt{u^2 + v^2} \qquad \tan\theta = \frac{v}{u}$$

$$= \sqrt{x^2 - y^2 + 4x^2y^2} \qquad = \frac{2xy}{x^2 - y^2}$$

$$w = f(z) = z^2$$

↑ 복소수

실수: x 허수 y

$$z = x + iy$$

이런 복소수 z에 대한

함수값 w도 복소수

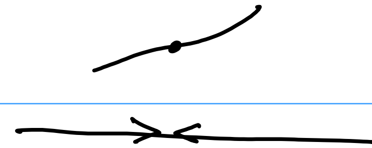
$$w = z^2$$

↑ 함수값 $(x+iy)^2 = x^2 - y^2 + 2xyi$

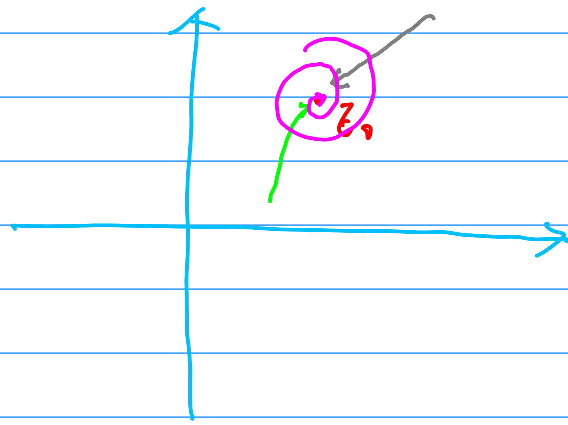
$x^2 - y^2$: 함수값의 실수

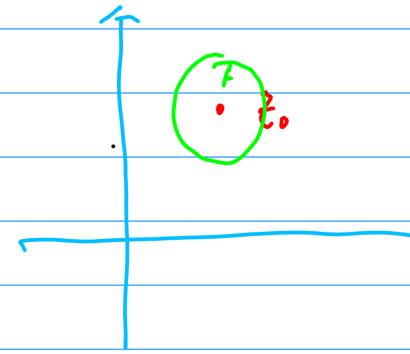
$2xy$: 함수값의 허수

$$u(x, y) + i\,v(x, y)$$



$$\lim_{z \to z_0} f(z) = L$$





$$\lim_{z \to z_0} f(z) = f(z_0)$$

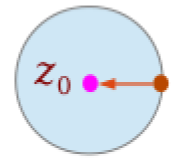
$$\Delta z = \Delta x + i\Delta y$$

horizontal approach $\Delta z \to 0 \Rightarrow \Delta x \to 0 \quad \Delta y = 0$

$$f'(z) = \lim_{\Delta z \to 0} \frac{u(x+\Delta x, y+\Delta y) + iv(x+\Delta x, y+\Delta y) - u(x,y) - iv(x,y)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x} + i\frac{v(x+\Delta x, y) - v(x,y)}{\Delta x}$$

$$= \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad = \frac{\partial f}{\partial x}$$



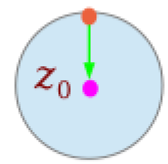
vertical approach $\Delta z \to 0 \Rightarrow \Delta y \to 0 \quad \Delta x = 0$

$$f'(z) = \lim_{\Delta z \to 0} \frac{u(x+\Delta x, y+\Delta y) + iv(x+\Delta x, y+\Delta y) - u(x,y) - iv(x,y)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{u(x, y+\Delta y) - u(x,y)}{i\Delta y} + i\frac{v(x, y+\Delta y) - v(x,y)}{i\Delta y}$$

$$= -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \quad = -i\frac{\partial f}{\partial y}$$

$$\frac{1}{i} = -i$$



$$z = r(\cos\theta + i\sin\theta)$$

$$f(z) = u(r,\theta) + i\, v(r,\theta)$$

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

$$\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

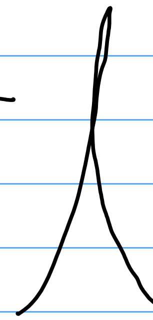
$f(z) = z^1$ continuos

z^2

z^3

z^5

$$f(z) = \frac{g(z)}{h(z)} = \frac{z^3 + z^2 + 2z + \dots}{z - 1}$$



$z = 1$ $\qquad \frac{c}{0} \to \infty$

$h(z) = 0 \qquad z \neq 1$

derivative

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{f(z_0+\Delta z) - f(z_0)}{\Delta z}$$

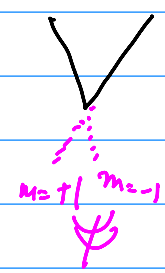
$$\frac{df}{dz}$$

$$f(z) = 3z^4 - 5z^3 + 2z$$

$$f'(z) = 12z^3 - 15z^2 + 2$$

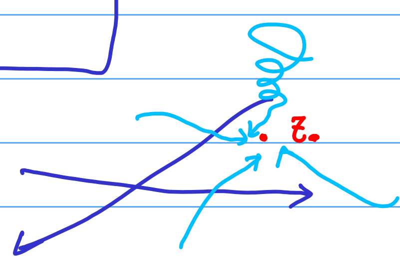
$$f(z) = \frac{z^2}{4z+1}$$

$$f'(z) = \frac{(z^2)'(4z+1) - z^2(4z+1)'}{(4z+1)^2}$$



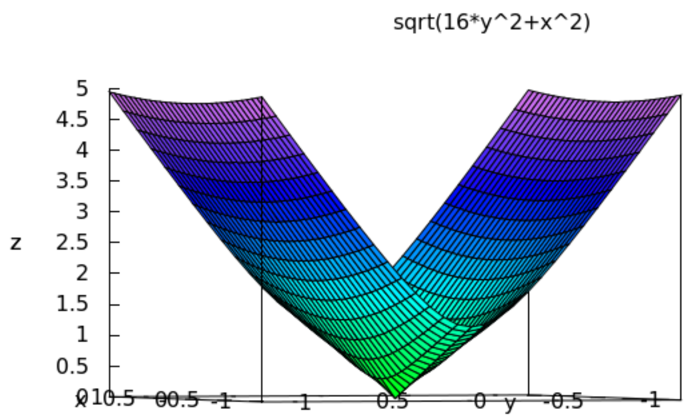
f : differentiable at $z = z_0$

$$\lim_{\Delta z \to 0} \frac{f(z_0+\Delta z) - f(z_0)}{\Delta z}$$



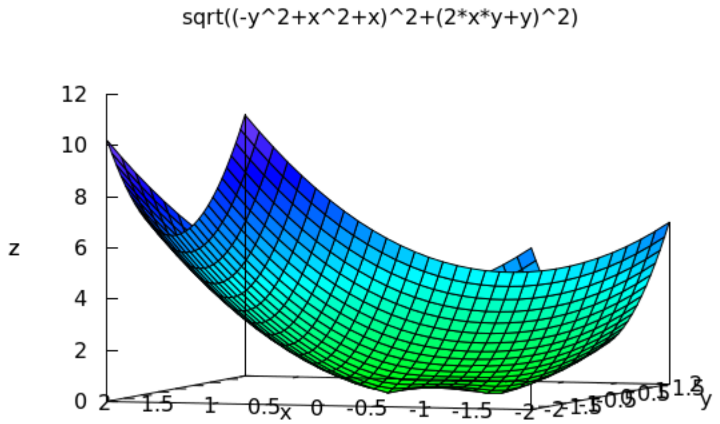
$f(z) = x + 4iy$

```
(%i9) plot3d(cabs(x + 4*%i*y), [x, -1.2, 1.2], [y, -1.2, 1.2]);
(%o9) /home/young/maxout.gnuplot_pipes
```



(예제1)

$$f(z) = z^2 + z \rightarrow \underbrace{(x^2 - y^2 + x)}_{u(x,y)} + i\underbrace{(2xy + y)}_{v(x,y)}$$



$z = x + iy \qquad z^2 = x^2 - y^2 + 2xy\,i$

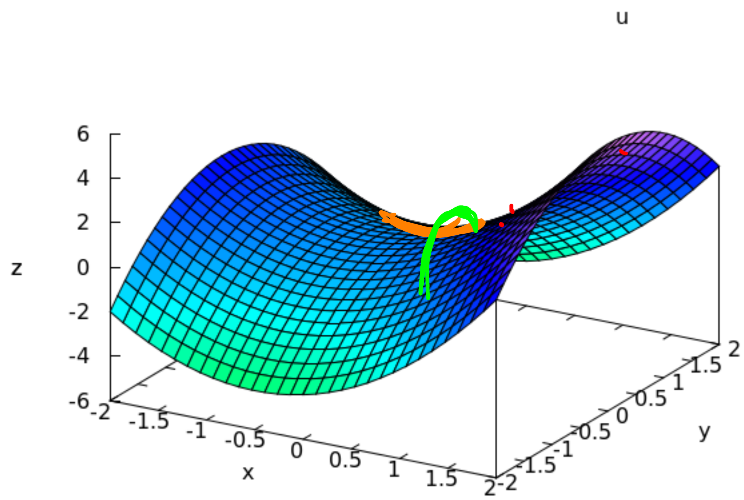
$$z^2 + z = \underbrace{x^2 + x - y^2}_{u} + i\,(\underbrace{2xy + y}_{v})$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad 2x + 1 = 2x + 1$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \qquad -2y = -(2y)$$

$$z^2+z = \underbrace{x^2+x-y^2}_{u} + i\,(\underbrace{2xy+y}_{v})$$

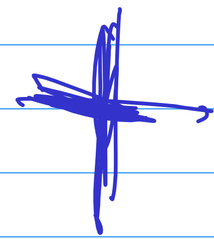
$$u(x,y) = x^2+x-y^2$$

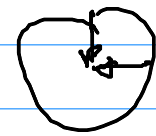


2x+1

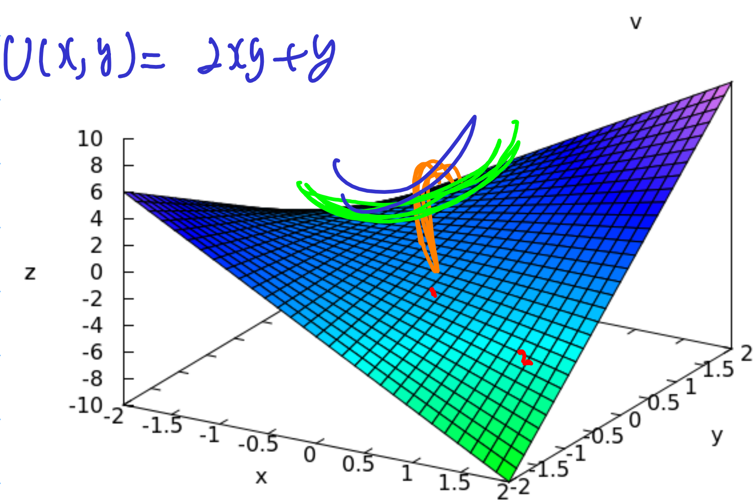
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

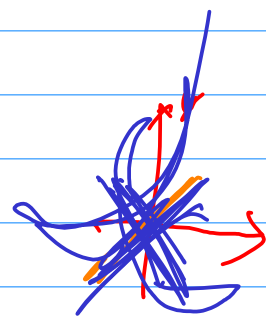
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

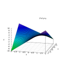




$$v(x,y) = 2xy+y$$

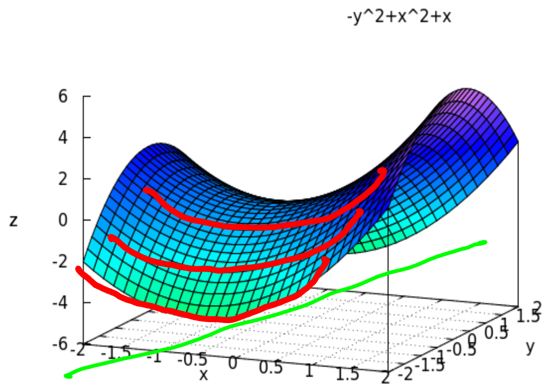






$u(x,y) = x^2 - y^2 + x$

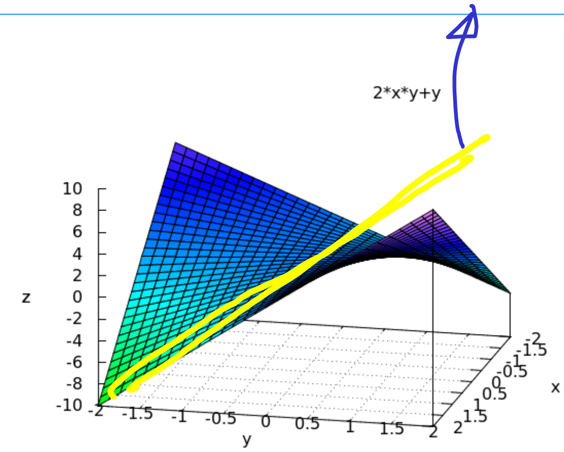
$\frac{\partial u}{\partial x} = 2x+1$



$v(x,y) = 2xy + y$

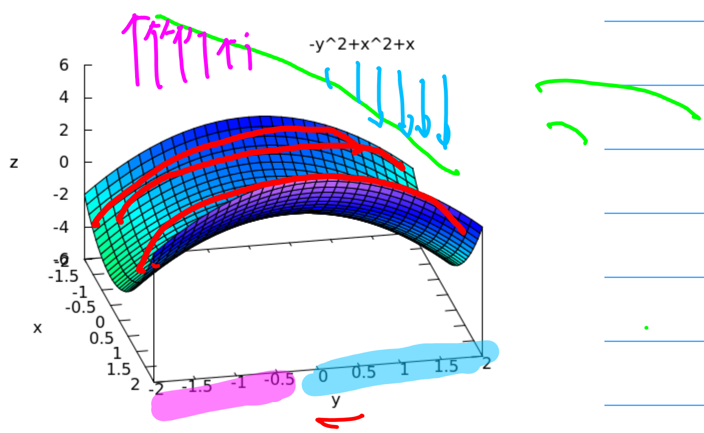
$\frac{\partial v}{\partial y} = 2x+1$

$m = 2x+1$



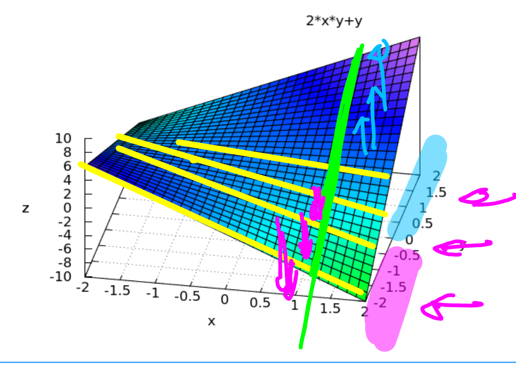
$u(x,y) = x^2 - y^2 + x$

$\frac{\partial u}{\partial y} = -2y$



$v(x,y) = 2xy + y$

$\frac{\partial v}{\partial x} = 2y$ → x 와 관계없는 상수



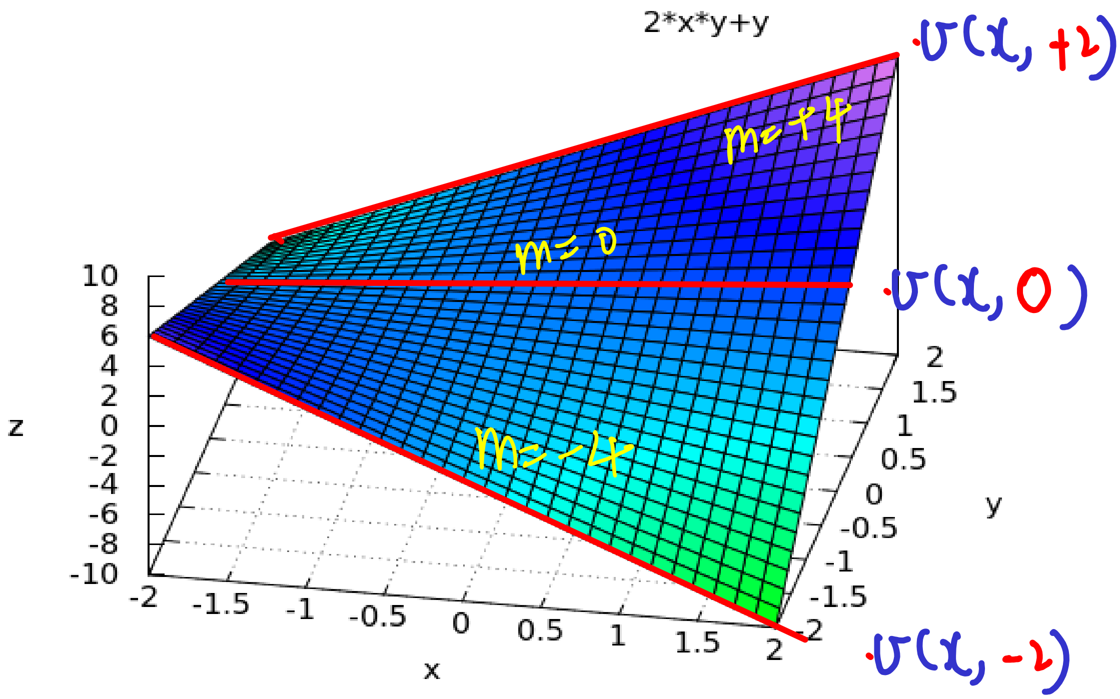
given v

$\frac{\partial v}{\partial x} = 2y$

x만 변수로 생각해서 differentiate

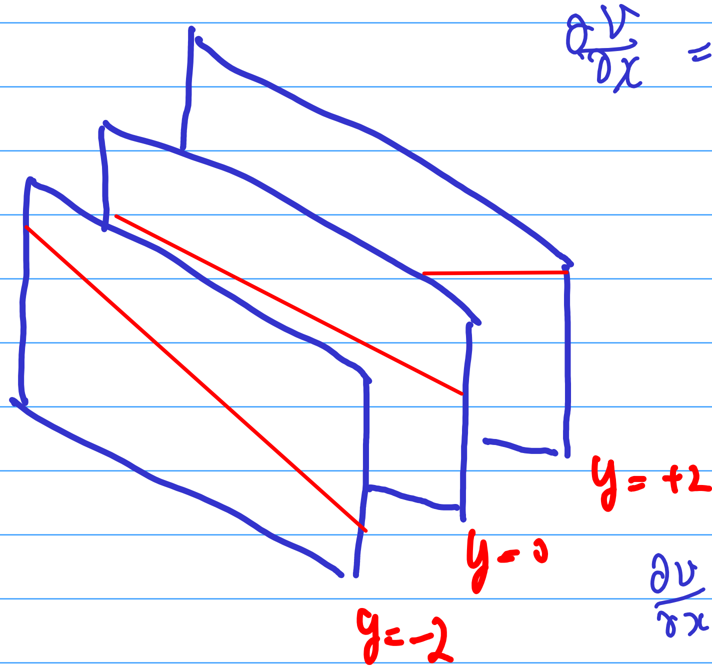
상수

given y 상수



$$v(x,y) = 2xy + y$$

$$\frac{\partial v}{\partial x} = 2y$$



$$\frac{\partial v}{\partial x}(x,+2) = 4$$

$$\frac{\partial v}{\partial x}(x,0) = 0.$$

$$\rightarrow \frac{\partial v}{\partial x}(x,-2) = -4$$

$U(x,y) = 2xy + y$

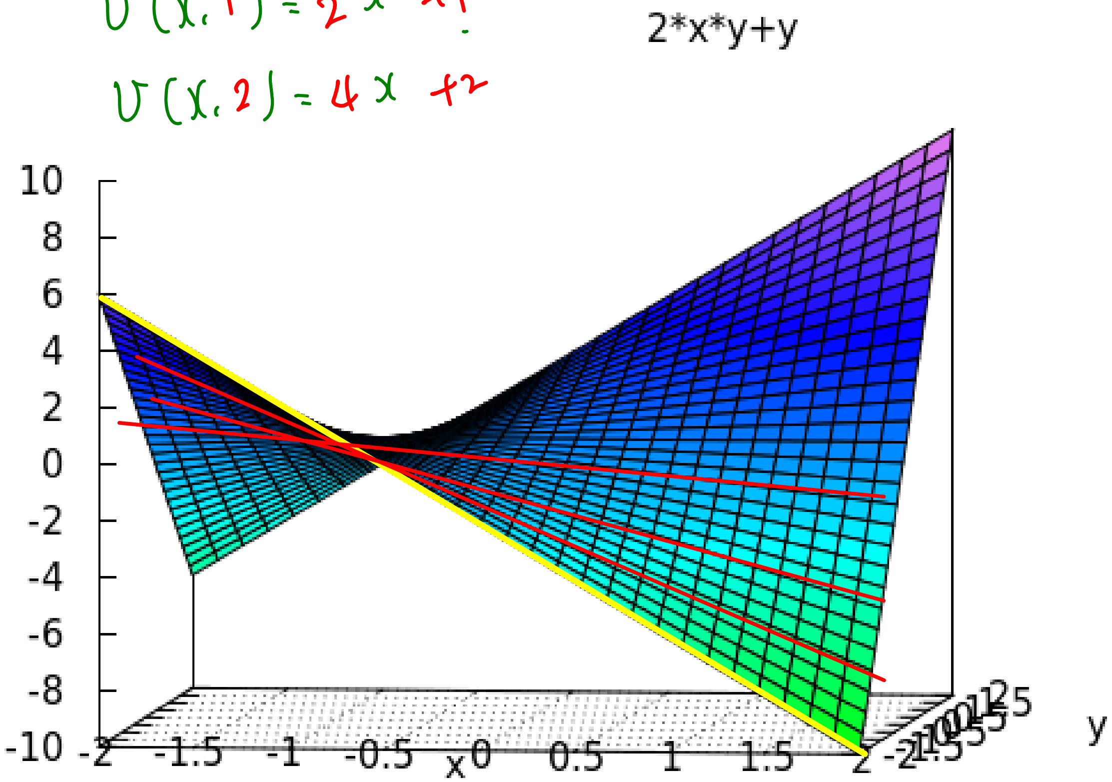
$U(x,-2) = -4x - 2$

$U(x,-1) = -2x - 1$

$U(x,0) = 0.x$

$U(x,1) = 2x + 1$

$U(x,2) = 4x + 2$



$U(x,y) = 2xy + y$

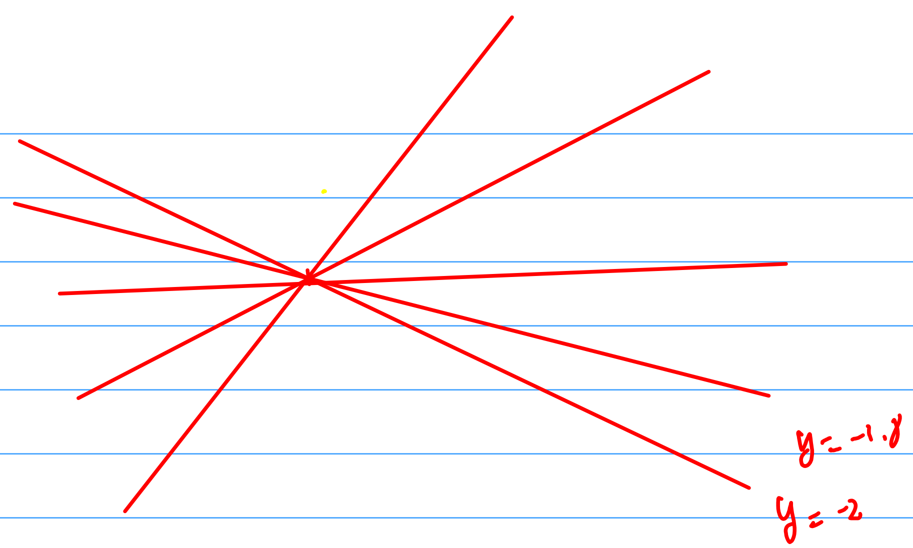
$U(x,-2) = -4x - 2$

$U(x,-1) = -2x - 1$

$U(x,0) = 0.x$

$U(x,1) = 2x + 1$

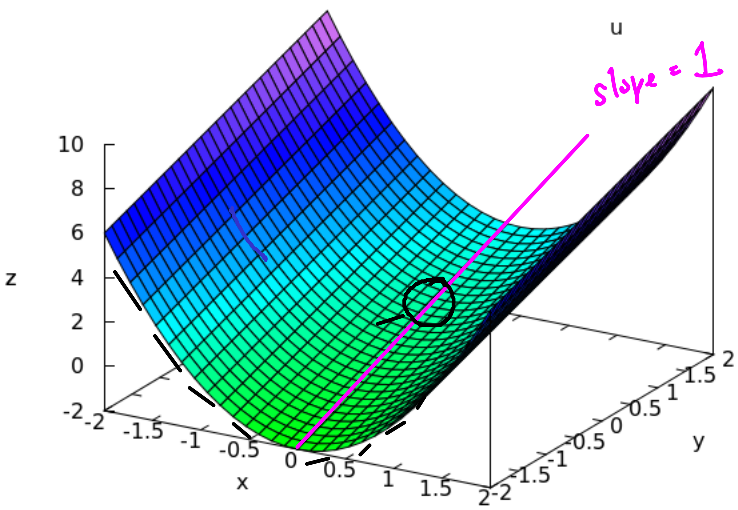
$U(x,2) = 4x + 2$

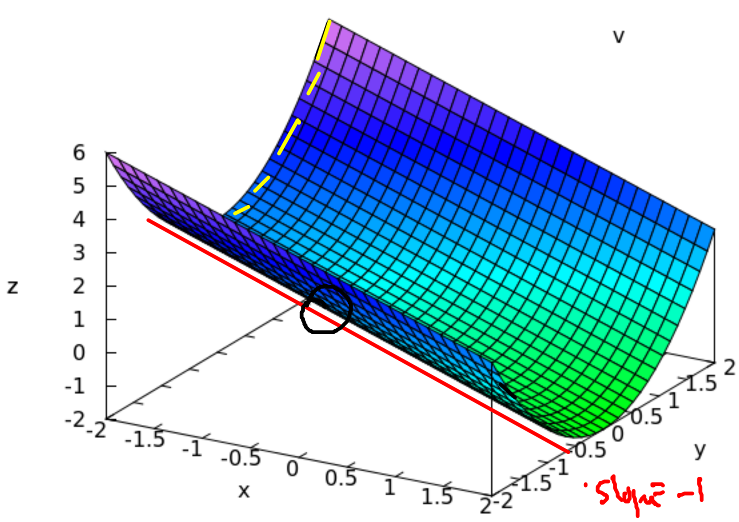


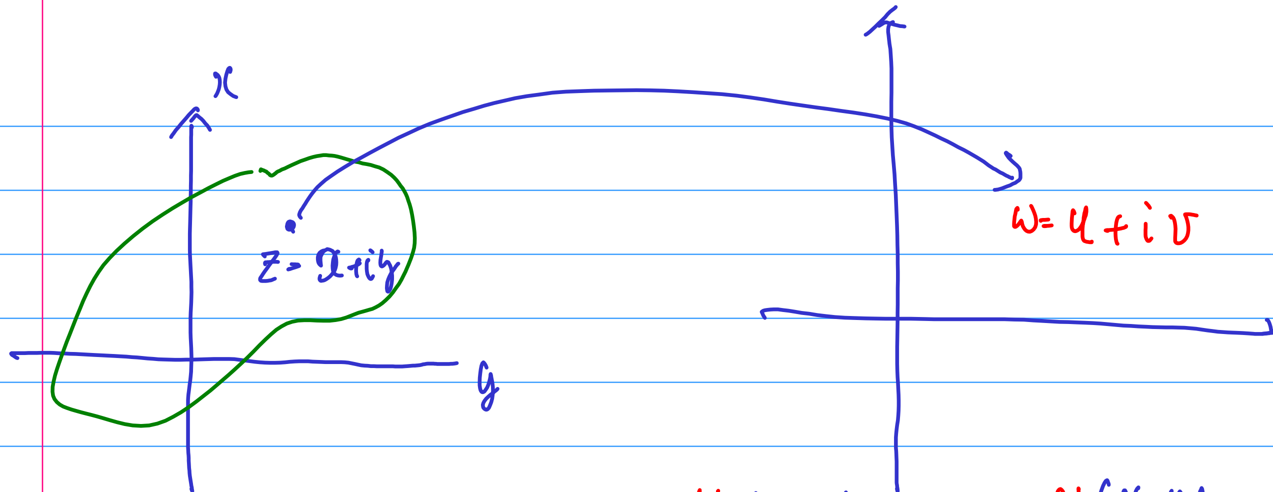
$$f(z) = \underbrace{(2x^2+y)}_{u} + i\,\underbrace{(y^2-x)}_{v}$$

$$\left(\frac{\partial u}{\partial x} = 2x\right) \neq \left(\frac{\partial v}{\partial y} = 2y\right)$$

$$\left(\frac{\partial u}{\partial y} = 1\right) = -\left(\frac{\partial v}{\partial x} = -1\right)$$







$u(x,y)$ $v(x,y)$

$u(x,y)$, $v(x,y)$

continuous

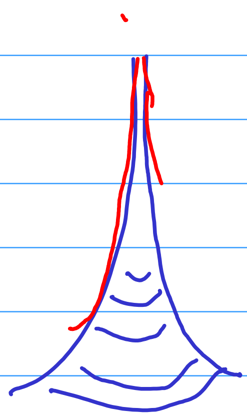
$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

continuous, existence

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -1 \frac{\partial v}{\partial x}$$

Cauchy-Rieman eq



$w = \frac{1}{z}$ $z=0$ $w = \frac{1}{0} \to \infty$

→ discontinuous at $z=0$

→ differentiable ✗

→ analytic ✗

(9) $f(z) = e^x \cos y + i \cdot e^x \sin y$

Cauchy-Rieman Eq

certain regin

* n-th degree polynomial

$$f(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_2 z^2 + a_1 z^1 + a_0$$

{ continuous
differentiable at any point
analytic

entire function

* rational function 유리함수

$$f(z) = \frac{g(z)}{h(z)} \quad \begin{matrix} \cdots\cdots \text{ polynomial} \\ \cdots\cdots \text{ polynomial} \end{matrix}$$

$h(z) = 0$ 만들어 주는 z 에서 discontinuous

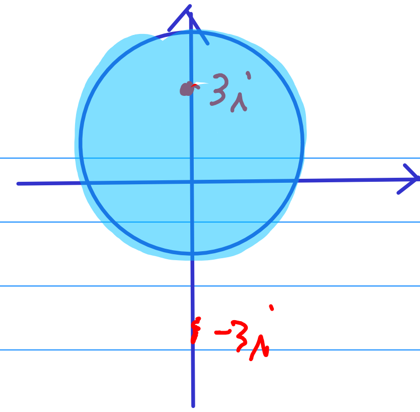
→ differentiable X

→ analytic X

* denominator polynomial $h(z)$가 0 되는 점들은 (pole, singular point)

continuous, differentiable, analytic

$$\oint \frac{z}{z^2+9}\, dz$$



$$\frac{z}{z^2+9} = \frac{z}{(z+3i)(z-3i)}$$

$$\oint \frac{\frac{z}{z+3i}}{z-3i}\, dz \qquad \frac{z}{z+3i} : f(z)$$

$$f(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{z-a}\, dz$$

Cauch Integration Formula I

$$f(z) = \frac{z}{z+3i}$$

z ← 1차 다항식
$z+3i$ ← 1차 다항식

) 유리함수

rational function

분모를 0으로 만들어주는 $z=-3i$ 를 제외한 모든 곳에서 analytic.

$$(a+b)(a-b) = a^2 - b^2$$

$$(a+ib)(a-ib) = a^2+b^2$$

$$f(z) = \frac{z}{z+3i} = \frac{x+iy}{x+iy+3i} = \frac{x+iy}{x+i(y+3)}$$

$$x+iy \longrightarrow u+iv$$

$$\frac{x+iy}{x+i(y+3)} \cdot \frac{x-i(y+3)}{x-i(y+3)} = \frac{(x^2+y^2+3y)+i(xy-xy-3x)}{x^2+(y+3)^2}$$

$$u = \frac{x^2+y^2+3y}{x^2+(y+3)^2} \qquad v = \frac{-3x}{x^2+(y+3)^2}$$

$$\frac{\partial u}{\partial x} = \qquad \frac{\partial v}{\partial y} =$$

```
--> ratsimp(diff((-3*x) / (x^2 + (y+3)^2), y, 1));
```

$\frac{\partial v}{\partial y}$

(%o7) $$\frac{6\,x\,y+18\,x}{y^4+12\,y^3+\left(2\,x^2+54\right)\,y^2+\left(12\,x^2+108\right)\,y+x^4+18\,x^2+81}$$

```
(%i6) ratsimp(diff((x^2+y^2+3*y)/(x^2+(y+3)^2), x, 1));
```

$\frac{\partial u}{\partial x}$

(%o6) $$\frac{6\,x\,y+18\,x}{y^4+12\,y^3+\left(2\,x^2+54\right)\,y^2+\left(12\,x^2+108\right)\,y+x^4+18\,x^2+81}$$

```
(%i8) ratsimp(diff((-3*x) / (x^2 + (y+3)^2), x, 1));
```

$\frac{\partial v}{\partial x}$

(%o8) $$-\frac{3\,y^2+18\,y-3\,x^2+27}{y^4+12\,y^3+\left(2\,x^2+54\right)\,y^2+\left(12\,x^2+108\right)\,y+x^4+18\,x^2+81}$$

```
(%i9) ratsimp(diff((x^2+y^2+3*y)/(x^2+(y+3)^2), y, 1));
```

$\frac{\partial u}{\partial y}$

(%o9) $$\frac{3\,y^2+18\,y-3\,x^2+27}{y^4+12\,y^3+\left(2\,x^2+54\right)\,y^2+\left(12\,x^2+108\right)\,y+x^4+18\,x^2+81}$$

$$\frac{x+iy}{x+i(y+3)} \cdot \frac{x-i(y+3)}{x-i(y+3)} = \frac{(x^2+y^2+3y)+i(xy-xy-3x)}{x^2+(y+3)^2}$$

$x=0$

$y=-3$.

$z=-3i$ 에서 분모를 0으로 만든다

$$f(z) = \frac{z}{z+3i}$$

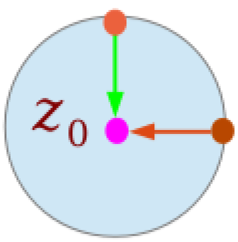
$$f'(z) = \left(\frac{z}{z+3i}\right)' = \frac{(z)'(z+3i) - (z)(z+3i)'}{(z+3i)^2}$$

$$= \frac{\cancel{z} + 3i - \cancel{z}}{(z+3i)^2} = \frac{3i}{(z+3i)^2}$$

if the real functions u(x,y) and v(x,y) are **continuous**
and have **continuous** first order partial derivatives
in a neighborhood of z,
and if u(x,y) and v(x,y) satisfy
the **Cauchy-Riemann equations** at the point z,

then the complex function f(z) = u(x,y) + iv(x,y)
is **differentiable** at z
and f'(z) is as belows.

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$



$$f(z) = u + iv$$

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\frac{1}{i}\frac{\partial f}{\partial y} = \left(\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right) \times \frac{1}{i}$$

$$\| \quad f'(z)$$

$$f(z) = u + i v$$

$$z = x + i y$$

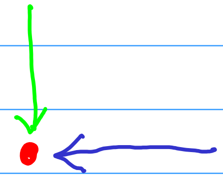
$$f'(z) = \frac{df}{dz} = \frac{\partial f}{\partial x} = \frac{1}{i} \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$$

$$\frac{\partial f}{\partial y} = \frac{\partial u}{\partial y} + i \frac{\partial v}{\partial y}$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dz$$

$$\left(f'(z) = \frac{1}{i}\frac{\partial f}{\partial y} \right) \quad \left(\begin{array}{l} \Delta x = 0 \\ \Delta y \to 0 \end{array} \right. \qquad \Delta z = i\Delta y \to 0$$



$$\left(\begin{array}{l} \Delta y = 0 \\ \Delta x \to 0 \end{array} \right)$$

$$f'(z) = \frac{\partial f}{\partial x}$$

$$\Delta z \to 0$$

Hyperbolic vs. Trigonometric Functions

Trigonometric Function		*Hyperbolic Function*
ix		x
$e^{+ix} = \cos x + i\sin x$	⟺	$e^{+x} = \cosh x + \sinh x$
$e^{-ix} = \cos x - i\sin x$	⟺	$e^{-x} = \cosh x - \sinh x$
$\cos x = \frac{1}{2}\left(e^{+ix} + e^{-ix}\right)$	⟺	$\cosh x = \frac{1}{2}\left(e^{+x} + e^{-x}\right)$
$\sin x = \frac{1}{2i}\left(e^{+ix} - e^{-ix}\right)$	⟺	$\sinh x = \frac{1}{2}\left(e^{+x} - e^{-x}\right)$
$\tan x = \frac{1}{i}\frac{\left(e^{+ix} - e^{-ix}\right)}{\left(e^{+ix} + e^{-ix}\right)}$	⟺	$\tanh x = \frac{\left(e^{+x} - e^{-x}\right)}{\left(e^{+x} + e^{-x}\right)}$

Trigonometric functions with imaginary arguments

$ix \rightarrow x$

$\cos x = \frac{1}{2}\left(e^{+ix} + e^{-ix}\right)$	⟺	$\cosh x = \frac{1}{2}\left(e^{+x} + e^{-x}\right)$
$\sin x = \frac{1}{2i}\left(e^{+ix} - e^{-ix}\right)$	⟺	$\sinh x = \frac{1}{2}\left(e^{+x} - e^{-x}\right)$
$\tan x = \frac{1}{i}\frac{\left(e^{+ix} - e^{-ix}\right)}{\left(e^{+ix} + e^{-ix}\right)}$	⟺	$\tanh x = \frac{\left(e^{+x} - e^{-x}\right)}{\left(e^{+x} + e^{-x}\right)}$

$\cos ix = \cosh x$	$\cos ix = \frac{1}{2}\left(e^{-x} + e^{+x}\right)$	$\cosh x = \frac{1}{2}\left(e^{+x} + e^{-x}\right)$
$\sin ix = i\sinh x$	$\sin ix = \frac{1}{2i}\left(e^{-x} - e^{+x}\right)$	$\sinh x = \frac{1}{2}\left(e^{+x} - e^{-x}\right)$
$\tan ix = i\tanh x$	$\tan ix = \frac{1}{i}\frac{\left(e^{-x} - e^{+x}\right)}{\left(e^{-x} + e^{+x}\right)}$	$\tanh x = \frac{\left(e^{+x} - e^{-x}\right)}{\left(e^{+x} + e^{-x}\right)}$

Hyperbolic functions with imaginary arguments

$$\cos x = \frac{1}{2}(e^{+ix} + e^{-ix}) \quad \Leftrightarrow \quad \cosh x = \frac{1}{2}(e^{+x} + e^{-x})$$

$$\sin x = \frac{1}{2i}(e^{+ix} - e^{-ix}) \quad \Leftrightarrow \quad \sinh x = \frac{1}{2}(e^{+x} - e^{-x})$$

$$\tan x = \frac{1}{i}\frac{(e^{+ix} - e^{-ix})}{(e^{+ix} + e^{-ix})} \quad \Leftrightarrow \quad \tanh x = \frac{(e^{+x} - e^{-x})}{(e^{+x} + e^{-x})}$$

$x \leftarrow i\,x$

$$\cos x = \frac{1}{2}(e^{+ix} + e^{-ix}) \qquad \cosh ix = \frac{1}{2}(e^{+ix} + e^{-ix})$$

$$\sin x = \frac{1}{2i}(e^{+ix} - e^{-ix}) \qquad \sinh ix = \frac{1}{2}(e^{+ix} - e^{-ix})$$

$$\tan x = \frac{1}{i}\frac{(e^{+ix} - e^{-ix})}{(e^{+ix} + e^{-ix})} \qquad \tanh ix = \frac{(e^{+ix} - e^{-ix})}{(e^{+ix} + e^{-ix})}$$

$\cosh ix = \cos x$

$\sinh ix = i \sin x$

$\tanh ix = i \tan x$

With imaginary arguments

$$\cos x = \frac{1}{2}(e^{+ix} + e^{-ix}) \quad \Leftrightarrow \quad \cosh x = \frac{1}{2}(e^{+x} + e^{-x})$$

$$\sin x = \frac{1}{2i}(e^{+ix} - e^{-ix}) \quad \Leftrightarrow \quad \sinh x = \frac{1}{2}(e^{+x} - e^{-x})$$

$$\tan x = \frac{1}{i}\frac{(e^{+ix} - e^{-ix})}{(e^{+ix} + e^{-ix})} \quad \Leftrightarrow \quad \tanh x = \frac{(e^{+x} - e^{-x})}{(e^{+x} + e^{-x})}$$

x

$\cos ix = \cosh x$

$\sin ix = i \sinh x$

$\tan ix = i \tanh x$

$\cosh ix = \cos x$

$\sinh ix = i \sin x$

$\tanh ix = i \tan x$

$i\,x$

Euler Formula

Euler Formula

$$e^{+ix} = \cos x + i \sin x$$
$$e^{-ix} = \cos x - i \sin x$$

Euler Formula

$$e^{+ix} = \cosh ix + \sinh ix$$
$$e^{-ix} = \cosh ix - \sinh ix$$

$$\cos ix = \cosh x \qquad \cosh ix = \cos x$$
$$\sin ix = i \sinh x \qquad \sinh ix = i \sin x$$
$$\tan ix = i \tanh x \qquad \tanh ix = i \tan x$$

Modulus of sin(z) – (1)

$$\begin{aligned}
\sin(z) &= \sin(x+iy) \\
&= \sin(x)\cos(iy) + \cos(x)\sin(iy) \\
&= \sin(x)\cosh(y) + i\cos(x)\sinh(y)
\end{aligned}$$

$$\begin{aligned}
|\sin(z)|^2 &= \sin(z)\,\overline{\sin(z)} \\
&= \tfrac{1}{2i}\left(e^{+i(x+iy)} - e^{-i(x+iy)}\right)\tfrac{-1}{2i}\left(e^{-i(x-iy)} - e^{+i(x-iy)}\right) \\
&= \tfrac{1}{4}\left(e^{-y+ix} - e^{+y-ix}\right)\left(e^{-y-ix} - e^{+y+ix}\right) \\
&= \tfrac{1}{4}\left(e^{-2y} - e^{+2ix} - e^{-2ix} + e^{+2y}\right) \\
&= \tfrac{1}{4}\left(e^{+2y} - 2 + e^{-2y} - e^{+2ix} + 2 - e^{-2ix}\right) \\
&= +\tfrac{1}{4}\left(e^{+2y} - 2 + e^{-2y}\right) - \tfrac{1}{4}\left(e^{+2ix} - 2 + e^{-2ix}\right) \\
&= \left[\tfrac{1}{2}\left(e^{+y} - e^{-y}\right)\right]^2 + \left[\tfrac{1}{2i}\left(e^{+ix} - e^{-ix}\right)\right]^2 \\
&= \sin^2(x) + \sinh^2(y)
\end{aligned}$$

$$e^{iy} = \cos y + i \sin y$$

$$e^{z} = e^{x+iy} = e^{x} \cdot e^{iy} = e^{x}(\cos y + i \sin y)$$

$$= \underbrace{e^{x}\cos y}_{u} + i\, \underbrace{e^{x}\sin y}_{v}$$

entire fn

$y' = y$ $\quad\quad$ $y' - y = 0$

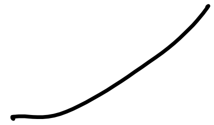
$y = e^{x}$ $\quad\quad$ $m - 1 = 0$ $\quad\quad$ e^{x}

$$(e^{x})' = e^{x}$$

$(e^{z})' = e^{z}$ $\quad\quad$ $\frac{d}{dz} e^{z} = e^{z}$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$$

$e^x \cdot \cos(y)$



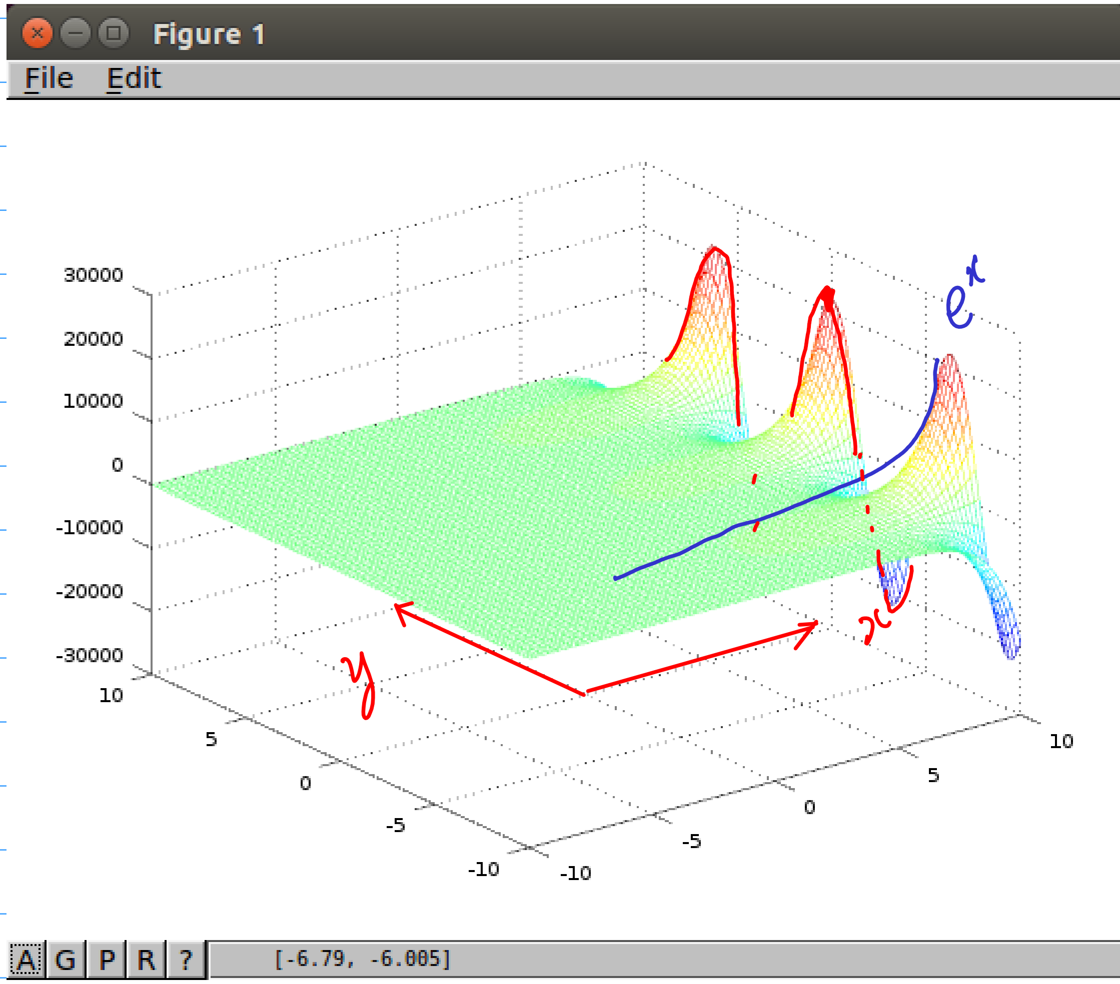
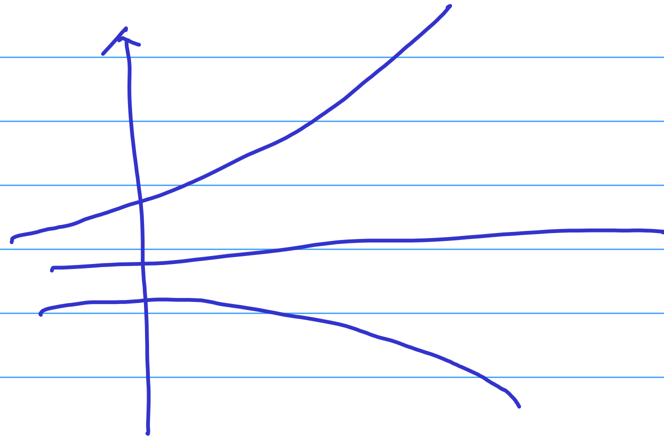
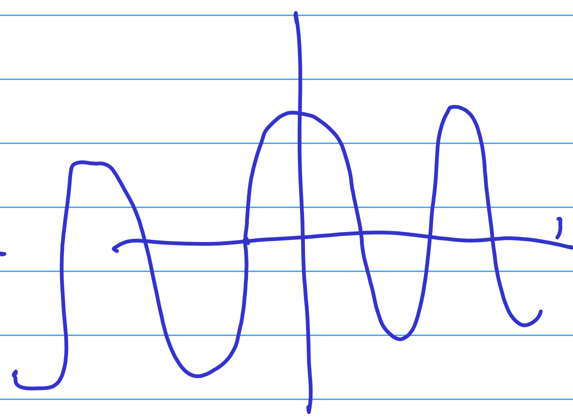


Figure 1
File
Edit
30000
20000
10000
0
-10000
-20000
-30000
10
5
0
-5
-10
-10
-5
0
5
10
e^x
y
x
A
G
P
R
?
[-6.79, -6.005]





$$z = x + iy$$

$$iz = ix - y$$

$$e^{iy} = \cos y + i \sin y$$

$$e^{z} = e^{x+iy} = e^{x} \cdot e^{iy} = e^{x}(\cos y + i \sin y)$$

$$= \underbrace{e^{x} \cos y}_{u} + i \, \underbrace{e^{x} \sin y}_{v}$$

$$e^{iz} = e^{ix-y} = e^{-y} \cdot e^{ix} = e^{-y}(\cos x + i \sin x)$$

$$e^{+ix} = \cos(x) + i\sin(x)$$

$$e^{-ix} = \cos(x) - i\sin(x)$$

$$\cos(x) = \frac{e^{+ix} + e^{-ix}}{2}$$

$$\sin(x) = \frac{e^{+ix} - e^{-ix}}{2i}$$

$$\cos(z) = \frac{e^{+iz} + e^{-iz}}{2}$$

$$\sin(z) = \frac{e^{+iz} - e^{-ixz}}{2i}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\begin{aligned}
\cos(z) &= \cos(x+iy) \\
&= \cos(x)\cos(iy) - \sin(x)\sin(iy) \\
&= \cos(x)\cosh(y) - i\sin(x)\sinh(y)
\end{aligned}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\begin{aligned}
\sin(z) &= \sin(x+iy) \\
&= \sin(x)\cos(iy) + \cos(x)\sin(iy) \\
&= \sin(x)\cosh(y) + i\cos(x)\sinh(y)
\end{aligned}$$

$$e^{+iz} = e^{ix-y} = e^{-y} \cdot e^{ix} = e^{-y}(\cos x + i \sin x)$$

$$e^{-iz} = e^{-ix+y} = e^{+y} \cdot e^{-ix} = e^{y}(\cos x - i \sin x)$$

$$e^{+iz} + e^{-iz} = \cos x \, (e^{y} + e^{-y}) - i \sin x (e^{y} - e^{-y})$$

$$\cos(z) = \cos(x) \cosh(y) - i \sin(x) \sinh(y)$$

$$e^{+iz} = e^{ix-y} = e^{-y} \cdot e^{ix} = e^{-y}(\cos x + i \sin x)$$

$$e^{-iz} = e^{-ix+y} = e^{+y} \cdot e^{-ix} = e^{y}(\cos x - i \sin x)$$

$$e^{+iz} - e^{-iz} = -\cos x \, (e^{y} - e^{-y}) + i \sin x (e^{y} + e^{-y})$$

$$i \sin(z) = -\cos(x) \sinh(y) + i \sin(x) \cosh(y)$$

$\frac{+}{i} \to +i$

$$\sin(z) = \sin(x) \cosh(y) + i \cos(x) \sinh(y)$$

$$\cos(z) = \frac{e^{+iz} + e^{-iz}}{2}$$

$$\sin(z) = \frac{e^{+iz} - e^{-ixz}}{2i}$$

sinh(z)   taylor series

sinh'(z)
sinh''(z)
⋮

$$\frac{d}{dz}\sin(z) = \frac{d}{dz}\frac{e^{iz}-e^{-iz}}{2i} = \frac{1}{2i}\left(ie^{iz}+ie^{-iz}\right)$$

$$= \frac{1}{2}\left(e^{iz}+e^{-iz}\right) = \cos(z)$$

$$\sin'(z) = \cos(z)$$

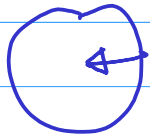
$$\cos(z) = -\sin(z)$$

$$\sin(-z) = -\sin(z)$$
$$\cos(-z) = \cos(z)$$

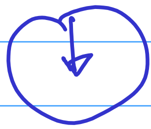
$$\sin^2(z) + \cos^2(z) = 1$$

$$\sin(z_1+z_2) = \sin(z_1)\cos(z_2) + \cos(z_1)\sin(z_1)$$
$$\cos(z_1+z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$$

$$f(z) = u + iv$$



$$\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$



$$\frac{\partial f}{\partial y} = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}$$

$$\boxed{f'(z)}$$

$$f'(z) = \frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$f(z) = e^z = e^x \cos y + i e^x \sin y$$

$$\frac{\partial f}{\partial x} = e^x \cos y + i e^x \sin y \qquad (e^x)' = e$$

① $f'(z) = f(z)$

$$\frac{d}{dz} e^z = e^z$$

② $f(z_1 + z_2) = f(z_1) \cdot f(z_2)$

logarithmic

$$\boxed{e^w = z}$$

$$w = \ln z$$

$$= \log_e z$$

$$e^{\ln z} = z.$$

$$z = e^w$$

$$x+iy = e^{u+iv} = e^u(\cos v + i\sin v) = \square \cdot e^v$$

$$= \underbrace{e^u \cos v}_{x} + i\,\underbrace{e^u \sin v}_{y}$$

$$z = x+iy$$

$$= re^{j\theta}$$

$$r = \sqrt{x^2+y^2}$$

$$\frac{y}{x} \qquad v$$

$$|z|^2 = r^2 \Rightarrow x^2 + y^2 \Rightarrow e^{2u}$$

$$e^{2u}\cos^2 v + e^{2u}\sin^2 v = e^{2u}$$

$$|z|^2 = e^{2u}$$

$$|z| = e^u \Rightarrow u = \log_e |z| = \ln|z|$$

실수

계산

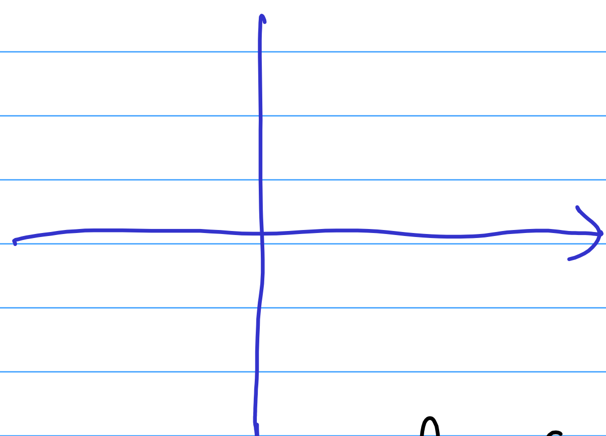
$\ln z$ (복소수) $= \log_e z$ (복소수)

$= \log_e |z| + i(\theta + 2k\pi)$

θ는 $\mathrm{Arg}(z)$

$\ln(x)$

$x > 0$



$\ln(-2) = \ln(2 \cdot e^{i\pi})$

$\ln(-2) = \log_e 2 + i(\pi + 2k\pi)$