Fourier Integral (H.1)

20151231

.

Fourier Series

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \sin(nx)\right)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)$$

$$= \frac{1}{2}\underbrace{\left[\boxed{\frac{1}{p}}\int_{-p}^{+p} f(x)\,dx\right]}_{=a_0}$$

$$+ \sum_{n=1}^{\infty}\left\{\underbrace{\left[\boxed{\frac{1}{p}}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx\right]}_{=a_n}\cos\left(\frac{n\pi}{p}x\right) + \underbrace{\left[\boxed{\frac{1}{p}}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx\right]}_{=b_n}\sin\left(\frac{n\pi}{p}x\right)\right\}$$

$$\alpha_n = \frac{n\pi}{p} \qquad \Delta\alpha = \alpha_{n+1} - \alpha_n = \frac{(n+1)\pi}{p} - \frac{n\pi}{p} = \frac{\pi}{p}$$

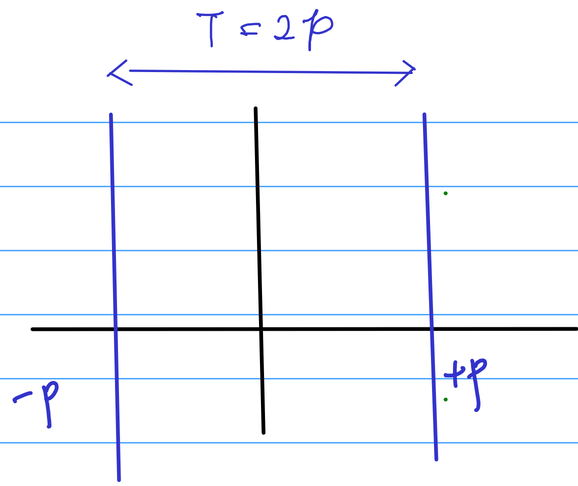
$$\boxed{\frac{1}{p} = \frac{\Delta\alpha}{\pi}}$$

$$\boxed{\alpha_n = \frac{n\pi}{p} \qquad \Delta\alpha = \alpha_{n+1} - \alpha_n = \frac{(n+1)\pi}{p} - \frac{n\pi}{p} = \frac{\pi}{p} \qquad \boxed{\frac{1}{p} = \frac{\Delta\alpha}{\pi}}}$$

$$f(x) = \frac{1}{2}\left[\frac{1}{\pi}\int_{-p}^{+p} f(x)\,dx\right]\Delta\alpha$$

$$+\sum_{n=1}^{\infty}\left\{\left[\frac{1}{\pi}\int_{-p}^{+p} f(x)\cos(\alpha_n x)\,dx\right]\cos(\alpha_n x) + \left[\frac{1}{\pi}\int_{-p}^{+p} f(x)\sin(\alpha_n x)\,dx\right]\sin(\alpha_n x)\right\}\Delta\alpha$$

$$\omega_n = 2\pi f n = \frac{2\pi}{T} n$$



$\boxed{P \to \infty}$ aperiodic signal

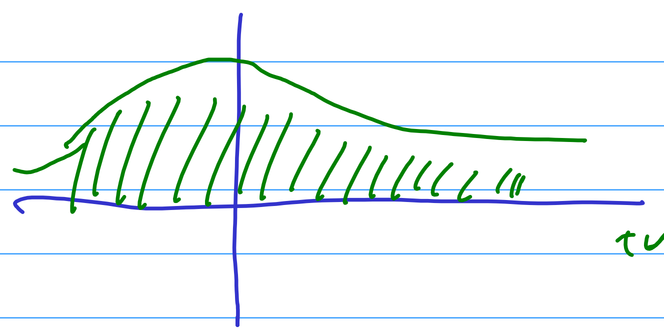
$\frac{1}{p} \to 0$ $\quad \boxed{\frac{1}{p} = \frac{\Delta\alpha}{\pi}}$ $\quad \Delta\alpha \to 0$ $\quad \Delta\alpha = \alpha_{n+1} - \alpha_n$

$$= \frac{(n+1)}{p}\pi - \frac{n\pi}{p} = \frac{\pi}{p}$$

$$\lim_{\Delta\alpha \to 0} \sum_{n=1}^{\infty} \{ \quad \} \Delta\alpha \longrightarrow \int_0^{\infty} \{ \quad \} d\alpha$$

$$\lim_{p\to\infty} \frac{1}{2}\left[\frac{1}{p}\int_{-p}^{+p} f(x)\,dx\right] < \infty \qquad \boxed{\int_{-\infty}^{+\infty} f(x)\,dx < \infty}$$

$$\lim_{p\to\infty} \frac{\int_{-p}^{+p} f(x)\,dx}{p} =$$



$$\lim_{p\to\infty} \frac{M^{<\infty}}{p} = 0$$

$$\boxed{\lim_{p \to \infty}}$$

$$f(x) = \frac{1}{2}\left[\frac{1}{\pi}\int_{-p}^{+p} f(x)\,dx\right]\Delta\alpha \rightarrow 0$$

$$+\sum_{n=1}^{\infty}\left\{\left[\frac{1}{\pi}\int_{-p}^{+p} f(x)\cos(\alpha_n x)\,dx\right]\cos(\alpha_n x) + \left[\frac{1}{\pi}\int_{-p}^{+p} f(x)\sin(\alpha_n x)\,dx\right]\sin(\alpha_n x)\right\}\Delta\alpha$$

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left\{\int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt \;\cos(\alpha x) + \int_{-\infty}^{\infty} f(t)\sin(\alpha t)\,dt \;\sin(\alpha x)\right\}d\alpha$$

$\Uparrow$

Fourier Integral.

$$\int_0^{\infty} \quad d\alpha \qquad\qquad \int_{-\infty}^{+\infty} \quad dt$$

positive freq only

both pos & neg time.

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left\{\begin{array}{l}\displaystyle\int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt \quad \cos(\alpha x) + \\ \displaystyle\int_{-\infty}^{\infty} f(t)\sin(\alpha t)\,dt \quad \sin(\alpha x)\end{array}\right\} d\alpha$$

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left[A(\alpha)\cos(\alpha x) + B(\alpha)\sin(\alpha x)\right] d\alpha$$

α 에 대해서 적분 → $\underline{\alpha : \text{freq.}}$

$$A(\alpha) = \int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt$$

주기 $-p \sim +p$ $\quad \int_{+p}^{-p} dt$

주기 $p \to \infty$ $\quad \Rightarrow \int_{-\infty}^{\infty} dt$

$$B(\alpha) = \int_{-\infty}^{+\infty} f(t)\sin(\alpha t)\,dt$$

↑

주파수

α.에 대하여

when $f(x)$: even

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \begin{array}{l} \int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt \quad \cos(\alpha x) + \\ \int_{-\infty}^{\infty} \underbrace{\underbrace{f(t)}_{\text{even}}\ \underbrace{\sin(\alpha t)}_{\text{odd}}}_{\text{odd}}\,dt \quad \sin(\alpha x) \end{array} \right\} d\alpha$$

odd $\longrightarrow$ should go to 0

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt \quad \cos(\alpha x) \right\} d\alpha$$

$$\boxed{\begin{array}{c} f(x) = \frac{2}{\pi} \int_0^{\infty} \left\{ \int_0^{+\infty} f(t)\cos(\alpha t)\,dt \quad \cos(\alpha x) \right\} d\alpha \\[2em] A(\alpha) = \int_0^{+\infty} f(t)\cos(\alpha t)\,dt \end{array}}$$

Fourier Cosine Integral

When $f(x)$: odd

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left\{\overbrace{\int_{-\infty}^{+\infty}\underbrace{f(t)}_{\text{odd}}\,\underbrace{\cos(\alpha t)}_{\text{even}}\,dt}^{\text{odd} \;\to\; \text{should go to } 0}\;\cos(\alpha x) + \int_{-\infty}^{\infty} f(t)\sin(\alpha t)\,dt\;\sin(\alpha x)\right\}d\alpha$$

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left\{\int_{-\infty}^{+\infty} f(t)\sin(\alpha t)\,dt\;\sin(\alpha x)\right\}d\alpha$$

$$\boxed{\begin{aligned} f(x) &= \frac{2}{\pi}\int_0^{\infty}\left\{\int_{0}^{+\infty} f(t)\sin(\alpha t)\,dt\;\sin(\alpha x)\right\}d\alpha \\ B(\alpha) &= \int_0^{+\infty} f(t)\sin(\alpha t)\,dt \end{aligned}}$$

Fourier Sine Integral

Fourier Integral

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt \quad \cos(\alpha x) + \int_{-\infty}^{\infty} f(t)\sin(\alpha t)\,dt \quad \sin(\alpha x) \right\} d\alpha$$

$$A(\alpha) = \int_{-\infty}^{+\infty} f(t)\cos(\alpha t)\,dt$$

$$B(\alpha) = \int_{-\infty}^{+\infty} f(t)\sin(\alpha t)\,dt$$

Fourier Cosine Integral

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \left\{ \int_0^{+\infty} f(t)\cos(\alpha t)\,dt \quad \cos(\alpha x) \right\} d\alpha$$

$$A(\alpha) = \int_0^{+\infty} f(t)\cos(\alpha t)\,dt$$

Fourier Sine Integral

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \left\{ \int_0^{+\infty} f(t)\sin(\alpha t)\,dt \quad \sin(\alpha x) \right\} d\alpha$$

$$B(\alpha) = \int_0^{+\infty} f(t)\sin(\alpha t)\,dt$$

Fourier Transform

$$F(\alpha) = \int_{-\infty}^{+\infty} f(x)\, e^{i\alpha x}\, dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\alpha)\, e^{-i\alpha x}\, d\alpha$$

$$F(\alpha) = \int_{-\infty}^{+\infty} f(x)\, e^{i\alpha x}\, dx$$

$$= \int_{-\infty}^{+\infty} f(x) \left[\cos(\alpha x) + i \sin(\alpha x)\right] dx$$

$$= \int_{\infty}^{+\infty} f(x) \cos(\alpha x) + i f(x) \sin(\alpha x) \quad dx$$

$$= \int_{\infty}^{+\infty} f(x) \cos(\alpha x)\, dx + i \int_{\infty}^{+\infty} f(x) \sin(\alpha x)\, dx$$

negative freq $\alpha < 0$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\alpha)\, e^{-i\alpha x}\, d\alpha$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \begin{array}{l} \int_{-\infty}^{+\infty} f(t) \cos(\alpha t)\, dt \;+ \\ i \int_{-\infty}^{+\infty} f(t) \sin(\alpha t)\, dt \end{array} \right\} \left\{ \begin{array}{l} \cos(\alpha x) + \\ -i \sin(\alpha x) \end{array} \right\} d\alpha$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \begin{array}{l} \left\{ \begin{array}{l} \int_{-\infty}^{+\infty} f(t) \cos(\alpha t)\, dt \;\; \cos(\alpha x) \\ \int_{-\infty}^{+\infty} f(t) \sin(\alpha t)\, dt \;\; \sin(\alpha x) \end{array} \right\} \\ \left\{ \begin{array}{l} -i \int_{-\infty}^{+\infty} f(t) \cos(\alpha t)\, dt \;\; \sin(\alpha x) \\ i \int_{-\infty}^{+\infty} f(t) \sin(\alpha t)\, dt \;\; \cos(\alpha x) \end{array} \right\} \end{array} \right\} d\alpha$$

15.3 Fourier Integral

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\left[A(\alpha)\,\underline{\cos(\alpha x)} + B(\alpha)\,\underline{\sin(\alpha x)}\right] d\alpha$$

$$A(\alpha) = \int_{-\infty}^{+\infty} f(x)\cos(\alpha x)\,dx$$

$$B(\alpha) = \int_{-\infty}^{+\infty} f(x)\sin(\alpha x)\cdot dx$$

Fourier Transform

Laplace Transform

$\mathcal{L}^{-1}\{F(s)\}$

$s = \sigma + i\omega$

$$\boxed{F(s)} = \int_0^{\infty} \boxed{f(x)}\, e^{-s\cdot x}\, dx$$

$\mathcal{L}\{f(x)\}$

$\mathcal{L}^{-1}\{F(\alpha)\}$

$$\boxed{F(\alpha)} = \int_{-\infty}^{+\infty} \boxed{f(x)}\, e^{i\alpha x}\, dx$$

$\mathcal{F}\{f(x)\}$

Fourier Sine, Cosine Transform

$$f(x) = \frac{2}{\pi}\int_0^{\infty} F_c(\alpha)\cos \alpha x \, d\alpha$$

$$f(x) = \frac{2}{\pi}\int_0^{\infty} F_s(\alpha)\sin \alpha x \, d\alpha$$

$$F_c(\alpha) = \int_0^{\infty} \boxed{f(x)} \cos \alpha x \, dx$$ Fourier Cosine Tranfrom

$$F_s(\alpha) = \int_0^{\infty} \boxed{f(x)} \sin \alpha x \, dx$$ Fourier Sine Transform

$$f(x) = \frac{2}{\pi}\int_0^{\infty} A(\alpha)\cos(\alpha x)\, d\alpha$$ Fourier Cosine Integral

$$f(x) = \frac{2}{\pi}\int_0^{\infty} B(\alpha)\sin(\alpha x)\, d\alpha$$ Fourier Sine Integral

$$A(\alpha) = \int_0^{+\infty} f(t)\cos(\alpha t)\, dt$$

$$B(\alpha) = \int_0^{+\infty} f(t)\sin(\alpha t)\, dt$$

Fourier Transform

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\alpha)\, e^{-i\alpha x}\, d\alpha$$

$$F(\alpha) = \int_{-\infty}^{+\infty} f(x)\, e^{i\alpha x}\, dx$$

Fourier Integral

$$f(x) = \frac{1}{\pi} \int_{0}^{\infty} \left[A(\alpha) \cos(\alpha x) + B(\alpha) \sin(\alpha x) \right] d\alpha$$

$\alpha \in [0, \infty]$

$$A(\alpha) = \int_{-\infty}^{+\infty} f(t) \cos(\alpha t)\, dt$$

$t \in [-\infty, \infty]$

$$B(\alpha) = \int_{-\infty}^{+\infty} f(t) \sin(\alpha t)\, dt$$