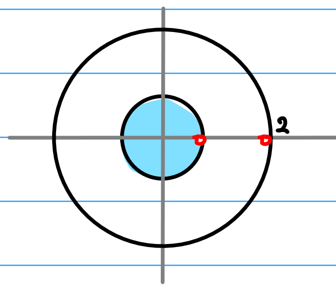
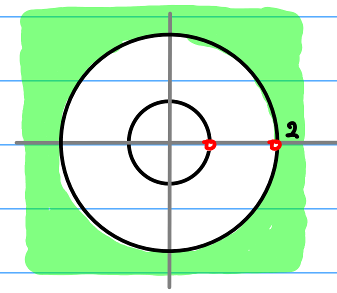
# Laurent Series and z-Transform Examples case 2.B

20171227

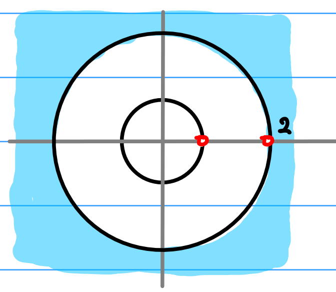
2. B

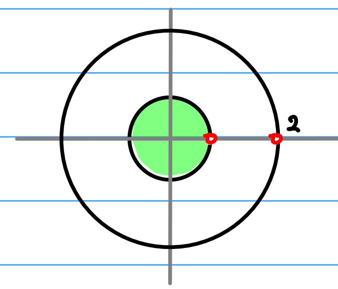
$$f(z) = \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$$



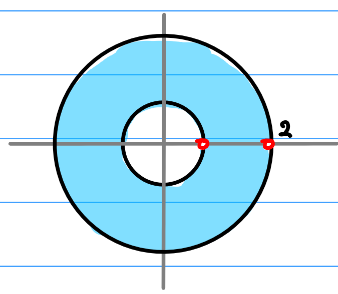



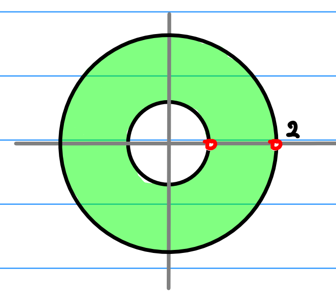

$$\sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}\right]z^{-n} = \sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}\right]z^{n}$$




$$\sum_{n=0}^{-\infty}\left[2^{n-1}-\left(\tfrac{1}{2}\right)^{n-1}\right]z^{-n} = \sum_{n=0}^{-\infty}\left[2^{n-1}-\left(\tfrac{1}{2}\right)^{n-1}\right]z^{n}$$




$$\sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^{-n}+\sum_{n=0}^{-\infty}2^{n-1}z^{-n} = \sum_{n=0}^{-\infty}2^{n-1}z^{n}+\sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^{n}$$

**2. B** $f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

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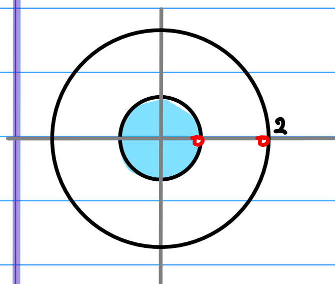
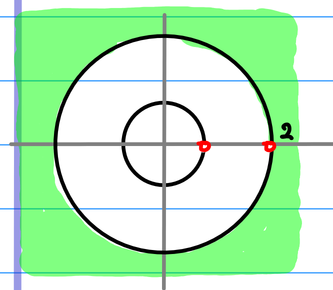



$$a_n = \begin{cases} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] & (n>0) \\ 0 & (n\leq 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^{-n}$$




$$x_n = \begin{cases} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] & (n>0) \\ 0 & (n\leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^{n}$$

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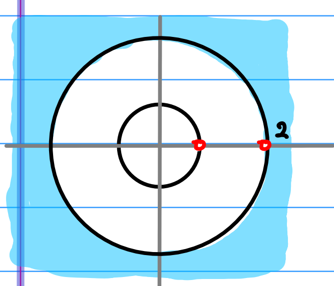
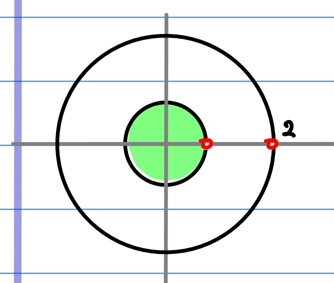



$$a_n = \begin{cases} 0 & (n>0) \\ \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] & (n\leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] z^{-n}$$




$$x_n = \begin{cases} 0 & (n>0) \\ \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] & (n\leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] z^{n}$$

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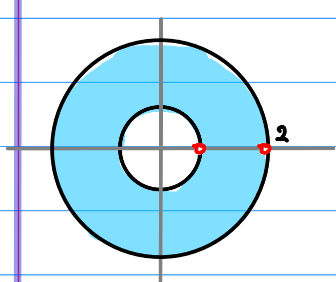
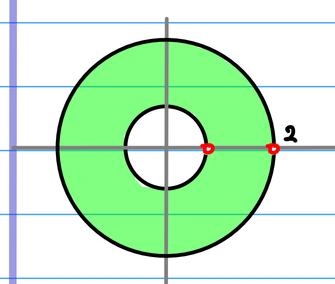



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n-1} & (n>0) \\ 2^{n-1} & (n\leq 0) \end{cases}$$

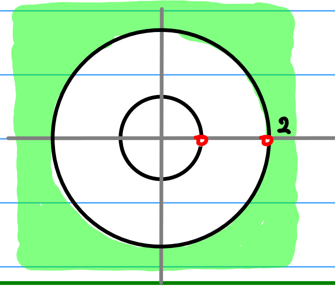
$$f(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^{-n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n}$$




$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n-1} & (n>0) \\ 2^{n-1} & (n\leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^{n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{n}$$

$$X(z) = \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$




$0.5 < |z|$

$2 < |z|$

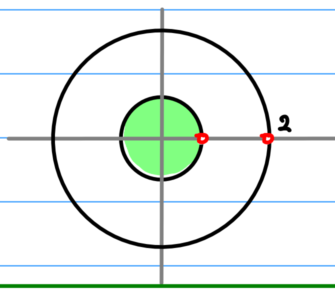
$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n-1} - \sum_{n=0}^{\infty} 2^n z^{-n-1}$$

$$= \sum_{n=1}^{\infty}\left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^{-n}$$

$$\boxed{\sum_{n=1}^{\infty}\left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^{-n}}$$




$|z| < 0.5$

$|z| < 2$

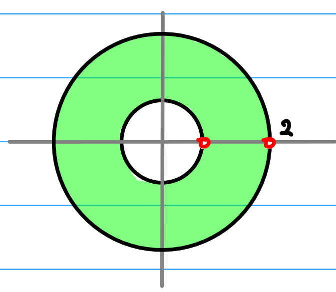
$$-\frac{(2)}{1-(2z)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty}(2)(2z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

$$= \sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^n$$

$$\boxed{\sum_{n=0}^{-\infty}\left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] z^{-n}}$$




$0.5 < |z|$

$|z| < 2$

$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n-1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

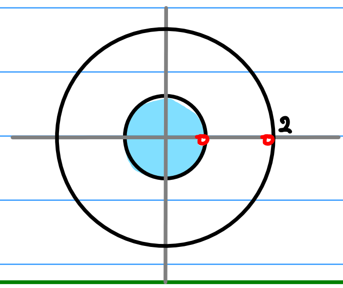
$$= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

$$\boxed{\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{-n}}$$

$$f(z) = \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$

$|z| < 2$




$$-\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (z)(2z)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

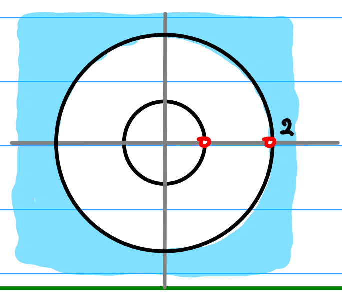
$$= -\sum_{n=0}^{\infty} 2^n z^{n+1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^n$$

$$\boxed{\sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}\right] z^n}$$

$0.5 < |z|$

$2 < |z|$




$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty} (2)\left(\frac{2}{z}\right)^n$$

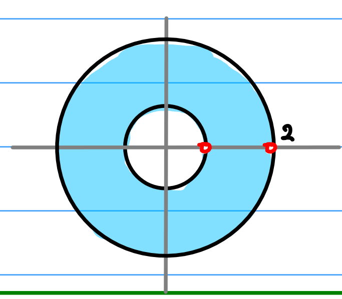
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} - \sum_{n=0}^{\infty} 2^{n+1} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^{-n}$$

$$\boxed{\sum_{n=0}^{-\infty} \left[2^{n-1} - \left(\frac{1}{2}\right)^{n-1}\right] z^n}$$

$0.5 < |z|$

$|z| < 2$




$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

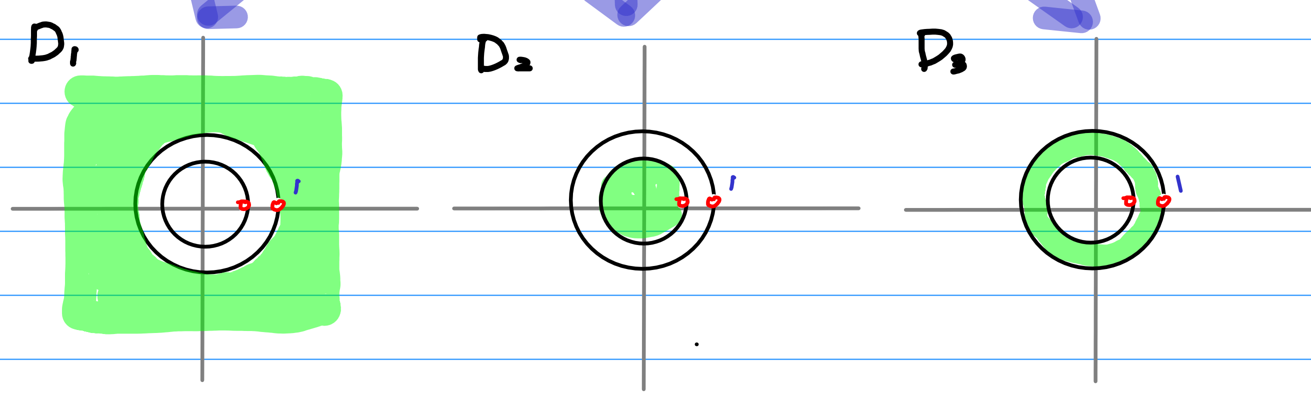
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

$$\boxed{\sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n}$$

Z.T. first

$$X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$




| | | |
|---|---|---|
| $x_n$? | $x_n$? | $x_n$? |
| $\Vert$ | $\Vert$ | $\Vert$ |
| $a_n$ | $a_n$ | $a_n$ |
| $f(z)$? | $f(z)$? | $f(z)$? |