

# Laurent Series and z-Transform Examples case 2.B

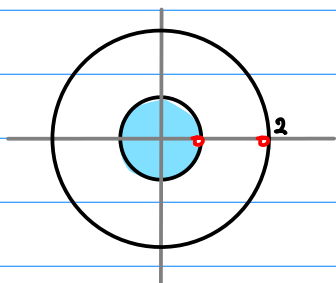
20171227

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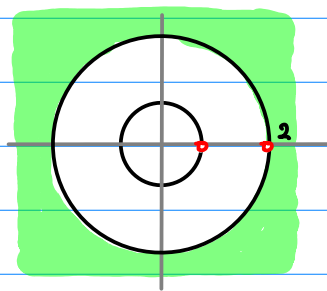
2. B

$$f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} \xleftrightarrow{z^{-1}} X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

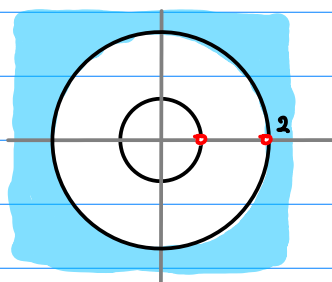


$$\sum_{n=-\infty}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$

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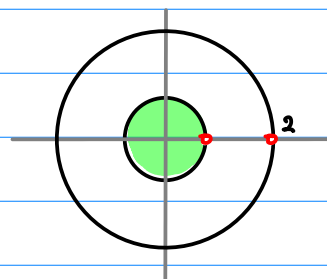


$$\sum_{n=-\infty}^{\infty} \left[ \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$

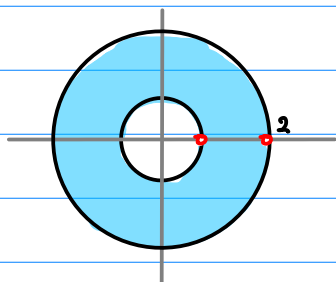


$$\sum_{n=0}^{\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$

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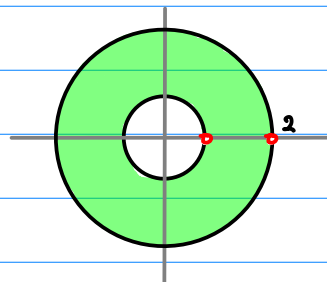


$$\sum_{n=0}^{\infty} \left[ 2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^n$$



$$\sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} 2^{n+1} z^{-n}$$

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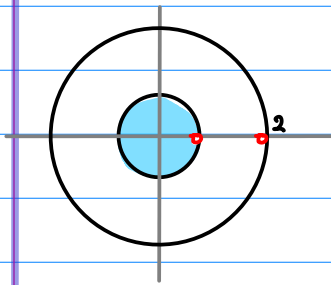


$$\sum_{n=0}^{\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

2. B

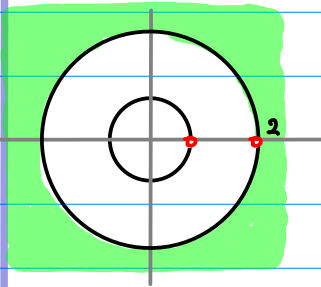
$$f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} \longleftrightarrow z^{-1} \longleftrightarrow X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$$

I



$$a_n = \begin{cases} [(\frac{1}{2})^{n-1} - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

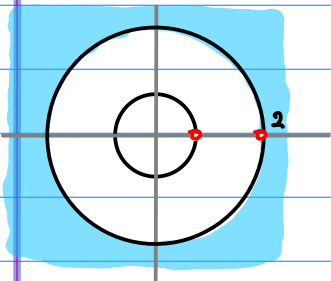
$$f(z) = \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^{-n}$$



$$x_n = \begin{cases} [(\frac{1}{2})^{n-1} - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

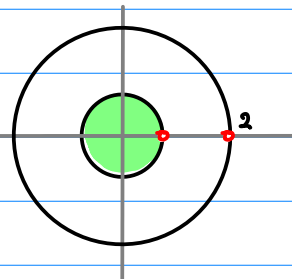
$$X(z) = \sum_{n=1}^{\infty} [(\frac{1}{2})^{n-1} - 2^{n-1}] z^n$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ [2^{n-1} - (\frac{1}{2})^{n-1}] & (n \leq 0) \end{cases}$$

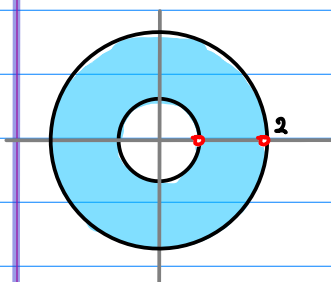
$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^{-n}$$



$$x_n = \begin{cases} 0 & (n > 0) \\ [2^{n-1} - (\frac{1}{2})^{n-1}] & (n \leq 0) \end{cases}$$

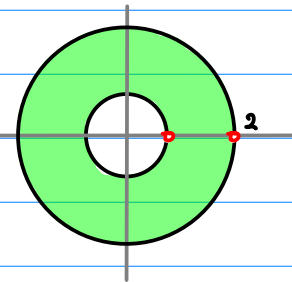
$$X(z) = \sum_{n=0}^{-\infty} [2^{n-1} - (\frac{1}{2})^{n-1}] z^n$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^{-n} + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n}$$

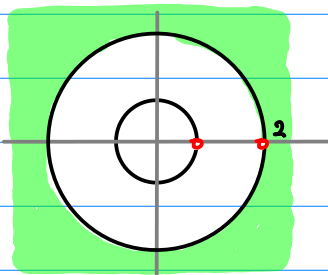


$$x_n = \begin{cases} (\frac{1}{2})^{n-1} & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n$$

$$X(z) = \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left( \frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$0.5 < |z|$   
 $2 < |z|$



$$\sum_{n=-\infty}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$

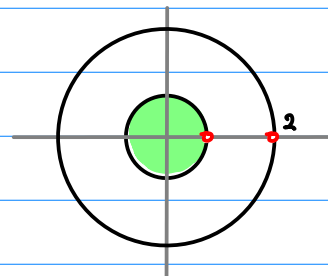
$$+ \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2z}\right)} - \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{2}{z}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n-1} - \sum_{n=0}^{\infty} 2^n z^{-n-1}$$

$$= \sum_{n=-1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$

$|z| < 0.5$   
 $|z| < 2$



$$\sum_{n=0}^{\infty} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$

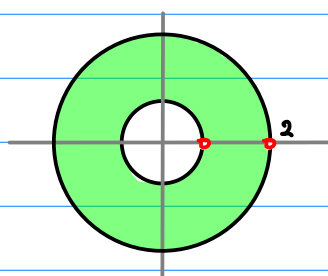
$$- \frac{(2)}{1 - (2z)} + \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{2}{z}\right)}$$

$$= - \sum_{n=0}^{\infty} (2)(2z)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{z}{2}\right)^n$$

$$= - \sum_{n=0}^{\infty} 2^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$= \sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^n$$

$0.5 < |z|$   
 $|z| < 2$



$$\sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} 2^{n+1} z^{-n}$$

$$+ \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2z}\right)} + \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{2}{z}\right)}$$

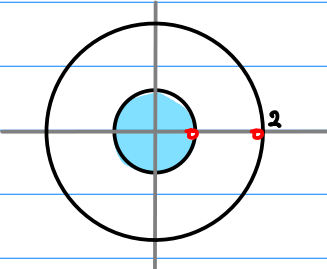
$$= + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n-1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$= \sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$f(z) = \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left( \frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right)$$

$|z| < 0.5$   
 $|z| < 2$



$$\sum_{n=1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$

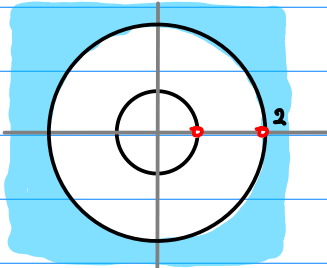
$$-\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (z)(2z)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} 2^n z^{n+1} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=1}^{\infty} \left[ \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \right] z^n$$

$0.5 < |z|$   
 $2 < |z|$



$$\sum_{n=0}^{-\infty} \left[ 2^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right] z^n$$

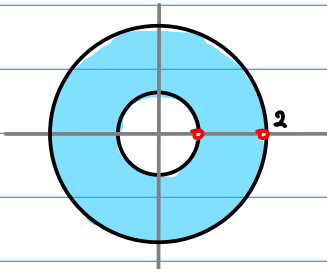
$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} - \sum_{n=0}^{\infty} 2^{n+1} z^{n+1}$$

$$= \sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \right] z^{-n}$$

$0.5 < |z|$   
 $|z| < 2$



$$\sum_{n=0}^{-\infty} 2^{n-1} z^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty} (z)\left(\frac{z}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{n+1}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^n$$

$$X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

