Complex Trig & TrigH (H.1)

20160906

Trigonometric Functions

real x

$$\boxed{\begin{aligned} e^{ix} &= \cos x + i \sin x \\ e^{-ix} &= \cos x - i \sin x \\ \sin x &= \frac{e^{ix} - e^{-ix}}{2i} \\ \cos x &= \frac{e^{ix} + e^{-ix}}{2} \end{aligned}}$$

complex $z = x + iy$ real x, real y

$$\boxed{\begin{aligned} \sin z &= \frac{e^{iz} - e^{-iz}}{2i} \\ \cos z &= \frac{e^{iz} + e^{-iz}}{2} \\ \tan z &= \frac{\sin z}{\cos z} \\ \cot z &= \frac{\cos z}{\sin z} \\ \sec z &= \frac{1}{\cos z} \\ \csc z &= \frac{1}{\sin z} \end{aligned}}$$

Analyticity

$e^{iz}, \ e^{-iz}$ entire function

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$ entire function

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$ entire function

$\sin z = 0$ only for real numbers $z = n\pi$

$\cos z = 0$ only for real numbers $z = (2n+1)\pi/2$

$\tan z = \dfrac{\sin z}{\cos z}$ $\quad \sec z = \dfrac{1}{\cos z}$ analytic except $z = (2n+1)\pi/2$

$\cot z = \dfrac{\cos z}{\sin z}$ $\quad \csc z = \dfrac{1}{\sin z}$ analytic except $z = n\pi$

Derivatives of Trigonometric Functions

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\frac{d}{dz}\sin z = \frac{ie^{iz} + ie^{-iz}}{2i} = \cos z$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\frac{d}{dz}\cos z = \frac{ie^{iz} - ie^{-iz}}{2} = -\sin z$$

$$\tan z = \frac{\sin z}{\cos z}$$

$$\frac{d}{dz}\tan z = \frac{\cos^2 z + \sin^2 z}{\cos^2 z} = \sec^2 z$$

$$\cot z = \frac{\cos z}{\sin z}$$

$$\frac{d}{dz}\cot z = \frac{-\sin^2 z - \cos^2 z}{\sin^2 z} = -\csc^2 z$$

$$\sec z = \frac{1}{\cos z}$$

$$\frac{d}{dz}\sec z = \frac{\sin z}{\cos^2 z} = \sec z \tan z$$

$$\csc z = \frac{1}{\sin z}$$

$$\frac{d}{dz}\csc z = \frac{-\cos z}{\sin^2 z} = -\csc z \cot z$$

Some Trigonometric Identities (1)

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\sin(-z) = \frac{e^{-iz} - e^{+iz}}{2i} = -\sin z$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\cos(-z) = \frac{e^{-iz} + e^{+iz}}{2} = \cos z$$

$$\tan z = \frac{\sin z}{\cos z}$$

$$\tan(-z) = \frac{-\sin z}{\cos z} = -\tan z$$

$$\sin^2 z = \frac{e^{i2z} + e^{-i2z} - 2}{-4}$$

$$\cos^2 z = \frac{e^{i2z} + e^{-i2z} + 2}{+4}$$

$$\sin^2 z + \cos^2 z = 1$$

Some Trigonometric Identities (2)

$$\cos(z_1+z_2) + i\sin(z_1+z_2) = e^{i(z_1+z_2)}$$

$$e^{iz_1} \cdot e^{iz_2}$$
$$= [\cos(z_1) + i\sin(z_1)][\cos(z_2) + i\sin(z_2)]$$
$$= [\cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)] + i[\cos(z_1)\sin(z_2) + \sin(z_1)\cos(z_2)]$$

$$\cos(z_1 \pm z_2) = [\cos(z_1)\cos(z_2) \mp \sin(z_1)\sin(z_2)]$$
$$\sin(z_1 \pm z_2) = [\sin(z_1)\cos(z_2) \pm \cos(z_1)\sin(z_2)]$$

$$\sin(z+z) = \sin(z)\cos(z) + \cos(z)\sin(z)$$
$$\sin(2z) = 2\sin(z)\cos(z)$$

$$\cos(z+z) = \cos(z)\cos(z) - \sin(z)\sin(z)$$
$$\cos(2z) = \cos^2(z) - \sin^2(z)$$

$\sin(z)$ as an angle sum

$$\sinh y = \frac{e^{y} - e^{-y}}{2}$$

$$\cosh y = \frac{e^{y} + e^{-y}}{2}$$

$$\sin(z) = \sin(x+iy) = \frac{e^{i(x+iy)} - e^{-i(x+iy)}}{2i}$$

$$2 * \left[\boxed{e^{ix} e^{-y}} - \boxed{e^{-ix} e^{y}} \right]$$

$$= \left[\begin{array}{l} \boxed{e^{ix} e^{y}} + \boxed{e^{ix} e^{-y}} \\ - \boxed{e^{-ix} e^{y}} - \boxed{e^{-ix} e^{-y}} \end{array} \right] - \left[\begin{array}{l} \boxed{e^{ix} e^{y}} - \boxed{e^{ix} e^{-y}} \\ \boxed{e^{-ix} e^{y}} - \boxed{e^{-ix} e^{-y}} \end{array} \right]$$

$$= (e^{ix} - e^{-ix})(e^{y} + e^{-y}) - (e^{ix} + e^{-ix})(e^{y} - e^{-y})$$

$$\sin(x+iy) = \frac{(e^{ix} - e^{-ix})}{2i} \frac{(e^{y} + e^{-y})}{2} - \frac{(e^{ix} + e^{-ix})}{2i} \frac{(e^{y} - e^{-y})}{2}$$

$$\boxed{\sin(x+iy) = \sin(x) \cosh(y) + i \cos(x) \sinh(y)}$$

$\cos(z)$ as an angle sum

$$\sinh y = \frac{e^{y} - e^{-y}}{2}$$

$$\cosh y = \frac{e^{y} + e^{-y}}{2}$$

$$\cos(z) = \cos(x+iy) = \frac{e^{i(x+iy)} + e^{-i(x+iy)}}{2}$$

$$2 * \left[\boxed{e^{ix} e^{-y}} + \boxed{e^{-ix} e^{y}} \right]$$

$$= \left[\begin{matrix} \boxed{e^{ix} e^{y}} + \boxed{e^{ix} e^{-y}} \\ \boxed{e^{-ix} e^{y}} + \boxed{e^{-ix} e^{-y}} \end{matrix} \right] - \left[\begin{matrix} \boxed{e^{ix} e^{y}} - \boxed{e^{ix} e^{-y}} \\ - \boxed{e^{-ix} e^{y}} + \boxed{e^{-ix} e^{-y}} \end{matrix} \right]$$

$$= \left(e^{ix} + e^{-ix}\right)\left(e^{y} + e^{-y}\right) - \left(e^{ix} - e^{-ix}\right)\left(e^{y} - e^{-y}\right)$$

$$\cos(x+iy) = \frac{\left(e^{ix} + e^{-ix}\right)}{2} \frac{\left(e^{y} + e^{-y}\right)}{2} - \frac{\left(e^{ix} - e^{-ix}\right)}{2} \frac{\left(e^{y} - e^{-y}\right)}{2}$$

$$\boxed{\cos(x+iy) = \cos(x) \cosh(y) - i \sin(x) \sinh(y)}$$

$|\sin(z)|^2$ & $|\cos(z)|^2$

$$\sinh y = \frac{e^y - e^{-y}}{2} \qquad \sinh^2 y = \frac{1}{4}\left(e^{2y} + e^{-2y} - 2\right)$$

$$\cosh y = \frac{e^y + e^{-y}}{2} \qquad \cosh^2 y = \frac{1}{4}\left(e^{2y} + e^{-2y} + 2\right)$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\boxed{\sin(x+iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y)}$$

$$|\sin(x+iy)|^2 = \underline{\sin^2(x)\cosh^2(y)} + \underline{\cos^2(x)\sinh^2(y)}$$

$$(1-\sin^2(x))(\cosh^2 y - 1)$$

$$\cosh^2 y - \sin^2(x)\cosh^2 y - 1 + \sin^2(x)$$

$$\underline{-\sin^2(x)\cosh^2 y} + \cosh^2 y - 1 + \sin^2(x)$$

$$|\sin z|^2 = \sin^2(x) + \sinh^2(y)$$

$$\boxed{\cos(x+iy) = \cos(x)\cosh(y) - i\sin(x)\sinh(y)}$$

$$|\cos(x+iy)|^2 = \underline{\cos^2(x)\cosh^2(y)} + \sin^2(x)\sinh^2(y)$$

$$(1-\cos^2(x))(\cosh^2 y - 1)$$

$$\cosh^2 y - \cos^2(x)\cosh^2 y - 1 + \cos^2(x)$$

$$\underline{-\cos^2(x)\cosh^2 y} + \cosh^2 y - 1 + \cos^2(x)$$

$$|\cos z|^2 = \cos^2(x) + \sinh^2(y)$$

Angle Sum Identities : real vs complex

complex

$z = x + iy$

$$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$$

$$\cos(x+iy) = \cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}$$

$$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$$

$$|\cos z|^2 = \cos^2(x) + \boxed{\sinh^2(y)}$$

real

x, y

$$\sin(x+y) = \sin(x)\boxed{\cos(y)} + \cos(x)\boxed{\sin(y)}$$

$$\cos(x+y) = \cos(x)\boxed{\cos(y)} - \sin(x)\boxed{\sin(y)}$$

zeros of $\sin(z)$ & $\cos(z)$

a complex number $z=0 \rightleftarrows |z|^2=0$

$$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$$

$$|\cos z|^2 = \cos^2(x) + \boxed{\sinh^2(y)}$$

$$\boxed{\sin z = 0 \quad \rightleftarrows \quad \sin^2(x) + \sinh^2(y) = 0}$$

$\sin(x) = 0 \qquad x = n\pi$

$\sinh(y) = 0 \qquad y = 0$

zero $\quad z = n\pi + i\cdot 0 = n\pi, \quad n = 0, \pm 1, \pm 2, \cdots$

$$\boxed{\cos z \quad \rightleftarrows \quad \cos^2(x) + \sinh^2(y)}$$

$\cos(x) = 0 \qquad x = (2n+1)\frac{\pi}{2}$

$\sinh(y) = 0 \qquad y = 0$

zero $\quad z = (2n+1)\frac{\pi}{2} + i\cdot 0 = (n+\frac{1}{2})\pi \quad n = 0, \pm 1, \pm 2, \cdots$

Hyperbolic Functions

real x

$$\boxed{\begin{aligned} \sinh y &= \frac{e^{x} - e^{-x}}{2} \\ \cosh y &= \frac{e^{x} + e^{-x}}{2} \end{aligned}}$$

for a complex number $z = x + iy$

$$\sinh z = \frac{e^{z} - e^{-z}}{2}$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\tanh z = \frac{\sinh z}{\cosh z}$$

$$\coth z = \frac{\cosh z}{\sinh z}$$

$$\operatorname{sech} z = \frac{1}{\cosh z}$$

$$\operatorname{csch} z = \frac{1}{\sinh z}$$

Derivatives of Hyperbolic Functions

$$\sinh z = \frac{e^{z} - e^{-z}}{2} \qquad \frac{d}{dz}\sinh z = \frac{e^{z} + e^{-z}}{2} = \cosh z$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2} \qquad \frac{d}{dz}\cosh z = \frac{e^{z} - e^{-z}}{2} = \sinh z$$

$$\tanh z = \frac{\sinh z}{\cosh z}$$

$$\frac{d}{dz}\tanh z = \frac{\cosh^2 z - \sinh^2 z}{\cosh^2 z} = \operatorname{sech}^2 z$$

$$\coth z = \frac{\cosh z}{\sinh z}$$

$$\frac{d}{dz}\coth z = \frac{\sinh^2 z - \cosh^2 z}{\sinh^2 z} = -\operatorname{csch}^2 z$$

$$\operatorname{sech} z = \frac{1}{\cosh z}$$

$$\frac{d}{dz}\operatorname{sech} z = \frac{-\sinh z}{\cosh^2 z} = -\tanh z \operatorname{sech} z$$

$$\operatorname{csch} z = \frac{1}{\sinh z}$$

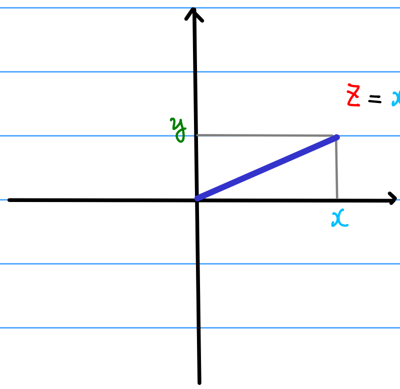
$$\frac{d}{dz}\operatorname{csch} z = \frac{-\cosh z}{\sinh^2 z} = -\coth z \operatorname{csch} z$$

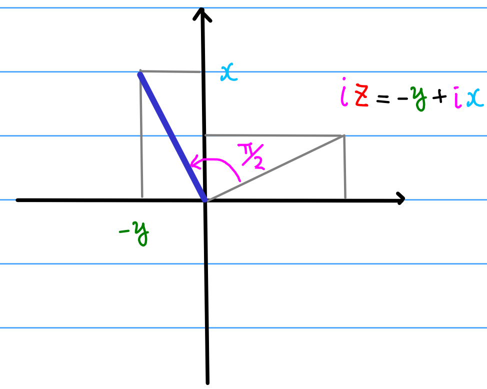
$\boxed{iz}$ function argument

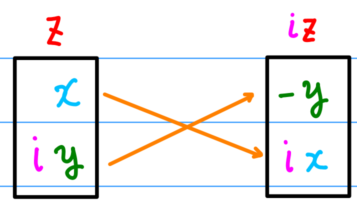
$\boxed{z} = x + iy$

$\boxed{iz} = ix - y = -y + ix$

ccw rotation by $\frac{\pi}{2}$







$$\sin(z) \xleftarrow{+i} \sin(iz)$$
$$\cos(z) \longleftarrow \cos(iz)$$
$$\sinh(z) \xleftarrow{+i} \sinh(iz)$$
$$\cosh(z) \longleftarrow \cosh(iz)$$

$$\sin(z) \xrightarrow{-i} \sin(iz)$$
$$\cos(z) \longrightarrow \cos(iz)$$
$$\sinh(z) \xrightarrow{-i} \sinh(iz)$$
$$\cosh(z) \longrightarrow \cosh(iz)$$

$\sinh(iz)$ & $\cosh(iz)$

$$\sinh(z) = \frac{e^{z} - e^{-z}}{2}$$

$$\cosh(z) = \frac{e^{z} + e^{-z}}{2}$$

$$\sinh(iz) = \frac{e^{iz} - e^{-iz}}{2} = i\sin(z)$$

$$\cosh(iz) = \frac{e^{iz} + e^{-iz}}{2} = \cos(z)$$

$$\boxed{\sinh(iz)} = i\sin(z) \qquad \sin(z) = -i\sinh(iz)$$

$$\boxed{\cosh(iz)} = \cos(z) \qquad \cos(z) = \cosh(iz)$$

$\sinh(iz)$ in terms of $i\sin(z)$

$\cosh(iz)$ in terms of $\cos(z)$

$\sin(iz)$ & $\cos(iz)$

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sin(iz) = \frac{e^{-z} - e^{z}}{2i} = -\frac{1}{i}\sinh(z)$$

$$\cos(iz) = \frac{e^{-z} + e^{z}}{2} = \cosh(z)$$

$$\boxed{\begin{aligned}\sin(iz)\\ \cos(iz)\end{aligned}}\begin{aligned}&= i\sinh(z)\\ &= \cosh(z)\end{aligned} \qquad \begin{aligned}\sinh(z) &= -i\sin(iz)\\ \cosh(z) &= \cos(iz)\end{aligned}$$

$\sin(iz)$ in terms of $i\sinh(z)$

$\cos(iz)$ in terms of $\cosh(z)$

$\sin(z), \cos(z) \longrightarrow \sinh(iz), \cosh(iz)$

$$\boxed{\begin{aligned} \sin z &= \frac{e^{iz} - e^{-iz}}{2i} \\ \cos z &= \frac{e^{iz} + e^{-iz}}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} \sinh z &= \frac{e^{z} - e^{-z}}{2} \\ \cosh z &= \frac{e^{z} + e^{-z}}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} \sinh iz &= \frac{e^{iz} - e^{-iz}}{2} \\ \cosh iz &= \frac{e^{iz} + e^{-iz}}{2} \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} -i \sinh iz &= \frac{e^{iz} - e^{-iz}}{2i} \\ \cosh iz &= \frac{e^{iz} + e^{-iz}}{2} \end{aligned}}$$

$\sinh(z)$, $\cosh(z) \longrightarrow \sin(iz)$, $\cos(iz)$

$$\boxed{\begin{aligned} \sin z &= \frac{e^{iz} - e^{-iz}}{2i} \\ \cos z &= \frac{e^{iz} + e^{-iz}}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} \sinh z &= \frac{e^{z} - e^{-z}}{2} \\ \cosh z &= \frac{e^{z} + e^{-z}}{2} \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \sin iz &= \frac{e^{-z} - e^{+z}}{2i} \\ \cos iz &= \frac{e^{-z} + e^{+z}}{2} \end{aligned}}$$

$$\longrightarrow$$

$$\boxed{\begin{aligned} -i\sin iz &= \frac{e^{+z} - e^{-z}}{2} \\ \cos iz &= \frac{e^{+z} + e^{-z}}{2} \end{aligned}}$$

$\sin(z)$, $\cos(z)$, $\sinh(z)$, $\cosh(z)$
$\sin(iz)$, $\cos(iz)$, $\sinh(iz)$, $\cosh(iz)$

$$\boxed{\begin{aligned} \sin(z) &= -i\sinh(iz) \\ \cos(z) &= \cosh(iz) \end{aligned}}$$

$$\boxed{\begin{aligned} \sinh(z) &= -i\sin(iz) \\ \cosh(z) &= \cos(iz) \end{aligned}}$$

$$\boxed{\begin{aligned} \sin(iz) &= i\sinh(z) \\ \cos(iz) &= \cosh(z) \end{aligned}}$$

$$\boxed{\begin{aligned} \sinh(iz) &= i\sin(z) \\ \cosh(iz) &= \cos(z) \end{aligned}}$$

$|\sinh(z)|^2$ & $|\cosh(z)|^2$

$$\sinh y = \frac{e^y - e^{-y}}{2} \qquad \sinh^2 y = \frac{1}{4}\left(e^{2y} + e^{-2y} - 2\right)$$

$$\cosh y = \frac{e^y + e^{-y}}{2} \qquad \cosh^2 y = \frac{1}{4}\left(e^{2y} + e^{-2y} + 2\right)$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\boxed{\sinh(x+iy) = \sinh(x)\cos(y) + i\cosh(x)\sin(y)}$$

$$|\sinh(x+iy)|^2 = \sinh^2(x)\cos^2(y) + \cosh^2(x)\sin^2(y)$$

$$(1+\sinh^2(x))(1-\cos^2 y)$$

$$1+\sinh^2(x) - \cos^2 y - \sinh^2(x)\cos^2 y$$

$$\sinh^2(x) + \sin^2(y)$$

$$|\sinh z|^2 = \sinh^2(x) + \sin^2(y)$$

$$\boxed{\cos\ (x+iy) = \cosh(x)\cos(y) + i\sinh(x)\sin(y)}$$

$$|\cosh(x+iy)|^2 = \cosh^2(x)\cos^2(y) + \sinh^2(x)\sin^2(y)$$

$$(1+\sinh^2(x))(1-\sin^2(y))$$

$$1+\sinh^2(x) - \sin^2(y) - \sinh^2(x)\sin^2(y)$$

$$\sinh^2(x) + \cos^2(y)$$

$$|\cosh z|^2 = \sinh^2(x) + \cos^2(y)$$

zeros of $\sinh(z)$ & $\cosh(z)$

a complex number $z=0 \rightleftarrows |z|^2=0$

$$|\sinh z|^2 = \sinh^2(x) + \boxed{\sin^2(y)}$$

$$|\cosh z|^2 = \sinh^2(x) + \boxed{\cos^2(y)}$$

$$\boxed{\sinh z = 0 \rightleftarrows \sinh^2(x) + \sin^2(y) = 0}$$

$\sinh(x) = 0 \qquad x = 0$

$\sin(y) = 0 \qquad y = n\pi$

zero $\quad z = 0 + i\cdot n\pi = n\pi i, \quad n = 0, \pm1, \pm2, \cdots$

$$\boxed{\cosh z = 0 \rightleftarrows \sinh^2(x) + \cos^2(y) = 0}$$

$\sinh(x) = 0 \qquad x = 0$

$\cos(y) = 0 \qquad y = (2n+1)\frac{\pi}{2}$

zero $\quad z = 0 + i\cdot(2n+1)\frac{\pi}{2} = (n+\frac{1}{2})\pi i, \quad n = 0, \pm1, \pm2, \cdots$

$$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$$

$$\cos(x+iy) = \cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}$$

$$|\sin z|^2 = \sin^2(x) + \boxed{\sinh^2(y)}$$

$$|\cos z|^2 = \cos^2(x) + \boxed{\sinh^2(y)}$$

zeros of $\sin z$ $\quad n\pi$

zeros of $\cos z$ $\quad (2n+1)\frac{\pi}{2}$

$$\sinh(x+iy) = \sinh(x)\boxed{\cos(y)} + i\cosh(x)\boxed{\sin(y)}$$

$$\cosh(x+iy) = \cosh(x)\boxed{\cos(y)} + i\sinh(x)\boxed{\sin(y)}$$

$$|\sinh z|^2 = \sinh^2(x) + \boxed{\sin^2(y)}$$

$$|\cosh z|^2 = \sinh^2(x) + \boxed{\cos^2(y)}$$

zeros of $\sinh z$ $\quad n\pi i$

zeros of $\cosh z$ $\quad (2n+1)\frac{\pi}{2} i$

real x, y

$$\sin(x+y) = \sin(x)\boxed{\cos(y)} + \cos(x)\boxed{\sin(y)}$$

$$\cos(x+y) = \cos(x)\boxed{\cos(y)} - \sin(x)\boxed{\sin(y)}$$

Complex z

$$\sin(x+iy) = \sin(x)\boxed{\cosh(y)} + i\cos(x)\boxed{\sinh(y)}$$

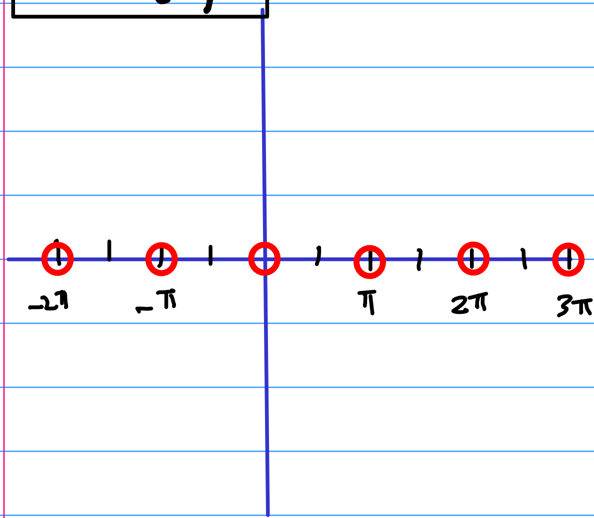
$$\cos(x+iy) = \cos(x)\boxed{\cosh(y)} - i\sin(x)\boxed{\sinh(y)}$$

complex z

$$\sinh(x+iy) = \sinh(x)\boxed{\cos(y)} + i\cosh(x)\boxed{\sin(y)}$$

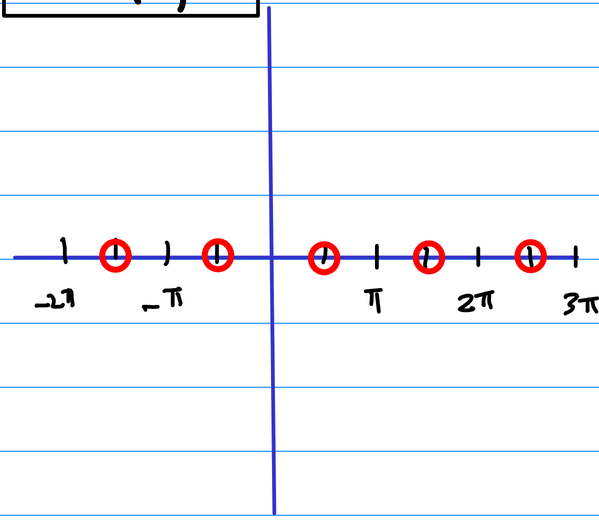
$$\cosh(x+iy) = \cosh(x)\boxed{\cos(y)} + i\sinh(x)\boxed{\sin(y)}$$

$\boxed{\sin(z)}$



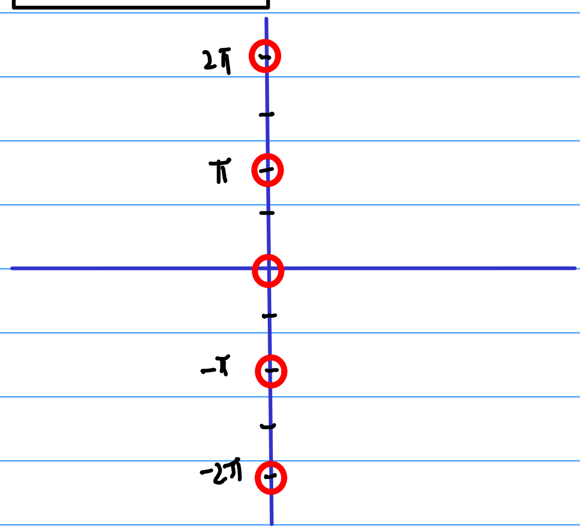
zeros of $\sin z$ $\quad n\pi$

$\boxed{\cos(z)}$



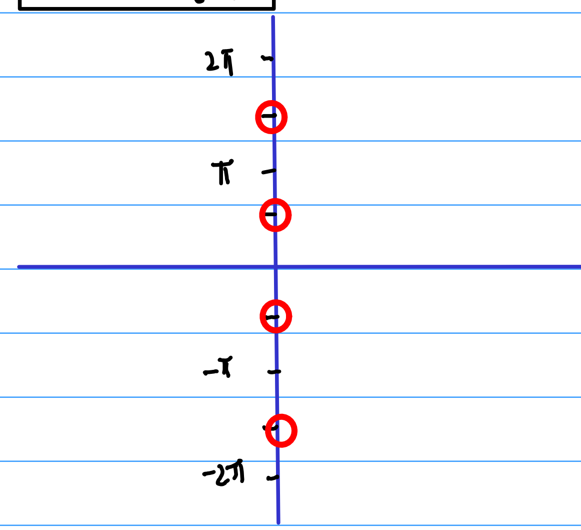
zeros of $\cos z$ $\quad (2n+1)\frac{\pi}{2}$

$\boxed{\sinh(z)}$



zeros of $\sinh z$ $\quad n\pi i$

$\boxed{\cosh(z)}$



zeros of $\cosh z$ $\quad (2n+1)\frac{\pi}{2} i$

Periodicity

$$\sin(z+2\pi) = \sin(x+2\pi+iy)$$
$$= \sin(x+2\pi)\cosh(y) + i\cos(x+2\pi)\sinh(y)$$
$$= \sin(x)\cosh(y) + i\cos(x)\sinh(y)$$
$$= \sin(x+iy) = \sin(z)$$

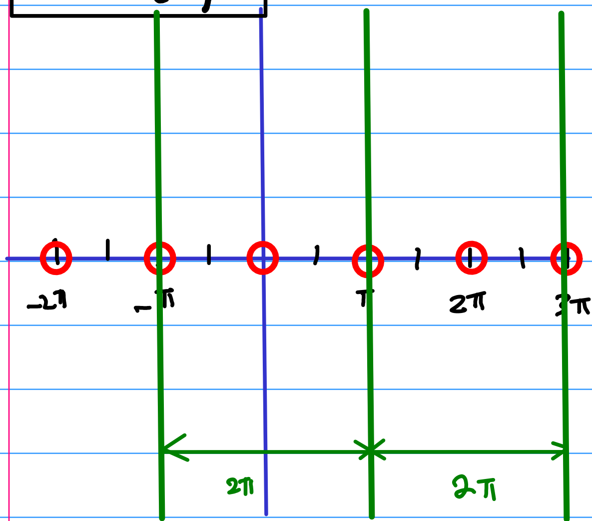
$$\cos(z+2\pi) = \cos(x+2\pi+iy)$$
$$= \cos(x+2\pi)\cosh(y) - i\sin(x+2\pi)\sinh(y)$$
$$= \cos(x)\cosh(y) + i\sin(x)\sinh(y)$$
$$= \cos(x+iy) = \cos(z)$$

$$\sinh(z+2\pi i) = \frac{e^{(z+2\pi i)} - e^{-(z+2\pi i)}}{2}$$
$$= \frac{e^{z} - e^{-z}}{2} = \sinh(z)$$

$$\cosh(z+2\pi i) = \frac{e^{(z+2\pi i)} + e^{-(z+2\pi i)}}{2}$$
$$= \frac{e^{z} + e^{-z}}{2} = \cosh(z)$$

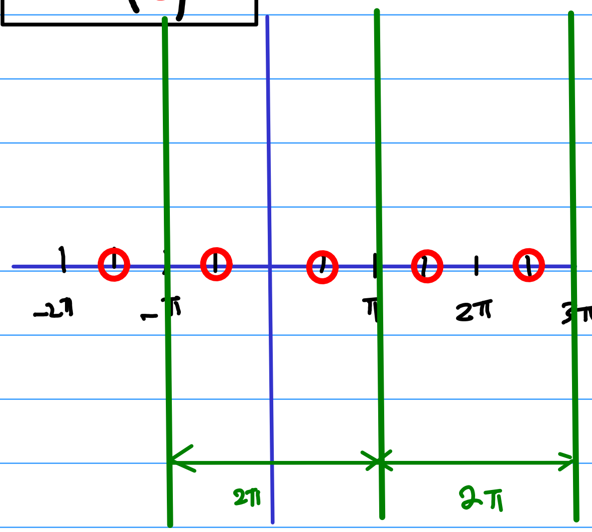
$$\boxed{\begin{aligned} e^{2\pi i} &= \cos(2\pi) + i\sin(2\pi) = 1 \\ e^{-2\pi i} &= \cos(2\pi) - i\sin(2\pi) = 1 \end{aligned}}$$

sin(z)



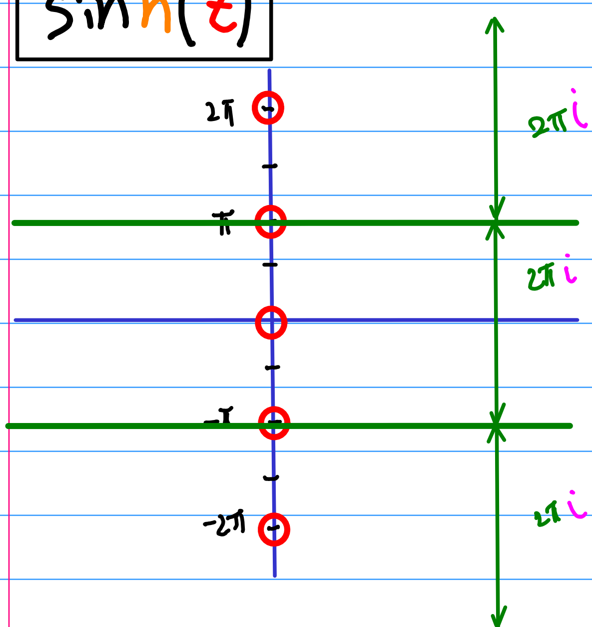
period of 2π

cos(z)



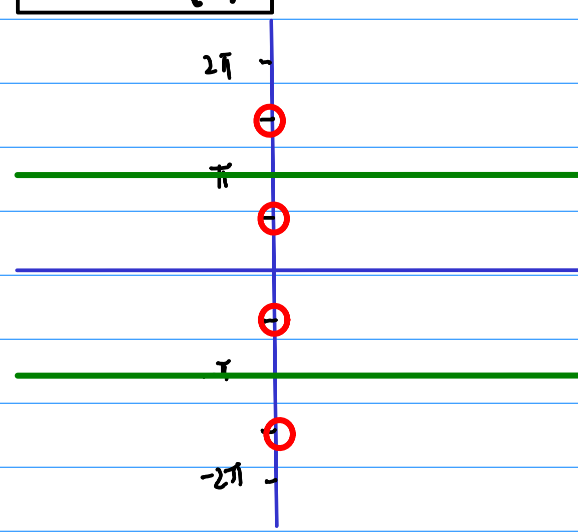
period of 2π

sinh(z)



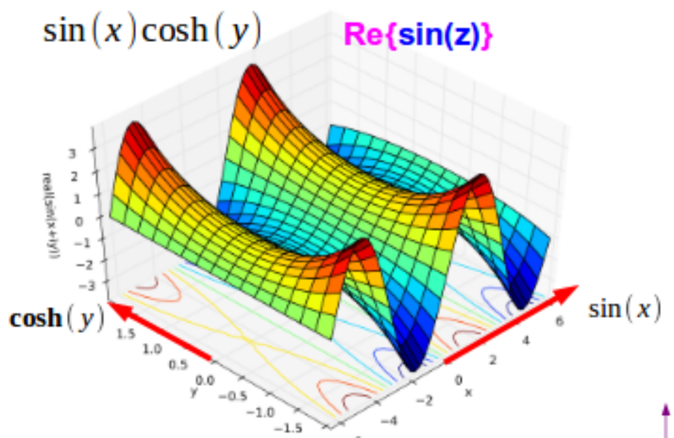
period of 2πi

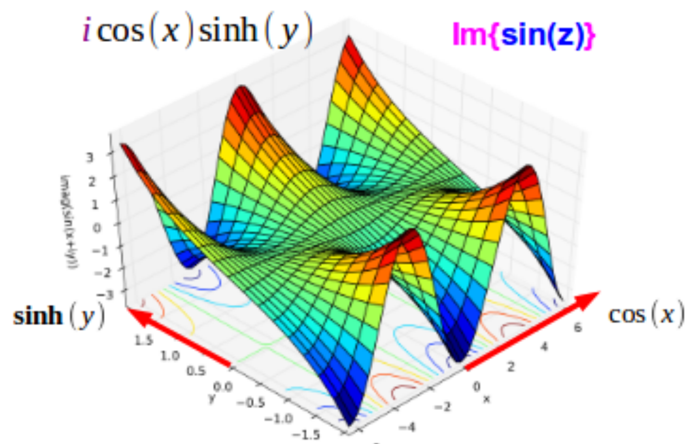
cosh(z)

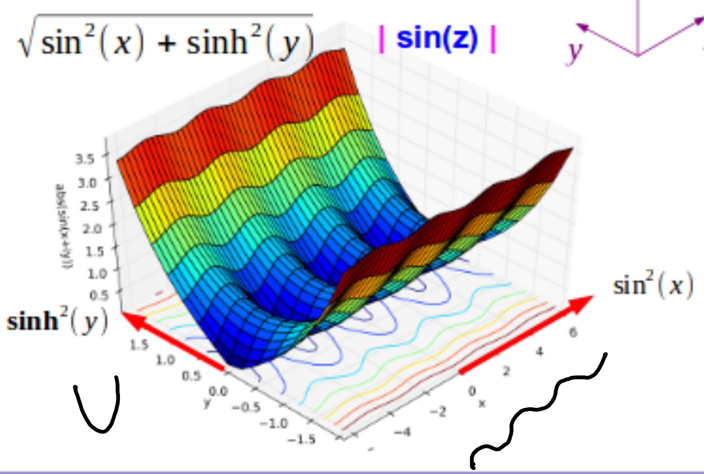


period of 2πi

Graphs of sin(z)







$$\sin(z) = \sin(x + i\,y)$$

$$= \sin(x)\cosh(y) + i\cos(x)\sinh(y)$$

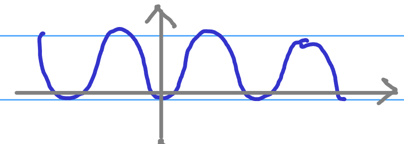
$$|\sin(z)|^2 = \sin^2(x) + \sinh^2(y)$$

http://en.wikipedia.org/

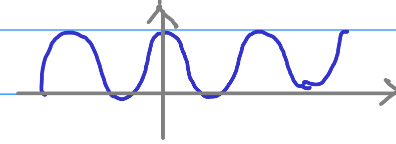
$$\sinh^2(y) = \frac{1}{4}\left(e^{+2y} + e^{-2y} - 2\right)$$

$$\tan\theta = \frac{\cos(x)\,\sinh(y)}{\sin(x)\,\cosh(y)} = \cot x \tanh y$$

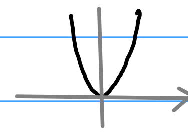
$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$



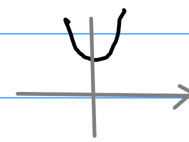
$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$



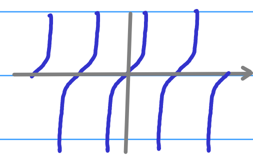
$$\sinh^2(x) = \frac{1}{4}(e^{+2x} + e^{-2x} - 2)$$



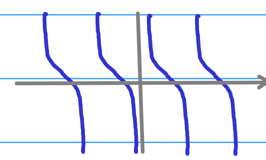
$$\cosh^2(x) = \frac{1}{4}(e^{+2x} + e^{-2x} + 2)$$



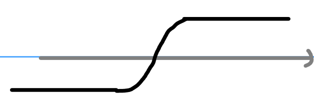
$\tan(x)$



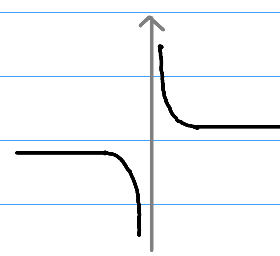
$\cot(x)$



$\tanh(x)$



$\coth(x)$



$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

zero $2x = 0, \pm 2\pi, \pm 4\pi, \cdots$

$x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \cdots$

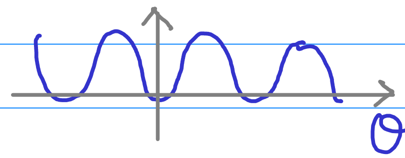
$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$

zero $2x = \pm\pi, \pm 3\pi, \pm 5\pi, \cdots$

$x = \pm\frac{1}{2}\pi, \pm\frac{3}{2}\pi, \pm\frac{5}{2}\pi, \cdots$

* $\sin^2(\arg z)$ plot

$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$



arg $\quad \theta = 0, 2\pi \rightarrow \sin^2\theta = 0$ $\boxed{\text{dominantly real}}$

$\theta = \frac{\pi}{2}, \frac{3}{2}\pi \rightarrow \sin^2\theta = 1$ $\boxed{\text{dominantly imag}}$

http://functions.wolfram.com/ElementaryFunctions/Sin/visualizations/5/

red $\quad \theta = \pm 2n\pi$

cyan $\quad \theta = \pm(2n+1)\pi$

the square of the sine of the argument of sin(z)

plot $\quad \boxed{\sin^2\theta}$, $\quad \tan\theta = \cot(x)\tanh(y)$

$$\theta = \arg\{\sin(z)\} = \tan^{-1}\{\cot(x)\tanh(y)\}$$

Domain Coloring

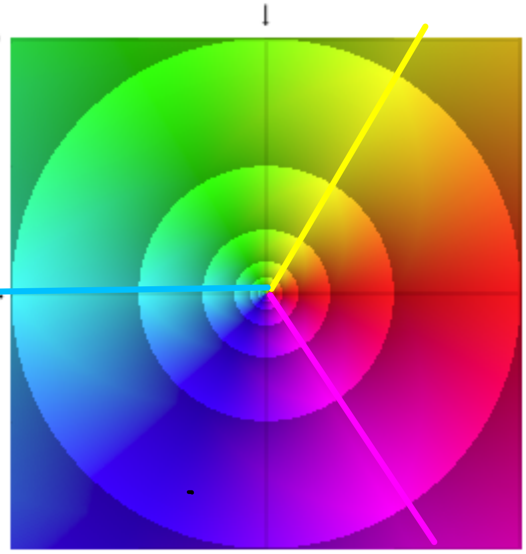
The Unit Circle

hue to phase/angle/argument legend:

hue	phase (radians)
red	$0 \bmod 2\pi$
yellow	$\pi/3 \bmod 2\pi$
green	$2\pi/3 \bmod 2\pi$
cyan	$\pi \bmod 2\pi$
blue	$4\pi/3 \bmod 2\pi$
magenta	$5\pi/3 \bmod 2\pi$

$\theta = \pi/2 \bmod 2\pi$

$\theta = \pi \bmod 2\pi$



$\theta = 0 \bmod 2\pi$

Each discontinuity in intensity occurs when $|z|=2^n$, for integer n $(0,-1,-2,..)$

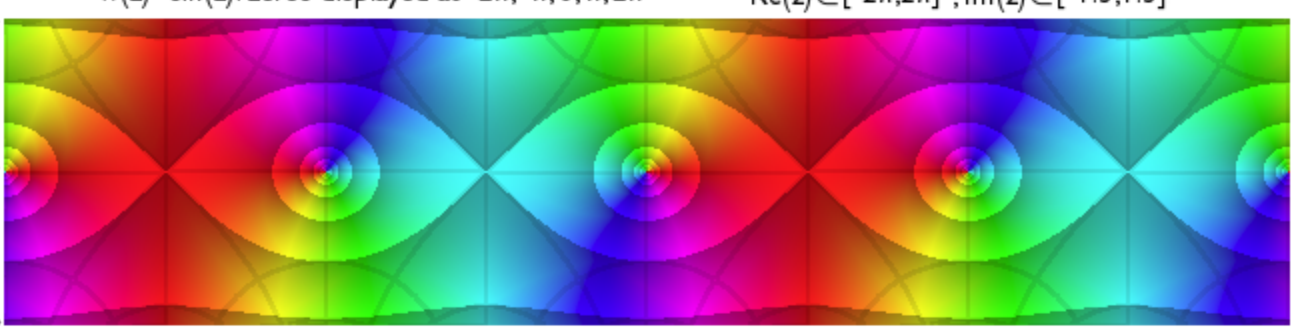
$\theta = 3\pi/2 \bmod 2\pi$

$w(z)=\sin(z)$. zeros displayed at $-2\pi, -\pi, 0, \pi, 2\pi$ $\qquad$ $Re(z)\in[-2\pi,2\pi]$, $Im(z)\in[-1.5,1.5]$

$Im(z)= 1.5$

$Im(z)=0$

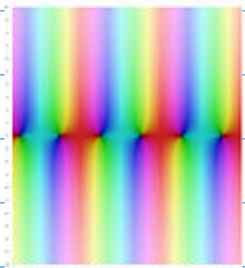
$Im(z)=-1.5$



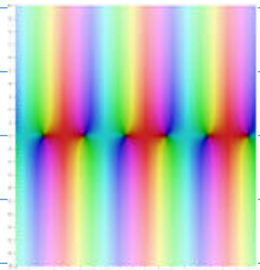
-2π $\quad$ $-3\pi/2$ $\quad$ $-\pi$ $\quad$ $-\pi/2$ $\quad$ 0 $\quad$ $\pi/2$ $\quad$ π $\quad$ $3\pi/2$ $\quad$ 2π

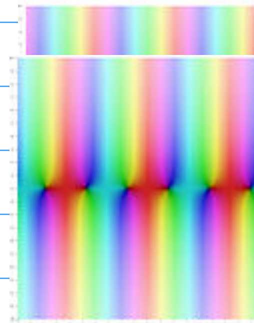
$Re(z)$

sin z



cos z

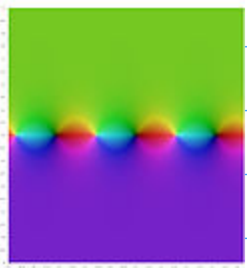




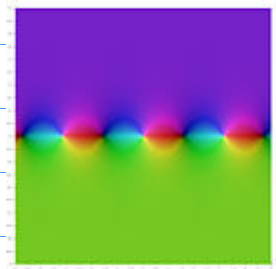
sin z

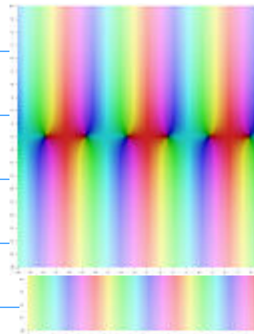
cos z

tan z



cot z



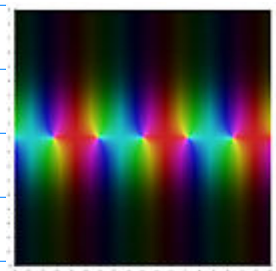


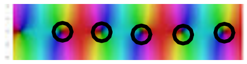
sin z

cos z

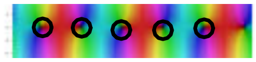
sec z

csc z





sin z



cos z

Complex Analysis in plain view

Contents [hide]

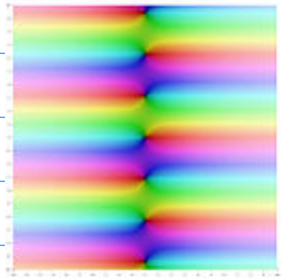
Complex Functions [edit]

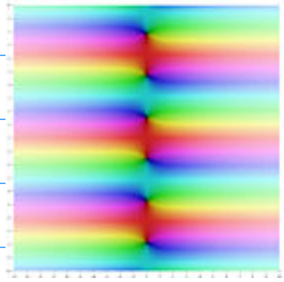
- Complex Functions (1.A.pdf, 1.B.pdf, 1.C.pdf)
- Complex Exponential and Logarithm (5.A.pdf, 5.B.pdf)
- Complex Trigonometric and Hyperbolic (7.A.pdf, 7.B.pdf)

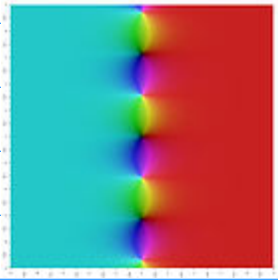
Complex Function Note

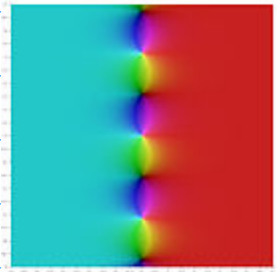
1. Exp and Log Function Note (H1.pdf)
2. Trig and TrigH Function Note (H1.pdf)
3. Inverse Trig and TrigH Functions Note (H1.pdf)

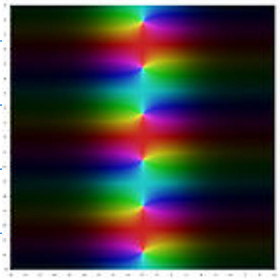
details to be moved here

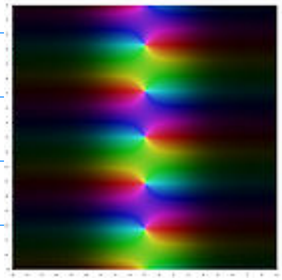












https://en.wikipedia.org/wiki/Hyperbolic_function