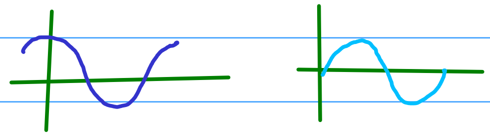
Fourier Series (H.1)

20151230

b

Taylor Series



$f(x) = \cos x$

$f'(x) = -\sin x$ $\quad$ $f'(0) = -\sin 0 = 0$

$f^{(2)}(x) = -\cos x$ $\quad$ $f^{(2)}(0) = -\cos 0 = -1$

$f^{(3)}(x) = +\sin x$ $\quad$ $f^{(3)}(0) = +\sin 0 = 0$

$f^{(4)}(x) = +\cos x$ $\quad$ $f^{(4)}(0) = +\cos 0 = +1$

$f^{(5)}(x) = -\sin x$ $\quad$ $f^{(5)}(0) = -\sin 0 = 0$

$f^{(6)}(x) = -\cos x$ $\quad$ $f^{(6)}(0) = -\cos 0 = -1$

$f^{(7)}(x) = +\sin x$ $\quad$ $f^{(7)}(0) = +\sin 0 = 0$

$f^{(8)}(x) = +\cos x$ $\quad$ $f^{(8)}(0) = +\cos 0 = +1$

$\vdots$ $\quad$ $\vdots$

$x=0$ 근방에서 Taylor series

$$\boxed{f(x) = f(0) + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4}$$

$$\cos x = +1 + \frac{0}{1!}x^1 + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{-1}{4!}x^4$$

$f'(0) = -\sin 0 = 0$

$f^{(2)}(0) = -\cos 0 = -1$

$f^{(3)}(0) = +\sin 0 = 0$

$f^{(4)}(0) = +\cos 0 = +1$

$$\cos x = 1 \underbrace{- \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}}_{\text{Series (급수)}} + \frac{x^{12}}{12!}$$

0 의 근방에서

Fourier Series

$$\boxed{\underset{\uparrow}{f(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n x) + b_n \sin(n x) \right)}$$

a given periodic function

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(n x)\, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(n x)\, dx$$

$$\begin{aligned}
a_0 &+ a_1 \cos(1x) + b_1 \sin(1x) \\
&+ a_2 \cos(2x) + b_2 \sin(2x) \\
&+ a_3 \cos(3x) + b_3 \sin(3x) \\
&\qquad \vdots \qquad\qquad\qquad \vdots \\
&+ a_n \cos(n x) + b_n \sin(n x) \\
&\qquad \vdots \qquad\qquad\qquad \vdots
\end{aligned}$$

$$\left\{ \left\{ \begin{matrix} \cos(1x) \\ \cos(2x) \\ \cos(3x) \\ \vdots \\ \cos(nx) \\ \vdots \end{matrix} \right\} \quad \left\{ \begin{matrix} \sin(1x) \\ \sin(2x) \\ \sin(3x) \\ \vdots \\ \sin(nx) \\ \vdots \end{matrix} \right\} \right\}$$

$\Downarrow$ orthogonal set

$\Downarrow$ orthogonal set

$$\langle \cos(mx), \cos(nx) \rangle = 0 \qquad \langle \sin(mx), \sin(nx) \rangle = 0$$

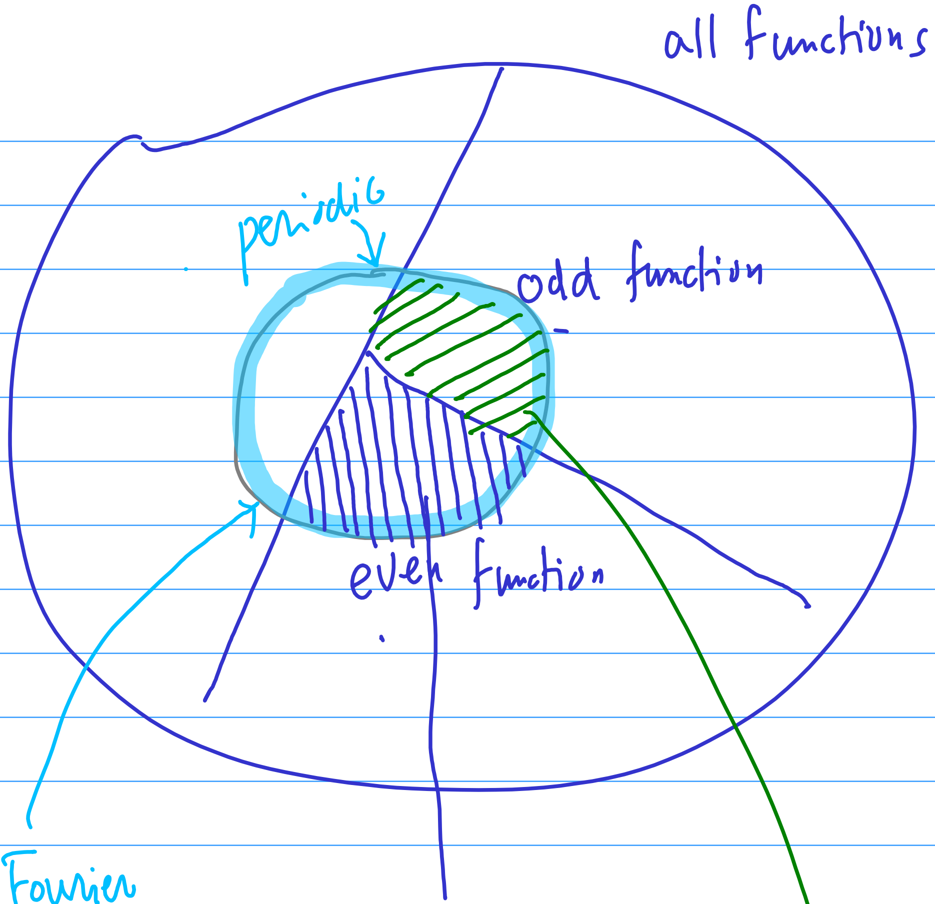
$$\langle \cos(mx), \sin(nx) \rangle = 0$$

$$\langle \sin(mx), \cos(nx) \rangle = 0$$

$$\langle f(x), g(x) \rangle \overset{\Delta}{=} \int_a^b f(x) \cdot g(x) \, dx$$

$$
\begin{array}{llll}
f(x) = & a_0 + & a_1 \cos(1x) & + \; b_1 \sin(1x) \\
 & + & a_2 \cos(2x) & + \; b_2 \sin(2x) \\
 & + & a_3 \cos(3x) & + \; b_3 \sin(3x) \\
 & & \vdots & \quad \vdots \\
 & + & a_n \cos(nx) & + \; b_n \sin(nx) \\
 & & \vdots & \quad \vdots
\end{array}
$$

$$
\cos(1x) \times f(x) = \boxed{\begin{array}{llll}
a_0 + & a_1 \cos(1x) & + \; b_1 \sin(1x) \\
+ & a_2 \cos(2x) & + \; b_2 \sin(2x) \\
+ & a_3 \cos(3x) & + \; b_3 \sin(3x) \\
 & \vdots & \quad \vdots \\
+ & a_n \cos(nx) & + \; b_n \sin(nx) \\
 & \vdots & \quad \vdots
\end{array}} \times \cos(1x)
$$



Fourier series

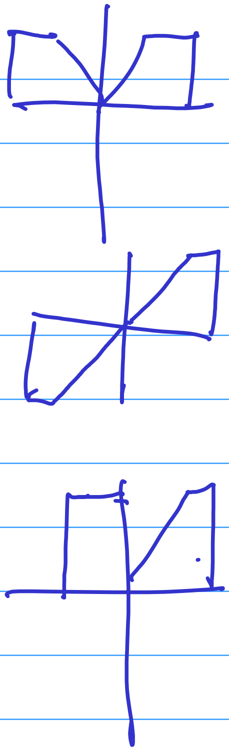
cosine series

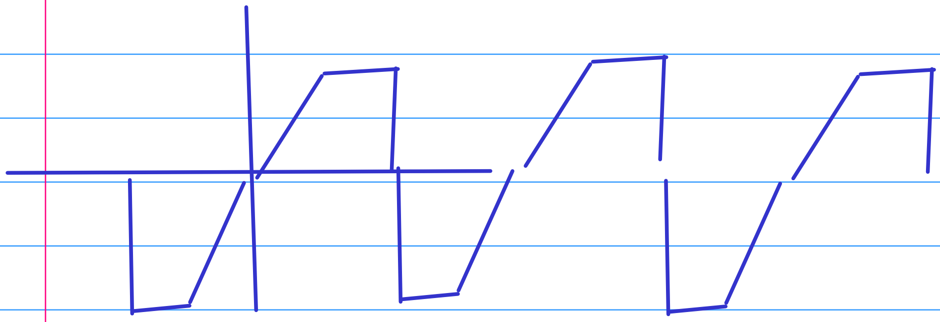
sine series

$\left\{\begin{matrix}\cos mx\\ \sin nx\end{matrix}\right\}$

$\{\cos mx\}$

$\{\sin mx\}$





discrete set of values

(

Continuous function

$\wedge$

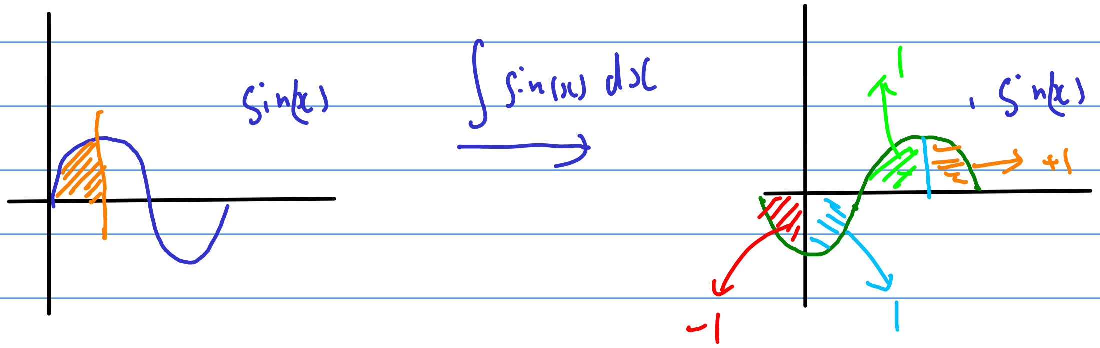
periodic

→ a_n

b_n

~

Fourier Series



$$\int_0^{\frac{\pi}{2}} \sin(x)\,dx = 1$$

$$\int_0^{2\pi} \sin(x)\,dx = 0$$

$$\int_0^{2\pi} \sin(2x)\,dx = 0$$

$n = 0, 1, 2, \ldots$

$$\int_{-\pi}^{+\pi} \cos(nx)\,dx = 0 \qquad \int_{-\pi}^{+\pi} \sin(nx)\,dx = 0$$

$$\int_{-\pi}^{+\pi} \cos(mx)\cos(nx)\,dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$$\int_{-\pi}^{+\pi} \sin(mx)\sin(nx)\,dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$$\cos\alpha \cos\beta = \frac{1}{2}\left\{+\cos(\alpha+\beta) + \cos(\alpha-\beta)\right\}$$

$$\sin\alpha \sin\beta = \frac{1}{2}\left\{-\cos(\alpha+\beta) + \cos(\alpha-\beta)\right\}$$

$$\cos(m x)\cos(n x) = \frac{1}{2}\left\{+\cos((m+n)x) + \cos((m-n)x)\right\}$$

$$\sin(m x)\sin(n x) = \frac{1}{2}\left\{-\cos((m+n)x) + \cos((m-n)x)\right\}$$

$$\underbrace{\cos(P x)}_{P=m+n} \qquad \underbrace{\cos(Q x)}_{Q=m-n}$$

$$\boxed{\begin{array}{lll} \displaystyle\int_{-\pi}^{+\pi} \cos(m x)\cos(n x)\,dx = 0 & \qquad & m \neq n \\ \displaystyle\int_{-\pi}^{+\pi} \sin(m x)\sin(n x)\,dx = 0 & \qquad & m \neq n \end{array}}$$

$$\boxed{\begin{array}{lll} \displaystyle\int_{-\pi}^{+\pi} \cos^2(m x)\,dx \neq 0 & \qquad & m = n \\ \displaystyle\int_{-\pi}^{+\pi} \sin^2(m x)\,dx \neq 0 & \qquad & m = n \end{array}}$$

$$\cos\alpha \cos\beta = \frac{1}{2}\left\{+\cos(\alpha+\beta) + \cos(\alpha-\beta)\right\}$$

$$\cos\theta \cos\theta = \frac{1}{2}\left\{+\cos(2\theta) + \cos(\ 0\ )\right\}$$

$$\cos^2\theta = \frac{1}{2}\left\{\cos(2\theta) + 1\right\}$$

$$\sin\alpha \sin\beta = \frac{1}{2}\left\{-\cos(\alpha+\beta) + \cos(\alpha-\beta)\right\}$$

$$\sin\theta \sin\theta = \frac{1}{2}\left\{-\cos(2\theta) + \cos(\ 0\ )\right\}$$

$$\sin^2\theta = \frac{1}{2}\left\{-\cos(2\theta) + 1\right\}$$

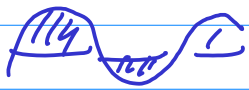
$$\sin^2\theta = \frac{1}{2}\left\{1 - \cos(2\theta)\right\}$$

$$\cos^2\theta = \frac{1}{2}\left\{1 + \cos(2\theta)\right\}$$

$$\int_{-\pi}^{+\pi} \cos^2(m\,x)\,dx = \int_{-\pi}^{\pi} \frac{1}{2}\left\{1 - \cancelto{0}{\cos(2mx)}\right\}dx = \pi \qquad m = n$$

$$\int_{-\pi}^{+\pi} \sin^2(m\,x)\,dx = \int_{-\pi}^{\pi} \frac{1}{2}\left\{1 + \cancelto{0}{\cos(2mx)}\right\}dx = \pi \qquad m = n$$

$$= \left[\frac{1}{2}x + \cancelto{0}{(\quad)}\right]_{-\pi}^{\pi} = \frac{1}{2}\cdot(2\pi) = \pi$$



$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left\{+\sin(\alpha+\beta) - \sin(\alpha-\beta)\right\}$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left\{+\cos(\alpha+\beta) + \sin(\alpha-\beta)\right\}$$

$$\cos(mx)\sin(nx) = \frac{1}{2}\left\{+\sin\left((m+n)x\right) - \sin\left((m-n)x\right)\right\}$$

$$\sin(mx)\cos(nx) = \frac{1}{2}\left\{+\sin\left((m+n)x\right) + \sin\left((m-n)x\right)\right\}$$

$$\boxed{\begin{aligned} \int_{-\pi}^{+\pi} \cos(mx)\sin(nx)\,dx &= 0 \\ \int_{-\pi}^{+\pi} \sin(mx)\cos(nx)\,dx &= 0 \end{aligned}}$$

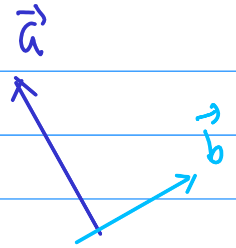
$$\sin(\alpha+\beta) = \cancel{\sin\alpha\cos\beta} + \cos\alpha\sin\beta$$

$$-)\ \underline{\sin(\alpha-\beta) = \cancel{\sin\alpha\cos\beta} - \cos\alpha\sin\beta}$$

$$\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\right\}$$

Def Orthogonal Functions



$\underbrace{\text{Orthogonal}}_{\text{직교}}$ $\vec{a} \cdot \vec{b} = 0$

$$(f_1, f_2) = \int_a^b f_1(x)\, f_2(x)\, dx$$

function f_1 & f_2 → orthogonal to each other on an interval $[a, b]$

$$\int_{-\pi}^{+\pi} \cos(1 \cdot x)\cos(2x)\, dx = 0$$

$\cos(1 \cdot x)$ & $\cos(2x)$ orthogonal to each other

$$\int_{-\pi}^{+\pi} \sin(1 \cdot x)\sin(2x)\, dx = 0$$

$\sin(1 \cdot x)$ & $\sin(2x)$ orthogonal to each other

Inner Product

$\vec{a} \cdot \vec{b}$ Vectors

$\langle f, g \rangle$ functions

R^2 $\vec{a} = (a_1, a_2)$

$\vec{b} = (b_1, b_2)$

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 b_2 = |\vec{a}| |\vec{b}| \cos\theta$

R^3 $\vec{a} = (a_1, a_2, a_3)$

$\vec{b} = (b_1, b_2, b_3)$

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 b_2 + a_3 b_3$

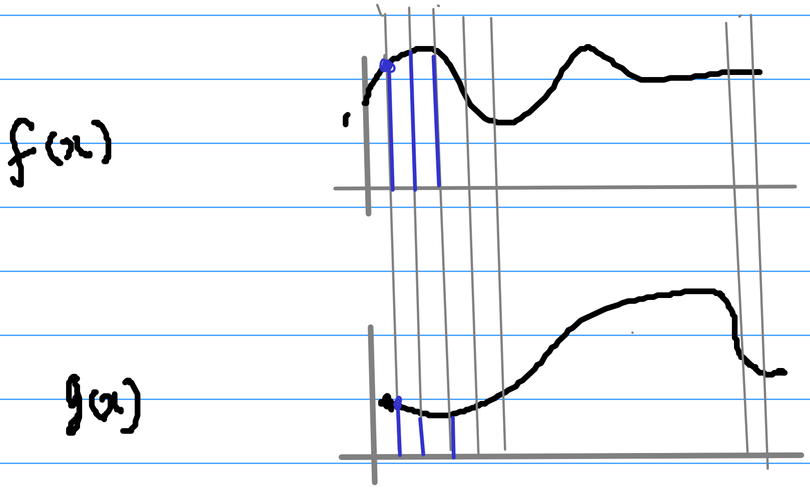
R^n $\quad \vec{a} = (a_1, a_2, a_3, \cdots, a_n)$

$\vec{b} = (b_1, b_2, b_3, \cdots, b_n)$

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n$$

$f(x_1), \ f(x_2), \ f(x_3), \ \cdots\cdots, \ f(x_n)$

$\Sigma \quad g(x_1), \ g(x_2), \ g(x_3), \ \cdots, \ g(x_n)$



$$\langle f, g \rangle = \int_{x_1}^{x} f(x) \cdot g(x) \, dx$$

Orthogonal set

$$\{ \phi_0(x), \quad \phi_1(x), \quad \phi_2(x), \quad \cdots\cdots \}$$

$$\langle \phi_0(x), \ \phi_1(x) \rangle = \int_a^b \phi_0(x)\ \phi_1(x)\ dx = 0$$

$$\langle \phi_0(x), \ \phi_2(x) \rangle = \int_a^b \phi_0(x)\ \phi_1(x)\ dx = 0$$

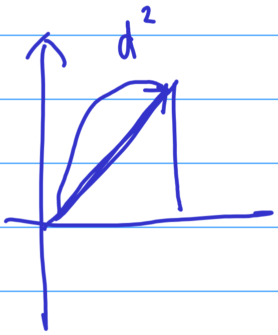
$$\langle \phi_0(x), \ \phi_3(x) \rangle = \int_a^b \phi_0(x)\ \phi_1(x)\ dx = 0$$

$$\langle \phi_m(x), \ \phi_n(x) \rangle = 0 \qquad m \neq n$$

$$\langle \phi_m(x), \ \phi_n(x) \rangle = \| \phi_m(x) \|^2 \qquad m = n$$

$$= \| \phi_n(x) \|^2$$

$$\vec{a} \cdot \vec{a} = \begin{matrix} (a_1, & a_2) \\ (a_1, & a_2) \end{matrix} = a_1^2 + a_2^2$$



$$\| \vec{a} \|^2 = \vec{a} \cdot \vec{a}$$

norm

$$\|\phi_n\|^2 = \langle \phi_n(x), \phi_n(x) \rangle = \int_a^b \phi_n^2(x)\,dx$$

$$\|\phi_n\| = \sqrt{\int_a^b \phi_n^2(x)\,dx}$$

Orthogonal set

$[-\pi, \pi]$

$\{ \cos(0\cdot x), \cos(1\cdot x), \cos(2\cdot x), \cdots \}$

$$\langle \cos(mx), \cos(nx) \rangle = \int_{-\pi}^{+\pi} \cos(mx) \cos(nx)\, dx = 0$$

$m \neq n$

$\{ \sin(0\cdot x), \sin(1\cdot x), \sin(2\cdot x), \cdots \}$

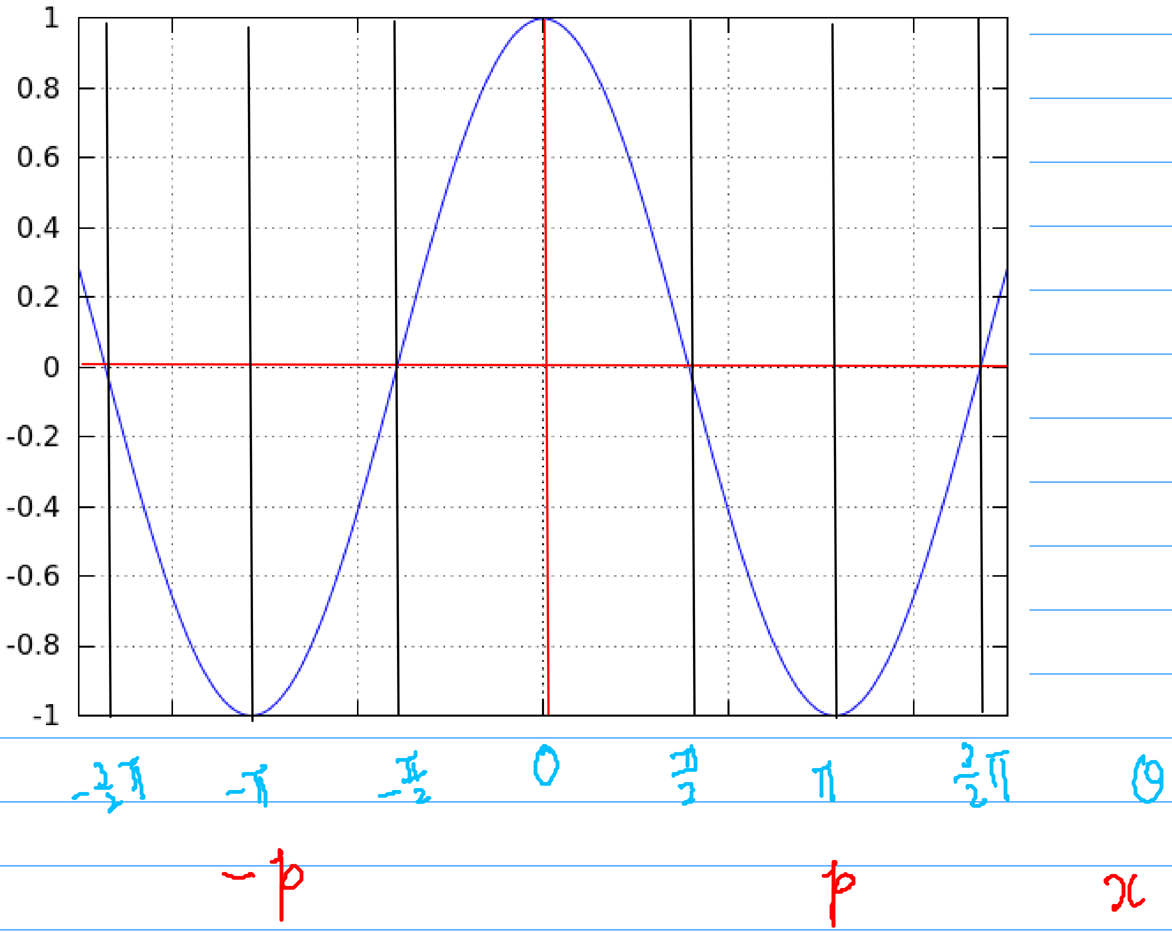
$$\langle \sin(mx), \sin(nx) \rangle = \int_{-\pi}^{+\pi} \sin(mx) \sin(nx)\, dx = 0$$

$m \neq n$

$$\left\{ \begin{matrix} \cos(0\cdot x), & \cos(1\cdot x), & \cos(2\cdot x), & \cdots \\ \sin(0\cdot x), & \sin(1\cdot x), & \sin(2\cdot x), & \cdots \end{matrix} \right\}$$

$$\cos\left(\frac{n\pi}{p}x\right) \xrightarrow{n=1} \cos\left(\frac{\pi}{p}x\right) = \cos\left(\boxed{\theta}\right)$$

$-\pi \sim \pi$



$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \sin(nx)\right)}$$

a_0 ?

a_n ? ← orthogonal function char.

b_n ?

Finding a_0

↙

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)}$$

$$\int_{-\pi}^{+\pi} f(x)\,dx = \int_{-\pi}^{+\pi} \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)dx$$

$$= \int_{-\pi}^{+\pi} \frac{a_0}{2}\,dx + \underbrace{\sum_{n=1}^{\infty}\left(\int a_n\cos(nx) + b_n\sin(nx)\,dx\right)}_{\substack{\parallel \\ 0}}$$

$$= \left[\frac{a_0}{2}x\right]_{-\pi}^{+\pi} = a_0\pi$$

$$\boxed{a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx}$$

Finding a_n

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(\overset{\swarrow}{a_n}\cos(nx) + b_n\sin(nx)\right)$$

$a_n, b_n \swarrow$

change index variables $k=1,2,3,\ldots\ldots$ $\textcircled{n}$: a specific number

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k\cos(kx) + b_k\sin(kx)\right)$$

$$f(x)\cos(nx) =$$

$$\frac{a_0}{2}\cos(nx) + \sum_{k=1}^{\infty}\left(a_k\cos(kx)\cos(nx) + b_k\sin(kx)\cos(nx)\right)$$

$$\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx = \underline{\int_{-\pi}^{\pi}\frac{a_0}{2}\cos(nx)\,dx}^{\nearrow 0}$$

$$\sum_{k=1}^{\infty}\left(\int_{-\pi}^{+\pi} a_k\cos(kx)\cos(nx) + \underline{b_n\sin(kx)\cos(nx)\,dx}^{\nearrow 0}\right)$$

$$= \pi\, a_n$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx$$

Finding b_n

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + \overset{\swarrow}{b_n} \sin(nx)\right)$$

a_n, b_n

change index variables $k = 1, 2, 3, \ldots\ldots$ $\boxed{n}$: a specific num

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos(kx) + b_k \sin(kx)\right)$$

$$f(x)\sin(nx) =$$

$$\frac{a_0}{2}\sin(nx) + \sum_{k=1}^{\infty}\left(a_k \cos(kx)\sin(nx) + b_k \sin(kx)\sin(nx)\right)$$

$$\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx = \overset{\nearrow 0}{\underline{\int_{-\pi}^{\pi} \frac{a_0}{2}\sin(nx)\,dx}}$$

$$\sum_{k=1}^{\infty}\left(\int_{-\pi}^{+\pi} \overset{\nearrow 0}{a_k \cos(kx)\sin(nx)} + \underline{b_n \sin(kx)\sin(nx)\,dx}\right)$$

$$= \pi b_n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \sin(nx)\right)}$$

$$\boxed{\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx \end{aligned}}$$

$-\pi \le x \le \pi$ $\quad\quad$ $-p \le x \le p$

Def Fourier Series

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

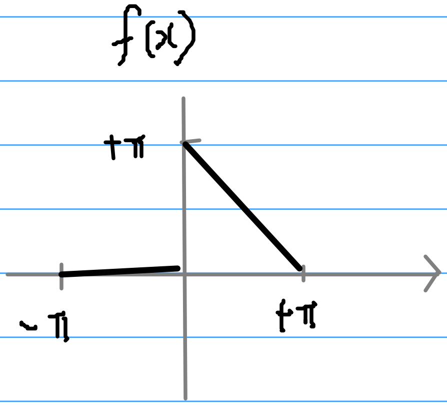
$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

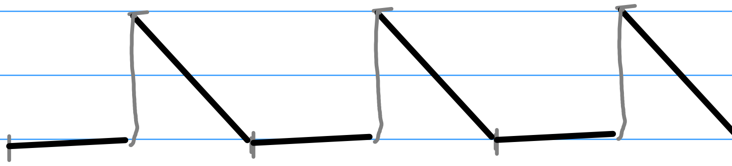
$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$$-p \leq x \leq p$$

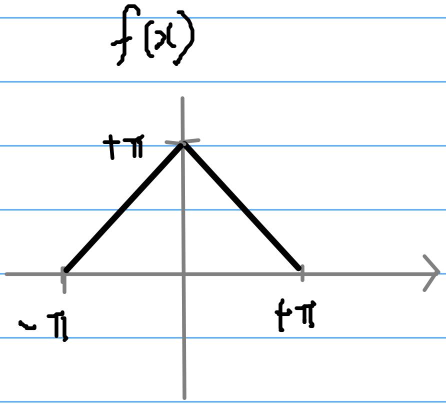
Find a_0, a_n, b_n

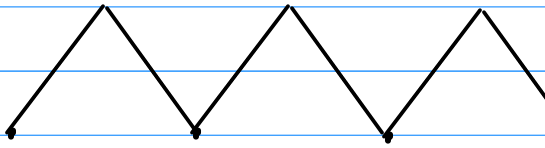
①



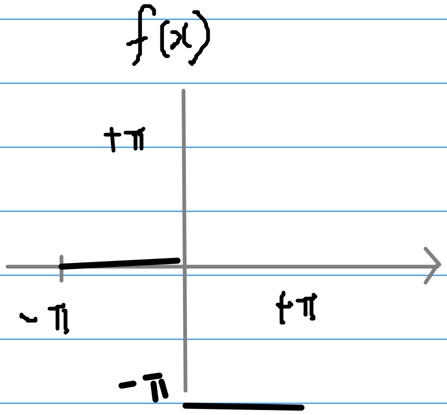


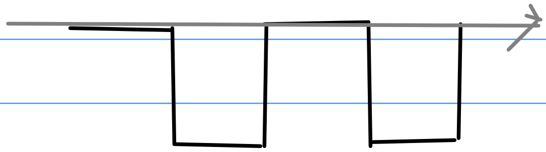
②



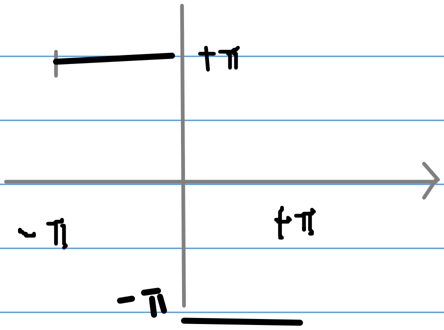


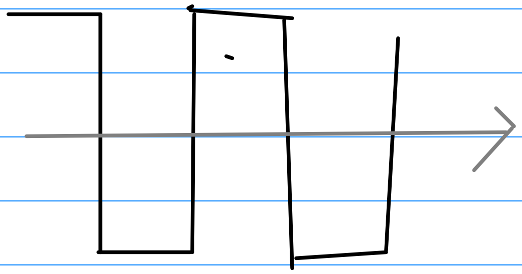
③



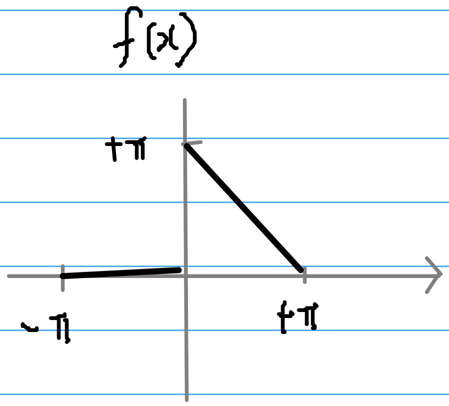


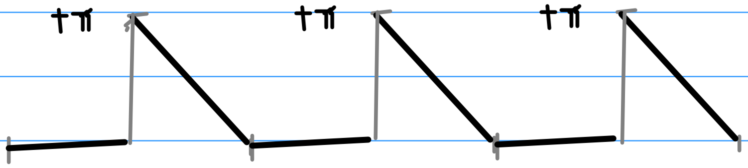
④





①





⇓

$a_0, \; a_n, \; b_n$

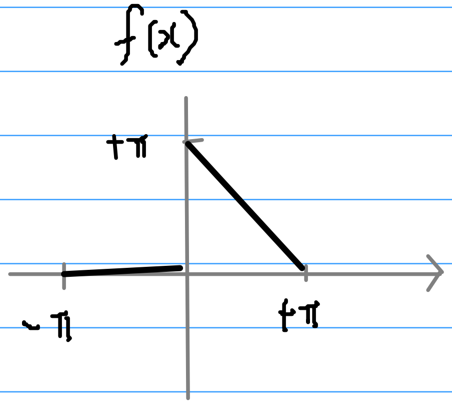
$$\begin{cases} f(x) = 0 & -\pi < x \leq 0 \\ f(x) = -x + \pi & 0 < x < \pi \end{cases}$$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)}$$

$$a_0 = \frac{1}{p} \int_{-p}^{+p} f(x)\, dx$$

$$a_n = \frac{1}{p} \int_{-p}^{+p} f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

$$b_n = \frac{1}{p} \int_{-p}^{+p} f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$



$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ -x+\pi & 0 \leq x < \pi \end{cases}$$

$$\boxed{a_0} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{\pi}\int_{0}^{\pi} (-x+\pi)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} -x+\pi\,dx = \frac{1}{\pi}\left[-\frac{1}{2}x^2+\pi x\right]_0^{\pi} = \frac{1}{\pi}\left[-\frac{\pi^2}{2}+\pi^2\right]$$

$$= \cancel{\frac{1}{\pi}}\cdot\frac{\pi^2}{2} = \boxed{\frac{\pi}{2}}$$

$$\boxed{a_n} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos\left(\frac{n\pi}{7}x\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\cdot\cos(nx)\,dx + \boxed{\frac{1}{\pi}\int_0^{\pi}\underline{(-x+\pi)}\,\underline{\cos(nx)}\,dx}$$

$$\boxed{\begin{aligned}&\int f g'\,dx\\ &= fg - \int f' g\,dx\end{aligned}}$$

$$\int \underline{\cos(nx)}\,dx = \frac{1}{n}\sin(nx)$$

$$= \frac{1}{\pi}\left[(-x+\pi)\frac{1}{n}\sin(nx) - \int(-1)\frac{1}{n}\sin(nx)\,dx\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[(-x+\pi)\frac{1}{n}\sin(nx) - \frac{1}{n^2}\cos(nx)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\left[(-x+\pi)\frac{1}{n}\overset{\nearrow 0}{\sin(nx)}\right]_0^{\pi} - \left[\frac{1}{n^2}\cos(nx)\right]_0^{\pi}\right]$$

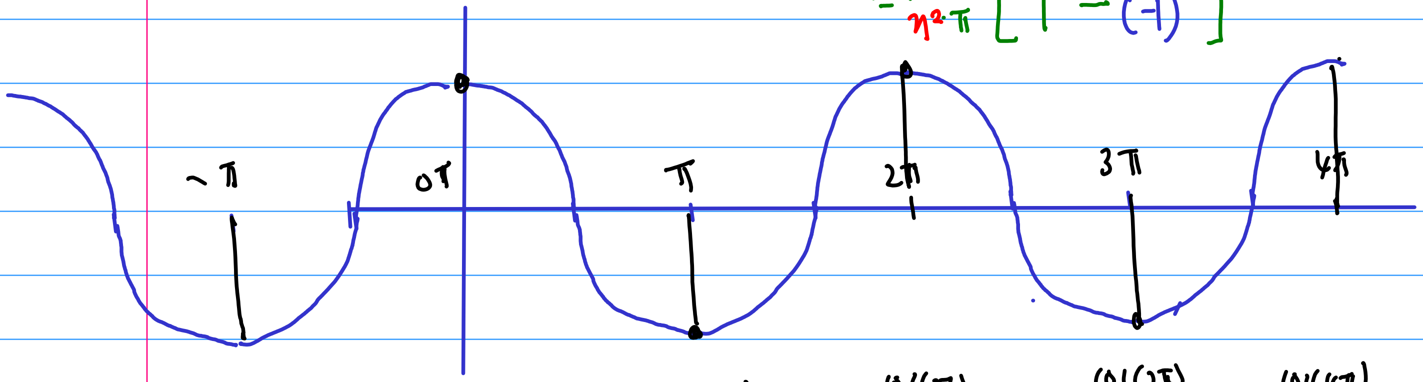
$\sin(n\pi) = 0$

$\sin(n \cdot 0) = 0$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}\cos(n\cdot\pi) - \frac{1}{n^2}\cos(n\cdot 0)\right]$$

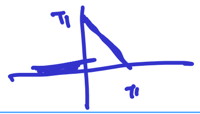
$$- \frac{1}{\pi}\left(\frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right)$$

$$= \frac{1}{n^2\pi}\left[1 - (-1)^n\right]$$



$\cos(0\pi)$	$\cos(1\pi)$	$\cos(2\pi)$	$\cos(3\pi)$	$\cos(4\pi)$
$+1$	-1	$+1$	-1	$+1$
$\cos(n\pi) = +1$	$\cos(n\pi) = -1$	$\cos(n\pi) = +1$	$\cos(n\pi) = -1$	
$n = 0$ $(-1)^0$	$n = 1$ $(-1)^1$	$n = 2$ $(-1)^2$	$n = 3$ $(-1)^3$	
$(-1)^n$	$(-1)^n$	$(-1)^n$	$(-1)^n$	

$\boxed{\cos(n\pi) = (-1)^n}$ $n \in$ integer



$$\boxed{b_n} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{+0} 0\cdot\sin(nx)\,dx + \underline{\frac{1}{\pi}\int_{0}^{+\pi}(\pi-x)\;\sin(nx)\,dx}$$

$$\int \sin(nx)\,dx = -\frac{1}{n}\cos(nx)$$

Integration by parts $\int f\,g'\,dx = fg - \int f'g\,dx$

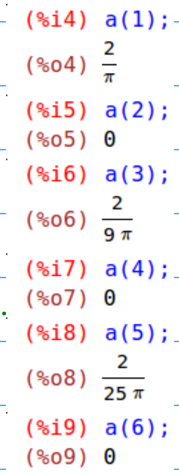
$$\frac{1}{\pi}\left[\left[(\pi-x)\frac{-1}{n}\cos(nx)\right]_0^{\pi} - \int_0^{\pi}(-1)\frac{-1}{n}\cos(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n} - \int_0^{\pi}\frac{1}{n}\cos(nx)\,dx\right] \; 0$$

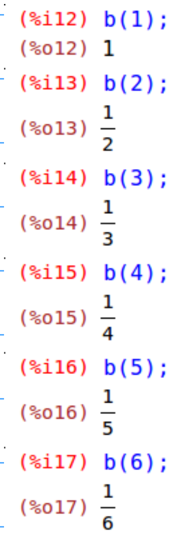
$$= \frac{1}{\pi}\left[\frac{\pi}{n} - \overset{\nearrow}{\left[\frac{1}{n^2}\sin(nx)\,dx\right]_0^{\pi}}\right]$$

$$= \frac{1}{n}$$

$a_0 = \frac{\pi}{2}$ $a_n = \frac{1-(-1)^n}{n^2 \pi}$ $b_n = \frac{1}{n}$

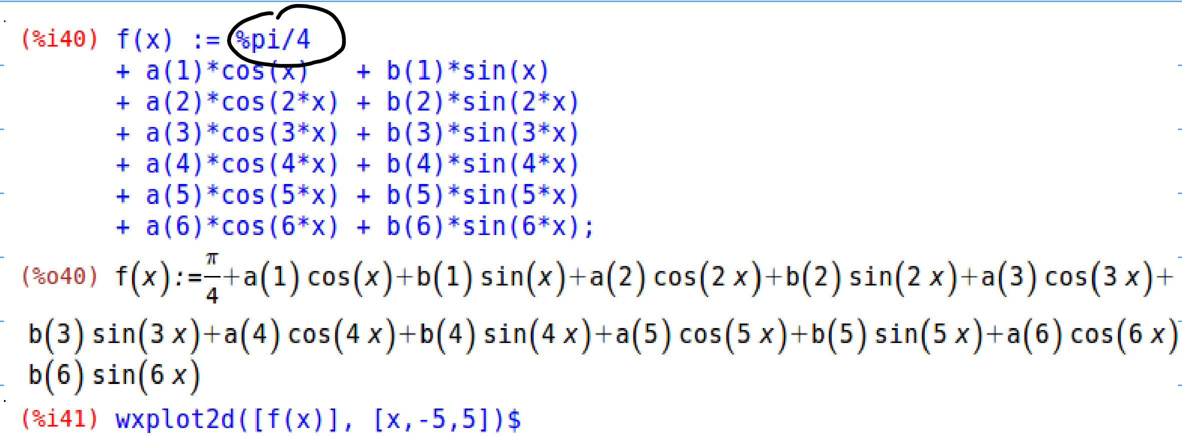


```
(%i4) a(1);
(%o4) 2/π
(%i5) a(2);
(%o5) 0
(%i6) a(3);
(%o6) 2/(9 π)
(%i7) a(4);
(%o7) 0
(%i8) a(5);
(%o8) 2/(25 π)
(%i9) a(6);
(%o9) 0
```



```
(%i12) b(1);
(%o12) 1
(%i13) b(2);
(%o13) 1/2
(%i14) b(3);
(%o14) 1/3
(%i15) b(4);
(%o15) 1/4
(%i16) b(5);
(%o16) 1/5
(%i17) b(6);
(%o17) 1/6
```

$a_0 = \frac{\pi}{2}$ $\frac{a_0}{2} = \frac{\pi}{4}$

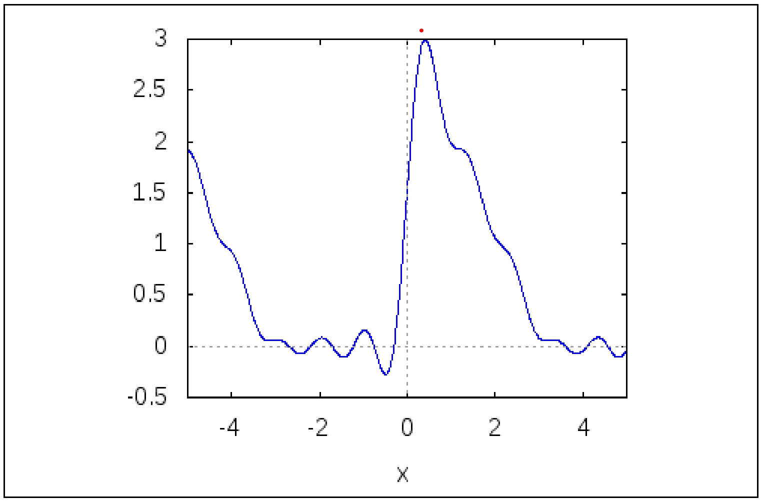


```
(%i40) f(x) := %pi/4
       + a(1)*cos(x)   + b(1)*sin(x)
       + a(2)*cos(2*x) + b(2)*sin(2*x)
       + a(3)*cos(3*x) + b(3)*sin(3*x)
       + a(4)*cos(4*x) + b(4)*sin(4*x)
       + a(5)*cos(5*x) + b(5)*sin(5*x)
       + a(6)*cos(6*x) + b(6)*sin(6*x);
```

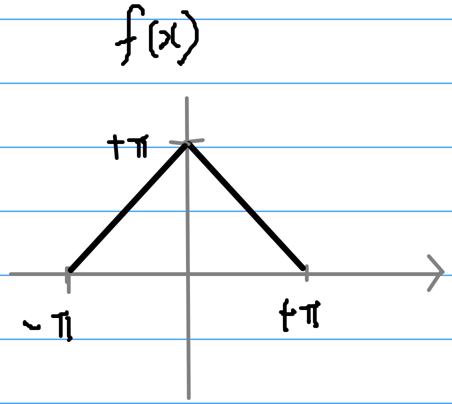
(%o40) $f(x) := \frac{\pi}{4} + a(1)\cos(x) + b(1)\sin(x) + a(2)\cos(2x) + b(2)\sin(2x) + a(3)\cos(3x) + b(3)\sin(3x) + a(4)\cos(4x) + b(4)\sin(4x) + a(5)\cos(5x) + b(5)\sin(5x) + a(6)\cos(6x) \, b(6)\sin(6x)$

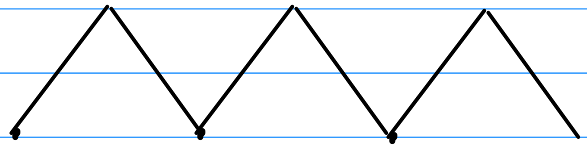
```
(%i41) wxplot2d([f(x)], [x,-5,5])$
```

(%t41)



②





a_0, a_n, b_n

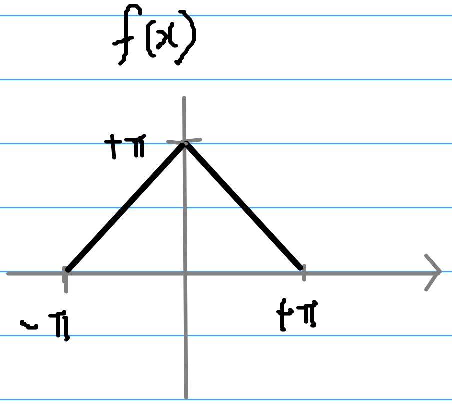
$$\begin{cases} f(x) = x + \pi & -\pi < x \leq 0 \\ f(x) = -x + \pi & 0 < x < \pi \end{cases}$$

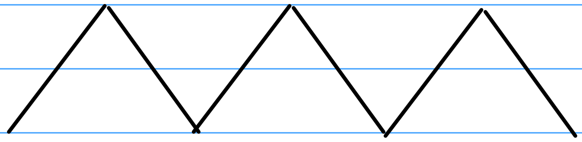
$$\boxed{f(x) = \frac{\textcircled{a_0}}{2} + \sum_{n=1}^{\infty} \left(\textcircled{a_n} \cos\left(\frac{n\pi}{p} x\right) + \textcircled{b_n} \sin\left(\frac{n\pi}{p} x\right) \right)}$$

$$\textcircled{a_0} = \frac{1}{p} \int_{-p}^{+p} f(x)\, dx$$

$$\textcircled{a_n} = \frac{1}{p} \int_{-p}^{+p} f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

$$\textcircled{b_n} = \frac{1}{p} \int_{-p}^{+p} f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$





$$\boxed{a_0} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(-x+\pi)\,dx$$

$$= \frac{1}{\pi}\left[\frac{1}{2}x^2+\pi x\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{1}{2}x^2+\pi x\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{1}{2}\pi^2+\pi^2\right] + \frac{1}{\pi}\left[-\frac{1}{2}\pi^2+\pi^2\right] = \pi$$

$$\boxed{a_n} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos\left(\frac{n\pi}{T}x\right)dx$$

$$\int \underline{\cos(nx)}\,dx = \frac{1}{n}\sin(nx)$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\cos(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi}\underline{(-x+\pi)}\,\underline{\cos(nx)}\,dx$$

$$= \frac{1}{\pi}\left[(x+\pi)\frac{1}{n}\sin(nx) - \int(+1)\frac{1}{n}\sin(nx)\,dx\right]_{-\pi}^{0}$$

$$+ \frac{1}{\pi}\left[(-x+\pi)\frac{1}{n}\sin(nx) - \int(-1)\frac{1}{n}\sin(nx)\,dx\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[(x+\pi)\frac{1}{n}\sin(nx) + \frac{1}{n^2}\cos(nx) \right]_{-\pi}^{0}$$

sin(0)=0 cos(0)=1

Sin (nπ)=0 cos(nπ)=(-1)^n

$$\frac{1}{\pi}\left[(-x+\pi)\frac{1}{n}\sin(nx) - \frac{1}{n^2}\cos(nx) \right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}\left(\cos(0) - \cos(-n\pi)\right)\right] + \frac{1}{\pi}\left[-\frac{1}{n^2}\left(\cos(n\pi) - \cos(0)\right)\right]$$

$$= 2\,\frac{1}{n^2\pi}\left(1-(-1)^n\right)$$

$$\int \sin(nx)\,dx = -\frac{1}{n}\cos(nx)$$

$$\boxed{b_n} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{+0}(\pi+x)\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{+\pi}(\pi-x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[(\pi+x)\frac{-1}{n}\cos(nx) - \int (+1)\frac{-1}{n}\cos(nx)\,dx \right]_{-\pi}^{0}$$

$$+ \frac{1}{\pi}\left[(\pi-x)\frac{-1}{n}\cos(nx) - \int (-1)\frac{-1}{n}\cos(nx)\,dx \right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[(\pi+x)\frac{-1}{n}\cos(nx) + \frac{1}{n^2}\sin(nx)\,dx \right]_{-\pi}^{0}$$

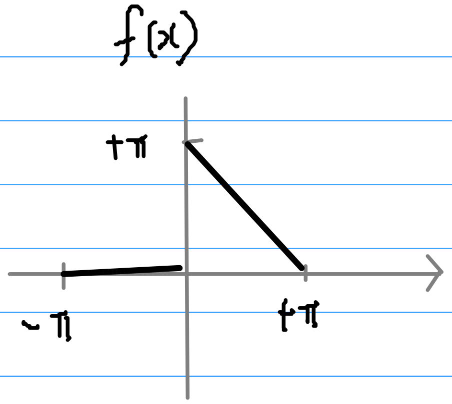
sin(0)=0 cos(0)=1

Sin (nπ)=0 cos(nπ)=(-1)^n

$$+ \frac{1}{\pi}\left[(\pi-x)\frac{-1}{n}\cos(nx) - \frac{1}{n^2}\sin(nx)\,dx \right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[\pi\frac{-1}{n} - \pi\frac{-1}{n} \right] = 0$$

①

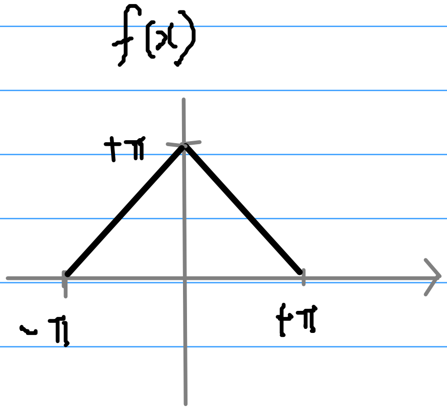


$$\boxed{a_0} = \boxed{\frac{\pi}{2}}$$

$$\boxed{a_n} = \frac{1}{n^2 \cdot \pi}\left[1 - (-1)^n\right]$$

$$\boxed{b_n} = \frac{1}{n}$$

②

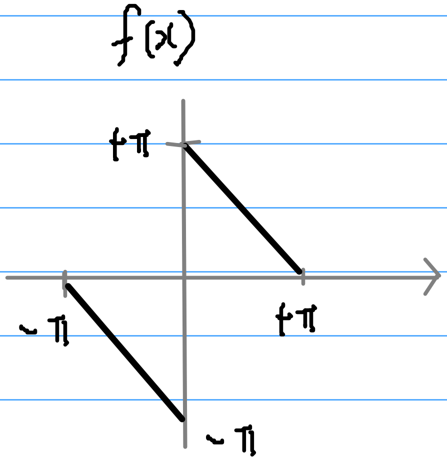


$$\boxed{a_0} = 2 \cdot \frac{\pi}{2}$$

$$\boxed{a_n} = 2 \cdot \frac{1}{n^2 \cdot \pi}\left(1 - (-1)^n\right)$$

$$\boxed{b_n} = 0$$

☆

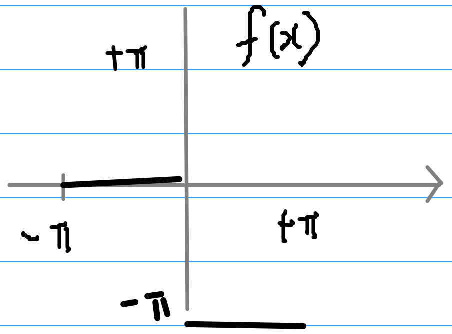


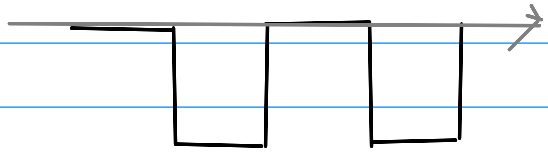
$$\boxed{a_0} = 0$$

$$\boxed{a_n} = 0$$

$$\boxed{b_n} \overset{?}{=} 2 \cdot \frac{1}{n}$$

③





$$a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{\pi}\int_{0}^{\pi} (-\pi)\,dx$$

$$= \frac{1}{\pi}\left[-\pi x\right]_0^{\pi} = \frac{1}{\pi}\left[-\pi^2\right] = \boxed{-\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos\left(\frac{n\pi}{\pi}x\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\cdot\cos(nx)\,dx + \boxed{\frac{1}{\pi}\int_{0}^{\pi}(-\pi)\,\underline{\cos(nx)}\,dx}$$

$$= \frac{1}{\pi}\left[\ (-\pi)\frac{1}{n}\sin(nx)\right]_0^{\pi} = \boxed{0}$$

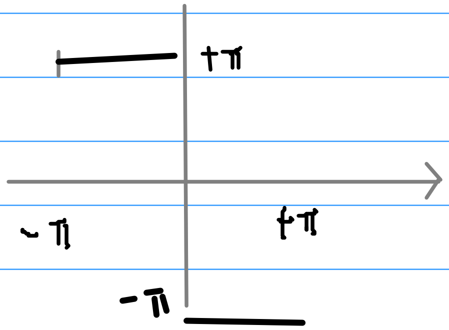
$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$

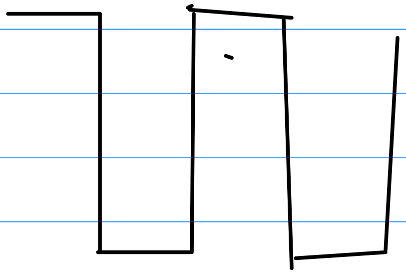
$$= \frac{1}{\pi}\int_{-\pi}^{+0} 0\cdot\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{+\pi}(-\pi)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[(-\pi)\frac{-1}{n}\cos(nx)\right]_0^{\pi} = \frac{1}{n}\cos(n\pi) - \frac{1}{n}\cos(0)$$

$$= \frac{1}{n}\left((-1)^n - 1\right)$$

④





$$\boxed{a_0} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(\pi)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(-\pi)\,dx$$

$$= \frac{1}{\pi}\left[+\pi x\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\pi x\right]_{0}^{\pi} = \pi - \pi = \boxed{0}$$

$$\boxed{a_n} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(\pi)\cdot\cos(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(-\pi)\underline{\cos(nx)}\,dx$$

$$= \frac{1}{\pi}\left[(+\pi)\frac{1}{n}\sin(nx)\right]_{-\pi}^{0} + \left[(-\pi)\frac{1}{n}\sin(nx)\right]_{0}^{\pi} = \boxed{0}$$

$$\boxed{b_n} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$

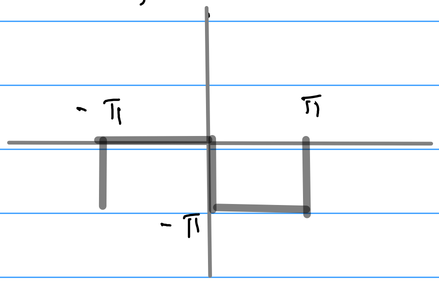
$$= \frac{1}{\pi}\int_{-\pi}^{+0}(\pi)\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{+\pi}(-\pi)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[(\pi)\frac{-1}{n}\cos(nx)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[(-\pi)\frac{-1}{n}\cos(nx)\right]_{0}^{\pi}$$

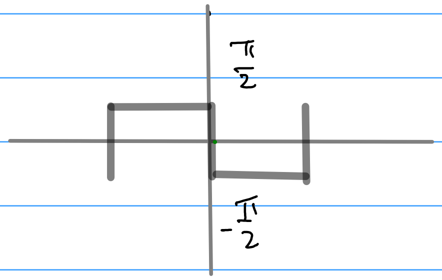
$$= 2\frac{1}{n}\left((-1)^n - 1\right) \qquad \frac{1}{n}\cos(n\pi) - \frac{1}{n}\cos(0)$$

$$\boxed{\frac{a_0}{2}} + \sum a_n \cos(\ ) + \sum b_n \sin(\ )$$

$f(x)$

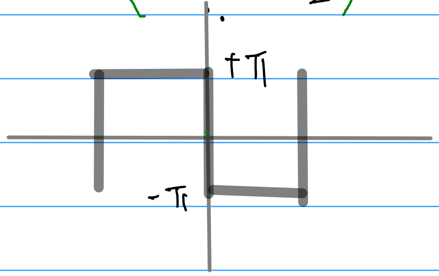


$f(x) + \frac{\pi}{2}$



?

$2 \cdot \left(f(x) + \frac{\pi}{2} \right)$



$\boxed{a_0} = -\pi$

$\boxed{a_n} = 0$

$\boxed{b_n} = \frac{1}{n}\left((-1)^n - 1\right)$

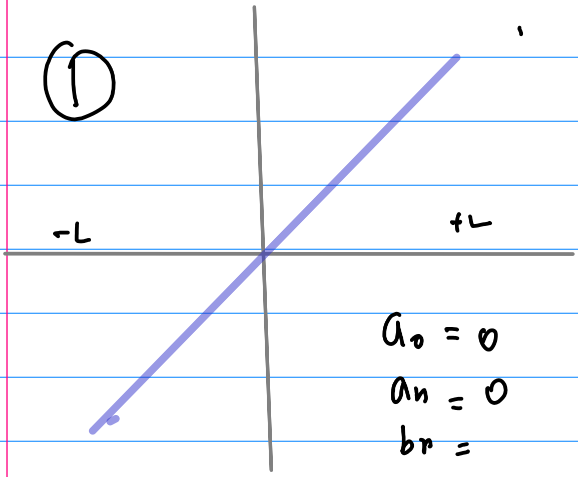
$\boxed{a_0} = 0$

$\boxed{a_n} = 0$

$\boxed{b_n} = 2\frac{1}{n}\left((-1)^n - 1\right)$

Example 1 Find the Fourier sine series for $f(x)=x$ on $-L\le x\le L$.

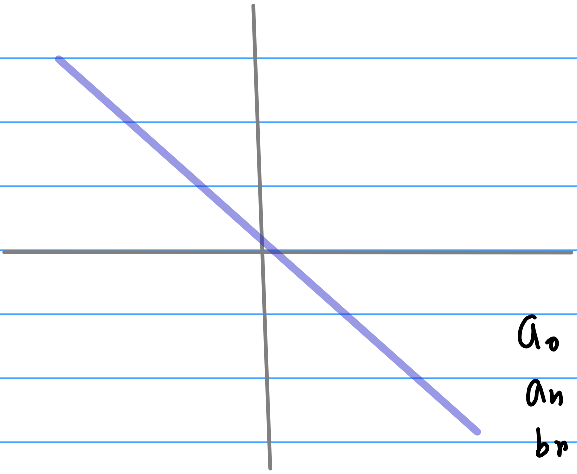
①



$a_0 = 0$

$a_n = 0$

$b_n =$



$a_0 = 0$

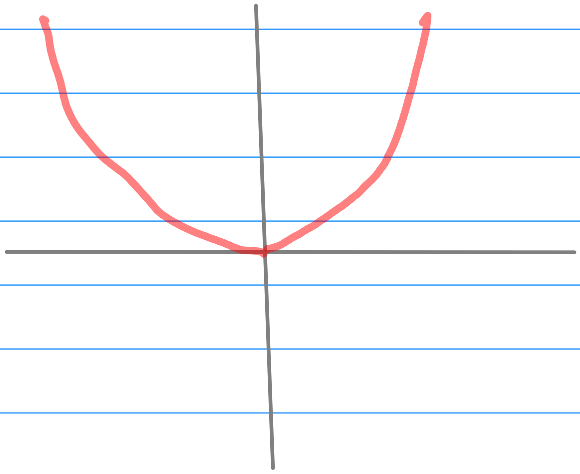
$a_n = 0$

$b_n =$

Example 1 Find the Fourier cosine series for $f(x)=x^2$ on $-L\le x\le L$.

Example 1 Find the Fourier series for $f(x)=L-x$ on $-L\le x\le L$.

②

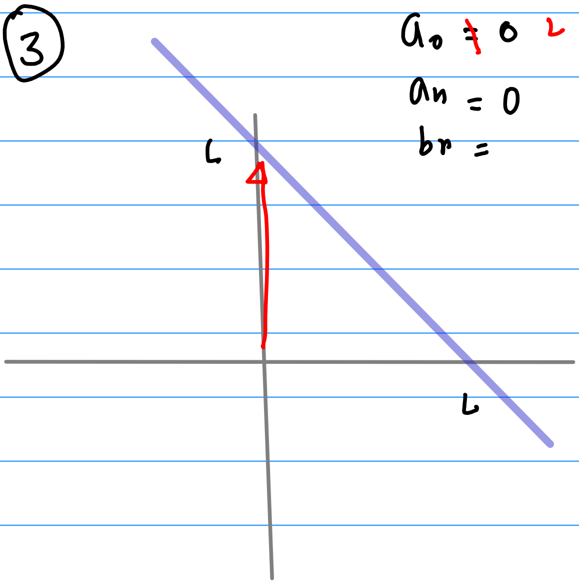


③

$a_0 \neq 0$ L

$a_n = 0$

$b_n =$



Taylor Series (McLaurin Series: $x=0$)

$$\boxed{f(x) = f(0) + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4}$$

Fourier Series

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

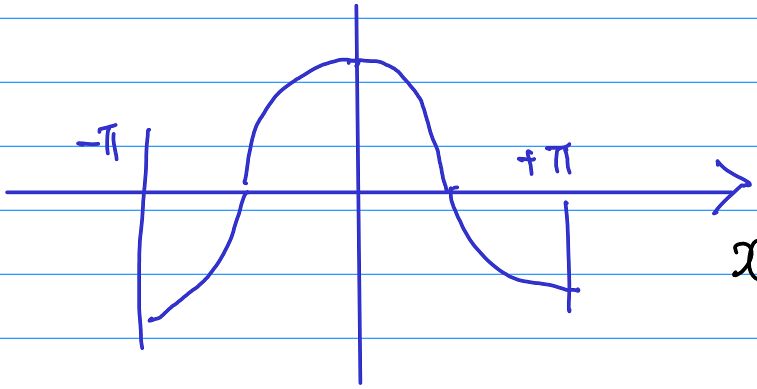
$x \in [-p, +p]$	$\cos\left(\frac{\pi}{p}x\right)$	$\sin\left(\frac{\pi}{p}x\right)$
$x \in \left[-\frac{p}{2}, +\frac{p}{2}\right]$	$\cos\left(\frac{2\pi}{p}x\right)$	$\sin\left(\frac{2\pi}{p}x\right)$
$x \in \left[-\frac{p}{3}, \frac{p}{3}\right]$	$\cos\left(\frac{3\pi}{p}x\right)$	$\sin\left(\frac{3\pi}{p}x\right)$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \sin(nx)\right)}$$

$x \in [-\pi, \pi]$	$\cos(1x)$	$\sin(1x)$	$n=1$
$x \in \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$	$\cos(2x)$	$\sin(2x)$	
$x \in \left[-\frac{\pi}{3}, +\frac{\pi}{3}\right]$	$\cos(3x)$	$\sin(3x)$	

$-\pi \leq x \leq \pi$ vs. $-p \leq x \leq p$

$\cos(\boxed{x})$ $-\pi \leq \boxed{x} \leq \pi$

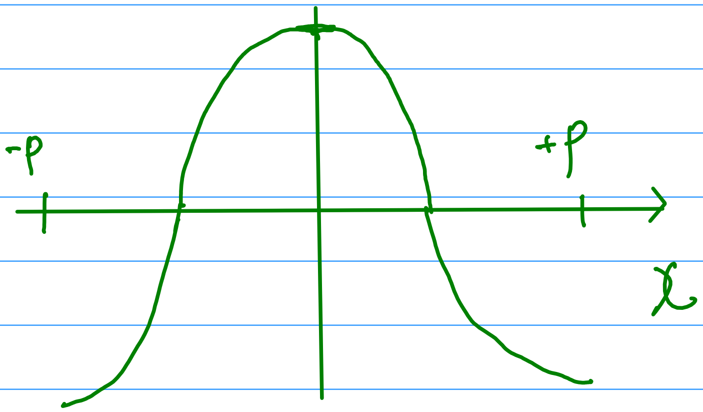


$\cos(2x)$ $-\pi \leq 2x \leq \pi$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\cos\left(\frac{\pi}{p}x\right)$ $-\pi \leq \frac{\pi}{p}x \leq \pi$

$-p \leq x \leq p$



$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx) \right)}$$

$\left(\begin{matrix} a_0, & a_1, & a_2, & a_3, & \cdots\cdots \\ & b_1, & b_2, & b_3 & \cdots \end{matrix} \right)$ 알 때

f(x) 를 만들수 있다.

$$\boxed{\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x)\,dx \\ a_n &= \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx)\,dx \\ b_n &= \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx)\,dx \end{aligned}}$$

원래 함수 f(x)로부터 계수구하기.

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$\left.\begin{matrix} a_0, & a_1, & a_2, & a_3, & \cdots\cdots \\ & b_1, & b_2, & b_3 & \cdots \end{matrix}\right)$ 알 때

$f(x)$를 만들수 있다.

$$\boxed{\begin{aligned} a_0 &= \frac{1}{p}\int_{-p}^{+p} f(x)\,dx \\ a_n &= \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx \\ b_n &= \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx \end{aligned}}$$

원래 함수 $f(x)$로부터 계수구하기.

* Paul's online math note

Fourier Series Definition

$$A_0 + \sum A_n \bigcirc + \sum B_n \bigcirc$$

$$f(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right)$$

$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$A_m = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{m\pi x}{L}\right)dx \qquad m = 1, 2, 3, \ldots$$

$$B_m = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{m\pi x}{L}\right)dx \qquad m = 1, 2, 3, \ldots$$

$$\boxed{f(x) = A_0 + \sum_{n=1}^{\infty}\left[A_n \cos\left(\frac{n\pi x}{L}\right) + B_n \sin\left(\frac{n\pi x}{L}\right)\right]}$$

$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\int_{-L}^{L}(L-x)\,dx = \frac{1}{2L}\left[Lx - \frac{1}{2}x^2\right]_{-L}^{L}$$

$$= \frac{1}{2L}\left(2L^2\right) = L$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}(L-x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} \underbrace{L\cos\left(\frac{n\pi x}{L}\right)}_{\text{odd x even = odd}} - x\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} -x\cos\left(\frac{n\pi x}{L}\right)dx = 0$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}(L-x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} \underbrace{L\sin\left(\frac{n\pi x}{L}\right)}_{\text{odd x odd = even}} - x\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} -x\sin\left(\frac{n\pi x}{L}\right)dx = 0$$

$$\int x \sin(kx)\,dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$= \frac{1}{L} \int_{-L}^{L} -x \sin\left(\frac{n\pi x}{L}\right) dx = 0$$

$$= -\frac{1}{L} \int_{-L}^{L} x \sin\left(\frac{n\pi}{L} x\right) dx$$

$$= \frac{1}{L} \left[-\frac{L}{n\pi} x \cos\left(\frac{n\pi}{L} x\right) + \underbrace{\left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L} x\right)}_{\rightarrow 0} \right]_{-L}^{L}$$

$$= -\frac{1}{L} \left[-\frac{L}{n\pi} L \cos\left(\frac{n\pi}{L} L\right) + \frac{L}{n\pi}(-L) \cos\left(\frac{n\pi}{L}(-L)\right) \right.$$

$$\left. \underbrace{\left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L} L\right)}_{\rightarrow 0} - \underbrace{\left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L}(-L)\right)}_{\rightarrow 0} \right]$$

$$= -\frac{1}{L} \left[-\frac{L}{n\pi} L \cos\left(\frac{n\pi}{L} L\right) + \frac{L}{n\pi}(-L) \cos\left(\frac{n\pi}{L}(-L)\right) \right]$$

$$= -\frac{1}{L} \left[-\frac{2L^2}{n\pi} (-1)^n \right] = \frac{2L}{n\pi} (-1)^n$$

$$\int f\, g'\, dx = f\, g - \int f'\, g\, dx$$

$$\int x \sin(kx)\, dx = x \left[\frac{-1}{k} \cos(kx) \right] - \int \left[\frac{-1}{k} \cos(kx) \right] dx$$

$$= -\frac{1}{k} x \cos(kx) + \frac{1}{k} \int \cos(kx)\, dx$$

$$= -\frac{1}{k} x \cos(kx) + \frac{1}{k} \left[\frac{1}{k} \sin(kx) \right]$$

$$= -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\int x \sin(kx)\, dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\Big[F(x) \Big]_{-a}^{+a} = F(a) - F(-a$$

$$\int f g' \, dx = f g - \int f' g \, dx$$

$$\int x \cos(k x) \, dx = x \left[\frac{1}{k} \sin(kx) \right] - \int \left[\frac{1}{k} \sin(kx) \right] dx$$

$$= \frac{1}{k} x \sin(kx) - \frac{1}{k} \int \sin(kx) \, dx$$

$$= \frac{1}{k} x \sin(kx) + \frac{1}{k} \left[\frac{1}{k} \cos(kx) \right]$$

$$= \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

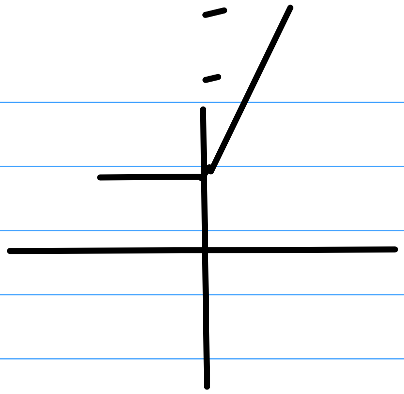
$$\int x \cos(k x) \, dx = \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

$$\int x \cos(k x) \, dx = \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

$$\int x \sin(k x) \, dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\int x^2 \cos(k x) \, dx = ?$$

$$\int x^2 \sin(k x) \, dx = ?$$



$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\left[\int_{-L}^{0} L\,dx + \int_{0}^{L} 2x\,dx\right]$$

$$= \frac{1}{2L}\left[L^2 + L^2\right] = \frac{2L^2}{2L} = L$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(kx)\,dx = \frac{1}{L}\left[\int_{-L}^{0} L\cos(kx)\,dx + \int_{0}^{L} 2x\cos(kx)\,dx\right]$$

$$= \frac{1}{L}\left[\left[\frac{L}{k}\sin(kx)\right]_{-L}^{0} + 2\left[\frac{1}{k}x\sin(kx) + \frac{1}{k^2}\cos(kx)\right]_{0}^{L}\right]$$

$$= \frac{1}{L}\left[\underbrace{\left[\frac{L}{k}\sin\left(\frac{n\pi}{L}x\right)\right]_{-L}^{0}}_{\to 0} + 2\left[\underbrace{\frac{L}{n\pi}\sin\left(\frac{n\pi}{L}x\right)}_{\to 0} + \left(\frac{L}{n\pi}\right)^2 \underline{\cos\left(\frac{n\pi}{L}x\right)}\right]_{0}^{L}\right]$$

$$= \frac{1}{L}\left[2\left(\frac{L}{n\pi}\right)^2\left((-1)^n - 1\right)\right]$$

$$\cos\left(\frac{n\pi}{\cancel{L}} \cdot x\right)$$

$$-\cos(0)$$

$$= \frac{2L}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(kx)\,dx = \frac{1}{L}\left[\int_{-L}^{0} L\sin(kx)\,dx + \int_{0}^{L} 2x\sin(kx)\,dx\right]$$

$$= \frac{1}{L}\left[\left[-\frac{L}{k}\cos(kx)\right]_{-L}^{0} + 2\left[\frac{-1}{k}x\cos(kx) + \frac{1}{k^2}\sin(kx)\right]_{0}^{L}\right]$$

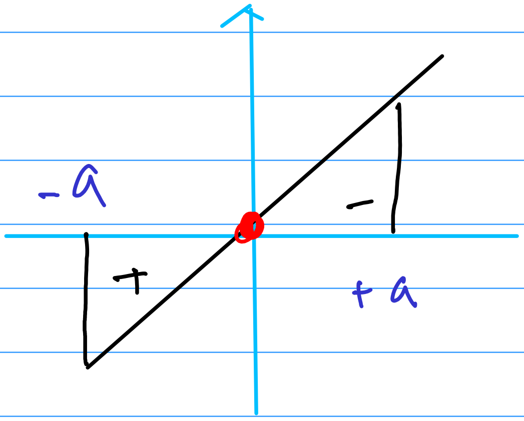
$$= \frac{1}{L}\left[\left[-\frac{L^2}{n\pi}\cos\left(\frac{n\pi}{L}x\right)\right]_{-L}^{0} + 2\left[-\frac{L}{n\pi}x\cos\left(\frac{n\pi}{L}x\right) + \underset{\longrightarrow 0}{\left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}x\right)}\right]_{0}^{L}\right]$$

$$= \frac{1}{L}\left[\left[-\frac{L^2}{n\pi}\left(1-(-1)^n\right) + 2\left[-\frac{L^2}{n\pi}(-1)^n\right]\right]\right]$$

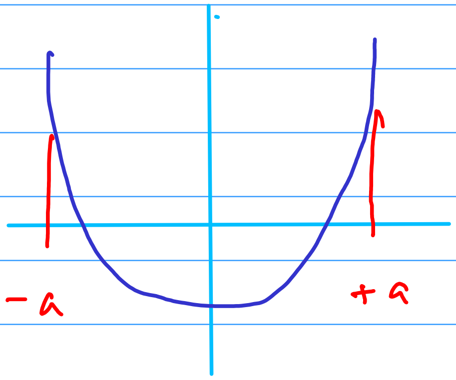
$$= \frac{1}{L}\left[\frac{L^2}{n\pi}\left(-1+(-1)^n\right) - \frac{2L^2}{n\pi}(-1)^n\right]$$

$$= \frac{1}{L}\left[\frac{L^2}{n\pi}\left(-1+(-1)^n - 2(-1)^n\right)\right]$$

$$= -\frac{L}{n\pi}\left(1+(-1)^n\right)$$



$$\int_{-a}^{+a} f_{odd}(x)\;dx = 0$$



$$\int_{-a}^{a} f_{even}(x)\;dx$$

$$=2\int_{0}^{a} f_{even}(x)\;dx$$

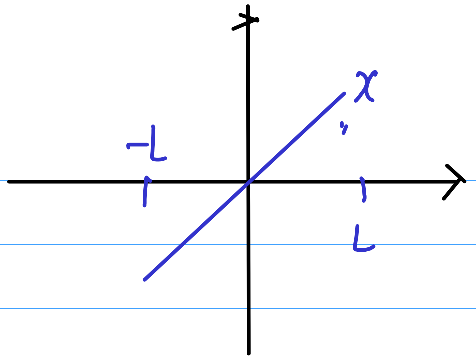
$\cos(\pi) = -1$

$\cos(2\pi) = +1$

$\cos(3\pi) = -1$

$\cos(4\pi) = +1$

$\underline{\cos(n\pi)} = \underline{(-1)^n}$



$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\int_{-L}^{L} x\,dx = 0$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(kx)\,dx = \frac{1}{L}\int_{-L}^{L} \underline{x}\cdot\underline{\cos(kx)}\,dx = 0$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(kx)\,dx = \frac{1}{L}\underline{\int_{-L}^{L} x\sin(kx)\,dx}$$

$$= \frac{1}{L}\left[-\frac{1}{k}x\cos(kx) + \frac{1}{k^2}\sin(kx)\right]_{-L}^{L}$$

$$= \frac{1}{L}\left[-\frac{L}{n\pi}x\cos\left(\frac{n\pi}{L}x\right) + \left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}x\right)\right]_{-L}^{L}$$

$$= \frac{1}{L}\left[-\frac{L}{n\pi}L\cos(n\pi) + \underbrace{\left(\frac{L}{n\pi}\right)^2\sin(n\pi)}_{\longrightarrow 0}\right.$$
$$\left. - \left(-\frac{L}{n\pi}(-L)\cos(-n\pi) + \underbrace{\left(\frac{L}{n\pi}\right)^2\sin(-n\pi)}_{\longrightarrow 0}\right)\right]$$

$$= \frac{1}{L}\left(\frac{L^2}{n\pi}\right)\left(-(-1)^n - (-1)^n\right)$$

$$= -\frac{2L}{n\pi}(-1)^n$$

Laplace Transform

$$F(s) = \int_0^{\infty} f(t)e^{-st}\,dt$$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{+st}\,ds$$

Fourier Serie

$$\boxed{C_n = \frac{1}{2p}\int_{-p}^{+p} f(x)\, e^{-i\left(\frac{n\pi}{p}x\right)}\, dx}$$

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} C_n\, e^{i\left(\frac{n\pi}{p}x\right)}}$$