

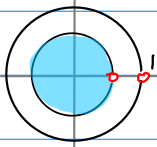
# Laurent Series and z-Transform Examples case 4.A

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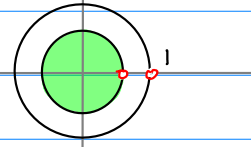
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I



$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

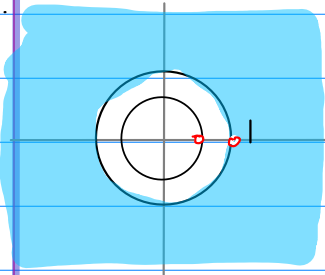
$$f(z) = \sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

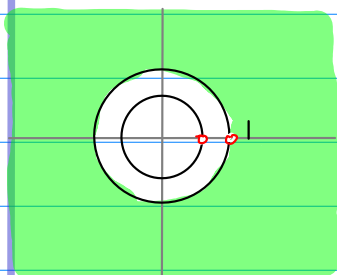
$$X(z) = \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

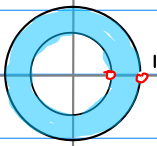
$$f(z) = \sum_{n=-1}^{\infty} [2^{n-1} - 1] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

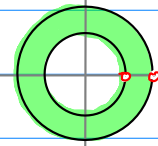
$$X(z) = \sum_{n=-1}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$

III



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = + \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n$$

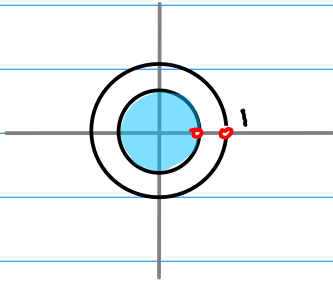


$$x_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

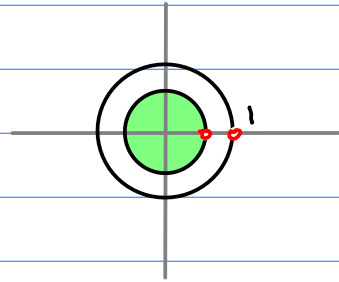
$$X(z) = + \sum_{n=-1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

4.A

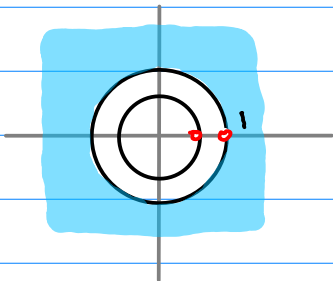
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \equiv X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



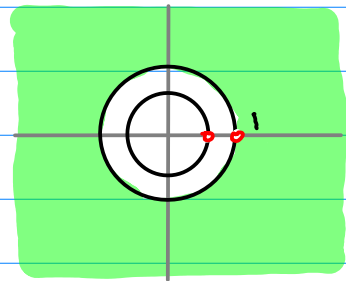
$$\sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n \xrightarrow{-n}$$



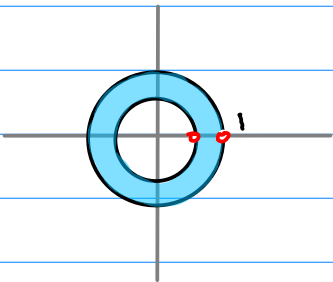
$$\sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$



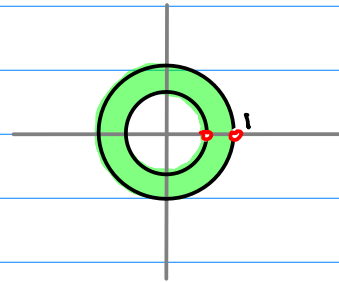
$$\sum_{n=-1}^{\infty} [2^{n-1} - 1] z^n \xrightarrow{-n}$$



$$\sum_{n=1}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$

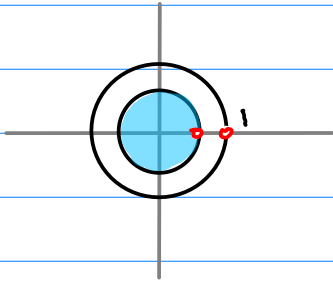


$$+\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n \xrightarrow{-n}$$

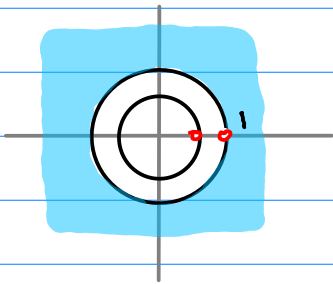


$$+\sum_{n=-1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

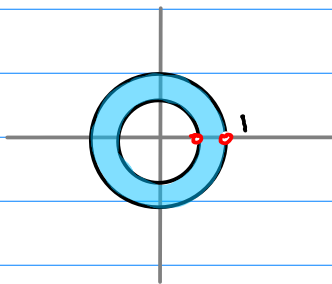
$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$



$$\begin{aligned} & + \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)} \\ & = + \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{2z}{1}\right)^n \\ & = + \sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1} \\ & = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n \end{aligned}$$

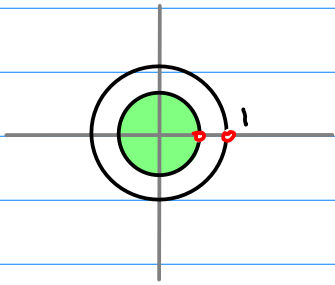


$$\begin{aligned} & - \frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} \\ & = - \sum_{n=0}^{\infty} (1) \left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n \\ & = \sum_{n=0}^{\infty} [2^{-n-1} - 1] z^{-n} \\ & = \sum_{n=1}^{\infty} [2^{n-1} - 1] z^n \end{aligned}$$



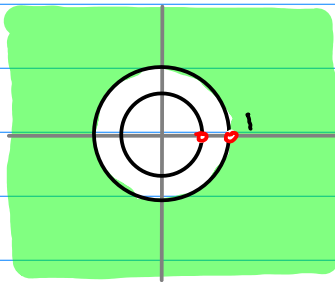
$$\begin{aligned} & + \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} \\ & = + \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n \\ & = + \sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n} \\ & = + \sum_{n=1}^{\infty} z^n + \sum_{n=1}^{\infty} 2^{n-1} z^n \end{aligned}$$

$$X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$



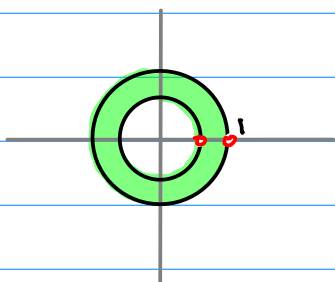
$$\sum_{n=-\infty}^{\infty} \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$

$$\begin{aligned} & + \frac{\left(\frac{z}{1}\right)}{1 - \left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1 - \left(\frac{2z}{1}\right)} \\ & = + \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{2z}{1}\right)^n \\ & = + \sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1} \\ & = \sum_{n=1}^{\infty} \left[ 1 - 2^{n-1} \right] z^n \end{aligned}$$



$$\sum_{n=-\infty}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$

$$\begin{aligned} & - \frac{(1)}{1 - \left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2z}\right)} \\ & = - \sum_{n=0}^{\infty} (1) \left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n \\ & = \sum_{n=0}^{\infty} \left[ 2^{-n-1} - 1 \right] z^{-n} \\ & = \sum_{n=-\infty}^{\infty} \left[ 2^{n-1} - 1 \right] z^n \end{aligned}$$



$$+ \sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

$$\begin{aligned} & + \frac{\left(\frac{z}{1}\right)}{1 - \left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2z}\right)} \\ & = + \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n \\ & = + \sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n} \\ & = + \sum_{n=1}^{\infty} z^n + \sum_{n=-\infty}^{\infty} 2^{n-1} z^n \end{aligned}$$

