

BVP in Rectangular Coordinates Oveview (H.1)

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Classical PDEs

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad k > 0 \quad \text{one-dim Heat eq}$$

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad \text{one-dim Wave eq}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{two-dim Laplace's eq}$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Initial Conditions

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad k > 0 \quad \text{one-dim heat eq}$$

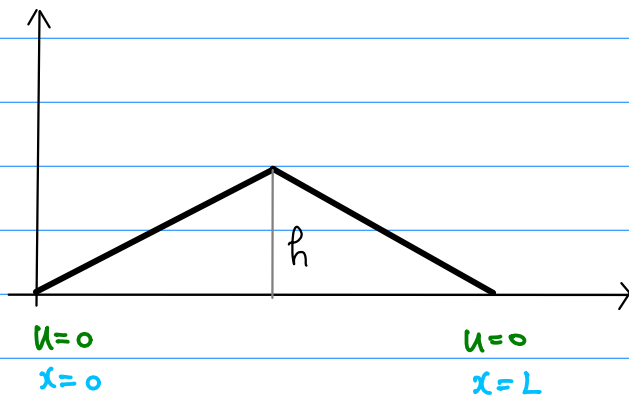
$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad \text{one-dim wave eq}$$

$$u(x, t) \longrightarrow u(x, 0) \quad \text{IC (Initial Conditions)}$$

$$\left\{ \begin{array}{l} u(x, 0) = f(x) \quad 0 < x < L \\ \frac{\partial}{\partial t} u(x, 0) = g(x) \end{array} \right.$$

$$\left\{ \begin{array}{l} u(x, t) \Big|_{t=0} = u(x, 0) = f(x) \\ \frac{\partial}{\partial t} u(x, t) \Big|_{t=0} = \frac{\partial}{\partial t} u(x, 0) = g(x) \end{array} \right.$$

Boundary Conditions



plucked string

$$u(0, t) = 0$$

$$u(L, t) = 0$$

$$t > 0$$

$$u(x, t) = f(x) \quad 0 < x < L$$

$$u(0, t) = f(0) = 0$$

$$u(L, t) = f(L) = 0$$

Three Types of BC

① u Dirichlet Condition

② $\frac{\partial u}{\partial n}$ Neuman Condition

③ $\frac{\partial u}{\partial n} + hu$ Robin Condition

$\frac{\partial u}{\partial n}$ normal derivative

directional derivative of u

in the direction perpendicular to the boundary

① u $u(L, t) = u_0$ $u_0: \text{const}$

② $\frac{\partial u}{\partial n}$ $\frac{\partial u}{\partial x} \Big|_{x=L} = 0$

③ $\frac{\partial u}{\partial n} + hu$ $\frac{\partial u}{\partial x} \Big|_{x=L} + h u(L, t) = h u_m \text{ const}$

$$\begin{cases} h > 0 & \text{const} \\ u_m & \text{const} \end{cases}$$

Boundary Value Problems

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad 0 < x < L, \quad 0 < t$$

$$(BC) \quad u(0, t) = 0, \quad u(L, t) = 0 \quad t > 0$$

$$(IC) \quad u(x, 0) = f(x), \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = g(x) \quad 0 < x < L$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad 0 < x < a \quad 0 < y < b$$

$$(BC) \quad \left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=a} = 0 \quad 0 < y < b$$

$$(BC) \quad u(x, 0) = 0, \quad u(x, b) = f(x) \quad 0 < x < a$$



Heat Equation

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad 0 < x < L, \quad t > 0$$

$$u(0, t) = 0 \quad u(L, t) = 0 \quad t > 0$$

$$u(x, 0) = f(x) \quad 0 < x < L$$

Wave Equation

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad 0 < x < L, \quad 0 < t$$

$$u(0, t) = 0, \quad u(L, t) = 0 \quad t > 0$$

$$u(x, 0) = f(x) \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = g(x) \quad 0 < x < L$$

Laplace Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad 0 < x < a, \quad 0 < y < b$$

$$\frac{\partial u}{\partial x} \Big|_{x=0} = 0, \quad \frac{\partial u}{\partial x} \Big|_{x=a} = 0 \quad 0 < y < b$$

$$u(x, 0) = 0, \quad u(x, b) = f(x) \quad 0 < x < a$$