BVP in Rectangular Coordinates Oveview (H.1)

20160520

Classical PDEs

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad k > 0$$ one-dim Heat eq

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$ one-dim Wave eq

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ two-dim Laplace's eq

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Initial Conditions

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad k > 0 \qquad \text{one-dim heat eq}$$

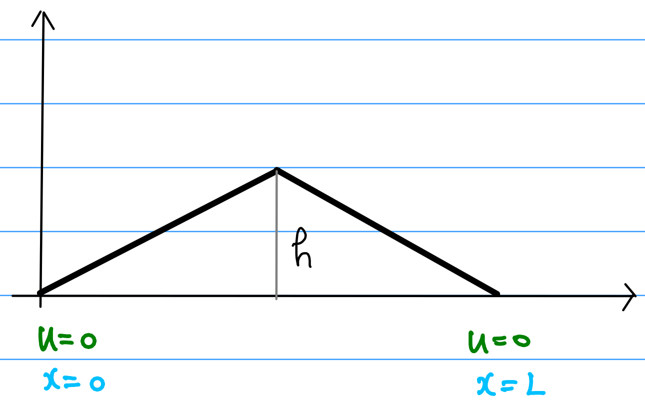
$$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad \text{one-dim wave eq}$$

$u(x,t) \longrightarrow u(x,0)$ IC (Initial Conditions)

$$\begin{cases} u(x,0) = f(x) & 0 < x < L \\ \frac{\partial}{\partial t}u(x,0) = g(x) \end{cases}$$

$$\begin{cases} u(x,t)\Big|_{t=0} = u(x,0) = f(x) \\ \frac{\partial}{\partial t}u(x,t)\Big|_{t=0} = \frac{\partial}{\partial t}u(x,0) = g(x) \end{cases}$$

Boundary Conditions



plucked string

$u(0,t)=0 \qquad u(L,t)=0 \qquad t>0$

$u(x,t) = f(x) \qquad 0<x<L$

$u(0,t) = f(0) = 0$

$u(L,t) = f(L) = 0$

Three Types of BC

① u Dirichelet Condition

② $\frac{\partial u}{\partial n}$ Neuman Condition

③ $\frac{\partial u}{\partial n} + hu$ Robin Condition

$\frac{\partial u}{\partial n}$ normal derivative

directional derivative of u

in the direction perpendicular to the boundary

① u $u(L, t) = u_0$ u_0: const

② $\frac{\partial u}{\partial n}$ $\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$

③ $\frac{\partial u}{\partial n} + hu$ $\left.\frac{\partial u}{\partial x}\right|_{x=L} + h\,u(L, t) = h\,u_m$ const

$\begin{cases} h > 0 & \text{const} \\ u_m & \text{const} \end{cases}$

Boundary Value Problems

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0 < x < L,\ 0 < t$$

(BC) $\quad u(0, t) = 0, \qquad u(L, t) = 0 \qquad t > 0$

(IC) $\quad u(x, 0) = f(x), \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x) \qquad 0 < x < L$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 < x < a \qquad 0 < y < b$$

(BC) $\quad \left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial u}{\partial x}\right|_{x=a} = 0 \qquad 0 < y < b$

(BC) $\quad u(x, 0) = 0, \qquad u(x, b) = f(x) \qquad 0 < x < a$

Heat Equation

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad 0 < x < L, \quad t > 0$$

$$u(0, t) = 0 \qquad u(L, t) = 0 \qquad t > 0$$

$$u(x, 0) = f(x) \qquad 0 < x < L$$

Wave Equation

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0<x<L, \quad 0<t$$

$$u(0,t)=0, \qquad u(L,t)=0 \qquad t>0$$

$$u(x,0)=f(x) \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x) \qquad 0<x<L$$

Laplace Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 < x < a, \quad 0 < y < b$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=a} = 0 \qquad 0 < y < b$$

$$u(x, 0) = 0, \qquad u(x, b) = f(x) \qquad 0 < x < a$$