Laurent Series and z-Transform
- Geometric Series
Causality B

20191026 Sat

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$\boxed{\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5}-\frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1}-\frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z}-\frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2}+\frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2}+\frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

2 formulas — Simple Pole Form

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each — Geometric Series Form

causal — anti-causal

$$\boxed{\frac{1}{z-p}} \to \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \triangleq f(z) = X(z^{-1})$$

$$\| \qquad \|$$

$$\boxed{\frac{1}{z-p}} \to \boxed{\frac{z^{-1}}{1-pz^{-1}}} \triangleq Y(z) = g(z^{-1})$$

causal — anti-causal

causal — anti-causal

$$\boxed{\frac{1}{z^{-1}-p}} \to \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq X(z) = f(z^{-1})$$

$$\| \qquad \|$$

$$\boxed{\frac{1}{z^{-1}-p}} \to \boxed{\frac{z}{1-pz}} \triangleq g(z) = Y(z^{-1})$$

causal — anti-causal

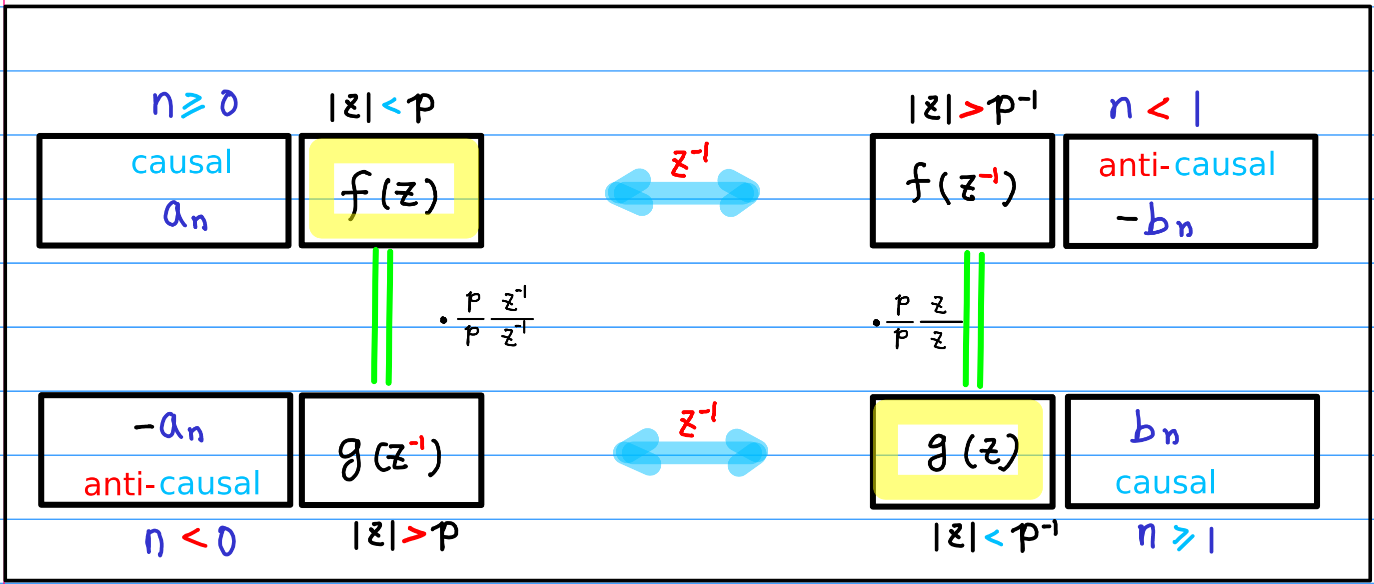
Simple Pole Form | Geometric Series Form

1

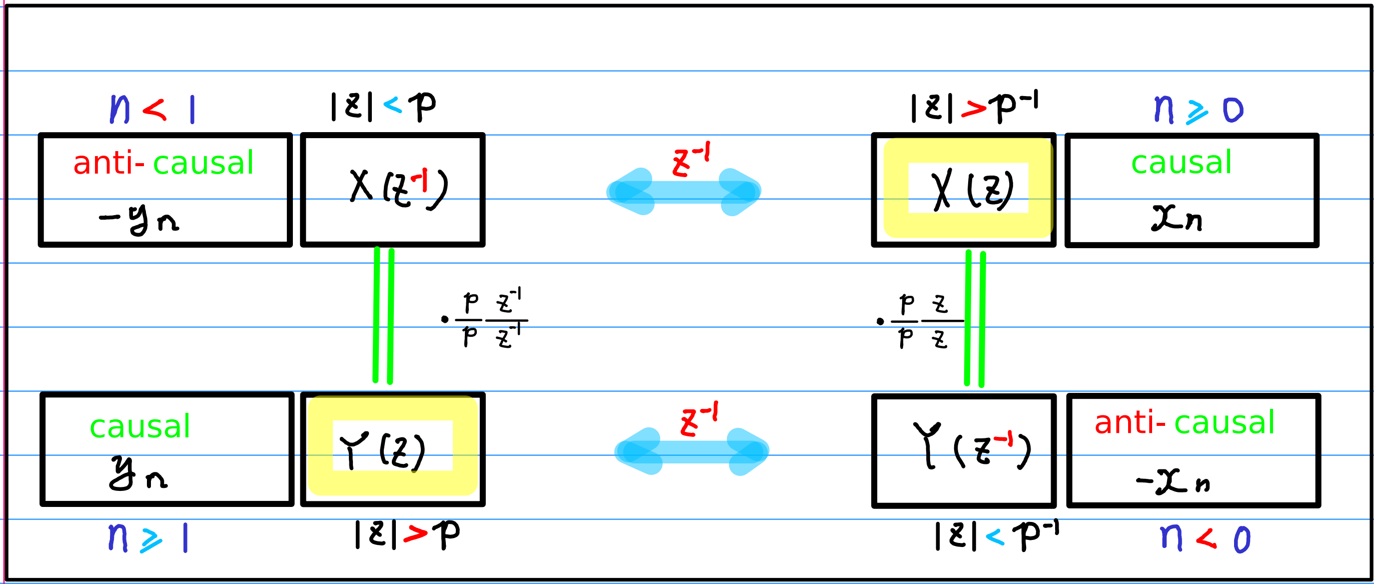
$f(z)$	$f(z^{-1})$
$g(z^{-1})$	$g(z)$

$X(z^{-1})$	$X(z)$
$Y(z)$	$Y(z^{-1})$

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$



Z- Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$



$f(z)$	$f(z^{-1})$
$f(z)$	$f(z^{-1})$

$X(z^{-1})$	$X(z)$
$X(z^{-1})$	$X(z)$

2

Laurent Series $a_n \leftrightarrow f(z)$

$n \geq 0$	$\lvert z\rvert < p$		$\lvert z\rvert > p^{-1}$	$n < 1$
causal a_n	$\boxed{f(z)}$	$\overset{z^{-1}}{\longleftrightarrow}$	$f(z^{-1})$	anti-causal a_{-n}
$-a_n$ anti-causal	$f(z)$	$\overset{z^{-1}}{\longleftrightarrow}$	$\boxed{f(z^{-1})}$	$-a_{-n}$ causal
$n < 0$	$\lvert z\rvert > p$		$\lvert z\rvert < p^{-1}$	$n \geq 1$

Z- Transform $X(z) \leftrightarrow x_n$

$n < 1$	$\lvert z\rvert < p$		$\lvert z\rvert > p^{-1}$	$n \geq 0$
anti- causal x_{-n}	$X(z^{-1})$	$\overset{z^{-1}}{\longleftrightarrow}$	$\boxed{X(z)}$	causal x_n
causal $-x_{-n}$	$\boxed{X(z^{-1})}$	$\overset{z^{-1}}{\longleftrightarrow}$	$X(z)$	anti- causal $-x_n$
$n \geq 1$	$\lvert z\rvert > p$		$\lvert z\rvert < p^{-1}$	$n < 0$

$\boxed{\begin{matrix} -(p^{-1}, p^{-2}, p^{-3}, \dots) & -(p^{-1}, p^{-2}, p^{-3}, \dots) \\ (p^{0}, p^{1}, p^{2}, \dots) & (p^{0}, p^{1}, p^{2}, \dots) \end{matrix}}$ $\boxed{\begin{matrix} -(p^{-1}, p^{-2}, p^{-3}, \dots) & -(p^{-1}, p^{-2}, p^{-3}, \dots) \\ (p^{0}, p^{1}, p^{2}, \dots) & (p^{0}, p^{1}, p^{2}, \dots) \end{matrix}}$

Laurent Series $a_n \leftrightarrow f(z)$

$$\begin{tikzcd}[column sep=small]
 & n \geq 0 & |z| < p & & |z| > p^{-1} & n < 1 & \\
a_n & \boxed{\begin{matrix}\text{causal} \\ -(p^{-1}, p^{-2}, p^{-3}, \dots)\end{matrix}} \arrow[d, leftrightarrow, "\cdot(-1)"] & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}} \arrow[d, equal] \arrow[rr, leftrightarrow, "z^{-1}"] & & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}} \arrow[d, equal] & \boxed{\begin{matrix}\text{anti-causal} \\ -(p^{-1}, p^{-2}, p^{-3}, \dots)\end{matrix}} \arrow[d, leftrightarrow, "\cdot(-1)"] & a_{-n} \\
b_n & \boxed{\begin{matrix}(p^{0}, p^{1}, p^{2}, \dots) \\ \text{anti-causal}\end{matrix}} & \boxed{\dfrac{z^{-1}}{1 - p z^{-1}}} \arrow[rr, leftrightarrow, "z^{-1}"] & & \boxed{\dfrac{z}{1 - p z}} & \boxed{\begin{matrix}(p^{0}, p^{1}, p^{2}, \dots) \\ \text{causal}\end{matrix}} & b_n \\
 & n < 0 & |z| > p & & |z| < p^{-1} & n \geq 1 &
\end{tikzcd}$$

Z- Transform $X(z) \leftrightarrow x_n$

$$\begin{tikzcd}[column sep=small]
 & n < 1 & |z| < p & & |z| > p^{-1} & n \geq 0 & \\
x_{-n} & \boxed{\begin{matrix}\text{anti-causal} \\ -(p^{-1}, p^{-2}, p^{-3}, \dots)\end{matrix}} \arrow[d, leftrightarrow, "\cdot(-1)"] & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}} \arrow[d, equal] \arrow[rr, leftrightarrow, "z^{-1}"] & & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}} \arrow[d, equal] & \boxed{\begin{matrix}\text{causal} \\ -(p^{-1}, p^{-2}, p^{-3}, \dots)\end{matrix}} \arrow[d, leftrightarrow, "\cdot(-1)"] & x_n \\
y_n & \boxed{\begin{matrix}(p^{0}, p^{1}, p^{2}, \dots) \\ \text{causal}\end{matrix}} & \boxed{\dfrac{z^{-1}}{1 - p z^{-1}}} \arrow[rr, leftrightarrow, "z^{-1}"] & & \boxed{\dfrac{z}{1 - p z}} & \boxed{\begin{matrix}(p^{0}, p^{1}, p^{2}, \dots) \\ \text{anti-causal}\end{matrix}} & y_{-n} \\
 & n \geq 1 & |z| > p & & |z| < p^{-1} & n < 0 &
\end{tikzcd}$$

$-\dfrac{p^{-1}}{1-p^{-1}z}$	$-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$
$\dfrac{z^{-1}}{1-pz^{-1}}$	$\dfrac{z}{1-pz}$

$-\dfrac{p^{-1}}{1-p^{-1}z}$	$-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$
$\dfrac{z^{-1}}{1-pz^{-1}}$	$\dfrac{z}{1-pz}$

Laurent Series $a_n \leftrightarrow f(z)$

	$n\in[0,\infty)$	$\lvert z\rvert<p$		$\lvert z\rvert>p^{-1}$	$n\in(-\infty,0]$	
a_n	causal $-(p^{-1})^{n+1}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$	$\xleftrightarrow{z^{-1}}$	$-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$	anti-causal $-(p^{-1})^{-n+1}$	a_{-n}
	$\updownarrow \cdot(-1)$	$\Vert$		$\Vert$	$\updownarrow \cdot(-1)$	
b_n	$(p^{-1})^{n+1}$ anti-causal	$\dfrac{z^{-1}}{1-pz^{-1}}$	$\xleftrightarrow{z^{-1}}$	$\dfrac{z}{1-pz}$	$(p^{-1})^{-n+1}$ causal	b_n
	$n\in(-\infty,-1]$	$\lvert z\rvert>p$		$\lvert z\rvert<p^{-1}$	$n\in[1,\infty)$	

Z- Transform $X(z) \leftrightarrow x_n$

	$n\in(-\infty,0]$	$\lvert z\rvert<p$		$\lvert z\rvert>p^{-1}$	$n\in[0,\infty)$	
x_{-n}	anti-causal $-(p^{-1})^{-n+1}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$	$\xleftrightarrow{z^{-1}}$	$-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$	causal $-(p^{-1})^{n+1}$	x_n
	$\updownarrow \cdot(-1)$	$\Vert$		$\Vert$	$\updownarrow \cdot(-1)$	
y_n	$(p^{-1})^{-n+1}$ causal	$\dfrac{z^{-1}}{1-pz^{-1}}$	$\xleftrightarrow{z^{-1}}$	$\dfrac{z}{1-pz}$	$(p^{-1})^{n+1}$ anti-causal	y_{-n}
	$n\in[1,\infty)$	$\lvert z\rvert>p$		$\lvert z\rvert<p^{-1}$	$n\in(-\infty,-1]$	

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift right, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[l, shift left] \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[u, shift right] &
\end{tikzcd}
$$

$$-\frac{p^{-1}}{1-p^{-1}z} = -(p^{-1})(p^{-1}z)^n = -(p^{-1})^{n+1} z^n$$

$$\frac{z^{-1}}{1-pz^{-1}} = -(1)(pz^{-1})^{k+1} = -(p)^{n-1} z^{-n}$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift right, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[l, shift left] \\
\boxed{\dfrac{z}{1-pz}} \arrow[u, shift right] &
\end{tikzcd}
$$

$$-\frac{p^{-1}}{1-p^{-1}z^{-1}} = -(p^{-1})(p^{-1}z^{-1})^n = -(p^{-1})^{n+1} z^{-n}$$

$$\frac{z}{1-pz} = -(1)(pz)^{k+1} = -(p)^{n-1} z^n$$

① focusing functions

$f(z)$	$g(z)$
$f(z)$	$g(z)$

$Y(z)$	$X(z)$
$Y(z)$	$X(z)$

Laurent Series $a_n \leftrightarrow f(z)$

$$\begin{tikzcd}
\boxed{f(z)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{g(z)} = f(z^{-1}) \arrow[d, equal] \\
\boxed{f(z)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{g(z)} = f(z^{-1})
\end{tikzcd}$$

Z- Transform $X(z) \leftrightarrow x_n$

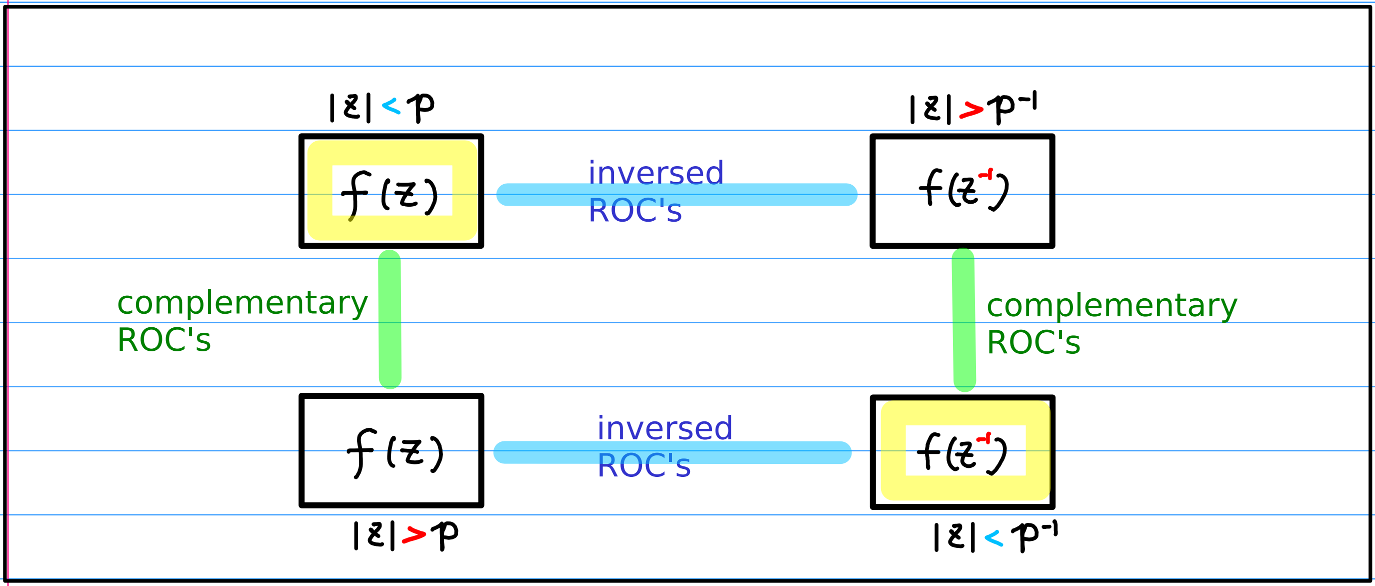
$$\begin{tikzcd}
X(z^{-1}) = \boxed{Y(z)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{X(z)} \arrow[d, equal] \\
X(z^{-1}) = \boxed{Y(z)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{X(z)}
\end{tikzcd}$$

② focusing ROC's

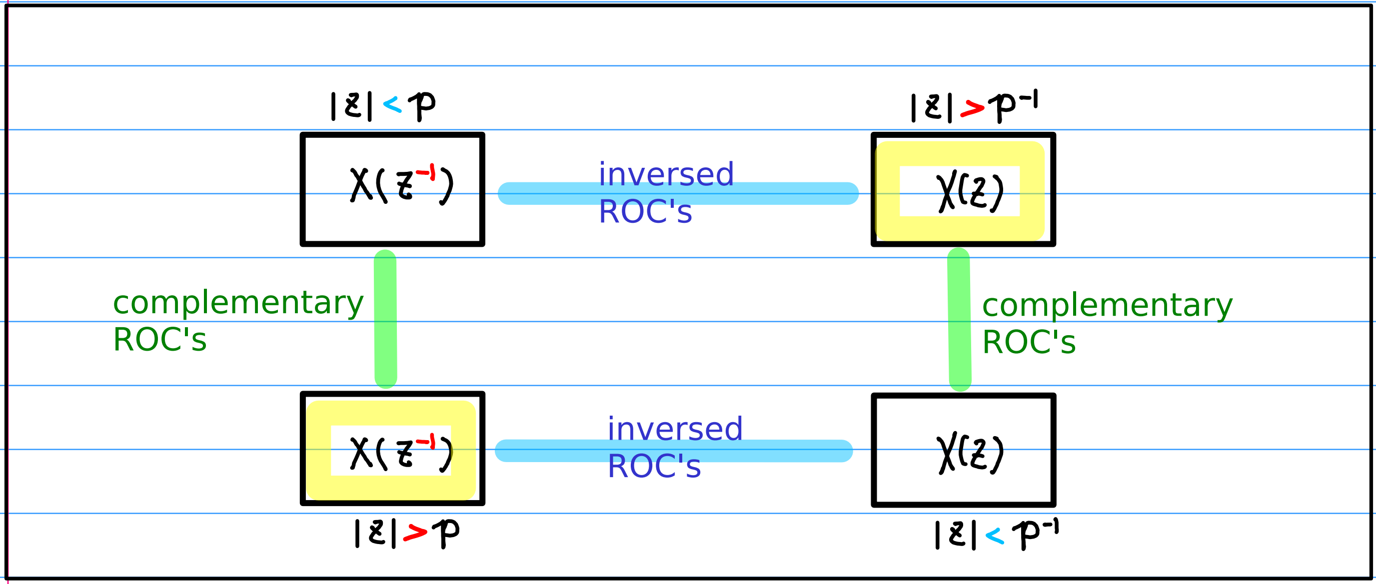
$\lvert z\rvert < \rho$	$\lvert z\rvert > \rho^{-1}$
$\lvert z\rvert > \rho$	$\lvert z\rvert < \rho^{-1}$

$\lvert z\rvert < \rho$	$\lvert z\rvert > \rho^{-1}$
$\lvert z\rvert > \rho$	$\lvert z\rvert < \rho^{-1}$

Laurent Series $a_n \leftrightarrow f(z)$



Z- Transform $X(z) \leftrightarrow x_n$



③ focusing ranges

$[0,\infty)$	$(-\infty,0]$
$(-\infty,-1]$	$[1,\infty)$

$(-\infty,0]$	$[0,\infty)$
$[1,\infty)$	$(-\infty,-1]$

Laurent Series $a_n \leftrightarrow f(z)$

$n \geq 0$
causal
a_n

symmetric ranges

$n < 1$
anti-causal
a_{-n}

complementary ranges

complementary ranges

$-a_n$
anti-causal
$n < 0$

symmetric ranges

$-a_{-n}$
causal
$n \geq 1$

Z- Transform $X(z) \leftrightarrow x_n$

$n < 1$
anti- causal
x_{-n}

symmetric ranges

$n \geq 0$
causal
x_n

complementary ranges

complementary ranges

causal
$-x_{-n}$
$n \geq 1$

symmetric ranges

anti- causal
$-x_n$
$n < 0$

4 focusing sequences

$-(p^{-1}, p^{-2}, p^{-3}, \dots)$

$(p^{0}, p^{1}, p^{2}, \dots)$

$-(p^{-1}, p^{-2}, p^{-3}, \dots)$

$(p^{0}, p^{1}, p^{2}, \dots)$

Laurent Series $a_n \leftrightarrow f(z)$

$n \geq 0$ causal

$-(p^{-1}, p^{-2}, p^{-3}, \dots)$

n= 0, 1, 2, 3,....

n= -1,-2,-3,....

$(p^{0}, p^{1}, p^{2}, \dots)$

$n < 0$ anti-causal

$n < 1$ anti-causal

$-(p^{-1}, p^{-2}, p^{-3}, \dots)$

n= 0,-1,-2,-3,....

n= 1, 2, 3,....

$(p^{0}, p^{1}, p^{2}, \dots)$

$n \geq 1$ causal

Z- Transform $X(z) \leftrightarrow x_n$

$n < 1$ anti-causal

$-(p^{-1}, p^{-2}, p^{-3}, \dots)$

n= 0,-1,-2,-3,....

n= 1, 2, 3,....

$(p^{0}, p^{1}, p^{2}, \dots)$

$n \geq 1$ causal

$n \geq 0$ causal

$-(p^{-1}, p^{-2}, p^{-3}, \dots)$

n= 0, 1, 2, 3,....

n= -1,-2,-3,....

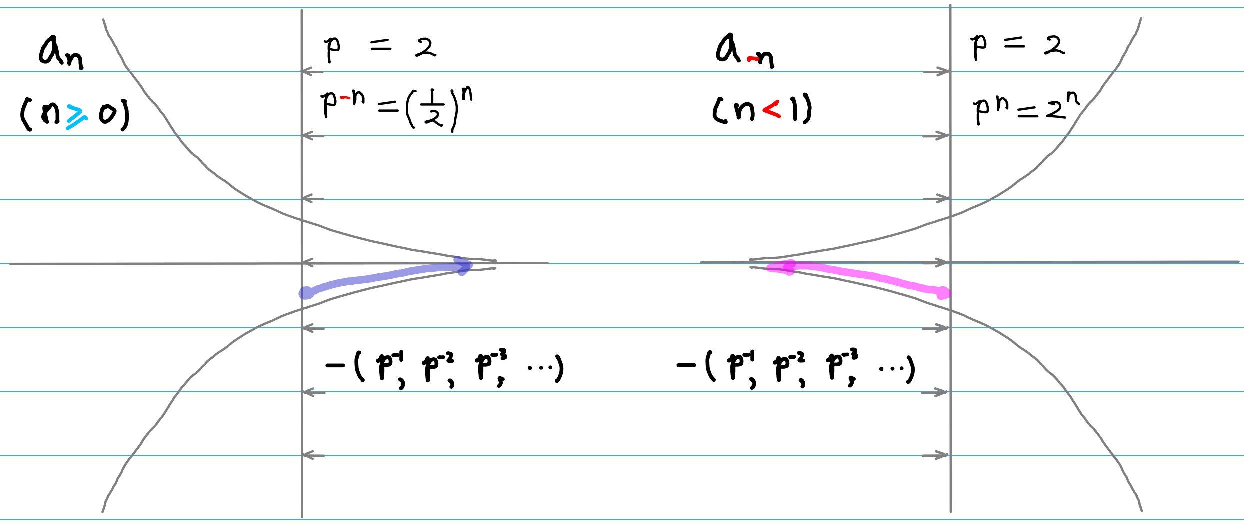
$(p^{0}, p^{1}, p^{2}, \dots)$

$n < 0$ anti-causal

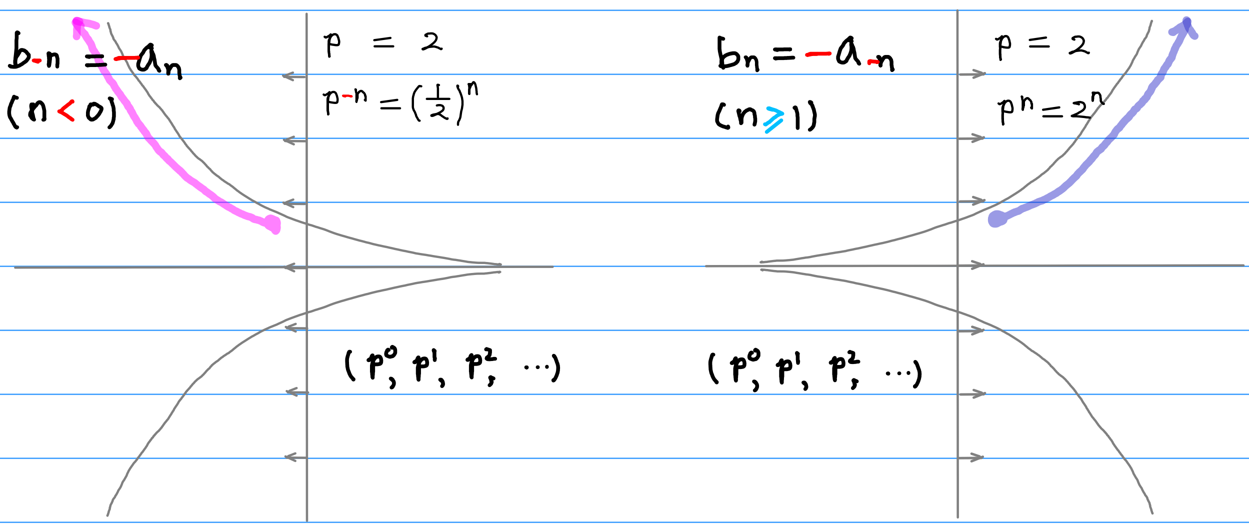
$$\boxed{\begin{matrix} a_n & a_{-n} \\ b_{-n} & b_n \end{matrix}}$$

a_n, b_n Laurent Series graphs of $f(z)$, $g(z)$

causal $f(z)$ anti-causal $f(z^{-1})$



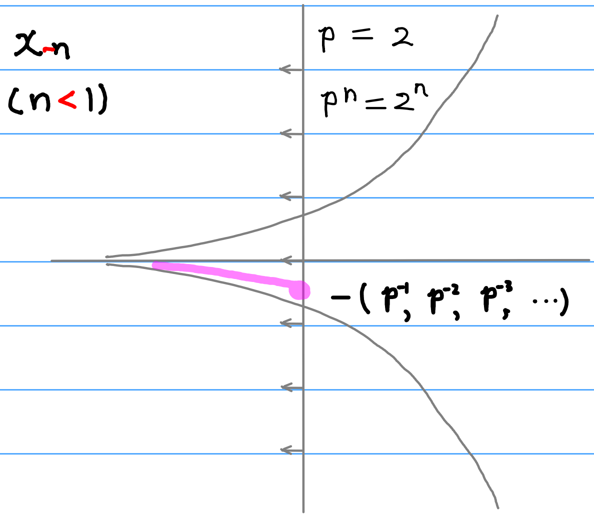
anti-causal $g(z^{-1})$ causal $g(z)$



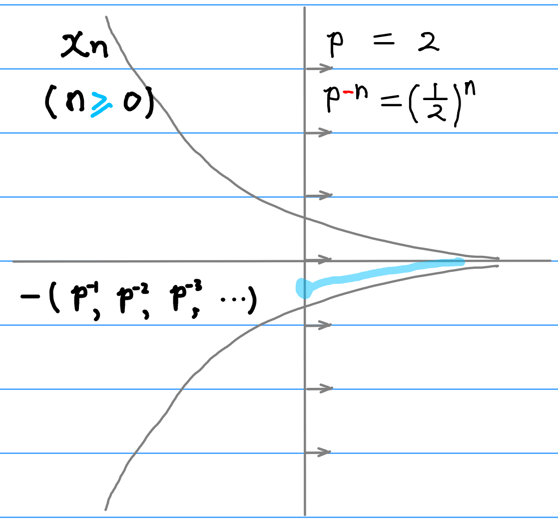
x_{-n} x_n
y_n y_{-n}

x_n, y_n graphs of z-transform $X(z)$, $Y(z)$

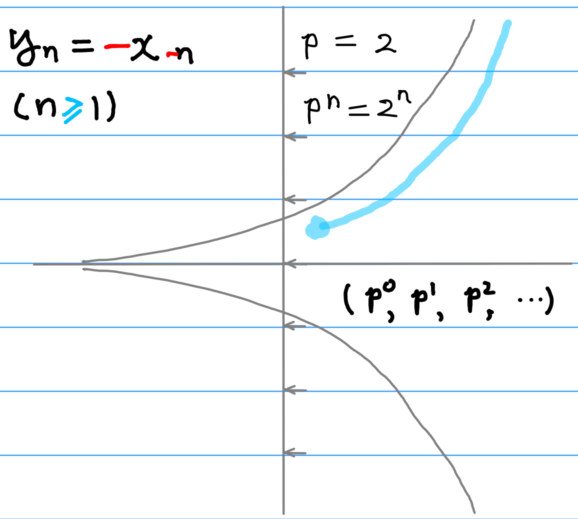
anti-causal $X(z^{-1})$



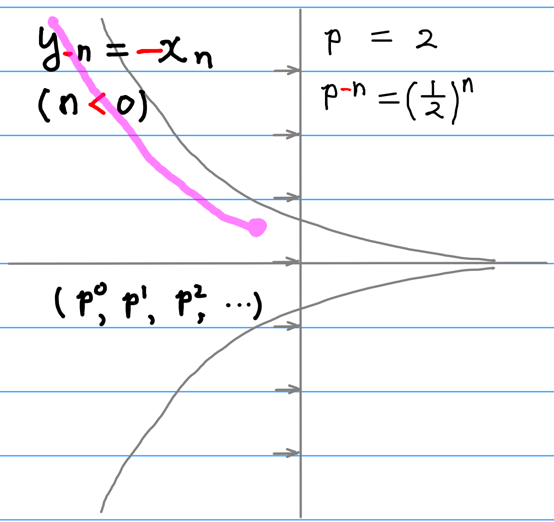
causal $X(z)$



causal $Y(z)$



anti-causal $Y(z^{-1})$



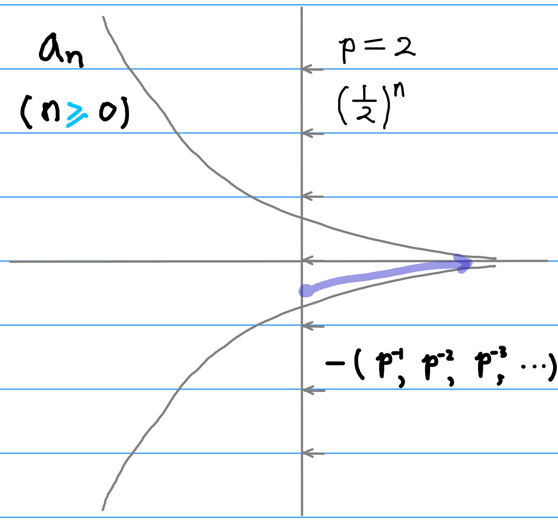
a_n a_{-n}

x_{-n} x_n

a_n, x_n graphs of $f(z)$, $X(z)$

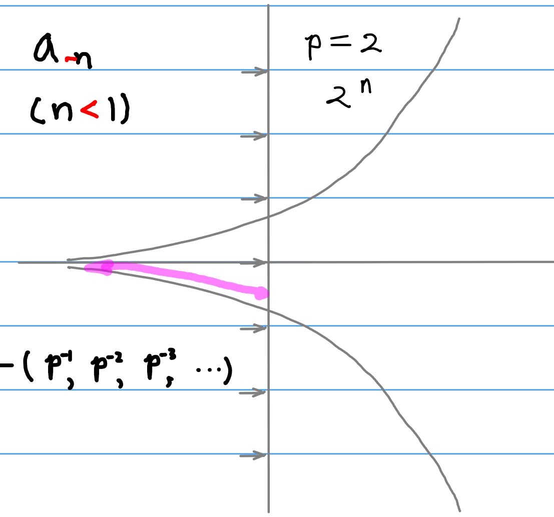
causal $f(z)$

a_n $(n \geq 0)$



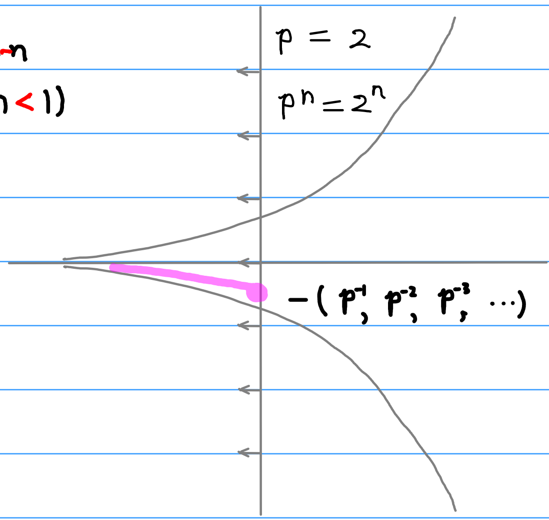
anti-causal $f(z^{-1})$

a_{-n} $(n < 1)$



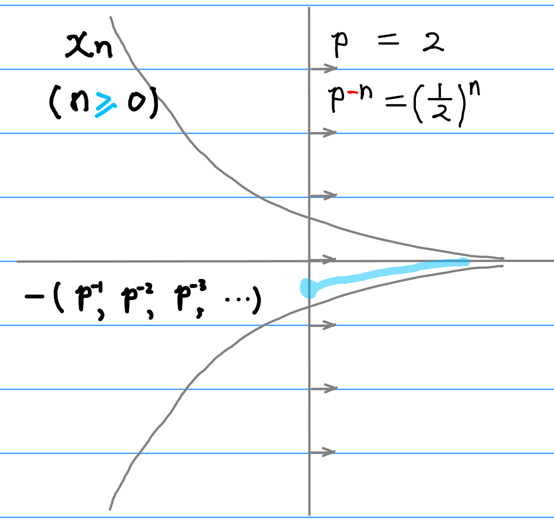
anti-causal $X(z^{-1})$

x_{-n} $(n < 1)$



causal $X(z)$

x_n $(n \geq 0)$



b_{-n}	b_n
y_n	y_{-n}

b_n, y_n graphs of $g(z)$, $Y(z)$

anti-causal $g(z^{-1})$

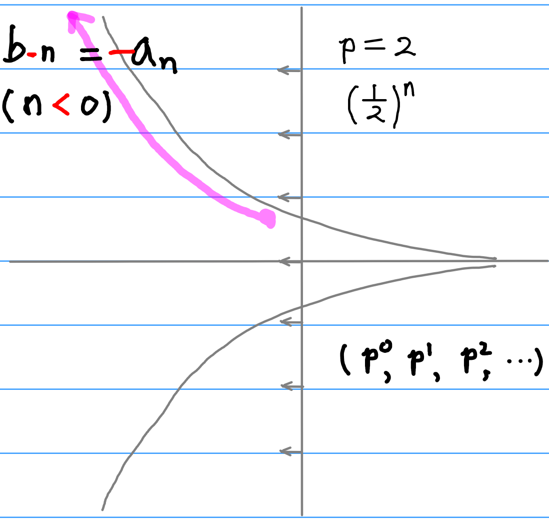
$b_{-n} = -a_n$

$(n < 0)$

$p = 2$

$(\frac{1}{2})^n$

$(p^0, p^1, p^2, \cdots)$



causal $g(z)$

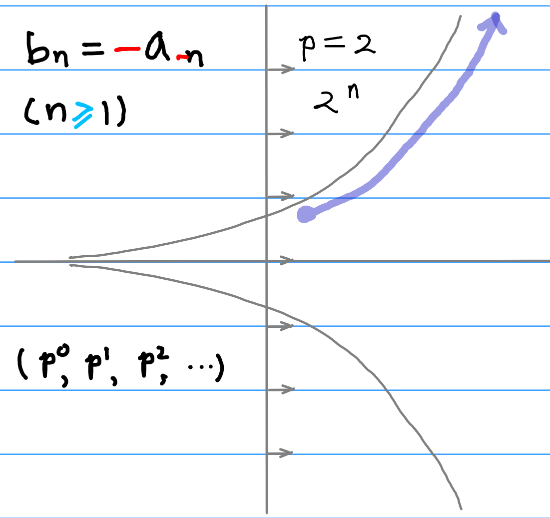
$b_n = -a_{-n}$

$(n \geq 1)$

$p = 2$

2^n

$(p^0, p^1, p^2, \cdots)$



causal $Y(z)$

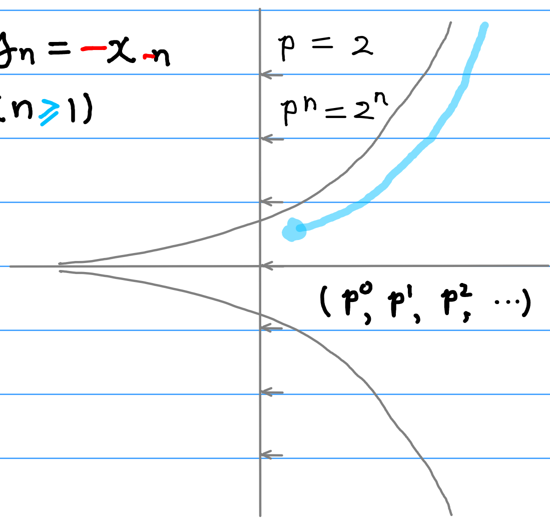
$y_n = -x_{-n}$

$(n \geq 1)$

$p = 2$

$p^n = 2^n$

$(p^0, p^1, p^2, \cdots)$



anti-causal $Y(z^{-1})$

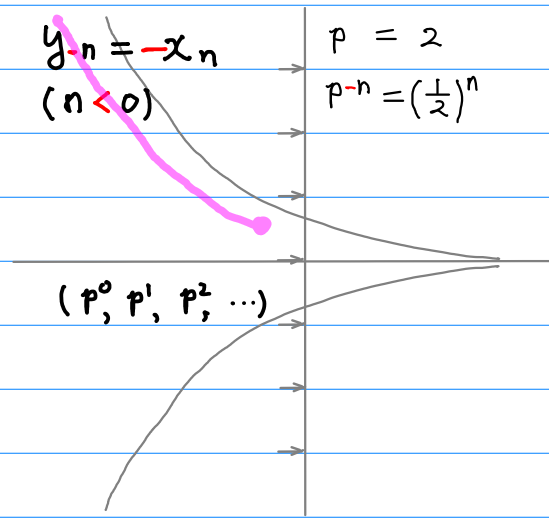
$y_{-n} = -x_n$

$(n < 0)$

$p = 2$

$p^{-n} = (\frac{1}{2})^n$

$(p^0, p^1, p^2, \cdots)$



inversed ROC's
symmetric ranges

$f(z)$ $(\lvert z\rvert < p)$	$f(z^{-1})$ $(\lvert z\rvert > p^{-1})$
a_n $(n \geq 0)$	a_{-n} $(n < 1)$
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$g(z^{-1})$ $(\lvert z\rvert > p)$	$g(z)$ $(\lvert z\rvert < p^{-1})$
b_{-n} $(n < 0)$	b_n $(n \geq 1)$
$(p^0, p^1, p^2, \cdots)$	$(p^0, p^1, p^2, \cdots)$

$X(z^{-1})$ $(\lvert z\rvert < p)$	$X(z)$ $(\lvert z\rvert > p^{-1})$
x_{-n} $(n < 1)$	x_n $(n \geq 0)$
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$Y(z)$ $(\lvert z\rvert > p)$	$Y(z^{-1})$ $(\lvert z\rvert < p^{-1})$
y_n $(n \geq 1)$	y_{-n} $(n < 0)$
$(p^0, p^1, p^2, \cdots)$	$(p^0, p^1, p^2, \cdots)$

$a_n \quad a_{-n}$

causal $f(z)$ $(|z|<p)$ anti-causal $g(z)$ $(|z|>p^{-1})$ $f(z)$ ①

$f(z) \leftrightarrow a_n \; (n \geq 0)$ $f(z^{-1}) \leftrightarrow a_{-n} \; (n<1)$

$n \geq 0$ $|z|<p$ $n = 0, 1, 2, \cdots$ $-p^{-n-1} z^n$

a_n | causal $f(z)$ $-(p^{-1})^{n+1}$ | $-\dfrac{p^{-1}}{1-p^{-1}z}$ | $-(p^{-1}+p^{-2}z^1+p^{-3}z^2+\cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^n \quad (n \geq 0)$

$n = 0, -1, -2, \cdots$ $-p^{n-1} z^n$ $|z|>p^{-1}$ $n<1$

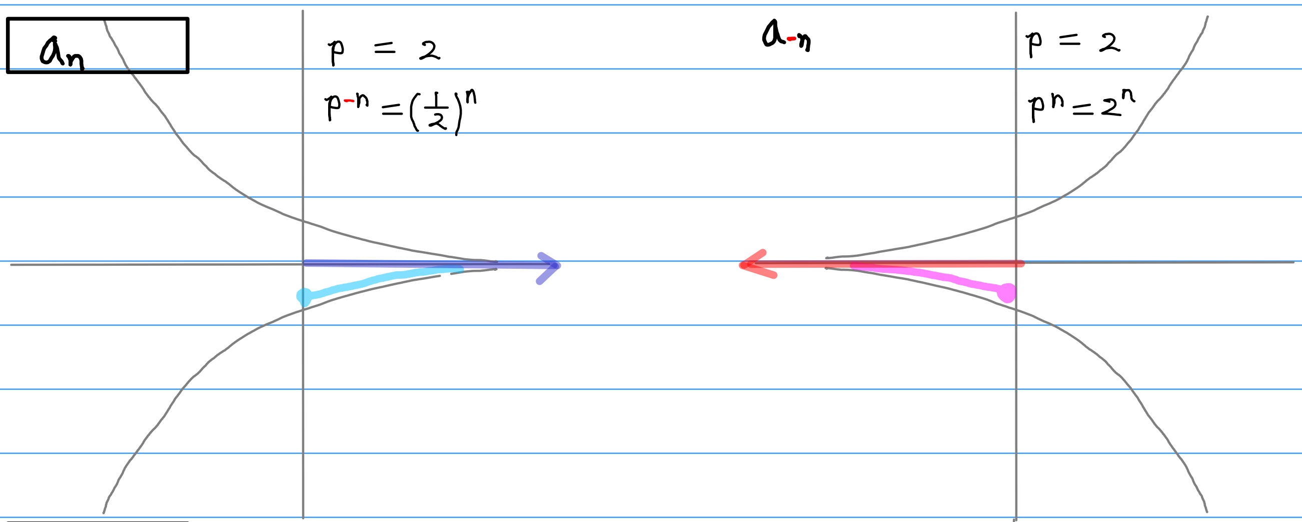
$-(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^n \quad (n<1)$ | $-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$ | anti- causal $g(z)$ $-(p^{-1})^{-n+1}$ a_{-n}

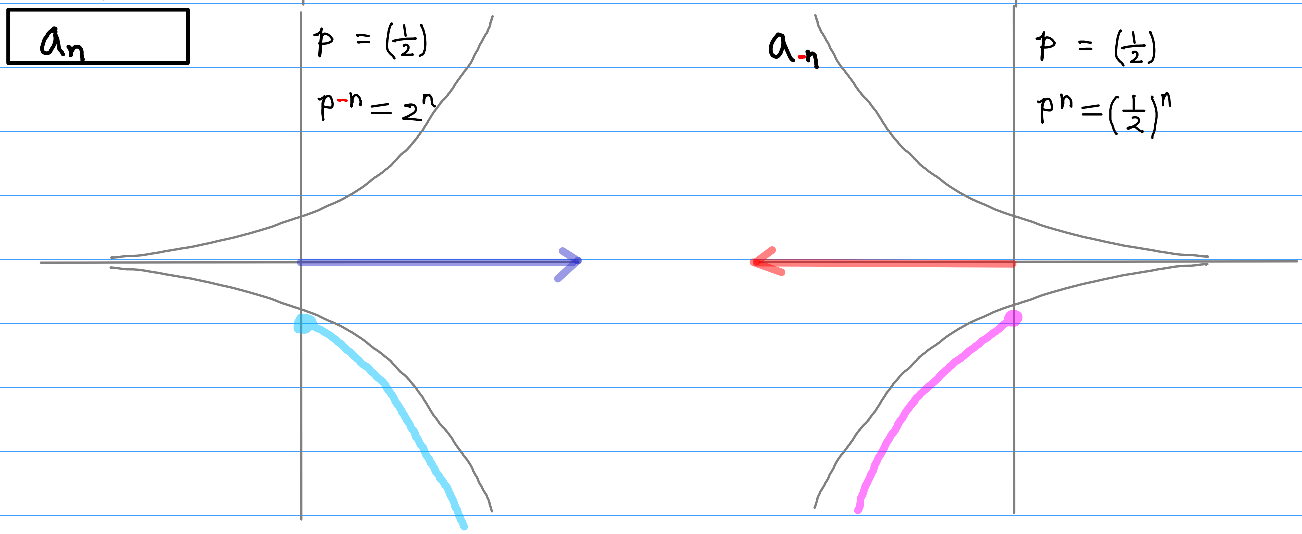
causal n=0,+1,+2,+3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

anti-causal n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$





$\boxed{b_{-n} \quad b_n}$

anti-causal $f(z)$ $(|z|>p)$ causal $g(z)$ $(|z|<p^{-1})$ $\boxed{f(z) \; ②}$

$g(z^{-1}) \longleftrightarrow b_{-n} \; (n<0)$ $\qquad g(z) \longleftrightarrow b_n \; (n \geq 1)$

---

b_{-n} $\boxed{(p)^{-n-1}}$ anti-causal f(z) $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\boxed{p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^n \quad (n<0)}$

$n<0$ $\qquad |z|>p$ $\qquad n=-1,-2,-3,\cdots$ $\qquad p^{-n-1} z^n$

$\boxed{p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^n \quad (n \geq 1)}$ $\boxed{\dfrac{z}{1-pz}}$ $\boxed{(p)^{n-1}}$ causal f(z) b_n

$n=1,2,3,\cdots$ $\qquad p^{n-1} z^n$ $\qquad |z|<p^{-1}$ $\qquad n \geq 1$

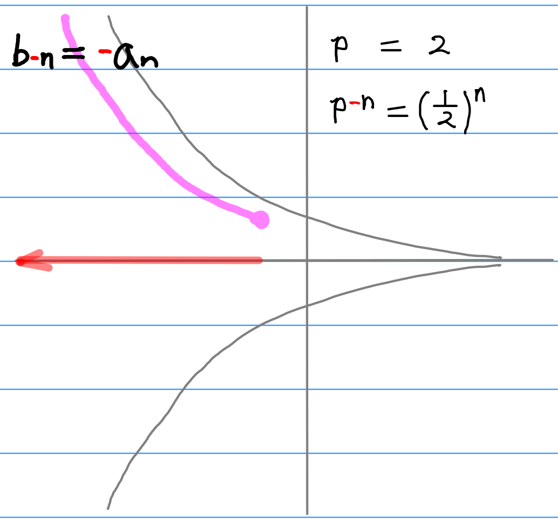
---

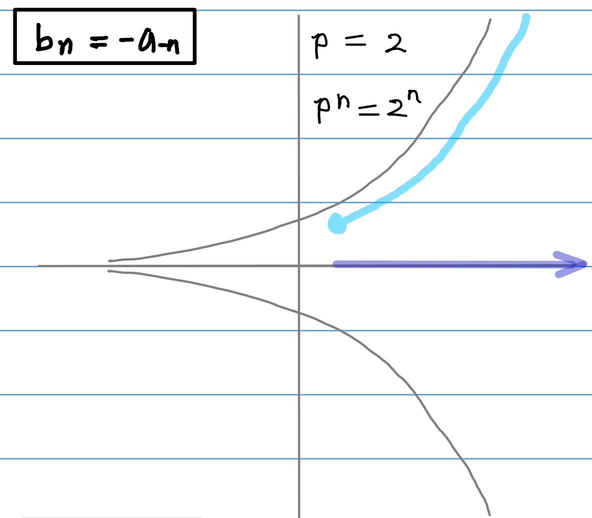
anti-causal $\quad n=-1,-2,-3,....$

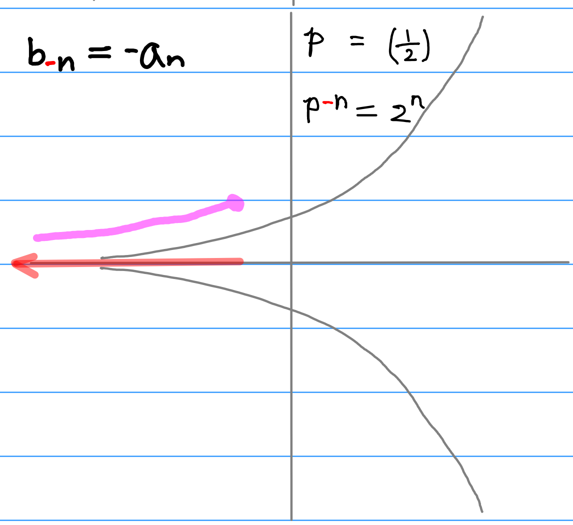
$(p^0, p^1, p^2, \cdots)$

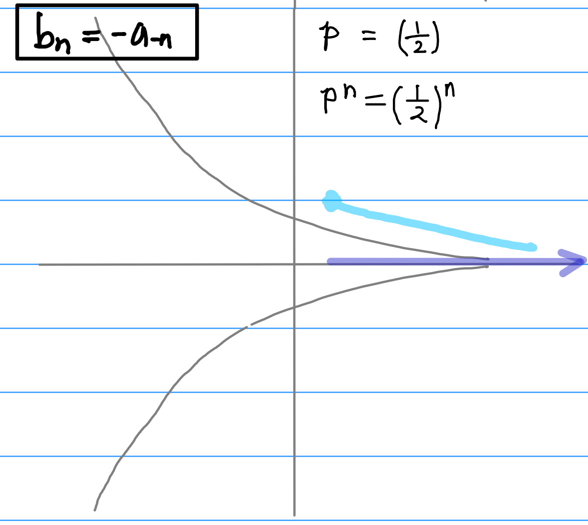
causal $\quad n=+1,+2,+3,....$

$(p^0, p^1, p^2, \cdots)$









a_n	a_{-n}
b_{-n}	b_n

$\boxed{\begin{array}{c} |z| < p \\ |z| > p^{-1} \end{array}}$ $\boxed{\begin{array}{c} |z| > p \\ |z| < p^{-1} \end{array}}$

inversed ROC's

$f(z)$ ③

anti-causal | causal

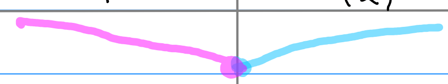
$g(z)$ $(|z| > p^{-1})$ | $f(z)$ $(|z| < p)$

a_{-n} $(n < 1)$ | a_n $(n \geq 0)$

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$ | $-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$p = 2$ | $p = 2$

$p^n = 2^n$ | $p^{-n} = (\frac{1}{2})^n$



anti-causal | causal

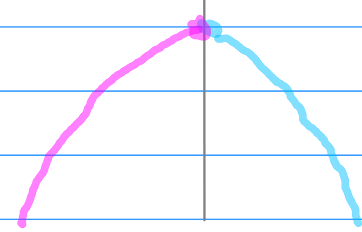
$g(z)$ $(|z| > p^{-1})$ | $f(z)$ $(|z| < p)$

a_{-n} $(n < 1)$ | a_n $(n \geq 0)$

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$ | $-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$p = (\frac{1}{2})$ | $p = (\frac{1}{2})$

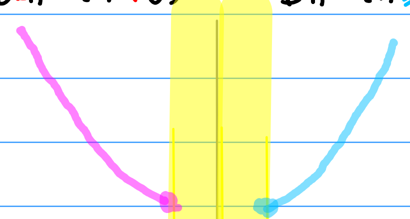
$p^n = (\frac{1}{2})^n$ | $p^{-n} = 2^n$



anti-causal | causal

$f(z)$ $(|z| > p)$ | $g(z)$ $(|z| < p^{-1})$

b_{-n} $(n < 0)$ | b_n $(n \geq 1)$



$p = 2$ | $p = 2$

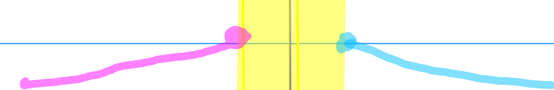
$p^{-n} = (\frac{1}{2})^n$ | $p^n = 2^n$

$(p^0, p^1, p^2, \cdots)$ | $(p^0, p^1, p^2, \cdots)$

anti-causal | causal

$f(z)$ $(|z| > p)$ | $g(z)$ $(|z| < p^{-1})$

b_{-n} $(n < 0)$ | b_n $(n \geq 1)$



$p = (\frac{1}{2})$ | $p = (\frac{1}{2})$

$p^{-n} = 2^n$ | $p^n = (\frac{1}{2})^n$

$(p^0, p^1, p^2, \cdots)$ | $(p^0, p^1, p^2, \cdots)$

x_{-n} x_n

anti-causal $Y(z)$ $(|z|<p)$ — causal $X(z)$ $(|z|<p^{-1})$ — $X(z)$ ①

$X(z^{-1}) \longleftrightarrow x_{-n}$ $(n<1)$ — $X(z) \longleftrightarrow x_n$ $(n \geq 1)$

$n<1$ $\quad |z|<p$ $\quad n = 0, -1, -2, \cdots$ $\quad -p^{n-1}z^{-n}$

x_{-n}: anti-causal Y(z) $\boxed{-(p^{-1})^{-n+1}}$ $\quad \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$

$$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots) = -\sum_{n=0}^{-\infty}(p^{-1})^{-n+1}z^{-n} \qquad n<1$$

$n = 0, 1, 2, \cdots$ $\quad -p^{-n-1}z^{-n}$ $\quad |z|>p^{-1}$ $\quad n \geq 0$

$$-(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{\infty}-(p^{-1})^{n+1}z^{-n} \qquad (n \geq 0)$$

$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\quad$ causal X(z) $\boxed{-(p^{-1})^{n+1}}$ $\quad x_n$

---

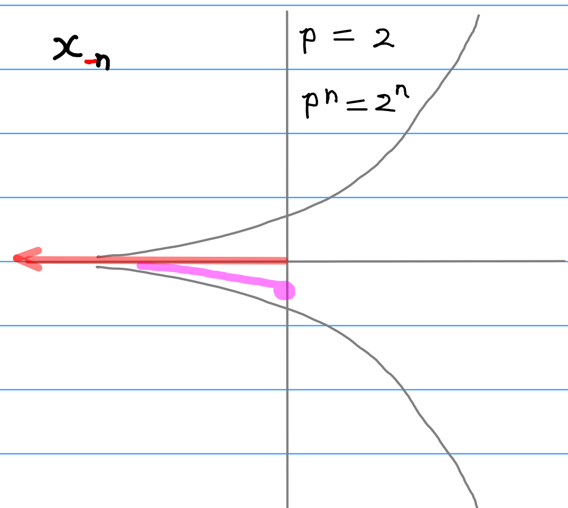
anti-causal $\quad$ n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

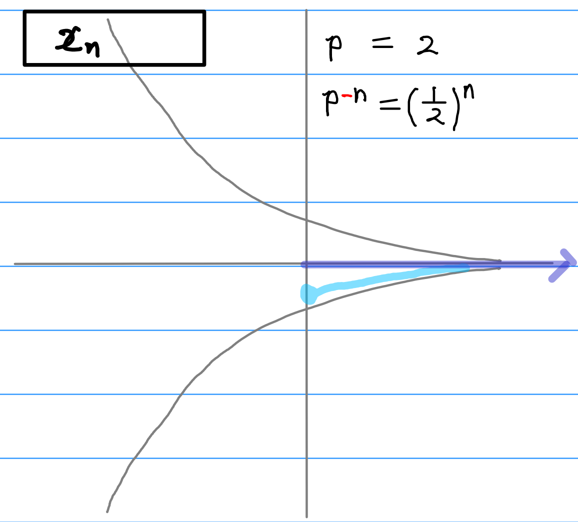
causal $\quad$ n=0,+1,+2,+3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

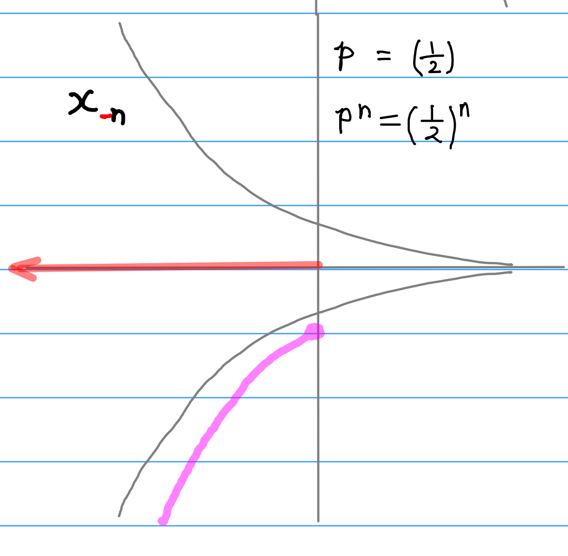
x_{-n}



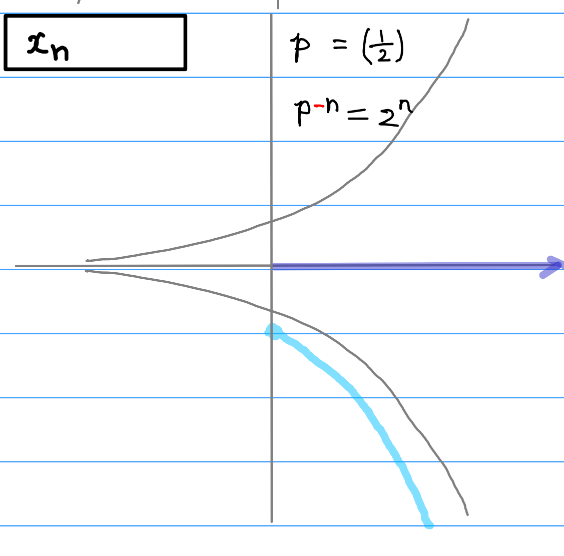
x_n



x_{-n}



x_n



$y_n \;\; y_{-n}$

causal $Y(z)$ $(|z|>p)$ anti-causal $X(z)$ $(|z|>p^{-1})$

$Y(z) \leftrightarrow y_n$ $(n \geq 0)$ $Y(z^{-1}) \leftrightarrow -y_n$ $(n<1)$

$X(z)$ ②

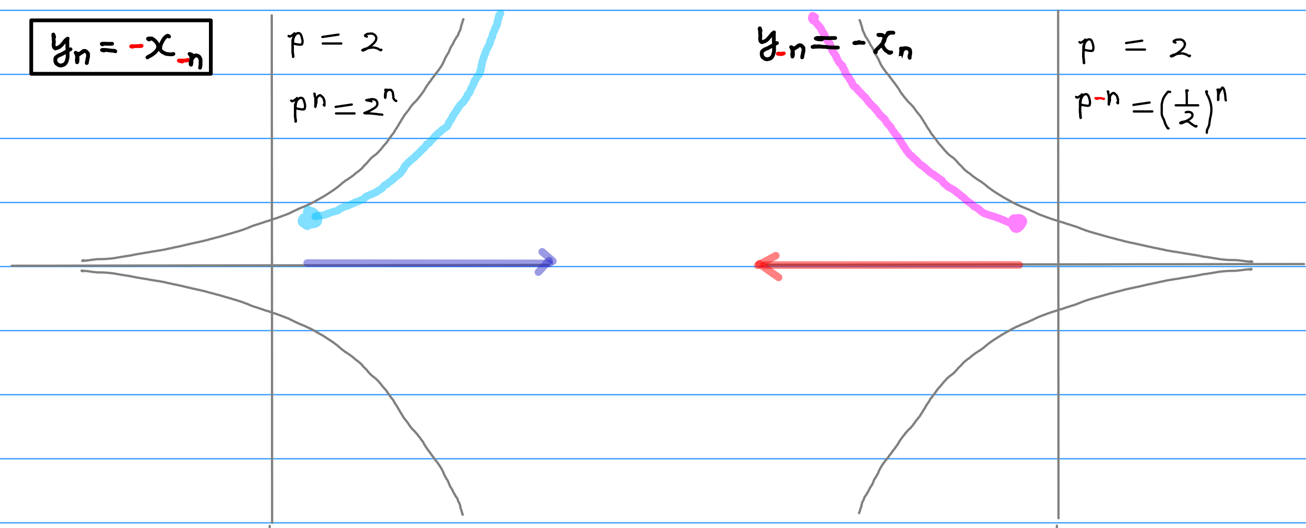
y_n	$(p)^{n-1}$ causal Y(z)	$\dfrac{z^{-1}}{1-pz^{-1}}$	$p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^{-n}$ $\quad n \geq 1$
	$n \geq 1$	$\lvert z\rvert > p$	$n = 1, 2, 3, \cdots$ $\quad p^{n-1} z^{-n}$

$p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=-1}^{-\infty} (p^{-1})^{n+1} z^{-n}$ $\quad (n<0)$	$\dfrac{z}{1-pz}$	$(p)^{-n-1}$ anti-causal X(z)	y_{-n}
$n = -1, -2, -3, \cdots$ $\quad p^{-n-1} z^{-n}$	$\lvert z\rvert < p^{-1}$	$n < 0$	

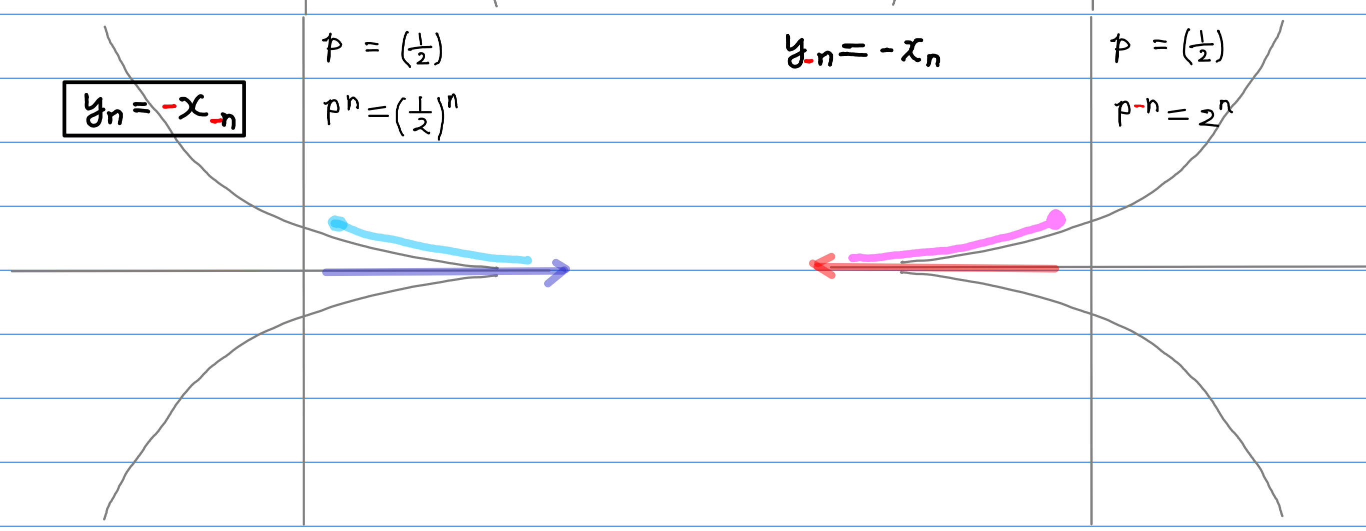
causal n=+1,+2,+3,.... anti-causal n=-1,-2,-3,....

$(p^0, p^1, p^2, \cdots)$ $\quad$ $(p^0, p^1, p^2, \cdots)$

$\boxed{y_n = -x_{-n}}$



$\boxed{y_n = -x_{-n}}$



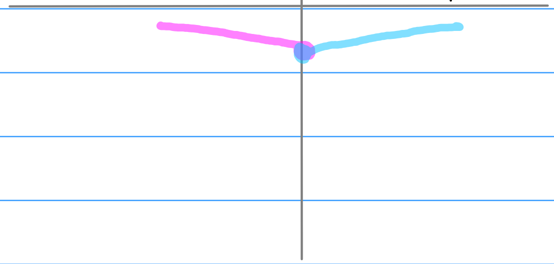
x_{-n} x_n
y_n y_{-n}

$$\boxed{\begin{array}{c}|z| > p \\ |z| < p^{-1}\end{array}} \qquad \boxed{\begin{array}{c}|z| < p \\ |z| > p^{-1}\end{array}}$$

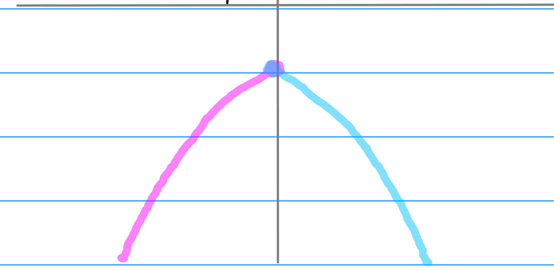
inversed ROC's

$\boxed{X(z) \text{ ③}}$

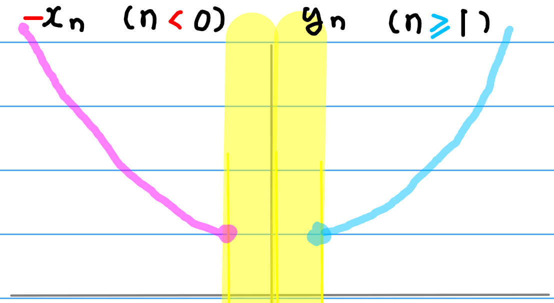
anti-causal	causal
$Y(z)$ $(\lvert z\rvert < p)$	$X(z)$ $(\lvert z\rvert > p^{-1})$
$-y_n$ $(n < 1)$	x_n $(n \geq 0)$
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$
$p = 2$	$p = 2$
$p^n = 2^n$	$p^{-n} = (\frac{1}{2})^n$



anti-causal	causal
$Y(z)$ $(\lvert z\rvert < p^{-1})$	$X(z)$ $(\lvert z\rvert > p)$
$-y_n$ $(n < 1)$	x_n $(n \geq 0)$
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$
$p = (\frac{1}{2})$	$p = (\frac{1}{2})$
$p^n = (\frac{1}{2})^n$	$p^{-n} = 2^n$

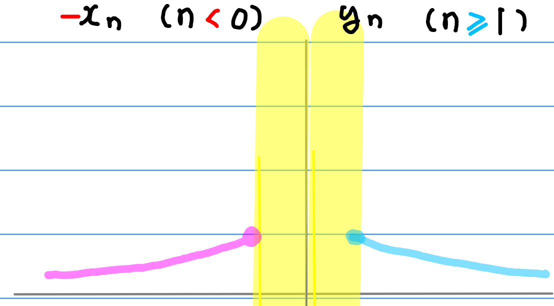


anti-causal	causal
$X(z)$ $(\lvert z\rvert < p^{-1})$	$Y(z)$ $(\lvert z\rvert > p)$
$-x_n$ $(n < 0)$	y_n $(n \geq 1)$



$p = 2$	$p = 2$
$p^{-n} = (\frac{1}{2})^n$	$p^n = 2^n$
$(p^0, p^1, p^2, \cdots)$	$(p^0, p^1, p^2, \cdots)$

anti-causal	causal
$X(z)$ $(\lvert z\rvert < p)$	$Y(z)$ $(\lvert z\rvert > p^{-1})$
$-x_n$ $(n < 0)$	y_n $(n \geq 1)$



$p = (\frac{1}{2})$	$p = (\frac{1}{2})$
$p^{-n} = 2^n$	$p^n = (\frac{1}{2})^n$
$(p^0, p^1, p^2, \cdots)$	$(p^0, p^1, p^2, \cdots)$

complementary ROC's

complementary ranges

$$f(z) \quad (|z| < p)$$
$$a_n \quad (n \geq 0)$$
$$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$$

$$g(z^{-1}) \quad (|z| > p)$$
$$b_{-n} \quad (n < 0)$$
$$(p^0, p^1, p^2, \cdots)$$

$$f(z^{-1}) \quad (|z| > p^{-1})$$
$$a_{-n} \quad (n < 1)$$
$$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$$

$$g(z) \quad (|z| < p^{-1})$$
$$b_n \quad (n \geq 1)$$
$$(p^0, p^1, p^2, \cdots)$$

$$X(z^{-1}) \quad (|z| < p)$$
$$x_{-n} \quad (n < 1)$$
$$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$$

$$Y(z) \quad (|z| > p)$$
$$y_n \quad (n \geq 1)$$
$$(p^0, p^1, p^2, \cdots)$$

$$X(z) \quad (|z| > p^{-1})$$
$$x_n \quad (n \geq 0)$$
$$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$$

$$Y(z^{-1}) \quad (|z| < p^{-1})$$
$$y_{-n} \quad (n < 0)$$
$$(p^0, p^1, p^2, \cdots)$$

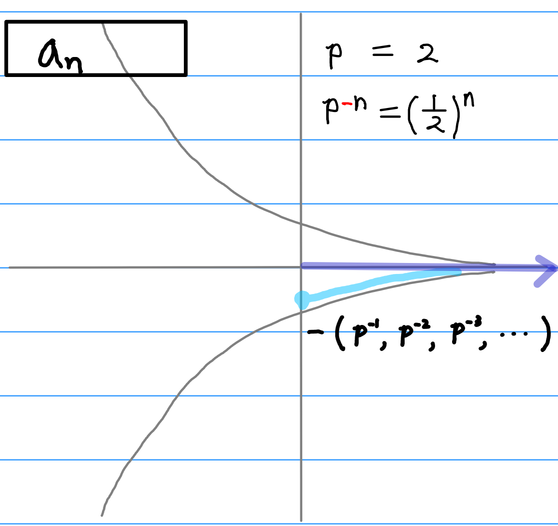
a_n
b_{-n}

causal $f(z)$ $(|z|<p)$ — anti-causal $f(z)$ $(|z|>p)$ — $f(z)$ ④

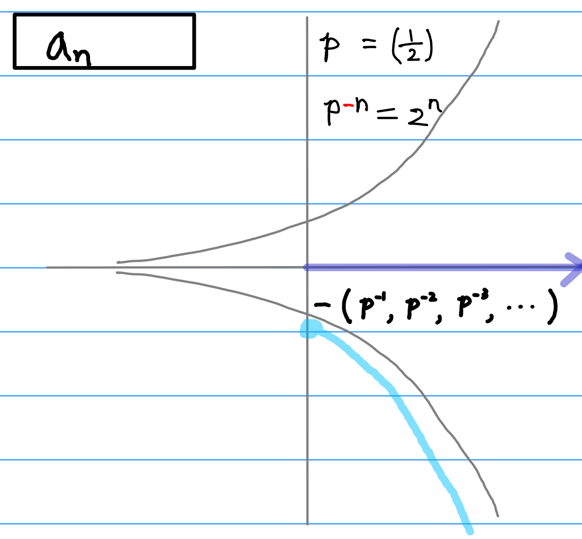
$f(z) \leftrightarrow a_n$ $(n \geq 0)$ — $g(z^{-1}) \leftrightarrow b_{-n}$ $(n<0)$

	$n \geq 0$	$\lvert z\rvert < p$	$n = 0, 1, 2, \cdots$	$-p^{-n-1} z^n$
a_n	causal f(z) $-(p^{-1})^{n+1}$	$-\frac{p^{-1}}{1-p^{-1}z}$	$-(p^{-1}+p^{-2}z^1+p^{-3}z^2+\cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^n$	$(n \geq 0)$
	$\cdot(-1)$	$\cdot \frac{p}{p} \frac{z^{-1}}{z^{-1}}$		
b_{-n}	$(p)^{-n-1}$ anti- causal f(z)	$\frac{z^{-1}}{1-pz^{-1}}$	$p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^n$	$(n<0)$
	$n<0$	$\lvert z\rvert > p$	$n = -1, -2, -3, \cdots$	$p^{-n-1} z^n$

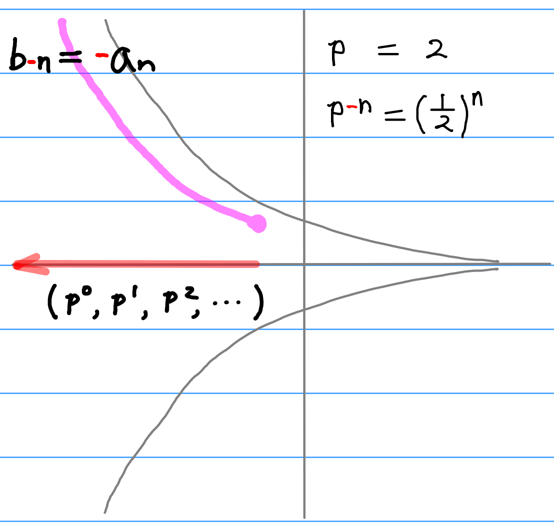
causal n=0,+1,+2,+3,....



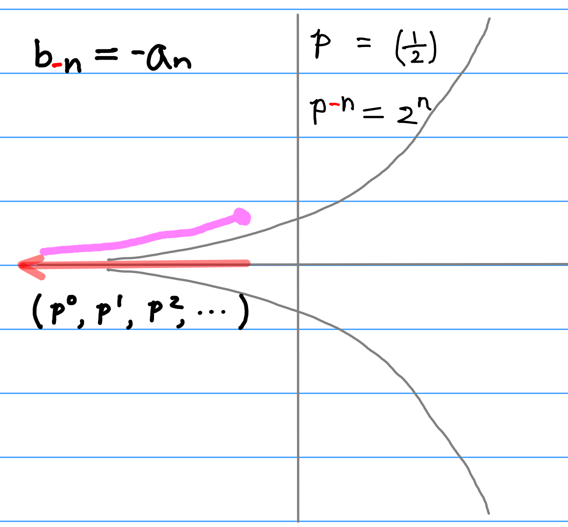
causal n=0,+1,+2,+3,....



anti-causal n=-1,-2,-3,....



anti-causal n=-1,-2,-3,....



a_{-n}

b_n

anti-causal $g(z)$ $(|z| > p^{-1})$ causal $g(z)$ $(|z| < p^{-1})$

$f(z^{-1}) \leftrightarrow a_{-n}$ $(n < 1)$ $g(z) \leftrightarrow b_n$ $(n \geq 1)$

$f(z)$ 5

$n = 0, -1, -2, \cdots$ $-p^{n-1} z^n$

$$\boxed{-(p^{-1} + p^{-2}z^{-1} + p^{-3}z^{-2} + \cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^n \quad (n < 1)}$$

$|z| > p^{-1}$ $n < 1$

$$\boxed{-\frac{p^{-1}}{1 - p^{-1}z^{-1}}} \quad \boxed{\begin{array}{c}\text{anti- causal } g(z) \\ -(p^{-1})^{-n+1}\end{array}} \; a_{-n}$$

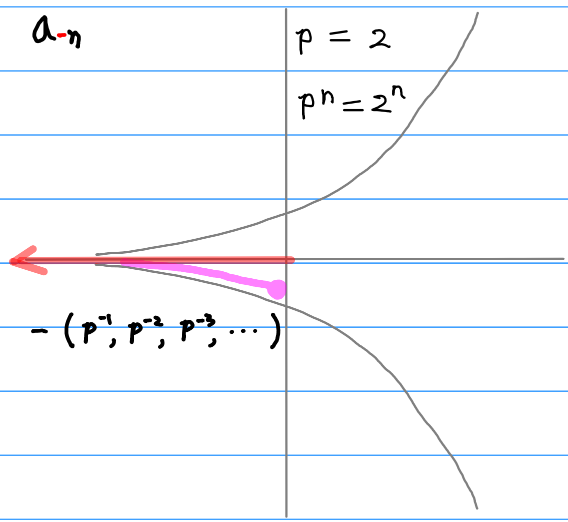
$\cdot \frac{p}{p}\frac{z}{z}$ $\cdot(-1)$

$$\boxed{p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^n \quad (n \geq 1)}$$

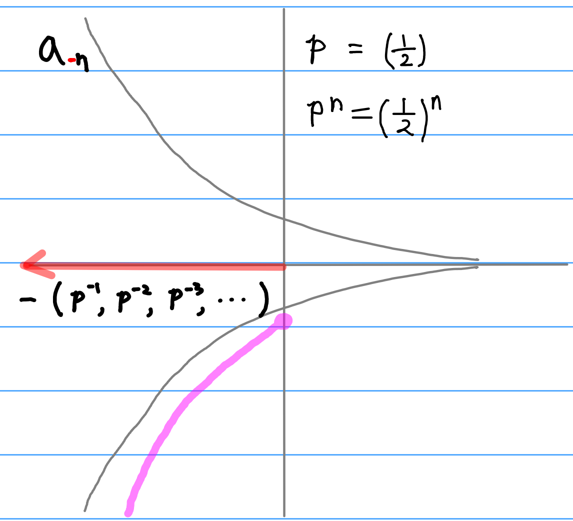
$$\boxed{\frac{z}{1 - pz}} \quad \boxed{\begin{array}{c}(p)^{n-1} \\ \text{causal } g(z)\end{array}} \; b_n$$

$n = 1, 2, 3, \cdots$ $p^{n-1} z^n$ $|z| < p^{-1}$ $n \geq 1$

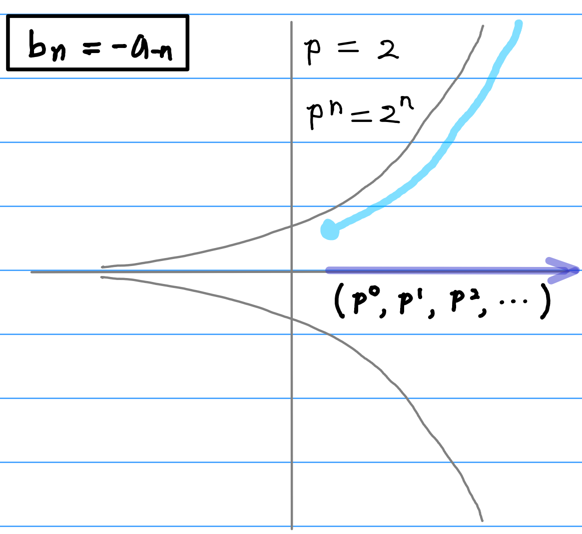
anti-causal $n = 0, -1, -2, -3, \ldots.$



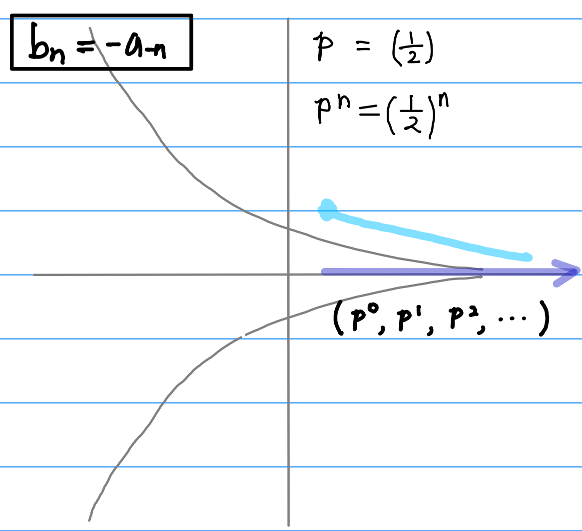
anti-causal $n = 0, -1, -2, -3, \ldots.$



causal $n = +1, +2, +3, \ldots.$



causal $n = +1, +2, +3, \ldots.$



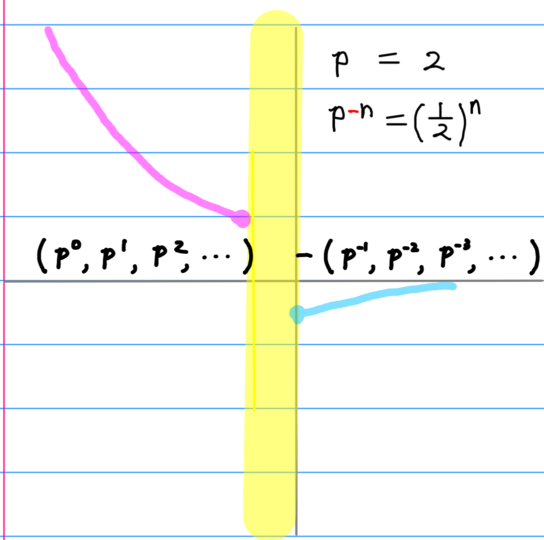
a_n	a_{-n}
b_{-n}	b_n

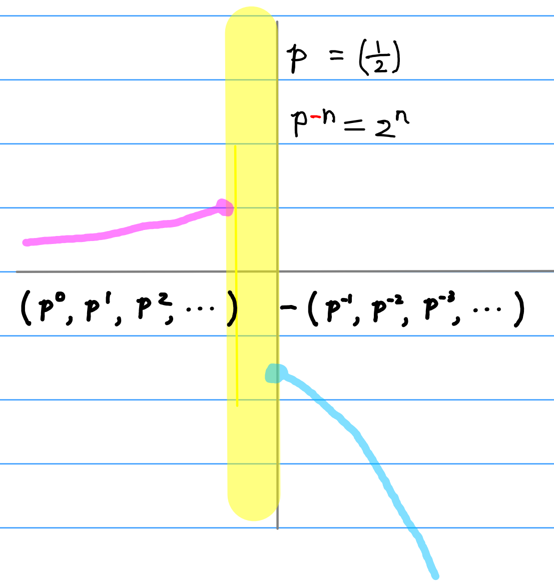
$$\boxed{\begin{array}{c}|z| < p \\ |z| > p\end{array}} \qquad \boxed{\begin{array}{c}|z| > p^{-1} \\ |z| < p^{-1}\end{array}}$$

complementary ROC's

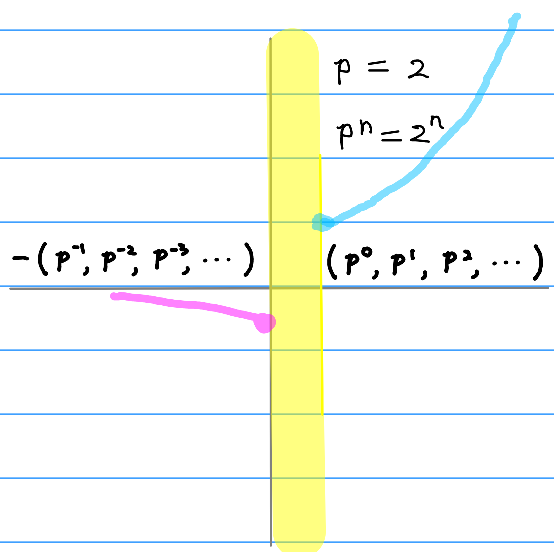
$f(z)$ ⑥

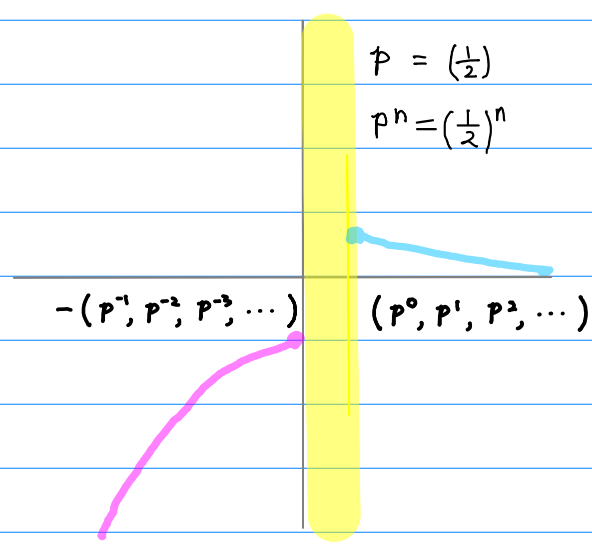
anti-causal	causal	anti-causal	causal
$f(z)$ $(\lvert z\rvert > p)$	$f(z)$ $(\lvert z\rvert < p)$	$f(z)$ $(\lvert z\rvert > p)$	$f(z)$ $(\lvert z\rvert < p)$
$-a_n$ $(n < 0)$	a_n $(n \geq 0)$	$-a_n$ $(n < 0)$	a_n $(n \geq 0)$





anti-causal	causal	anti-causal	causal
$g(z)$ $(\lvert z\rvert > p^{-1})$	$g(z)$ $(\lvert z\rvert < p^{-1})$	$g(z)$ $(\lvert z\rvert > p^{-1})$	$g(z)$ $(\lvert z\rvert < p^{-1})$
$-b_n$ $(n < 1)$	b_n $(n \geq 1)$	$-b_n$ $(n < 1)$	b_n $(n \geq 1)$





x_{-n}
y_n

anti-causal $Y(z)$ $(|z|<p)$ — $X(z^{-1}) \longleftrightarrow x_{-n}$ $(n<1)$

causal $Y(z)$ $(|z|>p)$ — $Y(z) \longleftrightarrow y_n$ $(n \geq 0)$

$X(z)$ ④

x_{-n}: $n<1$, $|z|<p$, $n = 0, -1, -2, \cdots$, $-p^{n-1}z^{-n}$

anti-causal Y(z): $-(p^{-1})^{-n+1}$ | $-\dfrac{p^{-1}}{1-p^{-1}z}$ | $-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots) = -\sum_{n=0}^{-\infty}(p^{-1})^{-n+1}z^{-n}$ $\quad n<1$

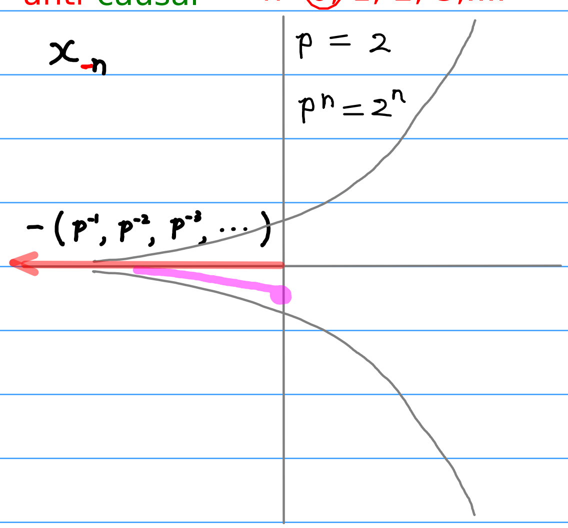
$\cdot(-1)$ $\qquad \cdot \dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$

y_n:

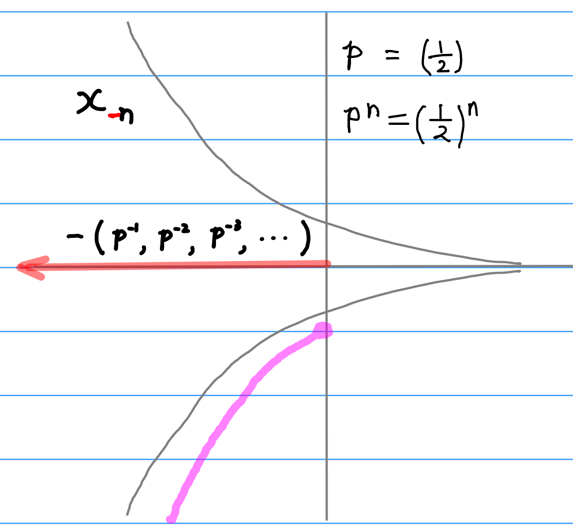
causal Y(z): $(p)^{n-1}$ | $\dfrac{z^{-1}}{1-pz^{-1}}$ | $p^{0}z^{-1}+p^{1}z^{-2}+p^{2}z^{-3}+\cdots = \sum_{n=1}^{\infty}(p)^{n-1}z^{-n}$ $\quad n \geq 1$

$n \geq 1$, $|z|>p$, $n = 1, 2, 3, \cdots$, $p^{n-1}z^{-n}$

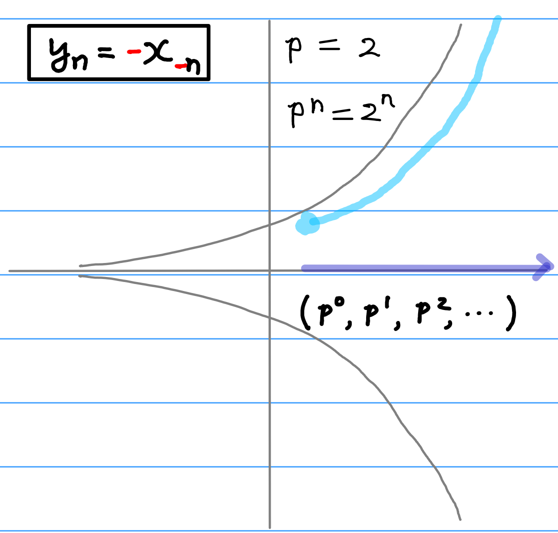
anti-causal $\quad$ n=0,-1,-2,-3,....



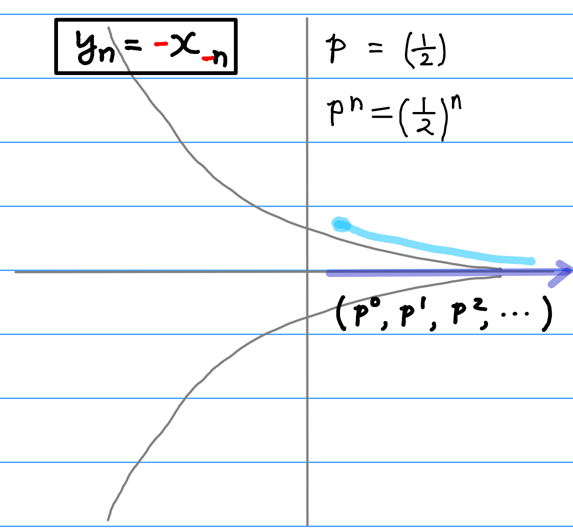
anti-causal $\quad$ n=0,-1,-2,-3,....



causal $\quad$ n=+1,+2,+3,....



causal $\quad$ n=+1,+2,+3,....



x_n
y_{-n}

causal $X(z)$ $(|z| < p^{-1})$ anti-causal $X(z)$ $(|z| > p^{-1})$

$X(z) \leftrightarrow x_n$ $(n \geq 1)$ $Y(z^{-1}) \leftrightarrow -y_n$ $(n < 1)$

$X(z)$ ⑤

$n = 0, 1, 2, \cdots$ $-p^{-n-1} z^{-n}$

$$\boxed{-(p^{-1} + p^{-2}z^{-1} + p^{-3}z^{-2} + \cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{-n} \quad (n \geq 0)}$$

$|z| > p^{-1}$ $n \geq 0$

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \quad \boxed{\text{causal } X(z) \quad -(p^{-1})^{n+1}} \quad x_n$$

$\cdot \frac{p}{p} \frac{z}{z}$ $\cdot(-1)$

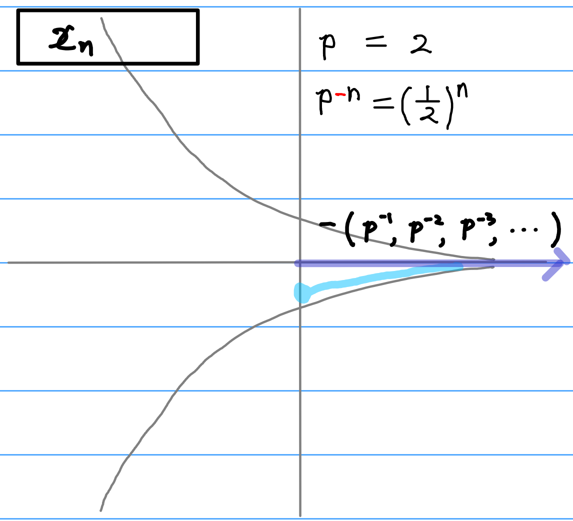
$$\boxed{p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=-1}^{-\infty} (p^{-1})^{n+1} z^{-n} \quad (n < 0)}$$

$$\boxed{\frac{z}{1-pz}} \quad \boxed{(p)^{-n-1} \quad \text{anti-causal } X(z)} \quad y_{-n}$$

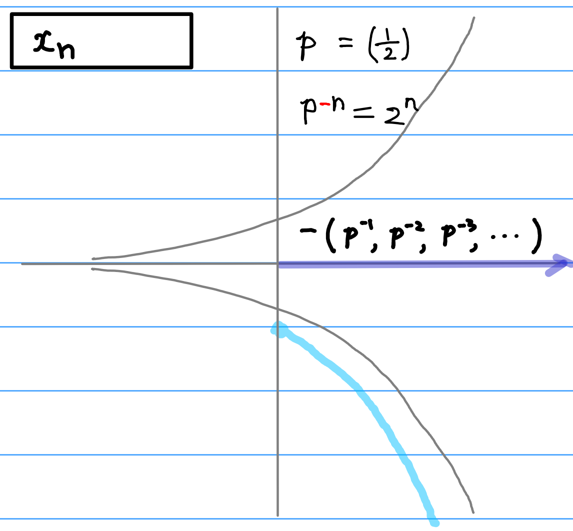
$n = -1, -2, -3, \cdots$ $p^{-n-1} z^{-n}$

$|z| < p^{-1}$ $n < 0$

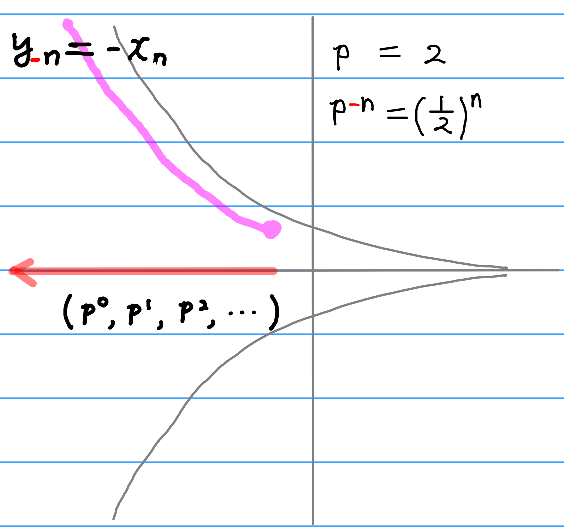
causal n=0,+1,+2,+3,....



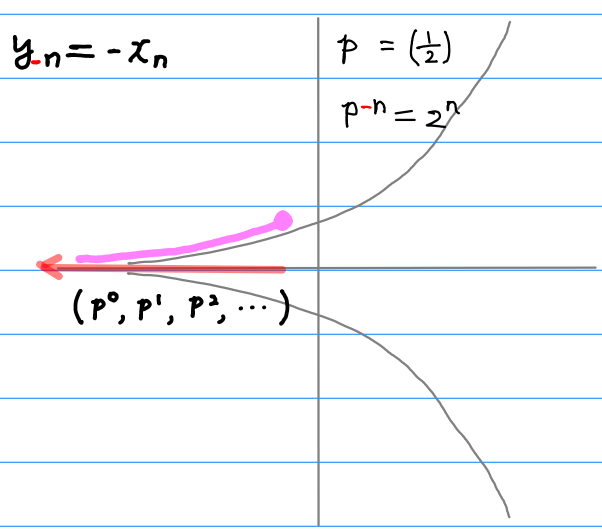
causal n=0,+1,+2,+3,....



anti-causal n=-1,-2,-3,....



anti-causal n=-1,-2,-3,....



x_{-n} y_n | x_n y_{-n}

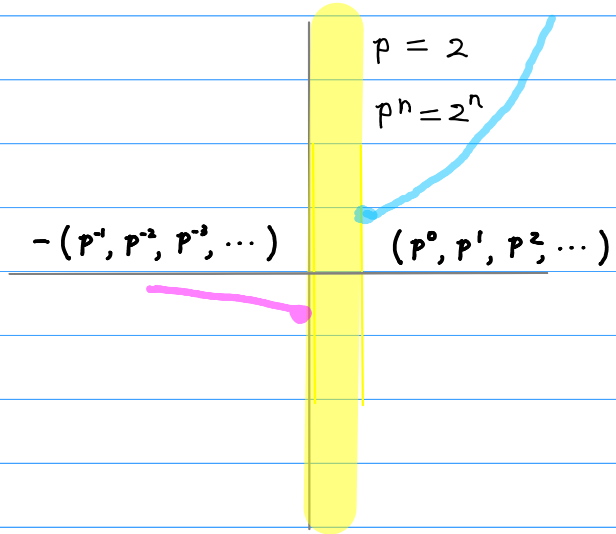
$|z| < p$
$|z| > p$

$|z| < p^{-1}$
$|z| > p^{-1}$

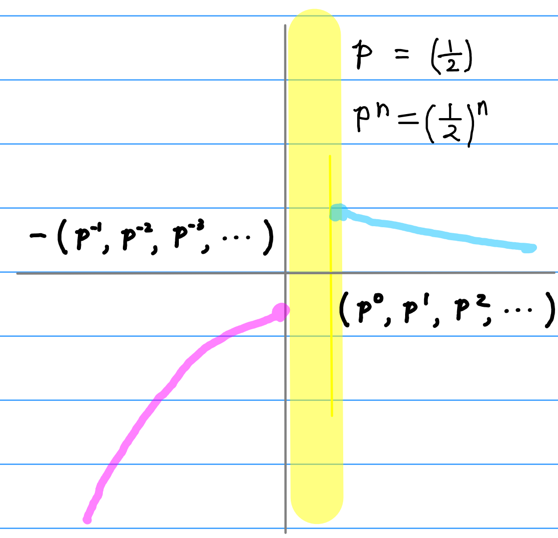
complementary ROC's

$X(z)$ 6

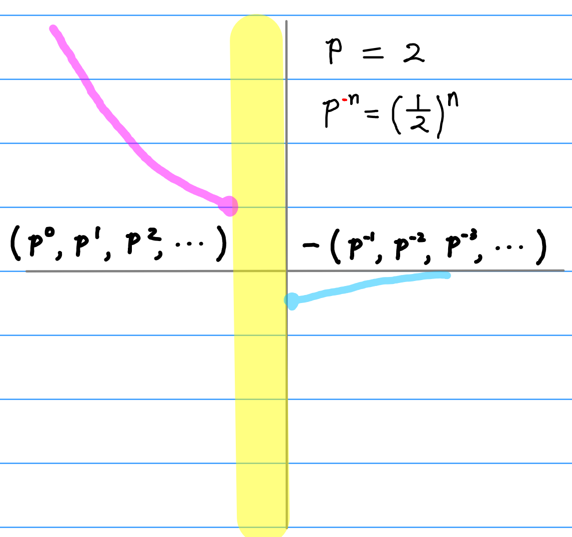
anti-causal: $Y(z)$ $(|z| < p)$, $-y_n$ $(n < 1)$
causal: $Y(z)$ $(|z| > p)$, y_n $(n \geq 1)$



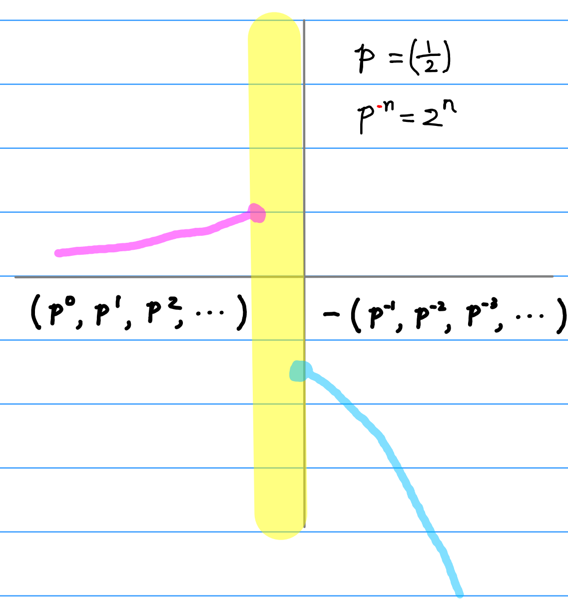
anti-causal: $Y(z)$ $(|z| < p)$, $-y_n$ $(n < 1)$
causal: $Y(z)$ $(|z| > p)$, y_n $(n \geq 1)$



anti-causal: $X(z)$ $(|z| < p^{-1})$, $-x_n$ $(n < 0)$
causal: $X(z)$ $(|z| > p^{-1})$, x_n $(n \geq 0)$



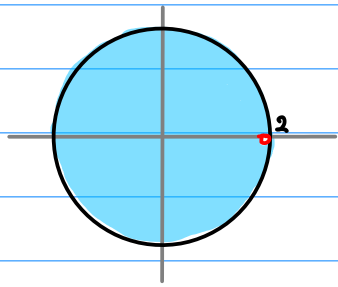
anti-causal: $X(z)$ $(|z| < p)$, $-x_n$ $(n < 0)$
causal: $X(z)$ $(|z| > p)$, x_n $(n \geq 0)$

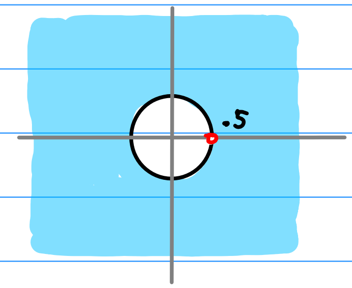


$\cdot(-1)$ $\cdot(-1)$

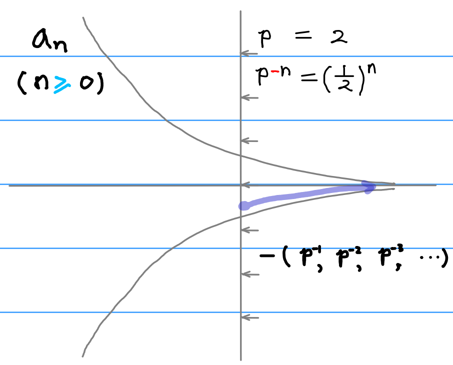
a_n

$n \in [0,\infty)$	$\lvert z\rvert < 2$	$\xleftrightarrow{z^{-1}}$	$\lvert z\rvert > 0.5$	$n \in (-\infty, 0]$
causal -0.5^{n+1}	$-\dfrac{0.5}{1-0.5z}$		$-\dfrac{0.5}{1-0.5z^{-1}}$	anti-causal -2^{+n-1}

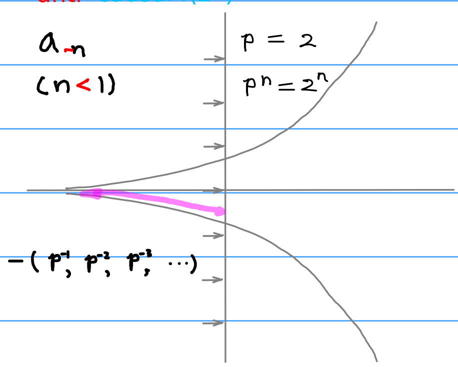




causal f(z)

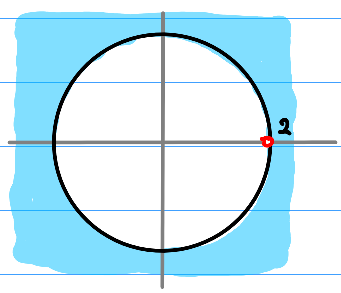


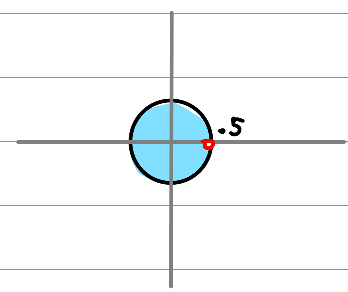
anti- causal f(z $^{-1}$)



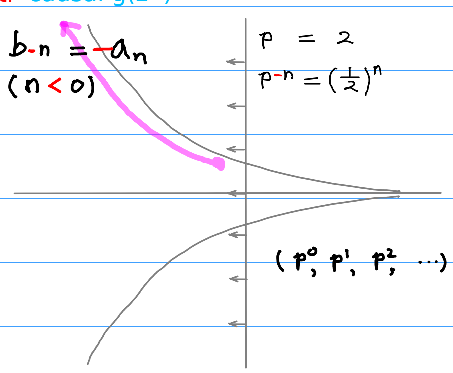
b_n

0.5^{n+1} anti-causal	$\dfrac{z^{-1}}{1-2z^{-1}}$	$\xleftrightarrow{z^{-1}}$	$\dfrac{z}{1-2z}$	2^{+n-1} causal
$n \in (-\infty, -1]$	$\lvert z\rvert > 2$		$\lvert z\rvert < 0.5$	$n \in [1,\infty)$

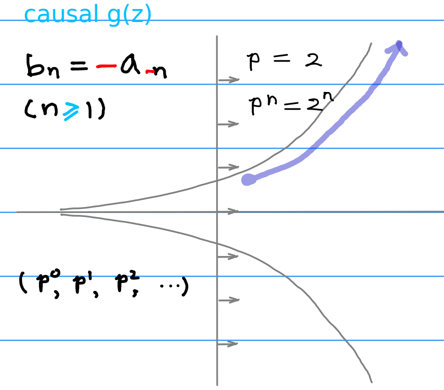




anti- causal g(z $^{-1}$)



causal g(z)



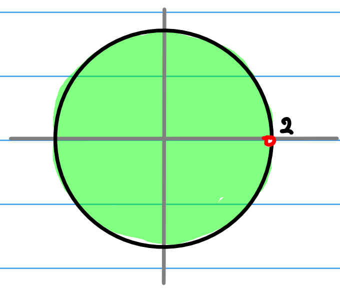
$n < 1$ $\quad |z| < 2$

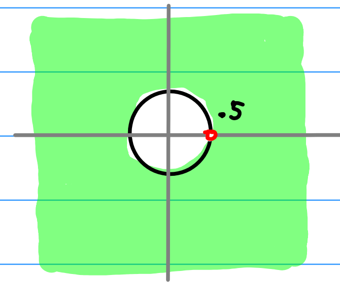
x_{-n}: anti-causal $\quad -(p^{-1}, p^{-2}, p^{-3}, \dots)$ $\quad \boxed{-\dfrac{0.5}{1-0.5z}}$

$\xleftrightarrow{z^{-1}}$

$|z| > 0.5$ $\quad n \geq 0$

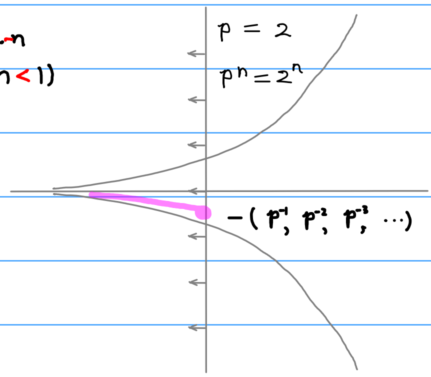
$\boxed{-\dfrac{0.5}{1-0.5z^{-1}}}$ $\quad$ causal $\quad -(p^{-1}, p^{-2}, p^{-3}, \dots)$: x_n





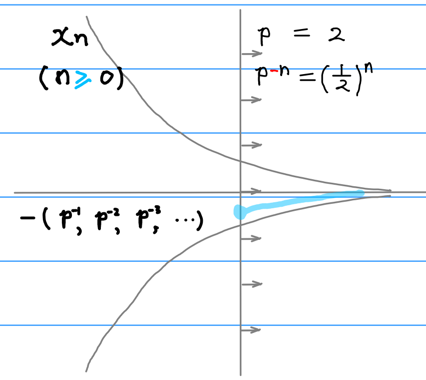
anti-causal $X(z^{-1})$

x_{-n} $(n<1)$



causal $X(z)$

x_n $(n \geq 0)$



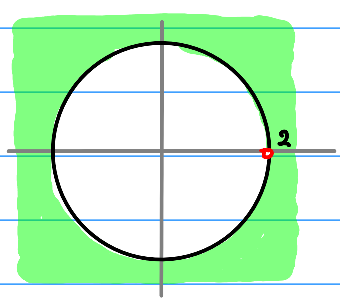
$n \geq 1$ $\quad |z| > 2$

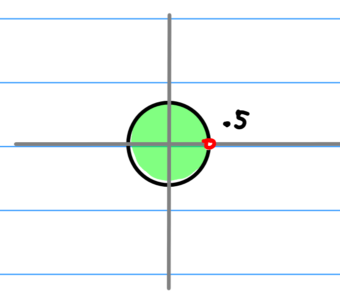
y_n: $(p^0, p^1, p^2, \dots)$ causal $\quad \boxed{\dfrac{z^{-1}}{1-2z^{-1}}}$

$\xleftrightarrow{z^{-1}}$

$|z| < 0.5$ $\quad n < 0$

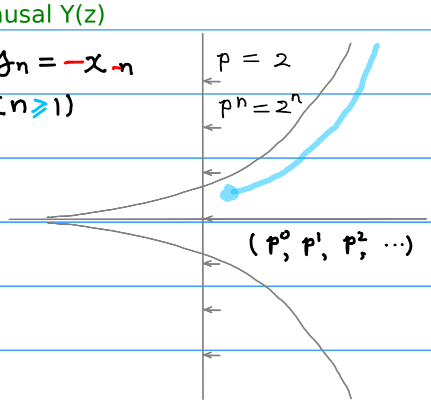
$\boxed{\dfrac{z}{1-2z}}$ $\quad (p^0, p^1, p^2, \dots)$ anti-causal : y_{-n}





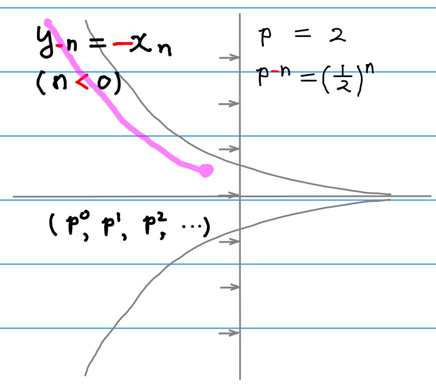
causal $Y(z)$

$y_n = -x_{-n}$ $(n \geq 1)$



anti-causal $Y(z^{-1})$

$y_{-n} = -x_n$ $(n < 0)$



Getting anti-causal sequence w/o memorizing

$$\begin{tikzcd}
f(z^{-1}) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[r, "1\ z^{-1}"] \arrow[d, leftrightarrow] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} = f(z) \arrow[d, leftrightarrow, "2"] \\
a_{-n} = \boxed{-(p^{-1})^{-n+1}} & \boxed{-(p^{-1})^{n+1}} = a_n \arrow[l, "3\ -n"']
\end{tikzcd}$$

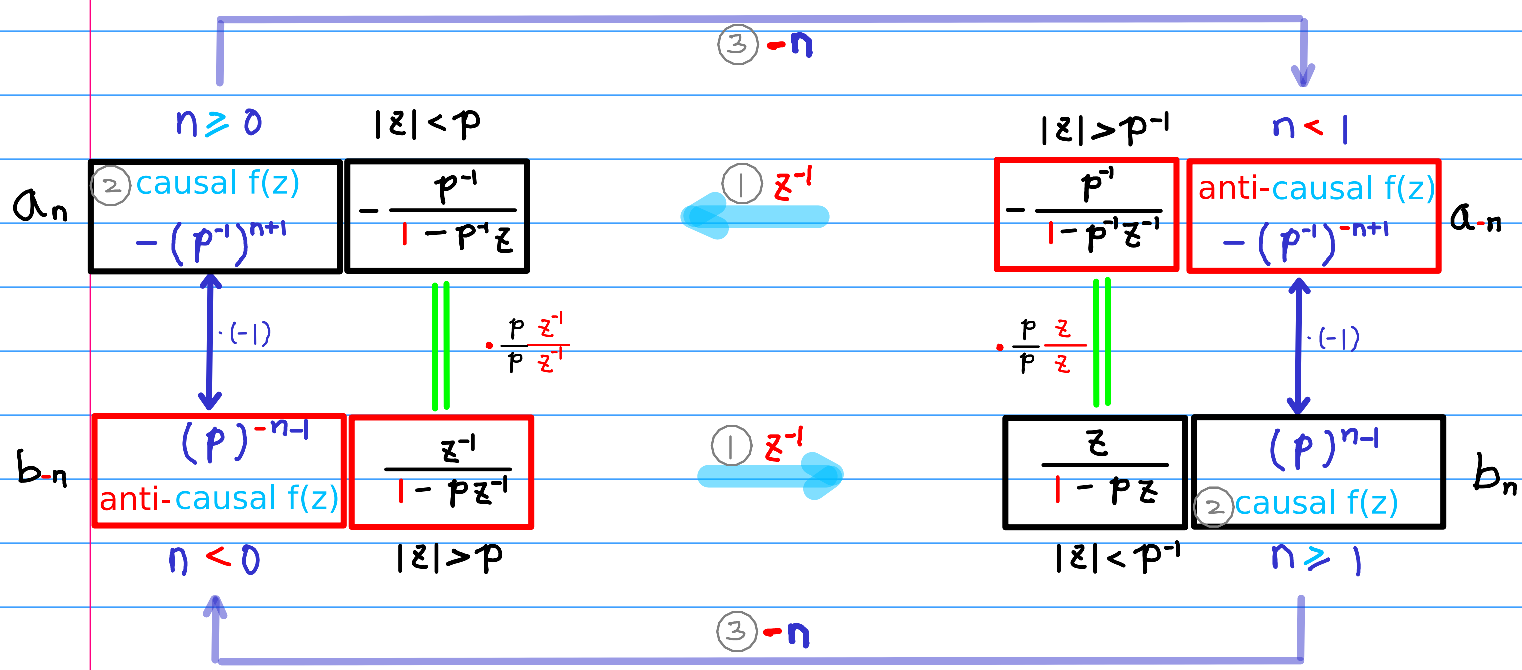
$$\begin{tikzcd}
g(z^{-1}) = \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[r, "1\ z^{-1}"] \arrow[d, leftrightarrow] & \boxed{\dfrac{z}{1-pz}} = g(z) \arrow[d, leftrightarrow, "2"] \\
b_{-n} = \boxed{(p)^{-n-1}} & \boxed{(p)^{n-1}} = b_n \arrow[l, "3\ -n"']
\end{tikzcd}$$

$$\begin{tikzcd}
Y(z^{-1}) = \boxed{\dfrac{z}{1-pz}} \arrow[r, "1\ z^{-1}"] \arrow[d, leftrightarrow] & \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} = Y(z) \arrow[d, leftrightarrow, "2"] \\
y_{-n} = \boxed{(p)^{-n-1}} & \boxed{(p)^{n-1}} = y_n \arrow[l, "3\ -n"']
\end{tikzcd}$$

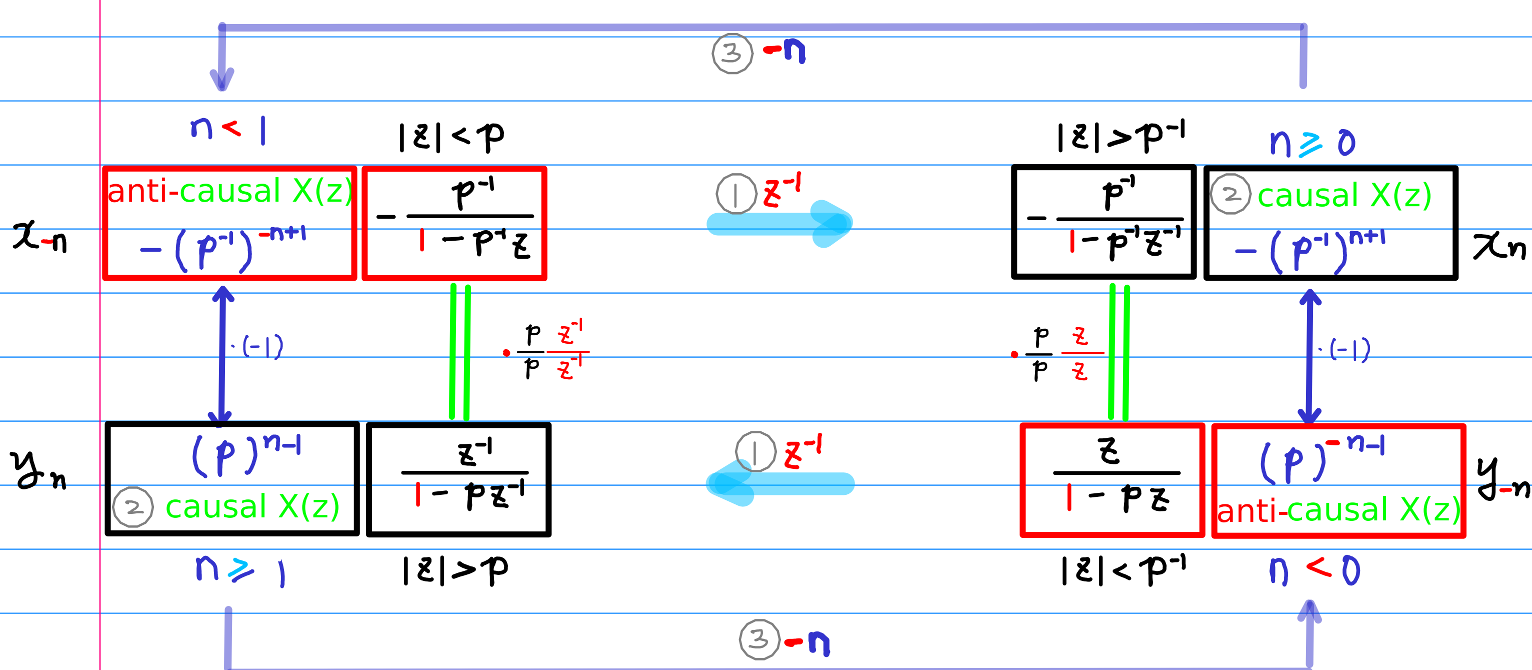
$$\begin{tikzcd}
X(z^{-1}) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[r, "1\ z^{-1}"] \arrow[d, leftrightarrow] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = X(z) \arrow[d, leftrightarrow, "2"] \\
x_{-n} = \boxed{-(p^{-1})^{-n+1}} & \boxed{-(p^{-1})^{n+1}} = x_n \arrow[l, "3\ -n"']
\end{tikzcd}$$

Getting anti-causal sequence

Laurent Series



z-Transform



① $\dfrac{-1}{(z-1)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$-\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z}$ $\quad$ $+\dfrac{z}{1-z}-\dfrac{z}{1-2z}$

$f(z)$ $|z|<1$ causal $\quad$ $g(z)$ $|z|<0.5$ causal

$Y(z)$ $|z|<1$ anti-causal $\quad$ $X(z)$ $|z|<0.5$ anti-causal

$+\dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$ $\quad$ $-\dfrac{1}{1-z^{-1}}+\dfrac{1}{1-0.5z^{-1}}$

$f(z)$ $|z|>2$ anti-causal $\quad$ $g(z)$ $|z|>1$ anti-causal

$Y(z)$ $|z|>2$ causal $\quad$ $X(z)$ $|z|>1$ causal

$-\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z}$

$f(z)$ $|z|<1$ causal

$Y(z)$ $|z|<1$ anti-causal

$-\dfrac{1}{1-z^{-1}}+\dfrac{1}{1-0.5z^{-1}}$

$g(z)$ $|z|>1$ anti-causal

$X(z)$ $|z|>1$ causal

$+\dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$

$f(z)$ $|z|>2$ anti-causal

$Y(z)$ $|z|>2$ causal

$+\dfrac{z}{1-z}-\dfrac{z}{1-2z}$

$g(z)$ $|z|<0.5$ causal

$X(z)$ $|z|<0.5$ anti-causal

causal f(z)

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$f(z) \qquad |z|<1$

causal g(z)

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$g(z) \qquad |z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$z^{-1}g(z) \qquad |z|<0.5$

causal Y(z)

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$Y(z) \qquad |z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$z\cdot Y(z) \qquad |z|>1$

causal X(z)

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$X(z) \qquad |z|>2$

anti-causal $f(z^{-1})$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$X(z^{-1})$ $|z|<1$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$X(z)$ $|z|>2$

anti-causal $g(z^{-1})$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$Y(z^{-1})$ $|z|<0.5$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$Y(z)$ $|z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$X(z)$ $|z|>1$

anti-causal $Y(z^{-1})$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$g(z^{-1})$ $|z|>1$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$g(z)$ $|z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$f(z)$ $|z|<0.5$

anti-causal $X(z^{-1})$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$f(z^{-1})$ $|z|>2$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$f(z)$ $|z|<1$