Multiobjective Behavioral Portfolio Selection with Efficient ESG Factors and Learning Network Estimation of Asset Returns

Gordon H. Dash[a] and Nina Kajiji[b]

[a]Finance and Decision Sciences Area, College of Business Administration and Interdisciplinary Neuroscience Program, University of Rhode Island, USA
[b]Department of Computer Science and Statistics, University of Rhode Island, USA, and The NKD-Group, Inc. USA

Email: ghdash@uri.edu and nina@nkd-group.com

JEL Codes: C45, C58, C61, G11, G13

Submitted for presentation consideration at the 2nd Frontiers of Factor Investing Conference, Lancaster University, UK, April 2-3, 2020.

Research support provided by the Office of Global Initiatives, University of Rhode Island, Kingston, RI, and The NKD-Group, Inc. Providence, RI

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DRAFT MANUSCRIPT

Abstract

This paper provides an aspirational multiple objective framework for behavioral portfolio modeling (BPM) for both the portfolio selection and rebalancing decision. The aim is to extend BPM to a framework that has the flexibility to incorporate newly extracted ESG factors that proxy investor sustainability bias. The proposed framework complements the transition from modern portfolio theory to behavioral portfolio theory with support for dynamic rebalancing. Assuming sustainability bias is proxied by investor attention for firm ESG production, we extract three new ESG factors from the Thompson Reuters large-cap ESG portfolio database. These same factors also serve as stylistic control variables in a machine-learning algorithm when modeling Fama-French asset returns. As such, this research incapsulates recent findings that challenge characteristic-based asset return prediction by implicating a shallow learning radial basis function neural network with production-theoretic signals. Lastly, we model loss aversion bias by dynamically estimating an option-priced CVaR metric. The results of solving the aspirational multiobjective BPM provide three important observations. We find that when only two hierarchical goal objectives are specified the BPM approximates the traditional mean-variance solution. Secondly, alternate model specifications demonstrate the importance of wealth goal setting when there is investor ESG sustainability bias. Lastly, research findings demonstrate the importance of dynamically estimating CVaR metrics to achieve aspirational goals related to loss-aversion bias.

Keywords: *Behavioral Portfolio Theory, Multiple Objective Portfolio Optimization, Factor Estimation, Option-theoretic CVaR, Artificial intelligence*

1 Introduction

Traditional finance models have long assumed economic agents are unbiased processors of relevant information and, therefore, make decisions in a manner consistent with utility maximization. But, as rational financial decision-making came under academic scrutiny, acceptance grew for the alternative notion some investors execute financial decisions guided by behavioral biases (Byrne & Brooks, 2008)). Behavioral portfolio theory (BPT) emerged as a plausible alternative to traditional financial models. The implication for advancing models predicated on the classic Markowitz mean-variance model (Markowitz, 1959) became evident. BPT offered a new, but yet efficient, way to approach portfolio selection when investors make decisions based on emotions and context-sensitive heuristics. When applied to the portfolio selection problem heuristic, or rule-based decision-making, is also referred to as goals-based wealth management (Das, Markowitz, Scheid, & Statman, 2010; Howard, 2014). Combining BPT and goals-based portfolio management offers a concise way to model behavioral factors. Behavioral portfolio factors are expressed by metrics that capture individualized characterization of loss aversion, hindsight bias, age, gender, recency bias and more (Frijns, Koellen, & Lehnert, 2008). Recently, new contributions present behaviorally biased portfolios as a layered mental account pyramid (MAL). In a study of investment beliefs of retail investors, (Giglio, Maggiori, Stroebel, & Utkus, 2019) provided evidence to support layered (hierarchical) MAL bias. The study also yielded incremental insight into how beliefs shape portfolio choice. The following two findings are particularly noteworthy: investor optimism is conditioned by individual heterogeneity; and, expected returns and the subjective probability of rare disasters are negatively related.

As contemporary behavioral fund managers turned their attention to judgment errors in financial investment decision-making, they also came to the understanding that in portfolio selection and rebalancing, behavioral errors (optimism, pessimism, depression, anxiety, etc.) steadily win the dispute against reasonable and rational behavior (Costa, Carvalho, & Moreira, 2019; Oprean, 2014). Multiple criteria decision analysis (MCDA) and its variants like multiobjective optimization emerged as an appropriate method to model hierarchical bias models under risk (Ogryczak, 2002). This research seeks to extend multiple objective behavioral portfolio modeling (MBPM) to include sustainability bias while explicitly considering the dynamic portfolio selection and rebalancing decision on different intervals (i.e., daily, monthly, quarterly, etc.). The interplay among optimal portfolio allocation, transaction costs and investment horizon defines the process of portfolio rebalancing. Active portfolio rebalancing implements some level of broadly defined periodic calendar schemes and *ad hoc* tolerance band methodologies. To begin the process of extending the dynamic selection and rebalancing MBPM framework to include a sustainability bias, we argue for a closer examination of what constitutes an efficient proxy for the ESG dimension.

1.1 ESG Dimensions and Firm Sustainability

Under the principle of shared value creation, also referred to as enlightened stakeholder theory, it is widely held that the firm's stakeholders (e.g., shareholders, employees, customers, suppliers, the environment, the community) all experience value enhancement from corporate social responsibility (CSR) activities (Cook, Romi, Sanchez, & Sanchez, 2018). Developing a commitment to transparency, corporate governance, life principles, ethical conduct, and giving back to communities leads firm managers to consider ESG criteria as a part of their CSR process (CFA Institute, curriculum support). Table 1 provides a representative list of the requirements that form the focus of the three sustainability dimensions:

Table 1: ESG Sustainable Impact Dimensions

Environmental	**Social**	**Governance**
Environment Policy	Human Rights	Corporate Governance
Environment Performance	Labor Standards	Code of Ethics
Climate Change	Health and Safety	Bribery and Corruption
Nuclear Energy	Employee Development	Death Penalty
Biodiversity	Supply Chain Standards	Military Expenses

Source: Invesco, Vigeo Eiris, https://www.invesco.com/corporate/about-us/esg

The problem facing investment managers is twofold: a) determining how a firm's commitment to ESG influences firm financial performance; and, b) reconciling the wide variation in methodology in use across the plethora of published scores (Chatterji, Durand, Levine, & Touboul, 2016; Chiu, 2010; Seubert, Jan 28. 2017; Windolph, 2011). By way of example, consider the evolution of the proprietary Thomson Reuters (TR) ESG factor score methodology (Thomson Reuters ESG Scores, 2017; note: the financial and risk division is now owned by Refinitiv, Inc.). The TR approach evaluates 10 main themes (Thomson-Reuters & S-Networks, 2018). After calculating relative sub-domain scores, the TR methodology reduces relative scoring to metrics for each ESG sub-domain as well as an overall ESG combined score. Subsequently, the combined score is discounted for news controversies that may materially impact firm performance. Passive portfolio managers adopt ESG metrics to align a portfolio with these stated values. Alternatively, active portfolio managers use these scores to dynamically summarize the financial materiality of sustainable corporate behavior (Tarmuji, Maelah, & Tarmuji, 2016). Post selection of a scoring representation, one question persists: does a firm's commitment to ESG factor scores translate into sustainable investment value?

1.2 ESG and Financial Performance

There is a growing literature investigating the relationship between ESG investments and firm financial performance. Overcoming prior research that failed to address the interrelationships among ESG with market conditions and corporate governance, Tseng et al., (2019) reported causal interrelationships among ESG and firm performance. The study examined ESG dimensions using the fuzzy set method DEMATEL (fuzzy linguistic modeling) to transform human judgments into fuzzy variables. Subsequently, the fuzzy variables were converted into crisp values of cause and effect groups. This approach found that investors expressed a preference for firms to improve ESG practices as they believed ESG-oriented firms perform best at asset deployment and generating risk-adjusted market returns.

Cho et al. (2019) examined the relationship between a single sustainability dimension, CSR performance, and corporate financial performance. Eschewing the use of ESG scores, the authors related a CSR proxy, the KEJI economic justice index, to firm financial performance. Financial performance proxies were the rate-of-return on assets ratio and the growth rate of assets. The authors found partial support for the hypotheses that CSR performance exerts positive effects on financial performance. Moreover, the reported results support the conclusion that CSR activities are investments that can enhance both corporate performance and value. Prior to Cho et al. (2019) contribution, Tarmuji, et al., (2016) researched the unique effect each ESG dimension on firm economic performance. Their study found that, over the long-run, sustainability investments directly impact firm size.

The primary aim of our research is to extend dynamic BPM by directly encapsulating investor aspirational bias towards sustainability. To achieve our object, we begin by delineating a set of efficiently derived pervasive ESG factors that maintain consistency with dynamic rebalancing objectives. Lastly, investor bias towards extreme shortfall is extended to a dynamic metric by incorporating an option-implied shortfall risk objective.

The paper proceeds as follows. In section 2, we identify three portfolios sorted by ESG scoring dimensions. Using these portfolios, we extract a set of orthogonal factors that form the foundation for creating the proposed pervasive ESG factors. In section 3, we invoke a well-known factor disentanglement algorithm to estimate efficient and ubiquitous ESG factors. Section 4 provides the theory and application of dynamic return and risk estimation. Also developed in section 4 is the networked-based production-theoretic asset pricing model, and the dynamic expected shortfall objective. Section 5 ties the manuscript sections together by presenting the dynamic MBPM optimization problem. A summary and conclusions are provided in section 6.

2 Pervasive ESG Factor Estimation

It has long been understood that most day-to-day variation in the returns of securities is due to the constant arrival of information through both priced and pervasive return generating factors. This explains why the academic finance literature focuses on the creation of factor portfolios by sorting on characteristics positively associated with expected returns (Daniel, Mota, Rottke, & Santos, 2018)

The first step in the process is to describe the taxonomy of pervasive ESG factors in security returns. In a comprehensive examination of over 300 factors (i.e., the factor zoo) reported in the existing literature, Harvey et al. (2016) make the point that a risk factor should have unpredictable variation through time as well as be able to explain cross-sectional return patterns. Previously, Cochrane (2011) had already argued for methods to identify prevalent and dominating risk factors. To test whether factors are priced efficiently and to overcome a data-mining bias in error specification, Ang, et al. (2009) refined the factor quest by arguing for the use of stocks over portfolios in studies of identification and number of risk factors. Lettau and Pelger (2018) provide an augmented principal component analysis (PCA) methodology for successfully estimating latent factors that explain covariance and expected returns structure in equity data. Under their approach, the extracted latent factor estimators work on projected data. A projection that is necessary to control for the time-variation in the loadings of individual stocks. The authors provide further evidence that their technique is superior to natural PCA or unobservable factor models when enumerating the optimal portfolio.

Backtested results published in academic outlets are mainly responsible for commercial products as well as exaggerated expectations based on inflated backtested results and are then disappointed by the live trading experience (Arnott, Harvey, Kalesnik, & Linnainmaa, 2019). Harvey and Liu (2019) attribute this to the out-of-control production of factors. These findings notwithstanding, the literature is in agreement that factor models provide three pillars of support – identifying risk premia; pricing behavioral biases; and, identifying structural impediments. While the production of factors may be excessive, what is needed is a clear statement on best practice for factor extraction. A best practice approach is found in the recent contribution from Pukthuanthong, Roll, and Subrahmanyam (2018), or PRS. The authors develop a protocol for palpable risk factor extraction based on the factor's relationship to the covariance matrix of asset returns, it's priced relationship in the cross-section of returns, and the factor's overall reward-to-risk ratio. Before deploying a modified PRS application, it is useful to abstract the arguments of Lettau and Pelger (2018). We state the following assumptions.

Assumption 1. Assume that excess returns follow the standard approximate factor model where the assumptions of arbitrage pricing theory are satisfied. In this case, asset $X_{j,t}$, have a systematic component captured by K factors and a nonsystematic, idiosyncratic component that captures asset-specific risk. Excess returns of J assets over T time periods are described as:

$$X_{j,t} = \mathbf{F}_t \Lambda_j^T + \varepsilon_{j,t}, j = 1, \dots, J; t = 1, \dots, T \tag{1}$$

$$\Leftrightarrow \underbrace{X}_{TxJ} = \underbrace{F}_{TxK} \underbrace{\Lambda^T}_{KxJ} + \underbrace{\varepsilon}_{TxJ} \tag{2}$$

Assumption 2. The factors and residuals are uncorrelated; hence, the covariance matrix of the returns consists of a systematic and idiosyncratic part.

$$Var(\mathbf{X}) = \Lambda Var(\mathbf{F})\Lambda^T + Var(\varepsilon) \tag{3}$$

The factors drive the largest eigenvalues of $Var(\mathbf{X})$; hence, PCA is available to estimate loadings and factors. In the next section, we estimate the unknown latent factors, **F**, and loadings Λ, from the Thomson Reuters constitutive ESG portfolios.

2.1 Portfolio Data

We refer to the three uniquely separated ESG portfolios maintained as part of the Thomson Reuters/S-Network ESG Best Practices Ratings and Indices (http://bit.ly/TRandSNetworkESG). Each collection (TR*E*NVUS, TR*S*CUS, and TRC*G*VUS) contains *n* vetted securities such that $B = \{n_E, n_S, n_G\}$. Tickers with incomplete data were removed to create a research sample set $N \subseteq B$ where $n_E = 245, n_S = 245$, and $n_G = 243$. For the market proxy (S&P 500) and all securities in *N*, we compute daily log-differenced returns from January 2015 through March 2018, inclusive ($T = 816$).

Following extant literature, we choose an enhanced beta estimate to capture systematic market variation in equity returns. Across all *j* securities, we implement the Vasicek (1973) adjusted market beta ($i.e., \beta_j^V$). Although Hollstein and Prokopczuk (2016) find that option-implied estimators of systematic risk consistently outperform all other approaches tested on both daily and monthly datasets, both Sarker (2013) and Cloete et al., (2002) report on the efficiency and robustness of Vasicek estimators compared to using unfiltered OLS methods. In a follow-up study, Wang et al., (2017) demonstrate improved stock return predictability using Vasicek-adjusted betas in both the CAPM and Fama-French three-factor model. Accordingly, for all securities after-market residuals are formed by equation (4).

$$\varepsilon_{i,t} = r_{i,t} - \alpha_{i,t} - \beta_i^v r_{M,t} \quad where,\ t=1..T,\ i=1..N \tag{4}$$

2.2 Latent ESG Factor Identification

Latent factors are extracted by applying PCA to the matrix of return residuals for each of the three portfolios. First, we test the return residuals for each set, $\boldsymbol{\varepsilon}_{n_E}, \boldsymbol{\varepsilon}_{n_S}, and\ \boldsymbol{\varepsilon}_{n_G}$, for PCA suitability. We test the hypothesis that the correlation matrix for each $\boldsymbol{\varepsilon}_{n_E}, \boldsymbol{\varepsilon}_{n_S}, and\ \boldsymbol{\varepsilon}_{n_G}$ is an identity matrix. Based on the results of applying Bartlett's test of sphericity (see table 2) the null hypothesis of no common factors is rejected at the 1% level. We reach the conclusion that an exploratory factor analysis (EFA) is statistically supportable. The Kaiser-Meyer-Olkin (KMO) measure is also applied to the three residual matrices. The results of the KMO test indicates a high proportion of the variance in the variables is caused by the underlying factors.

Table 2: Results from the Bartlett's Test and the KMO Test

Domain	**Bartlett's Test of Sphericity** ***Ho*: No Common Factors;** ***Ha*: At least one common factor**	**KMO Measure of Sampling Adequacy**
Environmental	$\chi^2_{29890} = 109690.280, p < 0.001$	0.85349
Social	$\chi^2_{29890} = 105587.052, p < 0.001$	0.85222
Governance	$\chi^2_{29403} = 99281.5095, p < 0.001$	0.83165

Following Han (2002), we calibrate the arbitrage return-generating framework based on equation 4 using an exploratory factor analysis (EFA) model on the after-market residuals (Jackson, 2005). Subsequently, after-market factors were rotated orthogonally. The results obtained from the rotation corroborated extant literature as far back as the mid-1970s (Fertuck, 1975). End-product industry effects were clearly separated (see figure 1). To the aim of this study, we observed, for example, that the factor labeled 'Banks and Bank Hldg' accounts for 46% of the after-market variation in the residuals. Extending these results to consider the ESG effects, we point to identifiable E- S-, and G sub-domains within the 'Banks and Bank Hldg' domain. The E-, S-, and G-domains account for 16-, 16-, and 15-percent of the after market variation, respectively. Similarly, for the second orthogonal factor ('Energy and Oil & Gas'), which accounts for 22% of the total after-market residual return variation, the ESG contribution is 6-, 8-, and 8-percent, respectively.

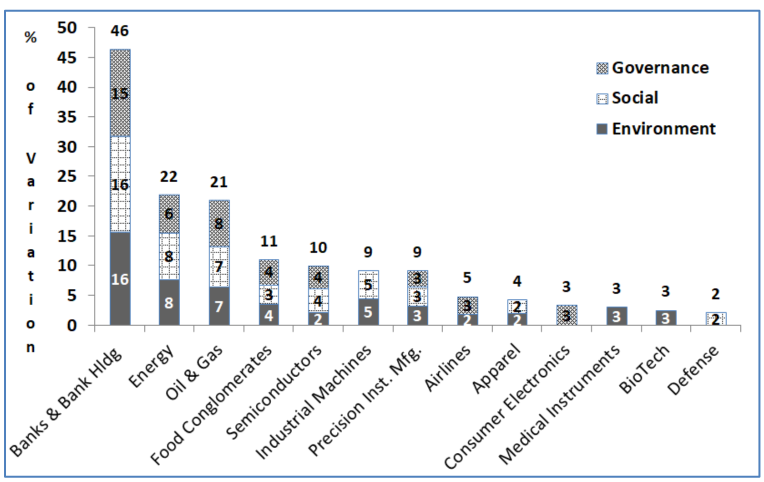

Figure 1: Percent of industry wide after-market variation explained by the E, S, & G domains.

Invoking the Kaiser-Guttman criteria leads us to retain 36 factors in each of the individual sub-domains, $C^E, C^S,$ and C^G. The percent total after-market variation explained is 81.14%, 80.66%, and 79.99%, respectively.

2.3 Determinate Factor Scores as Reproducible Factor Proxies

The next step in the algorithmic process is to create E, S, and G factor-based proxy variables. The index creation process requires transforming the rotated factors into hypothetical, but genuine, factor-policy variables. We compute a refined regression-based factor score estimates using SAS 9.4. The regression method proposed by Thurstone (1935) assures "maximum validity" or "highly determinate" estimates for a given analysis (Grice, 2001). Additionally, as shown by Beauducel (2007), Thurstone's calculations can reproduce the same covariance matrix. Although the problem of indeterminacy is resolved by the Thurston method, the scores are not correlation preserving. As amplified by Grice, the factor score estimates may be contaminated with variance from other orthogonal factors within the analysis. However, the ESG factor creation process is predicated on summing individual scores. Hence, we proceed with computing the index using the matrix of factor score estimates, ***f***. The formulae to calculate each sub-domain index value at time *t* is as shown below:

$$\mathrm{FSI}_t^E = \frac{\sum f_{ti}}{C^E} \quad i = (1, \ldots, C^E) \tag{5}$$

$$\mathrm{FSI}_t^S = \frac{\sum f_{ti}}{C^S} \quad i = (1, \ldots, C^S) \tag{6}$$

$$\mathrm{FSI}_t^G = \frac{\sum f_{ti}}{C^G} \quad i = (1, \ldots, C^G) \tag{7}$$

2.4 Stationarity Conditions

Continuing with the Lettau and Pelger (2018) procedure first introduced in section 2.1, we evaluate the stationarity condition of the three new hypothetical factors using the Philips-Perron (PP) test. The null hypothesis for the PP test states that the series has a unit root. When applied to the $FSI^E, FSI^S,$ and FSI^G, we reject the respective hypotheses for trend, single mean, and zero mean. Specifically, reported results are as follows: trend (E: $\tau = -7.88$, S: $\tau = -7.60$ and G: $\tau = -9.29$; all $p < 0.001$); single mean (E: $\tau = -7.96$, S: $\tau = -7.65$ and G: $\tau = -9.41$; all $p < 0.002$); and zero mean (E: $\tau = -8.06$, S: $\tau = -7.75$ and G: $\tau = -9.53$; all $p < 0.001$). By implication, when applied to each factor index, there is a high probability no unit root exists, a finding that each index is stationary with a zero mean.

3 The Augmented PRS Algorithm for Disentanglement

This section of the study is devoted to the disentanglement of embedded ESG factors in an investor-formed portfolio. To accomplish the task, we invoke the PRS protocol for identifying risk factors. The PRS protocol is used to identify factors associated with risk premia as well as 'pervasive' factors. Pervasive factors are unobserved and must be extracted from the asset returns of portfolios.

3.1 Derive Pervasive ESG Factors

The pervasive factor score variables computed above utilized daily residual returns data across the three E-, S- and G-portfolios. As the remainder of the analysis is focused on the behavior of investment portfolios, we follow the literature and use monthly returns from this point forward (Zibri & Kukeli, 2015). Accordingly, for each ESG domain derived in section 2.3, we average the daily factor score indices (FSI) observations into monthly observations.

3.2 The Investor Portfolio

The naively diversified investor portfolio in this study is owned and managed by a regional unit of the national non-profit *The Girl Scouts of the United States of America* (GSUSA). The national office transmits public policy and investment goals to its subordinate councils. In addition to earnings from current year operations, regional councils are expected to make investment decisions in a manner that is consistent with the organization's socially responsible narrative. The investor portfolio used in this study is naively diversified and is comprised of n=65 instruments representing 41 industries across 12 sectors.

3.3 Investor Portfolio Heterogeneity

The first step in the PRS algorithm is to identify an equity portfolio representing different industries with a 'good' level of heterogeneity. For the subject investor portfolio used in this research the industry and sector classifications are as shown in figure 2 (Source: Yahoo! Finance).

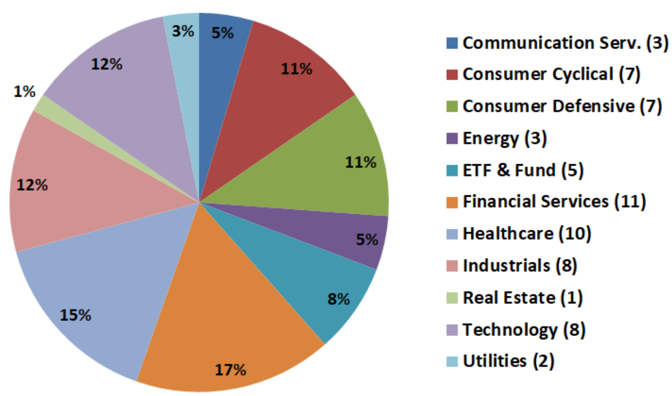

Figure 2: Diversification of investor portfolio

Two portfolio heterogeneity tests are applied: a) average correlation; and b) network analysis. Including the market index (S&P 500), we calculate monthly log-differenced returns for all 65-instruments in the investor portfolio from January 2015 through March 2018, inclusive (T=39).

Pollet and Wilson (2010) report that average correlation, $\bar{\rho}$, has predictive power for stock market returns. In their study, the authors find returns predictability from average correlation over the periods 1963-1974, 1974-1985, and 1996-2007. With some exceptions noted, the authors report that average correlations from the late 1980s forward are between 0.15 and 0.55. For the investor portfolio, we find an average correlation of $\bar{\rho}$ = 0.28. This finding lies within the bounds of extant research results.

As a further test of heterogeneity, we subject the correlation of asset returns to a Fruchterman-Reingold (FR) network analysis (Fruchterman & Reingold, 1991). The FR analysis is a force-directed network graph that distributes vertices evenly in a frame. As such, it is a useful method to examine the correlation structure. In the FR network, edges are similar in length and cross each other as little as possible. Nodes may be considered as electrically charged particles that repulse each other when they get too close. The edges act like springs that attract connected nodes closer together. As a result, nodes are evenly distributed through the graph and the layout is intuitive in that nodes that share more connections are closer to each other. A review of the FR network, figure 3, demonstrates the high intercorrelation among assets in the subject portfolio.

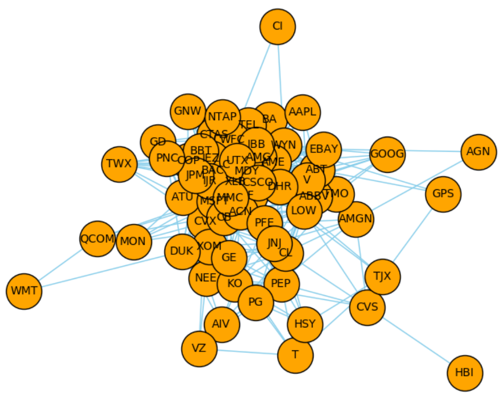

Figure 3: FR Network of Security Correlations

3.4 Extract L Principal Components

Step two of the PRS algorithm requires the extraction of L principal components from the asset return series computed from the investor portfolio. With T time-series units up to time t, we compute the TxT matrix, $\Omega_t = \left(\frac{1}{T}\right) RR'$, where R_{Txn} is the return matrix. As suggested by PRS, the cutoff point for the cumulative variance explained by the principal components is set to 90%. From this procedure, 16 principal components (PC) are retained. By way of example, figure 4 displays the cross-loading of the first three principal components (PC-1, PC-2, and PC-3). The 16 eigenvectors will form the dependent domain for subsequent canonical correlation analysis.

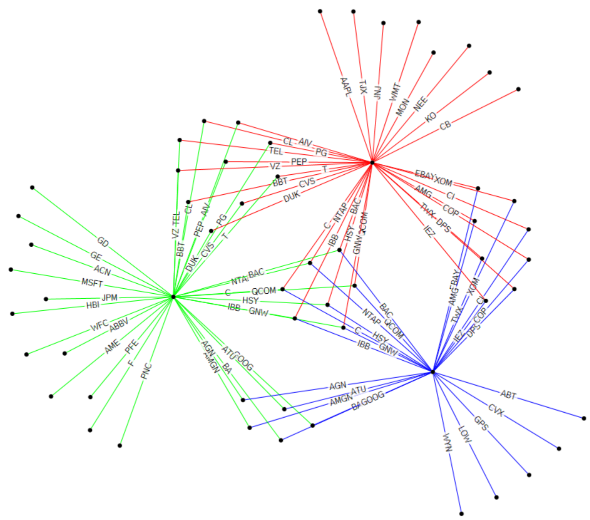

Figure 4: Excess Returns across Time.
Legend: PC-1=Green; PC-2=Blue; PC-3=Lime

3.5 The Canonical Correlation Between Pervasive Factors and the Investor Portfolio

The third step in the PRS protocol requires the identification of what is expected to be pervasive factor candidates. This step was completed as discussed in sections 2.2 and 2.3. Our aim to disentangle the latent after-market effects due to firm investments in sustainability (E, S, and G) are represented by the previously constructed genuine factor score policy variables (i.e., see: equations 5, 6, and 7).

The final step of the PRS algorithm, step four, requires conducting a canonical correlation analysis (CCA) between the set of pervasive ESG factors from step three and the corresponding 16 eigenvectors of the investor portfolio. To be labeled as the "best" (ESG) pervasive factors for the multifactor estimation of portfolio returns, the factors must exhibit a significant canonical correlation with the investment portfolio's best linear combination of eigenvectors. In the CCA model, the observed data $x \in \mathbb{C}^n$and $y \in \mathbb{C}^m$ are transformed into p-dimensional internal (latent) representations $\mathbf{a} = \mathbf{Sx}$ and $\mathbf{b} = \mathbf{Ty}$, where $p = \min(n, m)$. Using linear transformations described by the matrices $\mathbf{S} \in \mathbb{C}^{pxn}$ and $\mathbf{T} \in \mathbb{C}^{pxm}$ the key is to determine **S** and **T** such that most of the correlation between x and y is captured in a low-dimensional subspace (Song, Schreier, Ramírez, & Hasija, 2016). Specifically, we examine the null hypothesis that the surrogate E, S, G pervasive factors systematically influence the movement of portfolio asset prices.

The results from the CCA analysis of the first approximate F-value, indicates that as a group the factor candidates are conditionally related to the covariance matrix of market returns ($\lambda = 0.6411$, $F_{48,\ 63.253} = 2.0$, $p < 0.05$). The inference from the second approximate F-value (F=1.63; $p < 0.1$) is that the second and the third canonical correlations are equal zero. Lastly, the third approximate F-value (F=1.18, p-value > 0.1) suggests that, at the 90% level, the third correlate is not significant.

The canonical correlates of the three factors (E = -1.1622; S = -0.5230; and G = 1.0375) indicate that the contribution to the first canonical variate is primarily due to the E and G domains. The social factor shows an inverse relationship and, comparatively, at a much smaller level. To clarify, consider the following scenario. When all other variables in the model are held constant, an asset experiencing a one standard deviation increase in monthly returns in the environmental policy area (factor) would expect a -1.1622 standard deviation decrease in the score on the first canonical variate.

In a manner consistent with the PRS algorithm, this application of CCA yields statistical evidence that the multivariate E, S, and G, factor set is pervasive and linearly correlated with the set of asset returns in the investor portfolio.

In the next section, we demonstrate how pervasive factors produce asset returns. Instead of simple linear regression model on pooled cross-section time-series panel with real returns between dependent variables and betas on the pervasive factors, we exploit the interconnectedness of the pervasive ESG factors and the production of asset-level returns using a radial basis function artificial neural network.

4 Network Estimation of Cognitive Biases on Risk and Return

Evolutionary financial network theory seeks to understand asset return interconnectedness as a source of uncertainty in systematic risk using data science methodology (for a review, see (Priestley & McGrath, 2019) and (Roukny, Battiston, & Stiglitz, 2018)). This section of the study extends the traditional asset pricing model by inserting ESG asset pricing factors into the pricing equation. Under the evolutionary approach, the parameters of the pricing equation are estimated by an interconnected information network. We expect the interconnected factors to price complex asset returns in a manner contemplated by behaviorist decision-makers (Ozsoylev & Walden, 2011). Supporting evidence is provided by Hong et al., (2004) in a study on how fund managers exhibit collective (or, networked) behaviors. These findings are supported by Ivkovic and Weisbenner (2007) in a study that augments the hypothesis by providing results of collective action among individual investors.

In a comprehensive study of financial institutions, Billio et al., (2016) extend the classic factor-based asset pricing model to include network linkages of exogenous lagged and contemporaneous links across assets. In a related study, Horrace et al. (2016) provided evidence that peer effect networks interact with production functions to transform inputs into outputs. More recently, Herskovic (2018) provides an essential extension to asset pricing theory by uncovering a link between equilibrium asset prices and the two network attributes that drive systematic risk – network concentration and network sparsity. Herskovic observes how a sparse asset network has fewer but stronger linkages. By assuming firms experience a Cobb-Douglas shaped production technology, he reports innovations in the network factors are priced where the two production-based asset pricing factors of sparsity and concentration account for return spreads of 4.6% and -3.2% per year, respectively.

Guided by Sornett's (2017) vision of how *"…a complex system is the possible occurrence of coherent largescale collective behaviors with a very rich structure, resulting from the repeated nonlinear interactions among its constituents,"* the next section of the study presents a nonlinear production-theoretic return-generating model. In addition to the market excess returns (S&P 500), the model estimation includes the Fama-French SMB and HML factors as well as the three pervasive ESG factors. Under this formulation, the E-, S-, and G-factor elasticity estimates provide a measurement of how firm market returns respond to a unit change in a respective ESG factor.

4.1 Double-Log Production Functions

The efficiency of ESG utilization by management in a manner that influences asset return production depends on the ability of management to disaggregate and assimilate a noisy ESG set. Hence, we argue that the production of firm-level market returns requires management to identify the relative contribution of return-defining different factors.

4.1.1 A Cobb-Douglas ESG Model for Returns Production

For all *j* firms in the investor portfolio, we expect each *j-th* firm to combine capital (*k*) and labor (*l*) to produce output using a Cobb-Douglas production technology,

$$y_j = A_j k_j^{\alpha} l_j^{1-\alpha} \tag{8}$$

Without considering a firm's age or it's learning rate we further assume $A_j = e^{\beta_j \Delta_a}$, where $\Delta_a$is a common interconnected ESG shock that affects the returns productivity of all firms and $\beta_j$is the firm-specific exposure to the common shock Δ_a. Before implementing a firm-specific production decision, firm j observes a noisy ESG signal that is unique to the firm's market exposure: $sig_j = \beta_j + \epsilon_j$ where $\epsilon_j \sim i.i.d.,N(0,\frac{1}{\Delta_a}\tau^2)$. In this abstraction, the amount of noise in a firm's signal is captured by parameter τ^2. Perfect information occurs when $\tau = 0$ whereas as $\tau \to \infty$ the signal to firm j is not informative or firm management is numb to ESG factors. When observed and controlled for, the ESG signal sig_j helps firm j make efficient input factor choices.

We define the model as a nonlinear regression with a corresponding nonparametric estimation of model coefficients:

$$E(r_j) = f(\boldsymbol{X}) + \varepsilon_j \tag{9}$$

In equation 9 $\boldsymbol{X}$ denotes the factor input mix ($X = X_1, \dots, X_d$) where d is the dimensionality of the factor inputs and ε_j is a symmetric random noise term $\varepsilon_j \sim i.i.d.,N(\mu,\sigma)$. For the ESG-controlled asset-level returns production estimation model we use the establishment-specific equation (d = 4) to derive our factor elasticity estimates under the assumption that ESG signals are fully incorporated (i.e., $\tau \rightarrow 0$):

$$E(r_j) = \alpha_j + \beta_1 ln(1 + (r_M - r_f)) + \beta_2 ln(1 + FSI_E) + \beta_3 ln(1 + FSI_S) + \beta_4 ln(1 + FSI_G) + \beta_5 ln(1 + SMB) + \beta_6 ln(1 + HML) + \epsilon_j \quad (10)$$

where r_M is the total market portfolio return; r_f is the risk-free rate; FSI is the average return for each pervasive factor: E-, S-, and G, respectively; SMB is the size premium; and, HML is the value premium.

4.1.2 Neural Network Estimators and the Cobb-Douglas Production Function

Arreola et al., (2016) argue for new estimators based on modern machine learning algorithms for studies of complex observed (and statistically enumerated) datasets. In a comprehensive and comparative study of analytics across alternative machine learning methods, Gu et al., (2019) find that "shallow" learning networks perform best in studies of asset return estimation. Accordingly, this study introduces a "shallow" radial basis function artificial neural network (RANN) as a universal estimator to map asset returns in a Cobb-Douglas production function network. Artificial neural networks have previously provided a viable nonparametric alternative to fit nonlinear production functions and to describe the estimated technical efficiency (Santín, Delgado, & Valiño, 2007; Vouldis, Michaelides, & Tsionas, 2010). Specifically, we employ an augmented RANN known as the K4-RANN (Dash, Kajiji, & Vonella, 2018; Kajiji, 2001).

In the generalized RANN method, the optimal weighting values, w_j, are generally extracted by applying a supervised least-squares method to a subset (training set) of the data series. The supervised learning function is stated as, $\boldsymbol{y} = f(\boldsymbol{x})$ where $\boldsymbol{y}$, the output vector, is a function of the input vector $\boldsymbol{x}$ with p number of inputs. The function can be restated as:

$$f(x_i) = \sum_{j=1}^{m} w_j h_j(x) \quad (11)$$

where, m is the number of basis functions (centers), h is the number of hidden units, w is the weight vector, and $i = 1...p$ where p is the number of input vectors. As shown in equation (12) the K4-RANN minimizes a modified SSE cost function:

$$\frac{\text{argmin}}{k}\left(\varsigma \sum_{i=1}^{p} (y - f(x_i \mid \bar{k}))^2 + \sum_{j=1}^{m} k_j w_j^2 \right) \quad (12)$$

The result of applying the K4-RANN is the extraction of a set of weights (w_j) such that error (SSE) is minimized while simultaneously optimizing the accuracy of the predicted fit (smoothness). The estimated weights are analogous to nonlinear least-squares regression parameters. When applied to the networked production function, for all j in N (securities in the investor portfolio), the procedure maps the production of monthly returns, r_j, for all j-firms.

4.2 RANN Estimated ESG Returns-to-Scale

The K4-RANN weights associated with equation 10 are interpreted as the factor elasticity coefficients. The next section identifies the parameter settings applied to the K4-RANN algorithm. This is followed by a discussion of the model weights (4.2.2) and the associated production returns-to-scale (4.2.3).

4.2.1 Algorithmic Control Parameters

The K4-RANN algorithm requires several algorithmic parameters. In this study, the model parameters were identically applied to all *j* pricing models. Before invoking the algorithm, all data were standardized. The underlying transfer function chosen for the K4-RANN was Gaussian. The RANN radius was uniformly set to 1.0, and the error minimization rule was set to 'generalized cross-validation' (GCV). The GCV rule is known to perform well for both smooth and rough functions (Wahba, 1985).

4.2.2 Factor Elasticity Network Connectedness

The degree of asymmetric ESG connectedness among sectors and firms is shown in figure 5. The commonality in findings between the sector-limited Granger-causality networks reported by Billio et al. (2012) is evident. The network graph depicts the interrelated production of a standard ESG signal that binds the firms within the investor portfolio.

To understand the change in a firm's returns given a unit change in an ESG factor (any factor), we interpret the K4-RANN weights as quasi-factor elasticity metrics. Before explaining the K4-RANN weights it is useful to view the weights in two different network graphs. The left-side graph (figure 5) presents an overall view of interconnectedness among the ESG pervasive factors. The right-side network chart (figure 6) shows the unique directional impact on return production from each pervasive ESG factor.

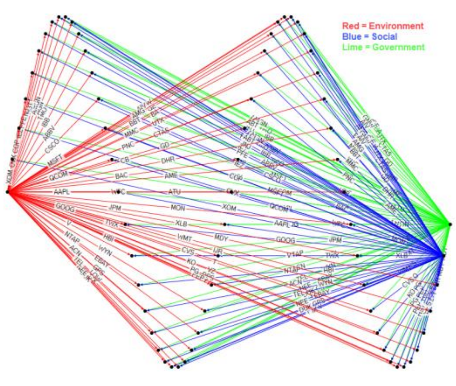

Figure 5: Interconnectedness of E, S, & G

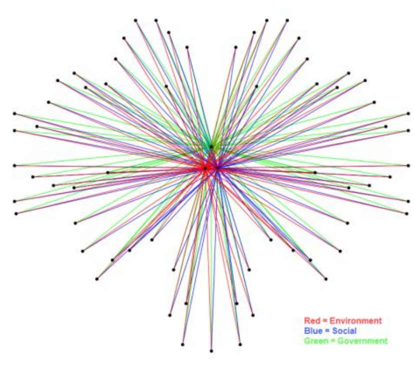

Figure 6: Directional impact of E, S, & G

From the common core ESG signal, the proportional impact of E-, S-, and G- pervasive factor effects are captured by the direction and length of the individual firm's spoke from the ESG centroid.

This proportional impact of ESG factors on return productivity differences among firms is also evident in the elasticity weights generated by the K4-RANN (figure 7; and table 3). We purposely abbreviate the K4-RANN weights to direct the analytical focus on the asymmetric connectedness of firms within the 'Financial Services' sector (see: Appendix A for complete results). Firm and sector connectedness was established in section 4.1.2; hence, we can utilize table 3 to identify the asymmetric influence of ESG factors.

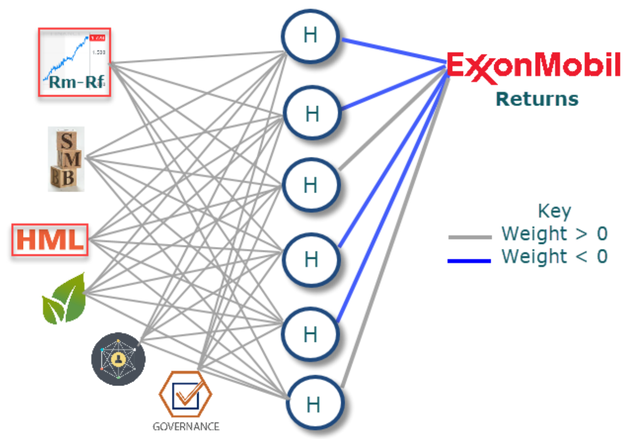

Figure 7: Network Elasticity Weights for ExxonMobil

Table 3: RANN Estimated Weights for Selected Assets

	Ticker	E	S	G	Scale
Banks – Global (Financial Services)	JPM	0.0101	0.3325	0.3058	0.6484
	WFC	-0.1957	0.2673	0.2562	0.3278
	BAC	0.0808	0.2226	0.1862	0.4896
	C	-0.2620	-0.0870	-0.1650	-0.5140
Oil&Gas (Energy)	XOM	-0.0943	-0.0388	0.7870	0.6538
	CVX	-0.2114	0.5529	0.7629	1.1045
	COP	0.4275	0.3749	0.7556	1.5580
iShares Oil Equip & Services ETF	IEZ	0.1224	0.0645	0.7072	0.8942

We begin with a detailed look at the ESG elasticity weights for the financial services industry. For Bank of America (BAC), the weights for E, S, and G are positive (0.0808; 0.2226; and, 0.1862, respectively). Conversely, all weights for Citigroup (C) are negative (-0.2620; -0.0870; -0.1650). We find clear evidence of how the common ESG signal is decomposed into a unique 'Financial Services' signal that is asymmetrically differentiated among individual sector-related firms. Given changes in sustainability performance, the expectation is for BAC to experience a positive increase to returns, while for the same change in sustainability investments, Citicorp will likely experience depreciated returns. Of further interest is the G dimension. Except for Citicorp, all firms displayed in table 3 are expected to benefit by firm responses to government sustainability changes. The asymmetric ESG weights within this sector offer further evidence of the need to understand the contribution of pervasive ESG factor variation to the returns producing process. In the next section, we extend the MBPM to include a dynamic loss-aversion metric.

4.3 Dynamic Option-Theoretic Shortfall Estimation

To meet the aim of this research, we focus on estimating a dynamic risk measure to proxy for 'loss aversion' bias. We limit our focus to the popular frequency-based conditional value-at-risk (CVaR) metric of Rockafeller and Uryasev (2002). CVaR captures the conditional expectation of losses in top (100 – β)% over a given investment horizon (e.g., β = 0.95 or 0.99).

$$CVaR_{\alpha}(X) = \frac{1}{\alpha}\int_{-\propto}^{X} VaR_{\beta}\, d\beta \tag{13}$$

CVaR is a coherent risk measure (Artzner, Delbaen, Eber, & Heath, 1999), and when used in the context of portfolio risk minimization the measure can be expressed as a continuous and convex function with respect to the optimization variables in a convex program (R.T. Rockafellar, Uryasev, & Zabarankin, 2006) and Rockafellar; Uryasev (2002); and, Krokhmal, et al. (2002)). CVaR in linear and multiple goal optimization models is also in evidence. Ogryczak (2002) was one of the first to contribute evidence on the incorporation of CVaR in a goal constraint. Kaminski et al. (2009) extended this line of research by providing a CVaR-based goal programming portfolio selection method to account for investor risk attitudes.

The implementation of CVaR (and VaR) is dependent on knowing the exact distribution information of market parameters. Often, these parameters are characterized by sampling error. There is a significant strain of literature devoted to the calculation of CVaR and its associated sensitivities. For example, Hong, Jeff, and Liu (2011) provide a detailed review of the performance of Monte Carlo methods used to estimate VaR and CVaR (including sensitives). Hsieh et al. (2014) extend the use of Monte Carlo methods by providing a fast algorithm to estimate VaR and CVaR. By contrast, Yao et al. (2013) employ nonparametric estimation of CVaR when applied to the portfolio selection problem.

In this paper, we adopt the put-option market algorithm of Barone-Adesi (2016). This method is consistent with our objective to develop a dynamic approach to the behavioral multiple objective portfolio selection and rebalancing model. Under the Barone-Adesi plan, for a given α, we estimate CVaR for an optionable asset by capturing the instantaneous spot price (S), the risk-free rate (r), and time to expiration *(T)*. Then for a given near-the-money strike ($X_{(put)}$), the algorithm calculates p = BSOPM$_{(put)}$ using the Black-and-Scholes price approximation. The algorithm proceeds by restating CVaR as the expected dollar loss beyond VaR given S. As such, it is affected by fatness in the tail of the distribution of S. In Barone-Adesi model CVaR is stated as:

$$CVaR = \frac{1}{\alpha}\int_{-\propto}^{X} L(S)f(S)dS \tag{14}$$

$$CVaR = e^{rT}\frac{p}{\alpha} + VaR \tag{15}$$

5 Modeling The ESG-Goal-Directed Investor Portfolio

Multi-criteria decision analysis (MCDA) is a featured method of decision support studies. In this section, we blend and formalize the behavioral specification by introducing a multiple criteria decision model (MCDM) that is commensurate with the aim of this research. Because multiobjective optimization methods are capable of handling various conflicting objectives at the same time, the methodology is well-suited for MBPM. When the number of conflicting hierarchical objectives is no more than two, the solutions generated by this model can form a Pareto optimal front – a set of compromised trade-off solutions from which the best possible compromise solution can be selected. Likewise, whenever the number of hierarchical objectives is higher than two, choosing an optimal compromise solution is not a trivial task (Ruotsalainen, 2010).

5.1 Quadratic Optimization

The Quadratic optimization (QO) problem where the objective function is convex quadratic, and the constraints remain linear is routinely applied to the portfolio selection model. The QO problem finds the global minimizers of a quadratic form over the standard simplex. That is, $p \coloneqq \min_{x \in \Delta_n} x^T Qx$. Where $Q \in S_n$ (the space of symmetric $n\ x\ n$ matrices), and $\Delta_n$is the standard simplex in $\mathbb{R}^n$, namely $\Delta_n = \left\{x \in \mathbb{R}^n : \sum_{j=1}^{n} x_j = 1, x \geq 0\right\}$.

Despite QO being NP-hard, it is amenable to a polynomial-time approximation scheme that has an exponentially sized linear programming reformulation (de Klerk, Pasechnik, & Schrijver, 2007). Various optimization approaches to the portfolio problem have exploited this fact. Mokhtar, Shib, and Mohamad (2014) survey over 40 related articles and classify solution algorithms according to their nature in heuristic and exact methods. They report that goal programming constitutes the highest number of mathematical programming techniques applied to the portfolio optimization problem. Based on the goal programming approach, Dash, Hanumara, and Kajiji (2003) proposed a methodology for the construction of a futures-hedged equity portfolio using a separable nonlinear mixed integer goal programming algorithm. The application focused on the ability of the separable programming algorithm to replicate the mean-variance approach to enumerating the efficient set. In the second stage, the authors demonstrate binary control over the decision to hedge the portfolio based on the next-period forecast of the market futures contract.

5.2 The Multiobjective Optimization Problem

$$min\{f_1(x), f_2(x), \dots, f_k(x)\}$$
$$subject\ to: x \in S,$$

Where x is a vector of continuous decision variables form the feasible set $S \subset \mathbb{R}^n$ defined by linear, nonlinear and box constraints $(k \geq 2)$. An objective vector is defined by $\boldsymbol{f}(x) = \left(f_1(x), f_2(x), \dots, f_k(x)\right)^T$. The image of a feasible set is denoted by $\boldsymbol{f}(S) = Z$ represents a feasible set. This is a subset of the objective space $\mathbb{R}^k$. The elements of Z are objective vectors denoted by $\boldsymbol{f}(\boldsymbol{x})$ or $\boldsymbol{z} = (z_1, z_2, \dots, z_k)^T$, where $z_i = f_i(\boldsymbol{x})$ for all $i = 1, \dots, k$ are objective function values.

When there is no conflict between the objective functions, then multiobjective optimization methods are not required as there is a single optimal solution. The multiobjective optimization problem is linear if all the objective functions and constraint functions are linear. Conversely, the nonlinear multiobjective optimization problem occurs if any of the objective or constraint functions are nonlinear.

5.3 Behavioral Portfolio Theory and Goal Programming

The MBPM is, in part, based on investor's utility maximization. As such part of the formulation is expressed as a parametric quadratic optimization problem. By this definition, the MBPM is a nonlinear optimization problem. Below, we specify the use of a nonlinear goal programming formulation to solve this model.

Before presenting the complete model, it is essential to review how the Markowitz mean-variance model is easily approximated by the Sharpe single-index model (Frankfurter, Phillips, & Seagle, 1976; Sharpe, 1971). The Sharpe single-index model (SIM) is known to produce an identical maximum rate of return portfolio (top-most portfolio on the efficient set) but a slightly inefficient replication of the global minimum variance portfolio. In the context of a bi-goal optimization of the SIM the first objective fixes the level of expected return and the second objective seeks to minimize the variance of portfolio returns. The alternative behavioral models presented below all share the foundation statement to assure the enumeration of efficient portfolios. Goal programming models are differentiated by the formulated goal hierarchy. The cardinal constraints/goals define the decision-making attributes of the portfolio selection model. In this section, four alternative models are formulated and solved.

5.3.1 Goal Hierarchy

The goal hierarchy for the first model, equation 16, is equivalent to solving the SIM portfolio optimization model. Hereafter referred to as M1, this model and the comparable efficiency solution is provided for reference. The remaining three models are present preliminary evidence on how to state and solve aspirational bias-driven portfolio selection.

The second model, M2, represented by the goal hierarchy, is stated in equation 17. This goal structure does not minimize portfolio return variance in the second objective. Instead, the model seeks to reduce 'loss aversion' bias (CVaR) before reducing portfolio return variation in the third hierarchical objective. Models 3 through 4 demonstrate the ability to specify returns-to-scale goals as a proxy for minimizing some part of both 'recency bias' and 'overoptimism bias.'

Model 3, M3, adds to the first objective, the desire to mimimize deviation from the client's focus on environmental returns-to-scale in the ESG dimension. The second goal has both the traditional risk minimization of portfolio returns and adds to that returns-to-scale on social policy. Lastly, model 4 demonstrates goal complexity. The first level objective represents the investor who places equal importance on achieving a fixed level of portfolio return while meeting complete ESG scale goals.

$$Model\ 1: Min\ Z = \{P_1[h_4^-], P_2[h_1^+], P_3[h_8^+]\} \tag{16}$$

$$Model\ 2: Min\ Z = \{P_1[h_4^-], P_2[h_8^+], P_3[h_1^+]\} \tag{17}$$

$$Model\ 3: Min\ Z = \{P_1[h_4^- + h_5^-], P_2[h_1^+ + h_6^-], P_3[h_8^+ + h_7^-]\} \tag{18}$$

$$Model\ 4: Min\ Z = \{P_1[h_4^- + h_5^- + h_6^- + h_7^-], P_2[h_1^+], P_3[h_8^+]\} \tag{19}$$

5.3.2 Portfolio Selection Goals

$$\sum_{j=1}^{n+1} \varepsilon_j^2 x_j - h_1^+ = 0 \tag{20}$$

$$\sum_{j=1}^{n} \beta_j^V x_j = \beta_M \tag{21}$$

$$\sum_{j-1}^{n} x_j = 1.0 \tag{22}$$

$$\sum_{j=1}^{n} E(r_j)\, x_j + h_4^- - h_4^+ = R_{R_p} \tag{23}$$

$$\sum_{j=1}^{n} RtS_j^E\, x_j + h_5^- - h_5^+ = RtS_p^E \tag{24}$$

$$\sum_{j=1}^{n} RtS_j^S\, x_j + h_6^- - h_6^+ = RtS_p^S \tag{25}$$

$$\sum_{j=1}^{n} RtS_j^G\, x_j + h_7^- - h_7^+ = RtS_p^G \tag{26}$$

$$\sum_{j=1}^{n} CVaR_j^{BSOPM_{put}} + h_8^- - h_8^+ = 0.0 \tag{27}$$

Where, $\sum_{j=1}^{n} E(r_j)\, x_j$ depends on equation (red eq); and, RS_p^E, RS_p^S RS_p^G= 1.0, respectively.

Equation 20 and 21 state the unsystematic and systematic risk goals, respectively. Equation 20 expresses the variance of the idiosyncratic risk (ε_j) for *n* investment securities and σ^2 the variance of returns for the market proxy as the *n+1* security. Structural systematic risk (β_j) is expressed by equation 21. The canonical form of the Sharpe market model requires equation 21; an expression that forces the portfolio beta to equal the weighted sum of the individual security beta coefficients. Equation 22 forces the portfolio to be fully invested (short-sales prohibited). Equation 23 is the goal constraint used to set the required return for the efficient portfolio, R_{R_p} . Individual security responses to pervasive ESG systemic risk production factors are modeled in equations 24 through 26. These goal constraints equate the *j-th* securities contribution to sustainable investing in return-to-scale units (*Scale*). The goal expression of the dynamically estimated option-priced CVaR for each *j-th* security is expressed in equation 27.

5.4 The Investor Equally Weighted Portfolio

For comparative purposes with the optimized ESG diversification plan, we create an equally weighted client portfolio (EqWg). As shown by equation 28, the equally weighted portfolio requires adding *n* additional constraints:

$$\sum_{j=1}^{n} x_j = (1/n) \quad where \quad x_j \begin{cases} 1 & if\,(i=j) \\ 0 & if\,(i \neq j) \end{cases}. \tag{28}$$

5.5 Comparative Investment Efficiency

The dominant Markowitz mean-variance efficient set is obtained by application of Lemke's complementary slackness algorithm (Cottle, Pang, & Stone, 1992). The efficient set produced by solving the approximate Sharpe diagonal model (model 1). This solution is also enumerated by an application of Lemke's algorithm and, for comparative purposes, the nonlinear goal programming algorithm. Reference is made to figure 8. By observation, and in a manner reflective of extant literature, material differences between the mean-variance and Sharpe diagonal efficient is most visible at low expected rates of portfolio return.

The Cartesian coordinates for the ESG portfolio, the non-optimized equally weighted portfolio (EqWg), and the initial, that is naively diversified, client diversification plan is plotted against the risk and return axis in figure 8. Interestingly, the EqWg and the investor's naïve selection scheme produce numerical coordinates that share approximately the same level of portfolio risk.

NOTE: Abbreviated results are presented in this draft version of the manuscript.

The solution results for the alternative models are presented in table 4. The notation differentiates the various solutions. As previously stated, M1, M2, M3, and M4 refer to goal-hierarchy statements already defined. To enhance the comparative analysis across the four-goal structures, three alternative solutions are generated for each model. By way of example, for model M1, alternate solutions are presented as M1-1, M1-2, and M1-3. This labeling convention is applied to the four-goal models. Graphical efficient set results are presented in figure 8.

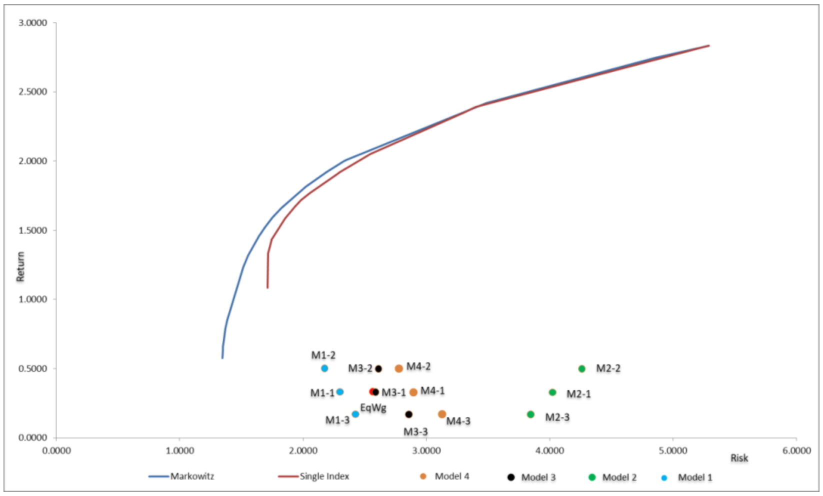

Figure 8: Comparative Efficient Frontiers

Reference is made to table 4. This table presents the parameter settings for each solution, the diversified selection plan, and the risk-return cartesian coordinates.

- The standard QP mean-variance and SIM dominate all behavioral portfolio selection results.
- Firm incorporation of ESG production opportunities results in increased portfolio risk to investors.
- ESG portfolio diversification ratios measured by the coefficient of variation (CV) demonstrates how, over moderate expected return levels, a firm’s commitment to ESG production opportunities can result in increased portfolio risk-adjusted return opportunities.
- Performance attributes of behavioral portfolios show a preference for eliminating ESG bias over that of ‘loss aversion bias’; or CVaR emphasis.

Table 4: Prioritized ESG Portfolio Selection

Tickers	E(R)	Risk	CVaR	M1-1	M1-2	M1-3	M2-1	M2-2	M2-3	M3-1	M3-2	M3-3	M4-1	M4-2	M4-3
NEE	1.24%	3.27%	18.79%	20.00%	20.00%	20.00%					2.26%				
DUK	0.08%	3.84%	11.00%	20.00%	20.00%	20.00%									
TJX	0.53%	3.01%	9.84%	19.65%	20.00%	7.78%				20.00%	20.00%	20.00%	20.00%	14.74%	19.77%
CVS	-0.82%	4.59%	8.09%	0.35%		12.22%							4.30%		8.13%
KO	0.19%	2.33%	5.13%	20.00%	20.00%	20.00%									
PG	-0.36%	3.61%	9.07%	20.00%		20.00%									
PEP	0.39%	2.58%	11.24%		18.42%					21.30%	5.59%	13.56%			
DPS	1.33%	4.34%	14.96%		1.58%					5.45%	20.00%		10.05%	15.04%	
F	-0.77%	5.10%	2.87%				21.49%	16.11%	26.55%						
EBAY	1.23%	5.57%	4.54%				56.56%	64.84%	48.77%						
HBI	-0.92%	6.07%	3.45%				21.95%	19.05%	24.68%						
WMT	0.11%	4.44%	10.02%							48.47%	51.39%	43.08%	55.53%	54.07%	58.13%
VZ	0.30%	4.43%	5.48%							4.78%		15.09%			4.25%
BA	2.74%	5.25%	33.80%								0.76%				
QCOM	-0.99%	6.83%	7.46%									8.27%			
V	1.55%	3.20%	13.32%										0.70%	7.16%	
LOW	0.56%	5.16%	9.89%										9.42%	8.99%	9.73%
Portfolio Return				0.3313%	0.5009%	0.1710%	0.3283%	0.4982%	0.1684%	0.3292%	0.4992%	0.1697%	0.3291%	0.4990%	0.1693%
Portfolio Risk				2.2982%	2.1742%	2.4237%	4.0213%	4.2603%	3.8440%	2.5877%	2.6113%	2.8571%	2.8945%	2.7747%	3.1279%
CV				693.73%	434.11%	1417.14%	1224.99%	855.09%	2282.93%	786.02%	523.12%	1683.95%	879.51%	556.05%	1847.58%

M1: smallest CV set but no ESG

ESG focus needs to be spread out

Quick Reference:
Model 1: R, V, C
Model 2: R, C, V
Model 3: RE, VS, CG
Model 4: RESG, V, C

These preliminary results extend our understanding of how to use the hierarchical goal programming approach to solving BPM decision problems. For the rational investor, the multiobjective model produces efficient portfolio selection while meeting bias goals as closely as possible.

6 Summary and Conclusions

The aim of this research was to extend the dynamic BPM to encapsulate aspirational bias towards sustainability. To achieve this aim this research employed a factor disaggregation model to derive new ESG pervasive factors. In the course of developing a multiple objective BPM for portfolio selection and rebalancing, machine learning was combined with a Cobb-Douglas production function specification for asset returns. The networked-based asset return model proved to offer new insight into new research seeking alternatives to the traditional capital asset-based pricing alternatives. Lastly, the dynamic estimation of 'loss aversion bias' was proxied by the introduction of an option-priced CVaR metric.

Upon solving alternative statements of the multiple goal behavioral model, we were able to establish the importance of ESG control in behavioral portfolio selection. We also report how an emphasis on 'loss aversion' is likely to produce less reward ratio efficient portfolios. These preliminary findings notwithstanding, there are several important areas where new research is needed. The time-variation of the new ESG factors received attention but deserves a more detailed study. Also, the examination of alternate priority structure models is warranted in further research. The question of a machine-learning estimation of a production-theoretic model of asset returns is new and now surfacing in the literature. The contribution provided by the application in this paper should provoke more pointed studies in this area.

Appendix A: Weights from the K4-RANN Analysis

		Rm-Rf	SMB	HML	E_FSI	S_FSI	G_FSI	ESG Elasticity	MSE
XOM	Exxon Mobil Corporation	-1.746	-0.737	0.585	-0.094	-0.039	0.787	0.654	0.00020
CVX	Chevron Corporation	-1.532	-0.001	0.554	-0.211	0.553	0.763	1.104	0.00028
COP	ConocoPhillips	-0.034	0.279	1.000	0.428	0.375	0.756	1.558	0.00032
IEZ	iShares US Oil Eqp & Srv ETF	-0.457	0.040	0.408	0.122	0.065	0.707	0.894	0.00012
PFE	Pfizer Inc	-0.284	0.016	0.000	-0.163	0.128	0.087	0.051	0.00000
JNJ	Johnson & Johnson	-1.407	-0.519	-0.518	-0.265	-0.159	-0.097	-0.520	0.00007
ABT	Abbott Laboratories	-0.034	0.266	-0.224	0.058	0.071	0.439	0.567	0.00002
CI	Cigna Corporation	-1.670	-2.011	-0.582	0.341	0.125	0.236	0.701	0.00046
AMGN	Amgen Inc	-0.340	0.008	-0.629	-0.661	0.000	0.316	-0.345	0.00006
AGN	Allergan plc	-0.712	-0.218	0.013	-0.819	-0.432	0.336	-0.915	0.00018
TMO	Thermo Fisher Scientific Inc	0.079	0.085	0.085	0.075	0.064	0.136	0.275	0.00000
IBB	iShares Nasdaq Biotech ETF	0.044	0.157	-0.026	-0.057	0.162	0.256	0.361	0.00009
ABBV	AbbVie Inc	0.925	-0.533	-1.805	0.196	0.134	0.778	1.108	0.00055
CSCO	Cisco Systems, Inc	0.612	0.715	0.048	0.729	0.924	0.998	2.652	0.00016
MSFT	Microsoft Corporation	0.180	0.223	0.063	0.756	0.125	0.642	1.524	0.00006
QCOM	QUALCOMM Incorporated	-0.223	-0.773	-1.052	2.315	0.037	0.524	2.876	0.00087
AAPL	Apple Inc	-0.338	0.279	-0.261	0.604	0.133	0.026	0.762	0.00005
GOOG	Alphabet Inc	-0.818	-0.311	-0.976	0.204	-0.372	0.562	0.395	0.00010
V	Visa Inc	-0.152	0.072	-0.045	0.400	0.243	1.043	1.685	0.00004
NTAP	NetApp, Inc	0.123	0.746	0.598	0.966	0.829	0.729	2.523	0.00052
ACN	Accenture plc	-0.067	-0.097	-0.261	0.185	0.159	0.186	0.530	0.00000
TEL	TE Connectivity Ltd	0.073	0.089	-0.012	0.194	0.049	0.159	0.402	0.00000
NEE	NextEra Energy, Inc	-0.106	0.287	0.193	0.196	0.140	-0.272	0.064	0.00001
DUK	Duke Energy Corporation	-0.667	-0.172	0.032	0.096	-0.318	-1.059	-1.281	0.00006
F	Ford Motor Company	-0.747	0.007	0.131	-0.081	-0.021	-0.157	-0.258	0.00004
TJX	The TJX Companies, Inc	0.513	0.673	0.567	0.477	0.712	0.623	1.812	0.00005
LOW	Lowe's Companies, Inc	-0.480	-1.682	-0.248	-0.363	1.187	1.689	2.513	0.00053

GPS	The Gap, Inc	0.063	0.236	-0.014	0.270	0.822	0.226	1.318	0.00021
EBAY	eBay Inc	0.659	-0.248	-0.556	-0.553	0.043	0.234	-0.276	0.00009
WYN	Wyndham Worldwide Corp	-0.079	-0.135	-0.158	0.007	0.074	0.253	0.335	0.00001
HBI	Hanesbrands Inc	-0.951	-0.565	-0.675	-1.949	0.691	0.561	-0.696	0.00059
TWX	Time Warner Inc	0.034	-0.108	-0.044	-1.196	-0.031	0.058	-1.169	0.00009
JPM	JPMorgan Chase & Co	0.044	-0.061	-0.247	0.010	0.333	0.306	0.648	0.00002
WFC	Wells Fargo & Company	-0.694	-0.603	-0.595	-0.196	0.267	0.256	0.328	0.00008
BAC	Bank of America Corp.	0.070	0.037	-0.108	0.081	0.223	0.186	0.490	0.00002
CB	Chubb Limited	-0.663	-0.908	-0.787	-0.446	-0.541	-0.402	-1.389	0.00006
PNC	PNC Financial Srv Grp, Inc.	0.109	-0.115	-0.816	-0.270	0.472	0.330	0.532	0.00006
MMC	Marsh & McLennan Co. Inc	-0.306	-0.294	-0.350	-0.330	-0.326	-0.327	-0.983	0.00001
BBT	BB&T Corporation	0.171	0.080	-0.154	0.084	0.402	0.553	1.039	0.00003
AMG	Affiliated Managers Group, Inc	-0.500	-0.031	-0.553	-0.242	0.216	-0.014	-0.040	0.00008
AIV	Apartment Investment and Management Company	-0.937	-0.105	0.061	-0.125	-0.230	-0.707	-1.062	0.00005
GNW	Genworth Financial, Inc	-0.116	-0.069	-0.101	-0.101	-0.088	-0.098	-0.287	0.00003
C	Citigroup Inc	-0.261	-0.345	-0.353	-0.262	-0.087	-0.165	-0.514	0.00004
GE	General Electric Company	-2.113	-0.936	-0.534	-2.518	-1.083	-0.297	-3.897	0.00110
BA	The Boeing Company	0.491	-0.341	-0.131	0.105	0.981	1.007	2.093	0.00018
UTX	United Technologies Corp.	-0.104	-0.049	-0.139	-0.017	0.098	0.243	0.324	0.00000
CTAS	Cintas Corporation	-0.186	0.382	-0.196	-0.243	0.445	0.169	0.371	0.00002
GD	General Dynamics Corporation	0.054	-0.046	-0.451	-0.629	-0.309	0.232	-0.706	0.00002
DHR	Danaher Corporation	-0.193	0.250	0.026	0.721	-0.166	0.802	1.358	0.00004
AME	AMETEK, Inc	0.082	0.224	-0.196	0.282	0.372	0.421	1.075	0.00002
ATU	Actuant Corporation	-0.430	-0.243	-0.102	-0.256	-0.137	-0.221	-0.615	0.00004
MON	Monsanto Company	0.059	0.011	0.158	-0.076	-0.015	0.148	0.057	0.00000
XLB	Materials Sector SPDR Fund	-0.333	-0.211	-0.209	-0.112	-0.077	0.114	-0.076	0.00001
WMT	Walmart Inc	-0.882	-2.064	-0.237	1.781	1.094	1.057	3.931	0.00054
CVS	CVS Health Corporation	-1.367	-1.106	0.019	-0.778	0.387	1.128	0.737	0.00027
KO	The Coca-Cola Company	-1.112	-0.550	-0.164	-0.217	-0.160	0.103	-0.274	0.00002
PG	Procter & Gamble Company	-1.820	-0.726	-0.685	-0.575	0.760	-0.341	-0.155	0.00018

PEP	PepsiCo, Inc	-1.878	-0.687	-1.071	0.408	0.828	0.058	1.294	0.00010
HSY	The Hershey Company	-1.231	-0.243	-0.570	-0.215	0.308	-0.370	-0.277	0.00012
CL	Colgate-Palmolive Company	-0.519	-0.309	-0.275	-0.309	-0.167	-0.193	-0.669	0.00001
DPS	Dr Pepper Snapple Group, Inc	2.540	0.059	0.333	-0.193	1.192	0.734	1.733	0.00043
VZ	Verizon Communications Inc	-0.867	-0.629	0.034	-0.725	1.795	0.157	1.227	0.00027
T	AT&T Inc	-0.296	-0.324	-0.498	-0.703	0.847	-0.194	-0.050	0.00007
IJR	iShares S&P SmallCap ETF	-0.482	0.064	-0.157	-0.170	-0.114	0.114	-0.170	0.00001
MDY	SPDR S&P MIDCAP 400 ETF	-0.868	0.059	-0.344	0.034	-0.014	0.370	0.391	0.00002

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