

Algorithms for Optimal Decisions

Tutorial 1

Questions

Exercise 1 Show that the intersection S of any numbers of convex sets S_i is a convex set.

Exercise 2 Show that if $f(x)$ and $g(x)$ are convex functions on a convex set S , then their sum

$$h(x) = f(x) + g(x) \tag{1}$$

is also a convex function on S .

Exercise 3 Show that if $f(x)$ is a convex function, then the set

$$L = \{x \in R^n \mid f(x) \leq b\} \tag{2}$$

is a convex set.

Exercise 4 Consider the non-linear problem:

$$\begin{aligned} \min_x \quad & f(x) = x_1^2 + x_2^2 - 4x_1 + 4 \\ \text{s.t.} \quad & g_1(x) = x_1 - x_2 + 2 \geq 0 \\ & g_2(x) = -x_1^2 + x_2 - 1 \geq 0 \\ & g_3(x) = x_1 \geq 0 \\ & g_4(x) = x_2 \geq 0. \end{aligned} \tag{3}$$

1. Show that the constraints define a convex set;
2. Show that the objective function $f(x)$ is convex.