Algorithms for Optimal Decisions
Tutorial 8
Questions

Solving Systems of Nonlinear Equations The equilibrium condition for the Nash Strategy:

$$\mathcal{W}(y) \equiv \nabla_{U^i} \mathcal{F}^i(U^1, U^2, ..., U^i, ..., U^N) = 0; \quad i = 1, 2, ..., N, \tag{1}$$

where $y \equiv [U^1, U^2, ..., U^i, ..., U^N]^t$.

We define $\mathcal{W}_j = \mathcal{W}(y_j)$ and, similarly, the Jacobian $\nabla \mathcal{W}_j = \nabla \mathcal{W}(y_j)$.

Pseudo code for the full quasi–Newton algorithm, as explained in the lecture notes:

1. set $j = 0$, choose y_0.
2. **do**
3. **if** $\|\mathcal{W}(y_j)\|_2 \leq \epsilon$ **exit**
4. $d_j = -(\nabla \hat{\mathcal{W}}_j)^{-1} \mathcal{W}_j$
5. Choose τ such that

$$\frac{1}{2}\|\mathcal{W}(y_j + \tau_j d_j)\|_2^2 - \frac{1}{2}\|\mathcal{W}(y_j)\|_2^2 \leq \rho \tau_j (\nabla \mathcal{W}^t(y_J) \mathcal{W}(y_J), d_j) \tag{2}$$

6. $y_{j+1} = y_j + \tau_j \cdot d_j$.
7. $\nabla \hat{\mathcal{W}}_{j+1} = \nabla \hat{\mathcal{W}}_j + \frac{[\mathcal{W}_{j+1} - \mathcal{W}_j - \nabla \hat{\mathcal{W}}_j (y_{j+1} - y_j)](y_{j+1} - y_j)^t}{\|y_{j+1} - y_j\|_2^2}, \quad j = j + 1$
8. **end do**

Exercise 1 *Solve the following problem using the interior point method:*

$$\begin{aligned} \min_x \quad f(x) &= x_2 \\ s.t. g_1(x) &= x_2 - \sin(x_1) - \frac{x_1}{2} \geq 0 \\ & x_1, x_2 \geq 0. \end{aligned} \tag{3}$$

Exercise 2 *Solve the following system of nonlinear equations:*

$$\mathcal{W}(y) = \begin{bmatrix} y_1 + y_2 - 3 \\ y_1^2 + y_2^2 - 9 \end{bmatrix} = 0, \tag{4}$$

starting the algorithm with the initial estimate $y_0 = (1, 5)$.

Exercise 3 *Find the appropriate stepsize parameter for the following system of equations:*

$$\mathcal{W}(y) = \begin{bmatrix} y_1^2 + y_2^2 - 2 \\ e^{y_1 - 1} + y_2^3 - 2 \end{bmatrix} = 0, \tag{5}$$

choosing $y_0 = (2, \frac{1}{2})$ *as a starting point.*