Integer Programming
Tutorial 1
Questions

Exercise 1 *Olympic Airways Wants to load n containers on one of its cargo air planes. Container j weighs a_j tons and its value is c_j dollars. The maximum capacity of the air plane is b tons. The airline wants to load the air plane in such a way that the value of its cargo is as large as possible. Formulate the problem as an integer programming problem.*

Exercise 2 *The owner of a big motor company wants to build $k = 10$ new factories in different areas. All factories make the same product. The owner has $n = 15$ customers. Customer i demands d_i units of the product. The operating cost of the factory j is $f_j \geq 0$ and the maximum number of units it can make is M_j. The cost of delivering 1 unit from factory i to customer j is $c_{i,j}$.*

Where should the owner build his new factories in order to minimise the delivery cost? Formulate the above problem as an I.P. programming problem.

Exercise 3 *Reformulate as IP problem the following problem:*

$$\begin{array}{rl} \min_{x_1,x_2} & 2x_1 - 7x_2 \\ s.t. & 0 \leq x_1 \leq 10 \\ & 0 \leq x_2 \leq 10, \end{array} \tag{1}$$

and at least one of the following holds:

$$\begin{aligned} -2x_1 + 3x_2 &\geq 0 \\ 5x_1 - 4x_2 &\geq 0. \end{aligned}$$

Exercise 4 *Solve the following problem:*

$$\begin{array}{rl} \min_x & c^t x \\ s.t. & Ax = b \\ & x \geq 0 \\ & x_1 \in \{r_1, r_2, ..., r_q\}. \end{array} \tag{2}$$

Exercise 5 *Formulate the following model as a mixed integer programming problem:*

$$\begin{array}{rl} \min_x & \sum_{j=1}^{n} \mathcal{C}_j(x_j) \\ s.t. & Ax \leq b \\ & x \geq 0 \\ & \mathcal{C}_j(x_j) = \left\{ \begin{array}{cc} 0 & x_j = 0 \\ k_j + c_j x_j & x_j > 0 \end{array} \right. \end{array} \tag{3}$$

where $c_j, k_j > 0$ and k_j are called fixed changes.