A lot of toast, with a side order of roast

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Good evening. It's a great pleasure and an honor for me to give this talk. As some of you will recall, Don Knuth was the after-banquet speaker at *my* one-million-first birthday conference a few years ago, and I'm happy to have the opportunity to retaliate – I mean – excuse me – to reciprocate.

Don Knuth is one of the most influential and creative theoretical computer scientists ever. If he had done nothing but create TEX, his permanent place in the pantheon of science would have been assured. But his accomplishments go far beyond that. His books, beginning with *The Art of Computer Programming*, have sales figures that rival those of – well, – Harry Potter. He has organized, pulled together, *and created*, large parts of theoretical computer science, and we are forever in his debt for that.

Don is one of the great communicators of the twentieth, and we all wish for him, the twenty-first centuries. Actually, that depends a little bit on which endpoint of the vector of communication Don is sitting at. If he is at the *initial* vertex of that arrow, he is supreme. His papers, books, web site, and other writings and talks, are brilliantly original, and crystal clear. All of us, very much including yours truly, have learned a lot about clarity of scientific communication from his example.

On the other hand – if Don is sitting at the terminal vertex of the arrow of communication, life is a bit different. You have to do a lot of things right in order to get your thoughts through to him.

First, abandon e-mail, all ye who seek to enter here. Although perhaps more than any other human being on the planet he has facilitated the communication of scientific results by electronic means, if you or I try to communicate with *him* by electronic means it will be rapidly unfacilitated. He doesn't have an e-mail address. Occasionally he will surface, with a *nom-de-guerre* of an e-mail address, in order to send some message quickly. But, forget that "Reply" button; the communication arrows point only outwards.

So you take the hint, and after writing 37 e-mail messages to various addresses that you thought might have gotten through to him, you decide on written communication. Of course, since you are about to write to the creator of TeX, you are *not* about to write in Microsoft Word, are you? Of course not. To show your respect for your addressee and his creations, you will write in TeX, and so you do. You spend a great deal of effort to get it to look pretty, and you send it off. Let's say that you're writing in order to describe a proof that $\mathbf{P} = \mathbf{NP}$ which you've recently found.

Unbeknownst to you, your letter will be placed on a stack that already has 5,379 letters that reached him before yours did, and which are waiting while he completes his latest additions to 47 new manuscripts and 311 revisions of already existing books. But one day, probably in the same year, your moment will come. He'll read your letter and give you his reply. You eagerly tear open the envelope that reaches you, and what you find inside is your own letter to him, on which he will have pencilled a number of pithy comments on your theorem. Really insightful comments, that make you pleased to have gotten the benefit of Don's knowledge and brilliance. For example, "It's best to use a backslash comma here, in order to get exactly .073 ems of space," or, "the backslash mathchardef doesn't belong here; see page 397 of the TeXBook for a better way," etc.

Well, that was a tad deflating, since what you *wanted* was to hear a comment about that $\mathbf{P} = \mathbf{NP}$ proof of yours. You ponder the situation for a while, wondering how to modify your communication strategy. You immediately discard the idea of reworking your letter in TeX in order to get one with perfect TeX-nique since you're quite certain that this goal of perfection is un-attainable by you. You take a clue from Don's reply, which was a pencilled scrawl on your original message. That's it! You'll sit down and write, by hand, in pen and ink, on fine paper, the whole $\mathbf{P} = \mathbf{NP}$ proof. Then, at least, you'll be past the TeX barrier and into real mathematical give-and-take. And so you do.

One more trip through the mile-high stack, and later that same year the answer arrives. Breathlessly you tear open the envelope once more, and again you read the now-familiar pencilled scrawl, this time wrapped around the words of your elegant handwritten letter. In one place it says "Good grief! The Euler numbers of the fourth kind cannot be denoted by $\mathbf{E}(\mathbf{n}, \mathbf{k})$! That notation went out in 1642 in the writings of Fermat, and since then everybody should be using the triple parentheses with the **n** upstairs and the **k** downstairs." Or "The Fibonacci numbers are never denoted by lower-case-**f**-sub **n**! Since July 3rd, 1308, following the speech of J. Kalashnikov to the St. Petersburg Academy, *everybody* has been using the capital-**F**-sub-**n** for those numbers." etc. Sigh. Congratulations! You've penetrated the TeX barrier but you have crashed and burned on the mathematical notation barrier. Don is very keen on mathematical notation. You didn't know that when you wrote that lovely handwritten missive, but he is. So before you make your next attempt at *communicationus interruptis*, you had better check that you are using the officially approved notation for everything in your message.

I could go on with a description of the complete Inbound Communication Algorithm, but I won't because there's a better way. The Collected Works of Monty Python's Flying Circus are well known to have the property, shared only by the Bible, by the works of Shakespeare, and by The Art of Computer Programming, that whatever it is that you would like to say, they have already said it, and in a more interesting way than you would have. So let me show you a video of the Monty Python group doing the Police Station skit, which summarizes such communication problems as I have been describing, much more interestingly than I would have been able to.

[Show video: *Monty Python's Flying Circus: The Police Station, with Silly Voices*]

Now let me give my recollection of an incident that happened in the 1980's and which Don spoke about at my birthday conference. In the 1980's, in the early days of the Journal of Algorithms, I was an Editor-in-Chief, and Don submitted a paper to me, authored by himself under the pseudonym of Ursula N. Owens, ostensibly from some small college in some small nonexistent town in Kansas. The reason was that he really wanted to get a tough and substantive referee's report on the paper, and he had been finding that sometimes referees had pulled their punches because of his name at the top of a paper. I was happy to agree, and sent it off to be refereed under that pseudonym. The report came back after a while, noting a few corrections that needed to be made, but recommending publication very warmly. I sent the report along to Don and told him the paper was accepted. When he submitted the final manuscript to me, his covering letter was written by and signed by one "Ursula N. Owens," and at the bottom of the letter there was a wonderful impression of female lips with lipstick. Someone who was wearing lipstick had kissed the letter, and now Ursula was sending it to me, without a doubt. I replied by calling a 1-800 number that gets flowers delivered, and I sent him – I mean, her – a half-dozen roses with a card that said "Your place, or mine?" We had a lot of fun with that one.

Here's a little story of math at 35,000 feet. Neil Calkin and I recently authored a paper entitled *Recounting the rationals*, in the American Mathematical Monthly. In it we showed that if $\mathbf{b}(\mathbf{n})$ is the number of partitions of the integer $\mathbf{n}$ into powers of 2, no power of 2 being used more than twice, then the fractions $\mathbf{b}(\mathbf{n})/\mathbf{b}(\mathbf{n}+\mathbf{1})$ take every rational number value once and only once, so they enumerate the rationals. I wrote to Don and sent him a few more related results that Neil and I had gotten in the hope of interesting Don in joining us for a research project. As a result, I got a letter from him that I found to very moving indeed. Of course it was in his familiar pencilled scrawl, written at the bottom of the note that I had sent him. It began as follows:

> Dear Herb, I am writing this on an airplane while flying to Austin to celebrate Dijkstra's 70th birthday. This whole subject is beautiful and still ripe for research (after more than 150 years of progress!), so I will tell you what I know about it (or can recall while on a plane) in hopes that it will be useful.

There followed four pages of tightly handwritten information that was a gold mine of the history of the sequence $\mathbf{b(n)}$ and related matters. It contained references to de Rham, Dijkstra, Carlitz, Neil Sloane, Stern (of Stern-Brocot trees fame), Eisenstein, Conway, Zagier, Schönhage, Hardy, Ramanujan, and so forth. It did not merely mention the names. It contained precise statements of the results of interest, of their relationship with the result of Calkin and myself and with each other. It contained a number of Don's own observations about related questions, and had several conjectures, with background, to offer.

It was a letter, ladies and gentlemen, that was written by a man who very much loves his subject and the history of his subject, and who knows more about that than any five other human beings that you could name. His enthusiasm, his background knowledge, and his mathematical values, that impelled him to write such a letter under such conditions, can be taken as examples for the rest of us to aspire to.

I would like to offer a small present to Don on the occasion of this birthday. I considered giving him a fountain pen, or a PlayStation 2, or a free e-mail account on an ISP of his choice, or a necktie, or a squirt gun, or a few other things that would no doubt have warmed the cockles of his heart. I settled on a theorem. It's a little theorem, not a big one. But it's nice, and it's in a subject that is very close to Don's heart: the subject of combinatorial sequences. I'm gift-wrapping it by surrounding it with the words of this talk, and inside the wrapping paper there is a small insight into Stirling numbers of the second kind. So, at the risk of using a bracket where I should be using a parenthesis, or a brace instead of an angle bracket, or of possibly being ignorant of Euclid's priority on this theorem, let me tell you what it says.

Theorem: [Dedicated to Don Knuth on his 64th birthday – from Herb Wilf] *Let* $\left\{ n \atop k \right\}$ *be the usual Stirling number of the second kind. Then, analogously to the well known generating function*

$$\sum_{n\geq k} \left\{ n \atop k \right\} x^n = \frac{x^k}{(1-x)(1-2x)(1-3x)\cdots(1-kx)},$$

we have the following generating function for the squares of these Stirling numbers:

$$\sum_{n\geq k} \left\{ n \atop k \right\}^2 x^n = x^k \sum_{r,s=1}^{k} \frac{A_{k,r}A_{k,s}}{1-rsx},$$

where
$$A_{k,r} = \frac{(-1)^{k-r} r^k}{k!} \binom{k}{r}.$$

To prove this you need only to write out the explicit formula

$$\left\{ {n \atop k} \right\} = \sum_{r=1}^{k} A_{k,r} r^{n-k},$$

for these Stirling numbers, square it, multiply by x^n, and notice that we'll still be able to sum over n since that will be a geometric series. $\square$

As an example, when $k = 3$ we get

$$\sum_{n \geq 3} \left\{ {n \atop 3} \right\}^2 x^n =$$

$$\frac{1 + 11\,x - 36\,x^2 - 36\,x^3}{(1-x)(1-2\,x)(1-3\,x)(1-4\,x)(1-6\,x)(1-9\,x)}.$$

Happy birthday, Don!

Thank you, everybody, and good night..