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CONTENTS

PI MU EPSILON JOURNAL

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'For manuscripts, authors ore requested to identify themselves as to their class or year if they are undergraduates or graduates, and the college or university they are attending, and as to position if they ore faculty members or in a non-ocodemic profession.

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EDITORIAL POLICY

As the newly elected editor of the Pi *Mu Epsilon Journal*, I would like to state our publication policy. The chief objectives of the *Journal* from its very beginning in 1949 have been to stimulate young mathematical minds, to encourage creativity and scholarship among its readers, and to provide an outlet for undergraduate writing in mathematics. Hence, we will not generally publish original research of a technical nature by experienced or more mature mathematicians. We do welcome expository papers from such individuals, or transcribes of talks presented to local chapters.

All papers will be evaluated on the basis of (1) clarity of style, (2) probable interest to readers, (3) suitability of level, and (4) creativity and originality. Our demands in each of these areas will be generally less severe for undergraduate authors than for graduate students or more experienced faculty members at a university, say, in line with our desire to encourage writing by beginning mathematicians. Our evaluation of papers will therefore depend very much on the experience and maturity of the authors, so we ask that all future authors inform us of their class if they are undergraduates or their year if they are graduates, and the school they are attending, and their position if they are either faculty members or in industry.

The format of the *Journal* is undergoing some changes with this issue, as will be obvious to our readers. We have been fortunate in obtaining a new typewriter, several symbol balls, and transfer symbol sheets which will enable us to put out a more attractive product. I pledge my efforts to make this journal not only attractive in appearance but also of literary worth in the field of mathematics, and I look forward to serving in this venture.

David C. Kay
Editor

WHAT IS THE MOST AMAZING APPROXIMATE INTEGER IN THE UNIVERSE?

By I. J. Good
Virginia Polytechnic Institute and State University

It is well known to modular functionaries [1, 2, 3, 4, 51 that there are some very striking approximations to integers involving the number $e^{\pi dm}$, especially when -m is a negative discriminant of a binary quadratic form having class number 1; that is, when m = 3, 4, 7, 11, 19, 43, 67 or 163. The proof that this is the whole set of negative discriminants for class number 1 was completed by Stark [5]. Although some other values of m give striking results for example [2]

$$e^{\pi\sqrt{58}} = 24591257751.99999982,$$

there is some reason to suppose that m = 163 provides the most striking approximations. The following example is mentioned in [3], and in [4] with a very slight inaccuracy:

$$e^{\pi\sqrt{163}} = 262537412640768743.999999999999250.. . \qquad (1)$$

(The rigorous proof depends on [6].) Although amazement is in the eyes of the beholder, and of course does not depend only on the degree of the approximation, but also on the simplicity and mathematical depth of the formula, I prefer

$$(e^{\pi\sqrt{163}} - 744)^{1/3} = 640319.9999999999999999999999993903... \qquad (2)$$

This formula is an extremely easy and elementary deduction from the remarkable fact that

$$x^{1/3} \prod_{\nu=1}^{\infty} (1 + x^{1-2\nu})^8 - \frac{256x^{-2/3}}{\prod_{\nu=1}^{\infty}(1 + x^{1-2\nu})^{16}} \qquad (3)$$

is an integer when $x = e^{\pi\sqrt{m}}$, where m is defined above, as proved by Weber [1], p. 461, who also lists the values of these integers for m = 11, 19, 43, 67, and 163.

From equation (2), we can derive the following result which is even better:

$$\left\{ \frac{1}{\pi} \log_e (640320^3 + 744)\right\}^2$$
$$= 163.00000000000000000000000000000000232... \qquad (4)$$

The question I wish to raise is whether there is any other approximate integer that is more amazing to the eye of an intelligent entity,

I wish to make it perfectly clear that I am laying claim to very little originality in this note, but I feel that the *people* are entitled to some cf the awe that tends to be reserved for the high functionaries, and to relish the possibility that (4) provides the most surprising possible approximate integer.

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DO YOU HAVE ANY UNSOLVED PROBLEMS?

Our Problems Editor informs us that a collection of unsolved problems comprehensible to the amateur is being compiled by Professor Dagmar Henney of George Washington University. If you have a problem which you think would be appropriate for this collection please send it, along with a short historical reference if possible, to:

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REPEATING AND NON-REPEATING SEQUENCES OF DIGITS IN DECIMAL FRACTIONS

By Kay P. Litchfield
Brigham Young University

Consider a/b where $0 < a < b$ $(a, b \in N)$. Expressing a/b as a decimal fraction, we have $0.\,d_1d_2d_3\ldots$ which has zero or more non-repeating digits, followed by zero (for terminating expansions) or more digits which are repeated without end.

Notation and Terminology

The letter n represents a positive integer greater than 1. The decimal expansion of $1/n$ has f_n non-repeating digits followed by r_n digits which are repeated. A string of symbols sym sym sym...sym containing k occurrences of the symbol sym shall be written:

$\underline{\mathrm{sym}\overset{k}{\ldots}\mathrm{sym}}$. (e.g. $\underline{1\overset{9}{\ldots}1} = 111111111$, $\underline{232\overset{3}{\ldots}232} = 232232232$, $\underline{9\overset{k}{\ldots}9} = 9\sum_{i=0}^{k-1} 10^i$)

Purpose

The purpose of this paper is to find f_n and r_n for any n in any number base (e.g. decimal, binary, ternary, duodecimal). The concept shall first be presented for decimal and then generalized to other bases.

Motivation

Consider $1/n$ and the elementary long division process for finding its decimal fraction (dividing into 1.000... until the fraction terminates or starts repeating). As each digit of the quotient is found a new remainder $(<n)$ is found. As soon as a remainder of zero is found, the fraction terminates. As soon as a remainder is found which equals a previously found remainder, the fraction starts repeating (from the point at which that remainder was previously found).

Result, with proof

For $1/n$ we have,

$$\frac{1}{n} = .d_1d_2\ldots d_{f_n}\overline{d_1'd_2'\ldots d_{r_n}'} = \frac{d_1d_2\ldots d_{f_n}}{10^{f_n}} + \frac{1}{10^{f_n}}\cdot\frac{d_1'd_2'\ldots d_{r_n}'\cdot 10^{-r_n}}{(10^{r_n}-1)\cdot 10^{-r_n}}$$

$$= \frac{d_1d_2\ldots d_{f_n}}{10^{f_n}} + \frac{1}{10^{f_n}}\cdot\frac{d_1'd_2'\ldots d_{r_n}'}{\underline{9\overset{r_n}{\ldots}9}} = \frac{(\underline{9\overset{r_n}{\ldots}9})(d_1d_2\ldots d_{f_n}) + d_1'd_2'\ldots d_{r_n}'}{(10^{f_n})\,(\underline{9\overset{r_n}{\ldots}9})},$$

and,

$$(\underline{9\overset{r_n}{\ldots}9})(d_1d_2\ldots d_{f_n}) + (d_1'd_2'\ldots d_{r_n}') = \frac{(\underline{9\overset{r_n}{\ldots}9})(10^{f_n})}{n}$$

Since the left hand side is an integer, we have $n \mid (\underline{9\overset{r_n}{\ldots}9})(10^{f_n})$. If we had used k/n instead of $1/n$, we would have arrived at the condition $n \mid k(\underline{9\overset{r_n}{\ldots}9})(10^{f_n})$. If $(k,n) = 1$ we have $n \mid (\underline{9\overset{r_n}{\ldots}9})\cdot(10^{f_n})$, as before.

Since the above process may be reversed, we have:

$$\frac{1}{n} = .d_1d_2\ldots d_{f_n}\overline{d_1'd_2'\ldots d_{r_n}'} \text{ if and only if } n \mid (\underline{9\overset{r_n}{\ldots}9})(10^{f_n})$$

and r_n and f_n are the least integers for which this is true.

To find f_n and r_n given n

Express n in the form $2^a f^b c$, where $(10,c) = 1$. Then it follows that $f_n = \max(a,b)$, and r_n is found by dividing c into 99... until a zero remainder is found (or into 1000... until a remainder of 1 is found).

Base other than 10

The section "Result, with proof" holds in any base m (where $2 < m < \infty$) if the digit 9 (wherever it occurs in the section) is considered to represent $10_m - 1$. (e.g. in binary—1, in duodecimal——E).

Expressing n as $a \cdot b$ where $(b,m) = 1$ and a divides some power of m, we see that $a \mid m^{f_n}$ and $b \mid (m^{r_n} - 1)$. (f_n and r_n refer to the fractional expansion in the base m.)

Table for base 10

Here is a table listing the prime factorization of $(10^k - 1)$ for $1 \leq k \leq 25$. The reciprocal of any factor of $(10^k - 1)$ has k repeating digits in base 10. The repeating digits of $1/n$ may be found by taking the product of the other factors of $(10^k - 1)$.

Similar tables for other bases may easily be produced. It may be relatively easy to produce more extensive tables.

Prime factorization of $(10^k - 1)$

k	prime factorization of $(10^k - 1) = \underbrace{9\ldots9}_{k}$
1	3,3.
2	3,3,11.
3	3,3,3,37.
4	3,3,11,101.
5	3,3,41,271.
6	3,3,3,7,11,13,37.
7	3,3,239,4649.
8	3,3,11,73,101,137.
9	3,3,3,3,37,333667.
10	3,3,11,41,271,9091.
11	3,3,21649,513239.
12	3,3,3,7,11,13,37,101,9901.
13	3,3,53,79,265371653.
14	3,3,11,239,4649,909091.
15	3,3,3,31,37,41,271,2906161.
16	3,3,11,17,73, 101 137, 5882353
17	3,3,2071723,5363222357.
18	3,3,3,3,7,11,13,19,37,52579,333667.
19	3~3~1111111111111111111.
20	3,3,11,41,101,271,3541,9091,27961.
21	3,3,3,37,43,239,1933,4649,10838689.
22	3,3,11,11,23,4093,8779,21649,513239.
23	3,3,11111111111111111111111.
24	3,3,3,7,11,13,37,73,101,137,9901,99990001.
25	3~3~41,271,21401~25601~182521213001.

BASIS FOR AN ALGEBRAIC SYSTEM

By J. E. Cain, Jr.
University of Oklahoma

A well known concept from linear algebra is every vector space has a basis. In this brief paper we will consider a generalization of this theorem to arbitrary algebraic systems. We will first define the concepts of **independence**, minimal generating **sets**, and basis, and then state and prove an existence theorem.

Let $a = (A;F)$ be an algebraic system and $B \subset A$. Then $a \in A$ is a ***combination of elements of B (combo of B)*** if $a \in W$, where W is defined **recursively** as follows:

(i) $B \subset W$

(ii) If $f_\zeta \in F$ and the degree of f_ζ is zero, then $f_\zeta \in W$.

(iii) If $f_\zeta \in F$, the degree of f_ζ is n, and $x_1, x_2, \ldots, x_n \in W$, then $f_\zeta(x_1, x_2, \ldots, x_n) \in W$.

For an algebra $a = (A;F)$, $B \subset A$ is a dependent set if there exists a $b \in B$ such that b is a combo of $B \setminus \{b\}$. B is an ***independent*** set if B is not a dependent set.

A generating set V for an algebra $a = (A;F)$ is called *minimal* if A is not generated by any proper subset of V.

Lemma: Let V be a generating set for an algebra $a = (A;F)$. Then V is independent iff V is minimal.

Proof: Assume V is an independent generating set. Let B be a proper subset of V and $b \in V \setminus B$. Then b is not a combo of B. Hence, B does not generate A and V is minimal.

Suppose V is a minimal generating set. If V is dependent, then there exists $b \in V$ such that b is a combo of $V \setminus \{b\}$. But then $V \setminus \{b\}$ generates A. Hence V must be independent. **Q.E.D.**

A ***basis*** for an algebra is an independent (minimal) **generating** set.

Theorem: Every algebra has a basis.

Proof: Let $\alpha = (A;F)$ be an algebra. Let P be the class of all independent subsets of A. The empty set is in P, hence P is nonempty. Then $\rho = (P;\subset)$ is a nonempty partially ordered set under ordinary set inclusion.

Let $\mathcal{C}$ be any chain in ρ. Then $\bigcup\mathcal{C}$ is a maximal element for $\mathcal{C}$. By Zorn's Lemma, there exists a maximal element V in P. Since V is a maximal independent subset for α, V is a minimal independent generating set, and hence a basis for a. Q.E.D.

COMMENT ON "COMMENTS ON THE PROPERTIES OF ODD PERFECT NUMBERS"

One of our readers informs us that a comment by Lee Ratzan in his article *Comments on the Properties of Odd Perfect Numbers* (this *Journal*, Vol. 5, No. 6, pp. 265-271) needs updating. According to Robert Prielipp, Wisconsin State University, it is now known that there are no odd perfect numbers less than or equal to 10^{36} (the number given by Lee Ratzan on p. 269 was 10^{20}). This result may be found in a paper by Bryant Tuckerman (IBM Research Paper RC-1925, Thomas J. Watson Research Center). Professor Prielipp and another reader, Henry J. Ricardo, Manhattan College, point out that the editorial comment at the bottom of page 265 is also out of date. Instead of the largest known even perfect number being $2^{11,212}(2^{11,213}-1)$, having 6751 digits, it is $2^{19,936}(2^{19,937}-1) = 9311445590\ldots0271942656$, having 12,003 digits, discovered only last year (1971) by Professor Bryant. In addition to the above reference, see also George E. Andrews, *Number Theory* W. B. Saunders Company, 1971, page 112).

ON HOLDING A RAFFLE (WITHOUT MAKING N-1 PEOPLE UNHAPPY)[1]

By Edward J. Wegman
University of North Carolina

Introduction

Let us assume that we have a prize valued at k dollars to be given away to one of n people. Usually one person is chosen at random and awarded the prize and the remaining n-1 people are sent on their way with nothing to show. If one is unwilling to accept the whims of Tyche (the goddess of chance), schemes may be devised to outwit her and let everyone walk away with something for nothing. One scheme which we will describe here is of interest because of an unusual combination of an intuitively simple idea with elementary concepts in statistics and techniques of advanced calculus. The calculus student may avoid another routine exercise and the statistics student has an opportunity to apply a relatively sophisticated technique to a simple problem.

The Scheme

Our scheme will involve a tradeoff: the participants must be willing to buy some security. Our raffle will proceed through n-1 rounds. For the first round everyone pays the admission price of p_1k dollars and the prize is awarded to one person at random. The winner drops out and the remaining participants pay a second admission price of p_2k dollars. Again, the second round winner is awarded the k dollar prize. The procedure is simply carried out in this manner until the $(n-1)^{st}$ round. In this round each of the two remaining participants pay the premium of $p_{n-1}k$ dollars and each is awarded the prize with no raffle, that is, both are declared winners. Table I summarizes.

---

[1]The motivation for this research was twofold. First was that the author has a long history of losing raffles, the most recent being one where he had a probability of .5 of winning. Second was the dilemma of awarding a journal subscription to several equally deserving graduate students. Implementing this scheme alleviated the second, but hasn't helped the first at all.

Table I

	amount paid this round by each remaining participant	total amount the winner must pay	total paid by all participants this round
round 1	kp_1	kp_1	nkp_1
round 2	kp_2	$k(p_1+p_2)$	$(n-1)kp_2$
.	.	.	.
.	.	.	.
.	.	.	.
round (n-2)	kp_{n-2}	$k(p_1+p_2+\ldots+p_{n-2})$	$3kp_{n-2}$
round (n-1)	kp_{n-1}	$k(p_1+p_2+\ldots+p_{n-1})$	$2kp_{n-1}$

Several constraints on the selection of the p_i's are obvious, Clearly $0 \leq p_i < 1$, $i = 1, 2, \ldots n-1$. In addition, we want no one to pay more than k dollars, Hence, $k\sum_{i=1}^{n-1} p_i \leq k$ or $\sum_{i=1}^{n-1} p_i \leq 1$. Equality means that the winners of the $(n-1)^{st}$ round have had to buy their own prize. However, if $p = \sum_{i=1}^{n-1} p_i < 1$, then even those $(n-1)^{st}$ round winners still "get something for nothing".

The scheme works, of course, because the premiums paid purchase the additional (n-1) prizes, Hence, we must have (totalling the third column of Table I),

$$k\sum_{i=1}^{n-1} (n-i+1)p_i = (n-1)k.$$

Rewriting

(1)
$$\sum_{i=1}^{n-1} ip_i = (n+1)p - n + 1 = n(p-1) + p + 1.$$

In the special case that $p = 1$, this equation becomes:

$$\sum_{i=1}^{n-1} ip_i = 2.$$

That is, the p_i's form a probability distribution with mean 2.

Several questions of interest arise. The first is the probability of winning on the jth round and the second is the expected amount of payment a participant must make. To answer the first, let $1 < j < n-1$. If the participant is to win on the jth round he must have lost on all previous rounds and then won on the jth. Hence that probability is:

$$\frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \ldots \cdot \frac{n-j}{n-j+1} \cdot \frac{1}{n-j} = \frac{1}{n}$$

Obviously the probability of winning on the first round is $\frac{1}{n}$ and thus the probability of winning on the jth round for $1 \leq j < n-1$ is $\frac{1}{n}$. To win on the $(n-1)^{st}$ round means to have lost on all previous rounds. Hence that probability is:

$$\frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \ \ldots \ \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{n} \ .$$

Next, let X be the amount of money a participant must pay. The expected value of X is

$$E(X) = kp_1 \cdot \frac{1}{n} + k(p_1+p_2)\frac{1}{n} + \ldots + k(p_1+p_2+\ldots+p_{n-2})\frac{1}{n} +$$

$$k(p_1+p_2+\ldots+p_{n-1})\frac{2}{n} \ .$$

Rewriting

$$E(X) = kp_1 + \left(\frac{n-1}{n}\right)kp_2 + \ldots + \left(\frac{3}{n}\right)kp_{n-2} + \left(\frac{2}{n}\right)kp_{n-1} = k\sum_{i=1}^{n-1}\left(\frac{n-i+1}{n}\right)p_i$$

Thus,

$$E(X) = k\left(1+\frac{1}{n}\right)p - \frac{k}{n}\sum_{i=1}^{n-1} ip_i.$$

Using equation (1) for $\sum^{n-1} ip_i$, we find,

$$E(X) = \frac{n-1}{n} \cdot k$$

Hence, independent of the choice of the p_i's, the participant expects to pay $\left(\frac{n-1}{n}\right)k$ dollars. This, of course, means the expected winnings are $\frac{k}{n}$ dollars, which is the same as with the ordinary raffle. The purpose of introducing the present scheme in place of the ordinary raffle was to eliminate some of the variability or uncertainty. Hence, let us compute var(X). Clearly,

$$\operatorname{var}(X) = (kp_1)^2 \frac{1}{n} + (kp_1 + kp_2)^2 \frac{1}{n} + \ldots + (kp_1 + \ldots + kp_{n-1})^2 \frac{2}{n} - k^2\left(\frac{n-1}{n}\right)^2.$$

Rewriting

$$\operatorname{var}(X) = \frac{k^2}{n}\left[p^2 + \sum_{j=1}^{n-1}\left(\sum_{i=1}^{j} p_i\right)^2 - \frac{(n-1)^2}{n}\right]$$

To minimize the variance, one must minimize

$$p^2 + \sum_{j=1}^{n-1}\left(\sum_{i=1}^{j} p_i\right)^2$$

subject to the constraints

$$\sum_{i=1}^{n-1} p_i = p$$

and

$$\sum_{i=1}^{n-1} ip_i = (n+1)\,p-n+1.$$

Using Lagrangian multipliers, one finds the following set of simultaneous equations

$$2\sum_{j=\ell}^{n-1}\sum_{i=1}^{j} p_i + 2\sum_{i=1}^{n-1} p_i + \lambda(\ell - (n+1)) = 0 \qquad \ell = 1, 2, \ldots, n-1.$$

$$\sum_{i=1}^{n-1} ip_i - (n+1)\sum_{i=1}^{n-1} p_i + n-1 = 0.$$

Solving these n equations simultaneously leads to the minimum variance (in fact, the zero variance) solution

$$p_1 = \frac{n-1}{n}, \quad p_i = 0 \qquad\qquad i = 2, 3, \ldots, n-1.$$

This degenerate situation corresponds to giving each participant a prize of $\frac{k}{n}$ dollars, that is, splitting the prize up equally. The solution need not be degenerate; however.

In the previous case, we were letting p be variable and finding the minimum variance solution subject to a variable p. We may fix p however and find the minimum variance solution. For example, suppose $p = 1$. Then to minimize the variance, we must minimize

$$\sum_{j=1}^{n-1}\left(\sum_{i=1}^{j} p_i\right)^2$$

subject to

$$\sum_{i=1}^{n-1} p_i = 1 \text{ and } \sum_{i=1}^{n-1} ip_i = 2.$$

Again using Lagrangian multipliers, we have

$$2\sum_{j=\ell}^{n-1}\sum_{i=1}^{j} p_i + \lambda j + \mu = 0 \qquad\qquad \ell = 1, 2, \ldots, n-1.$$

$$\sum_{i=1}^{n-1} p_i = 1 \text{ and } \sum_{i=1}^{n-1} ip_i = 2.$$

Solving these (n+1) equations simultaneously yields

$$p_1 = \frac{n-3}{n-2}, \quad p_2 = 0, \quad p_3 = 0, \ldots, p_{n-2} = 0, \quad p_{n-1} = \frac{1}{n-2}.$$

Of course, using a minimum set of p_i's makes the raffle least sporting, but insures the most satisfaction among the customers. Larger variance then corresponds to a more sporting raffle. In particular, if the initial p_i's are small and the terminal ones large, one approximates the usual raffle.

Finally, we observe that making the payments, p_ik, need not be regarded as actually having to take place. Since the prize is k dollars and the amount paid is something less than or equal to k dollars, the net prize is just k minus the total payment. Thus, all and all, everyone gets at least a little for nothing.

THE CURVES OF PERSEUS

G. Mavrigian, Youngstown State. University
S. Sarikelle, University of. Akron

Introduction

The production of Greek mathematics that followed Apollonius of Perga was described as a reflection or re-examination of prior works. Among later developments in geometry we find the classification of curves [6,8]: Manaechmus (who flourished around 350.B.C.) was credited with the discovery of the conic sections - ellipse, hyperbola, and parabola; and Perseus (who presumably flourished after Euclid, around 150 B.C.) is generally credited with the discovery and characterization of **spiric** sections.

The. Curves of. Perseus

It was Proclus the philosopher who stated, around the year 460, that the following mathematicians discovered and/or described properties of various curves [8]:

Apollonius of the conic sections,
Hippias of the quadratrix,
Nicomedes of the conchoids, and
Perseus of the spirals.

Perseus, a geometer, treated the sections of the spire, or torus (also called the anchor-ring). The work of Perseus unfortunately is wholly lost, and no extracts from his works appear in the writing of later mathematicians [4,5,7]! Scientists have expressed doubt that he (Perseus) laid the foundation to mathematical astronomy. Historians say that it appears quite uncertain that Perseus discovered such curves as the ovals (nee, Ovals of **Cassini**), the lemniscate, and **spiral** sections (as the helix). It is-fannery (1843-1904) who identifies five types of planar cross section, of the open spire, as the ***five curves*** of ***Perseus.*** The analytic description of these curves follows.

We consider the rotation of a circle about the xaxis; wherein, in the x-y rectangular Cartesian plane, the circle has radius of

a units, and has its center at the point $(x,y) = (0,c)$; $c > a$. Because of symmetry, in the generated surface of revolution, we consider only the respective semi-circle and the resultant surface in the initial or reference octant of three dimensional space. The torus, or surface of revolution, is then cut by planes $z = d$ (that is, by planes parallel to the x-y plane); where d = constant. The five curves of Perseus are then identified as those planar curves of intersection between the torus and the planes $z = d$ = constant, for the following cases:

$$
\begin{aligned}
&1.\quad c < d < (c + a)\,,\\
&2.\quad d = c,\\
&3.\quad (c - a) < d < c\,,\\
&4.\quad d = (c - a)\,,\\
&5.\quad 0 < d < (c - a).
\end{aligned}
\tag{1}
$$

Curves defined by sections 1, 2, and 3, each describe an oval; the curve defined by section 4 is a **hippopede**, or 'horse-fetter' (i.e., a figure eight) [3]; and the trace defined by section 5 consists of two symmetrical ovals (distorted circles). Figure 1 reveals

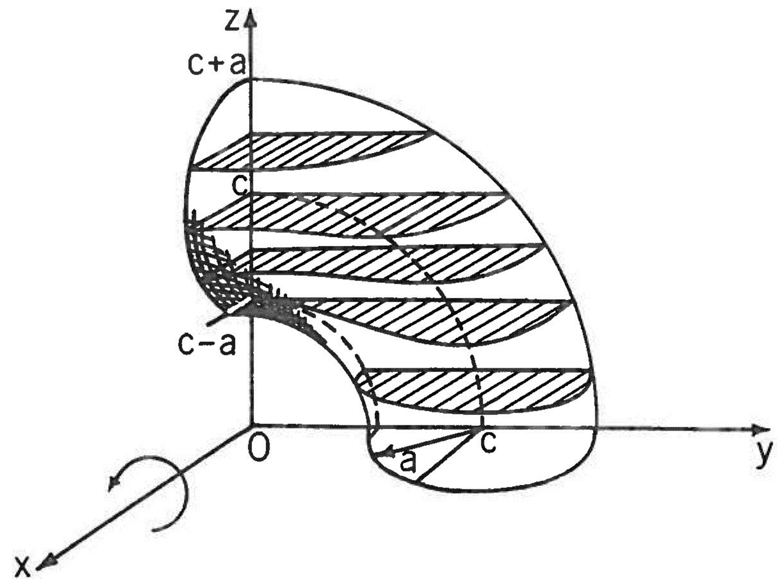

FIGURE 1

the segments of the torus and the five planar cross sections, appearing in the first octant.

The equation of the directrix, or generating planar curve, is

$$x^2 + (y - c)^2 = a^2 \ , \ z = 0 \ , \ c > a > 0 \ ; \qquad (2)$$

and, with an axis of revolution given by the **x** axis, the resultant torus, or surface of revolution, is:

$$(x^2 + y^2 + z^2 + c^2 - a^2)^2 = 4c^2 \cdot (y^2 + z^2). \qquad (3)$$

Additional Notes on the Spiric Sections

The peculiar spiral sections, called the curves of Perseus, appear to be the same as the hippopede curves of Eudoxus (Eudoxus of Cnidos, 408-355 B.C.). These same curves were also described by Heron of Alexandria, who lived after Apollonius and before Pappus. The lemniscate was described by Jakob Bernoulli (1654-1705); the general lemniscate, or Cassinian ovals, were invented and described by Jean-Dominique Cassini (1625-1712) [6]. The Darboux curves, represented by complex numbers, generalized the spiric curves, and were attributed to Jean Gaston Darboux (1842-1917) [9].

Many of the section properties of the curves of Perseus, and also of volumes of the truncated segments of the torus, remain most ellusive to mathematical computation. The exact evaluation of definite integrals, representative of arc length, surface area, and volume, for example, are generally impossible to obtain. To illustrate, let us re-write the equation of the torus, as given by Equation (3), in vector or parametric form, as follows:

$$\bar{r} = \bar{r}(\mu, v) \equiv x\bar{i} + y\bar{j} + z\bar{k}$$

$$= \{(a \cdot \mathrm{Sin}\ \mu)\bar{i} + [(c + a \cdot \mathrm{Cos}\ \mu)\ \mathrm{Sin}\ v]\bar{j} + [(c + a \cdot \mathrm{Cos}\ \mu) \cdot \mathrm{Cos}\ v]\bar{k}\} \qquad (4)$$

Note that $\bar{r}$ is our position vector, locating any point on the spiric curve. If we let $0 \leq d \leq (c - a)$, in our illustration, then the length of the curve of Perseus, as shown in Figure 2 is:

$$\mathscr{L} = \widehat{P_1 P P_2} = \int_{\mu_1}^{\mu_2} \left|\frac{d\bar{r}}{d\mu}\right| \cdot d\mu = a \cdot \int_{\mu=0}^{\pi} f(\mu) \cdot d\mu \ ; \qquad (5)$$

$$f(\mu) = \left[\frac{(c + a \cdot \mathrm{Cos}\ \mu)^2 - d^2 \cdot \mathrm{Cos}^2\ \mu}{(c + a \cdot \mathrm{Cos}\ \mu)^2 - d^2}\right]^{\frac{1}{2}}$$

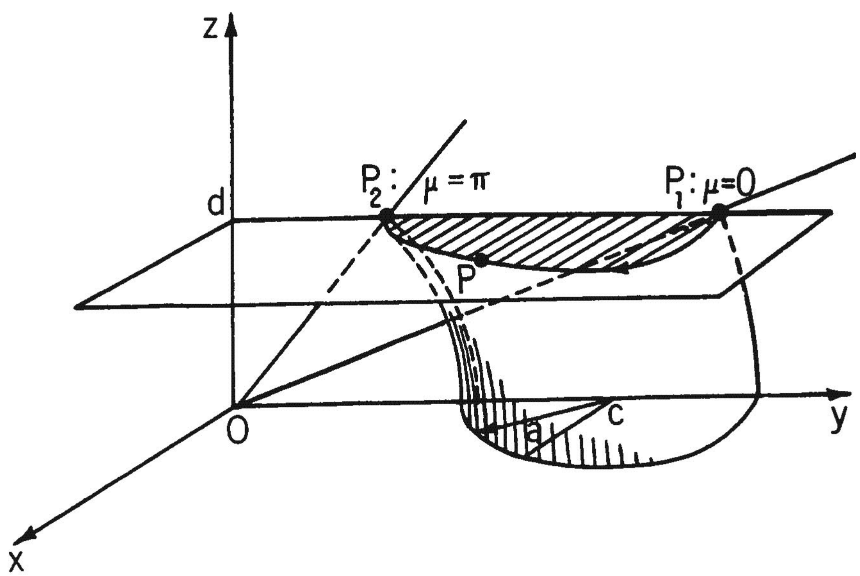

FIGURE 2

The definite integral, represented in Equation (5), is referred to as a hyper-elliptic integral of Jacobi or Legendre type (Carl G. J. Jacobi, 1804-1851; Adrien-Marie Lengendre, 1752-1833) [2]. Such integrals reduce, in special cases, to elementary functions and/or special higher mathematical functions of applied mathematics, as Jacobian elliptic integrals of the first, second, and third kinds. However, for an immediate evaluation of such hyper-elliptic integrals, one usually resorts to direct numerical integration and/or use of extensive series representations.

Illustrative Integrals

We consider herein numerical treatment for the length of various curves of Perseus.

1. Let $d = 0$. The curve of Perseus gives a semi-circular trace in the initial octant. From Equation (5),

$$\mathcal{L} = a \cdot \int_0^{\pi} d\mu = \pi a \approx 3.1416 \cdot a \text{ units.}$$

2. Let $c = 2a$, and $d = \frac{c-a}{2} \equiv \frac{a}{2}$. The curve of Perseus gives an oval. From Equation (5),

$$\mathcal{L} = a \cdot \int_{-1}^{1} \left[\frac{(4+w) \cdot (4+3w)}{(1-w^2) \cdot (3+2w) \cdot (5+2w)} \right]^{\frac{1}{2}} \cdot dw; \quad w \equiv \operatorname{Cos} \mu,$$

The ellusiveness is now apparent. Employing numerical integration, the approximation of this higherr-elliptic integral gives,

$$\mathcal{L} \approx 3.1723 \cdot a \text{ units}$$

3. Let $c = 2a$, and $d = c - a \equiv a$. The curve of Perseus describes a lemniscate. From Equation (5),

$$\mathcal{L} = 2a \cdot \int_0^{\pi} \frac{d\mu}{\sqrt{3 + \operatorname{Cos} \mu}} = 2a \cdot F(\tfrac{\pi}{2},k) \equiv 2a \cdot K(k) \;;\; k = \frac{1}{\sqrt{2}}$$

where F and K refer, respectively, to the incomplete and complete Jacobian elliptic integrals of the first kind. Herein, $\mathcal{L} \approx 3.7082 \cdot a$ units.

Some interesting characteristics of the locus in example 3 should be noted. If $\hat{r}$ denotes the polar co-ordinate for radial length, and S denotes the length of arc of the lemniscate, from its center (the nodal point) to any additional point in the reference quadrant, then one can write the relationship [1]

$$\hat{r} = \hat{r}(S) = 2 \cdot \sqrt{2} \cdot a \cdot C_n \left\{ K\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2a} \cdot S \right\} ,$$

where $C_n(\mu)$ denotes a special Jacobian elliptic function. In the neighborhood of the nodal point, the lemniscate shares the unique property of the klothoid; namely, that the product of the radius of curvature and arc length holds constant. This inherited property is most important to highway builders when they trace bends in a roadway. [9].

Today, we marvel at the application of these curves of Perseus to many technical disciplines; illustratively, in filamentary pressure vessel design, groundwater seepage, highway engineering, optical diffraction, and vehicular tire mechanics.

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In 1545 the Italian algebraist H. Cardan gave a general formula for the solution of a cubic equation $x^3 + ax^2 + bx + c = 0$ involving the complex numbers $(1 \pm i\sqrt{3})/2$ (cube roots of unity). Historically, this result was responsible for the introduction of complex numbers in mathematics.

COMPOSITION OF CONVERGENT SEQUENCES

By Richard K. Williams
Southern Methodist University

If two sequences of functions $\{f_n\}$ and $\{g_n\}$ converge to f and g respectively, it is natural to wonder whether $\{f_ng_n\}$ converges to fg, where, of course, the compositions are assumed to be meaningful. In this note, we give two conditions, each of which is sufficient for $\{f_ng_n\}$ to converge to fg. Two consequences of this result are deduced, and an example is given to show that $\{f_ng_n\}$ need not converge to fg in general.

Throughout this paper, the notation $f: A \to B$ will mean that f is a continuous function from A into B. Also, $f_n \to f$ will mean that the sequence $\{f_n\}$ converges pointwise to f.

<u>Theorem</u>: *Let X, Y, and Z be metric spaces. Let $f_n: Y \to Z$, $g_n: X \to Y$, $f_n \to f$, $g_n \to g$. Then $f_ng_n \to fg$ if either of the following is satisfied:*

(a) $\{f_n\}$ is equicontinuous in some neighborhood of each point in the range of g;

(b) $\{f_n\}$ converges uniformly to f in some neighborhood of each point in the range of g.

Proof: Under Hypothesis (a), the result follows from the inequality $d(f_ng_n(x), fg(x)) \leq d(f_ng_n(x), f_ng(x)) + d(f_ng(x), fg(x))$, where d is the metric on Y.

Under Hypothesis (b), the result follows from the continuity of f and the inequality

$$d(f_ng_n(x), fg(x)) \leq d(f_ng_n(x), fg_n(x)) + d(fg_n(x), fg(x)).$$

<u>Corollary</u>: *If in the hypotheses of the theorem, we take $f_n = g_n$, $f = g$, and $X = Y = Z$, then either of conditions (a) or (b) is sufficient for $f_n^k \to f^k$ for $k = 1, 2, \cdots$.*

Proof: If the sequence $\{f_n\}$ satisfies condition (a), then since $d(f_n^{k+1}(x), f_n^{k+1}(y)) = d(f_n(f_n^k(x)), f_n(f_n^k(y)))$, it follows by induction that $\{f_n^k\}$ also satisfies condition (a) for $k = 1, 2, \cdots$. The result follows from the theorem.

If $\{f_n\}$ satisfies condition (b), then since

$$d(f_n^{k+1}(x), f^{k+1}(x)) \leq d(f_nf_n^k(x), ff_n^k(x)) + d(ff_n^k(x), ff^k(x)),$$

it follows by induction that $\{f^k(x)\}$ also satisfies condition (b) for $k = 1, 2, \cdots$. Again, the result follows from the theorem.

The following example shows that the corollary (and hence the theorem) is not true if one assumes neither condition (a) nor (b).

Example: *Let*

$$X = \{0\} \cup \{\tfrac{1}{n} : n = 1, 2, \cdots\},$$

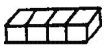
and let d be the usual metric on the reals. Define $\{f_n\}$ as follows: Let f_1 interchange 1 and 1/2, keeping everything else fixed; let f_2 interchange 1 and 1/3, 1/2 and 1/4, keeping everything else fixed; let f_3 interchange 1 and 1/4, 1/2 and 1/5, 1/3 and 1/6, keeping everything else fixed. Continue this process to define each f_n. Clearly, each f_n maps X into X, and $f_n \to f \equiv 0$. Also, each f_n^2 is the identity, so f_n^2 converges to the identity. Hence $f_n \to f$, but $f_n^2 \not\to f^2$.

As a minor application of the corollary, if x_0 is periodic of period k under each f_n (i.e., $f_n^k(x_0) = x_0$), then x_0 is periodic of period k under f. Of course, if each f_n is periodic of period k (i.e., $f_n^k(x) \equiv x$), then so is f.

A 2x2x1 SOLUTION TO "INSTANT INSANITY"

By Kay P. Litchfield
Brigham Young University

The usual goal with "Instant Insanity" is to form a 4x1x1 prism

 such that: the four upper faces are different, the four lower faces are different, the four near faces are different, and the four far faces are different.

The standard "Instant Insanity" puzzle may also be used to form one of two 2x2x1 prisms as follows, satisfying a larger number of conditions, the individual cubes being displayed in the form of a cross:

B	G		far			B	G		
R W W	R B R	left	upper	right		R W W	B R W		
R	R		near			R	G		
G	W		lower			G	B		
W	G					W	R		
W G G	B R W					W G G	R B R		
B	G					B	G		
R	B					R	W		

(A) The four upper faces are different,
(B) The four lower faces are different.
(C) The four side faces that can be seen from each of the corners are different.
(D) The four faces of each cube which are not touching other cubes are different.
(E) Separating the two left most cubes from the right most cubes reveals four different faces.

Conditions (A) though (D) are the more significant ones.

B.L. Schwartz presents a very efficient method for finding all 4x1x1 solutions to "Instant Insanity" puzzles in *An Improved Solution to "Instant Insanity"*, Mathematics Magazine, 43, (1970), 20-23. His method may readily be applied to "Instant Insanity" puzzles other than the standard kind consisting of four cubes with four distinct kinds of faces.

The purpose of this paper is to present the 2x2x1 solution to the four cube puzzle, and a method for determining all solutions of this type. As this is, essentially, just a continuation of the method given by Schwartz, his method for the regular solutions is briefly presented first, for the sake of completeness.

To *find all 4x1x1 solutions to "Instant Insanity" puzzles:*

As an example, to clarify the description, here is a nonstandard "Instant Insanity" puzzle.

W	R	B	W
R B B	G B R	G R W	G W G
G	B	B	R
G	W	W	B

1. List the pairs of opposite faces of each cube.

WG	RB	BB	WR
BG	BW	RW	WB
RB	GR	GW	GG

2. Choose a pair of opposite faces from each cube so that the set of four pairs chosen contains each color twice. Find and list all such sets of four pairs. (There are at most 81 possibilities to check, but seldom that many.)

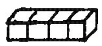
(1)	WG	GR	BB	WR
(2)	BG	RB	GW	WR
(3)	BG	GR	RW	WB
(4)	RB	BW	RW	GG
(5)	RB	GR	GW	WB

3. Pair these sets together if they have no pair of faces in common on the same cube.

(1)	WG	GR	BB	WR
(4)	RB	BW	RW	GG
(2)	BG	RB	GW	WR
(4)	RB	BW	RW	GG

Any of these pairs of sets gives at least one solution.

4. Putting the cubes on a surface, use one set of a pair of sets to determine the upper and lower faces of the cubes, inverting where necessary so that each color appears once as an upper face (and therefore once

as a lower face). Then, without lifting the cubes from the surface, use the other set of the pair of sets to determine the near and 'far faces, this time rotating where necessary so that each color appears once on the near side (and therefore once on the far side). Put the cubes in a row 

Note that, in the example, the BB pair and the GG pair may be placed in two directions each. There are, therefore, six solutions to the example.

The faces of the most commonly sold "Instant Insanity" cubes may often be separated without breaking (although glued) and reassembled to create new "Instant Insanity" puzzles.

To *find all 2x2x1 solutions to* a *four cube "Instant Insanity" puzzle:*

1.,2. Do steps 1 and 2 of the previous method. Each set of four pairs, found in 2 may yield 2x2x1 solutions. A set of four pairs will provide the upper and lower faces of the solution as in step 4 of the previous method.

3. Examine each cube to decide if condition (D) can be satisfied.

Example (from above)	***(1) and (4) give no solution as the upper and lower faces of one cube are the same. Sometimes the two other colors are not adjacent. (2), (3), and (5) remain possibilities.***

4. For each remaining possibility place the cubes on a surface, satisfying (A) and (B) (as in step 4 above). In continuing, do not raise the cubes from the surface, To satisfy (C) the two near faces must be the same as the two far faces, and the two left faces must be the same as the two right faces. Noting which faces may be away from the adjacent cubes, to satisfy (D), attempt to orient the cubes to achieve condition (D) and the-variation of condition (C) simultaneously. It should require only a few moments to determine if this is possible, or if there is more than one way to do it.

(2) W	B	(3) R	B	(5) W	R
R B B	W G R	W G G	W W G	G R B	G B G
G	B	B	B	G	W
G	W	B	R	B	W
W	B	W	R	W	B
G W G	R R G	G B G	B R W	B W B	R G B
B	W	R	B	R	W
R	B	W	G	G	R

Requiring condition (E) also, leaves only the first of these solutions.

Five minutes should generally be sufficient to completely solve any "Instant Insanity" puzzle.

1970-71 MANUSCRIPT CONTEST WINNERS

The judging for the three best papers submitted for the contest during the 1970-71 school year has now been completed. We congratulate the following winners:

FIRST PRIZE ($200): ***Sidney Graham,*** University of Oklahoma for his paper "On 'Almost Unitary Perfect' Numbers" (this *Journal,* Vol. 5, No. 6, pp. 272-275).

SECOND PRIZE ($100): ***Martin Swiatkowski,*** John Carroll University (Cleveland, Ohio), for his paper "Wronskian Identities" (this *Journal,* Vol. 5, No. 4, 1971, pp. 191-194.

THIRD PRIZE ($50): ***Thomas R. Bingham,*** State University of New York College at Fredonia for his paper "Newton and the Development of the Calculus" (this *Journal,* Vol. 5, No. 4, 1971, pp. 171-181).

1972-73 CONTEST

We take this opportunity to remind you that the contest for this year has begun and to send us your paper if you want to participate. Papers submitted to the *Journal* for publication will automatically be entered if the author is an undergraduate, but we must receive a total of at least ten papers during the year in order to conduct the contest. ***In order to be eligible, authors must not have received a Master's degree at the time they submit their paper.***

UNDERGRADUATE RESEARCH PROJECTS

1. An Amazing Parity Theorem

Proposed by Morton C. Schwartz, Brookline, New York

Take any number of zeros, and any number of ones and place them in a circle, in any order. Reproduce the circle a second time, concentrically with the first. Rotate either circle, and any number of places. The number of zeros opposite ones will always be even. Find other numerical properties related to the number of zeros and ones used.

2. An Algorithm for Reducing the Size of an Integer

Proposed by the Editor

Let n be any positive integer, and consider the number $k(n)$ defined in the following manner:

$$k(n) = \frac{n}{2} \text{ if } n \text{ is even,}$$
$$= 3n + 1 \text{ if } n \text{ is odd.}$$

It has been conjectured that if we define $k^1(n) = k(n)$ and for $i \geq 1$ $k^{i+1}(n) = k(k^i(n))$, the sequence $\{k(n), k^2(n), k^3(n), \ldots, k^i(n), \ldots\}$ ultimately ends in 1. The proof or verification of this conjecture appears to be rather difficult (verification has been carried out by a computer for $n \leq 10{,}000$).

Suppose we generalize this by defining for positive integers p, q, and r,

$$k(n) = \frac{n}{p} \text{ if } p \text{ divides } n$$
$$= qn + r \text{ if } p \text{ does not divide } n.$$

Is there any choice for p, q, and r for which the above conjecture could be settled (conveniently)?

GLEANINGS FROM CHAPTER REPORTS

OHIO EPSILON CHAPTER at Kent State University announces the winner of their Pi Mu Epsilon Award. He was *Michael J. Kotowski* and won $25.00 for the purchase of mathematics books and a plaque with the appropriate inscription.

FLORIDA ZETA CHAPTER at Florida Atlantic University sponsored a series of lectures during the year, one of which had a most puzzling title: "On the Equation $1 + 2 + 4 + \ldots = -1$". The speaker was *Professor Irving Reiner*, University of Illinois at Urbana, who is a former Councillor of Pi Mu Epsilon.

GEORGIA BETA CHAPTER at Georgia Institute of Technology reports that students in mathematics who graduate with a grade point average greater than 3.6 (4.0 perfect) in all mathematics courses receive book awards. Those students during the past year were: *Alton L. Godbold, John A. Lessl, Clark L. March, Kyle T. Siegrist,* and *Thomas J. Tosch.*

NEW YORK EPSILON CHAPTER at St. Lawrence University conducted a two county wide high school mathematics contest in April, 1972 (St. Lawrence and Franklin Counties) in which 16 schools participated.

MISSOURI GAMMA CHAPTER at St. Louis University conducted a *Junior-Senior Problem Solving Contest* in 1972 and awarded the contest winners $50.00 each. The winners were: *Stephen Davis* (senior) and *K. L. Chu Keith* (junior).

WEST VIRGINIA ALPHA CHAPTER at West Virginia University awarded a membership in the Mathematical Association of America to *Marshal Onellion* for the presentation of his paper "Topological Groups" and to *Michael Mays* for his paper "Color Groups".

TENNESSEE BETA CHAPTER at the University of Tennessee at Chattanooga sponsored a *Math Opportunity Day* in which representatives from six companies (DuPont, Provident Life, Hamilton National Bank, Combustion, IBM, and TVA) came and discussed the needs of mathematics in business and industry.

BOOK REVIEWS

Edited by Roy B. Deal
University of Oklahoma Health Sciences Center

Tomorrow's Math - Unsolved Problems for the Amateur - 2nd Edition. *By C. Stanley Ogilvy.* Oxford University Press, N. Y., N. Y. 1972. 198 pages. $ 7.50.

For those from all fields who enjoy the hobby of working on unsolved problems, it is nice to know that Ogilvy has a new edition of his intriguing book written ten years ago. Since then some of the problems have been solved and many new ones have arisen and are presented here.

Mathematics: A Humanistic Approach. *By G. Joseph Wimbish, Jr.* Wadsworth Publishing Company, Inc., Belmont, California. 1972. 169 pages.
Reading for Mathematics: A Humanistic Approach. *By G. Joseph Wimbish, Jr.* Wadsworth Publishing Company, Inc., Belmont, California. 1972. 194 pages.

The first of these two books is a collection of topics written in an interesting manner for presentation to liberal arts students, but the second is what makes the pair particularly unique. It is a collection of essays or articles by well-known people on a variety of subjects in mathematics with a well thought-out collection of questions following each article.

Fundamentals of Elementary Mathematics: Geometry, *By Merlyn J. Behr and Dale G. Jungst.* Academic Press, Inc. N. Y., N. Y. 1972 xix + 326 pages. $ 9.50.

Written to provide a prospective teacher of modern elementary geometry with background material designed to be directly useful to this end.

An Introduction to Plane Geometry. *By H.F. Baker.* Chelsea Publishing Company, Bronx, N. Y. 1971. viii + 382 pages. $ 9.50.

This a reprint, with corrections of a work originally published in 1943 and written mostly before 1939. It is of interest to those who would like an extensive collection of examples and exercises on elementary projective geometry.

Projective Planes. *By Frederick W. Stevenson.* W.H. Freeman and Company, San Francisco, California. 1972. x + 416 pages. $ 13.50.

Written for the advanced undergraduate student to study some subject in depth, this is an excellent textbook on "Everything You Always Wanted to Know About Projective Planes..."

Algebraic Numbers. *By Paulo Ribenboim.* Wiley-Interscience, N. Y., N. Y. 1972. xi + 300 pages. $ 17.95.

Written throughout as an introduction for the advanced undergraduate or first year graduate student who has had the customary modern algebra background with some Galois theory, and who would like to go on in algebra with a subject which not only has historical interest, but which leads naturally to many other fields in algebra.

Algebraic Methods in Statistical Mechanics and Quantum Field Theory. *By Gerard G. Emch.* John Wiley and Sons, Inc., N. Y., N. Y. 1972. xiv + 333 pages. $ 19.95

A modern intensive book on what has become a new discipline in the history of the short, but fruitful marriage of mathematics and quantum mechanics. "The level of the presentation has been determined by the criterion that it should be explicit enough to be within reach of the graduate students in mathematics or physics, and advanced enough to sustain their respective interests." Certainly facility and perhaps motivation would be enhanced for the reader who has a prior knowledge of Dirac's quantum mechanics, even Focks spaces, group theory, and functional analysis. The book carefully describes the inadequacies which led to the algebraic formulation presented here and the physical reasons behind this approach.

Functional Analysis. *By Michael Reed and Barry Simon.* Academic Press, Inc., N. Y., N. Y. 1972. xvii + 325 pages. $ 12.50.

This is the first of a three volume series written at the first year graduate level and devoted to an exposition of functional analysis methods in modern mathematical physics. Although Volume Two is on the analysis of operators and Volume Three on operator algebras this volume contains the fundamentals of both bounded and unbounded operators and the spectral theorem.

Approximation of Elliptic Boundary - Value Problems. *By Jean-Pierre Aubin.* Wiley-Interstate, N. Y., N. Y. 1972. xvii + 360 pages. $ 17.95

An excellent book for advanced graduate students or researchers in numerical analysis, functional analysis, or mathematical physics who are interested in highly efficient numerical features for solving non-homogenous boundary-value problems. The book is intended to imbed the finite element and variational methods into the framework of functional analysis and to explain its applications to approximation of non-homogenous boundary-value problems for elliptic operators.

Listed Books

Algebra, An Introduction for College Students. *By Bruns, Bryant, Matejcic, Melton.* Cummings Publishing Company, Menlo Park, California 1972. xi + 302 pages. $ 8.25.

A Table of the Inverse Sine - Amplitude Function in the Complex Domain. *By Henry E. Fettis and James C. Caslin.* Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio. 1972. iv + 174 pages.

Ten Place Tables of the Jacobian Ellipic Functions. *By Henry E. Fettis and James C. Caslin.* Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio. 1972. iv + 398 pages.

PROBLEM DEPARTMENT

Edited by Leon Bankoff
Los Angeles, California

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity. Occasionally we shall publish problems that should challenge the ability of the advanced undergraduate or candidate for the Master's Degree. Old problems characterized by novel and elegant methods of solution are also acceptable. Proposals should be accompanied by solutions, if available, and by any information that will assist the editor. Contributors of proposals and solutions are requested to enclose a self-addressed postcard to expedite acknowledgement.

Solutions should be submitted on separate sheets containing the name and address of the solver and should be mailed before May 31, 1973.

Address all communications concerning problems to Dr. Leon Bankoff, 6360 Wilshire Boulevard, Los Angeles, California 90048.

Problems for Solution

281. *Proposed by Solomon W. Golomb, University of Southern California.*

We define an "average" number to be a real number for which the average of the digits in its decimal expansion is $(0+1+2+3+4+5+6+7+8+9)/10 = 4.5$. Prove that the number $1/p$, for p prime, is an "average" number if and only if the period of its decimal expansion has an even number of digits.

282. *Proposed by Charles W. Trigg, San Diego, California.*

Four differently colored isosceles right triangles can be assembled to form a square in six essentially different ways (not counting rotations). By joining these tetrachrome squares domino-like with like-colored sides meeting, a variety of configurations can be formed. Show that (a) they can be so assembled into a 2×3 rectangle with solid colors along each side and that (b) they can *not* be so assembled into a 2x3 rectangle with its four sides differently colored.

283. *Proposed by David L. Silverman, Los Angeles, California.*

Let a_1 = sin 1 and, for every positive integer n, let a_{n+1} = sin a_n. Does $\sum a_n$ (n = 1, 2, 3, ..., ∞) converge?

284. *Proposed by Gregory Wulczyn, Bucknell University, Lewdburg, Pennsylvania.*

A polygonal number can be defined:

$$\overset{m}{P}(n) = \frac{n}{2}[(m-2)n - (m-4)].$$

An r-digit automorph integer base b can be defined:

$$(n_1, n_2, \ldots, n_r)_b^2 = (\ldots, n_{r+1}, n_1, n_2, \ldots, n_r)_b.$$

If b = 2m = 2(2r + 1), show that the last two digits of $\overset{m}{P}(b+1)_b$ is a two-digit automorph.

Example in base 10: $\overset{5}{P}(11)_{10}$ = 176. In base 10, 76 is a two-digit automorph since 76^2 = 5776.

285. *Proposed by Murray S. Klamkin, Fotd Motor Company Scientific Laboratory, Dearborn, Michigan.*

Solve the equations:

$$a(x^2 - y^2) - 2bxy + cx - dy + e = 0$$
$$b(x^2 - y^2) + 2axy + dx + cy + f = 0.$$

286. *Proposed by Adeline hl. Gustafson, University of Utah.*

Given decimals $x = .x_1x_2x_3 \ldots$, $y = .y_1y_2y_3 \ldots$ in [0,1], define $x*y = .x_1y_1x_2y_2x_3y_3 \ldots$, and let $P_x = \{x*y: y \in [0,1]\}$. (The only decimal ending in 9's is 1.)

(i) Use the sets P_x, $0 \le x \le 1$, to write [0,1] as the union of c pairwise disjoint perfect sets.

(ii) There are many ways to write [0,1] as the union of c pairwise disjoint perfect sets P_x, $0 \le x \le 1$. Let T be any family of such decompositions $\{P_x: 0 \le x \le 1\}$ such that no two decompositions in T have a set in common. Prove that the cardinal number of T cannot exceed c.

(iii) Modify (i) to obtain a family T of the kind considered in (ii) with cardinal number c.

References: E. Hewitt, *Real and Abstract Analysis*, Springer, N.Y.; I. P. Natanson, *Theory of Functions of a Real Variable*, Frederick Ungar Publishing Co., N.Y., (in particular, see Exercise 5, p. 54).

287. *Proposed by Erwin Just, Bronx Community College.*

For each real number x, prove that

$$-x^n + \sum_{k=0}^{2n} (-1)^k x^k \ge 0.$$

288. *Proposed by Leon Bankoff and Alfred E. Neuman, Mu Alpha Delta Fraternity.*

If $\alpha + \beta + \gamma = \pi$ show that

(1) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma \le \sin\alpha + \sin\beta + \sin\gamma$

(2) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma \le$

$$\sin\alpha + \sin\beta + \sin\gamma + \sin 3\alpha + \sin 3\beta + \sin 3\gamma$$

equality holding if and only if $\alpha = \beta = \gamma$.

289. *Proposed by R. S. Luthar, University of, Wisconsin, Waukesha.*

If $p_1, p_2, \ldots, p_n$ are the first n primes, prove that for n > 2,

$$p_n < p_1 + p_2 + \cdots + p_{n-1}$$

and hence show that between p_n and $p_1 + p_2 + \cdots + p_n$, there always lies a prime number.

290. *Proposed by Solomon W. Golomb, University of, Southern California.*

Let M be an a×b matrix of ab distinct real numbers, with ab > 1. Show that there exists a real number μ such that either every row of M or every column of M (or possibly both) has an entry less than μ and an element greater than μ.

291. *Proposed by C. W. Trigg, San Diego, California.*

How may a square card be folded into a tetrahedron? What is the volume of the tetrahedron in terms of the side of the square?

Solutions

258. *[Fall 1971] Proposed by Charles W. Trigg, San Diego, California.*

Tetrahedral numbers constitute the fourth row (or column) of the arithmetic triangle as Pascal wrote it (a horizontal row of 1's on top and a vertical column of 1's on the left). Only one of these numbers is a permutation of nine consecutive digits. Find it and show it to be unique.

I. Solution by Jeanette Bickley, St. Louis, Missouri.

On the next page is a BASIC program and output from an XDS 940 computer (accessed by a time-sharing terminal). The sequence of tetrahedral numbers is 1, 4, 10, 20, 35, 56, ..., with the nth term equal to

$$\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n^3 + 3n^2 + 2n}{6}$$

The smallest number which is a permutation of nine consecutive digits is 102345678, and the largest number which is a permutation of nine consecutive digits is 987654321. Hence all integral values of n between 843 and 1820 must be tested.

The output for the program gives each value of n and the corresponding tetrahedral number for those cases where the nine digits are each different. We are looking for the tetrahedral number which contains either the digits 0 through 8 or the digits 1 through 9. Of the four tetrahedral numbers which are listed as output only the second one meets the conditions of the problem. Therefore, the required unique tetrahedral number (corresponding to n = 1026) is 180,534,276.

II. Solution by the Proposer.

If the tetrahedral number $T_n = n(n+1)(n+2)/3!$ contains nine consecutive digits, then $T_n \equiv 0 \pmod 9$. This occurs only when n has one of the forms 27k, 27k − 1 or 27k − 2. Furthermore, since T_n has nine digits, $T_{843} \leq T_n \leq T_{1816}$. Of the 108 tetrahedral numbers within this range for which n is of the proper form, only the one for k = 27(38) is composed of nine consecutive digits, namely T_{1026} = 180534276.

Also solved by Marc Kaufman, *Mountain View, California, who observes that there are no tetrahedral numbers which are permutations of all ten digits.*

259. [Fall 1971] *Proposed by John Bender, Rutgers University.*

Prove that the product of the eccentricities of two conjugate hyperbolas is equal to or greater than 2.


```
1 FOR N=843 TO 1820
2 LET X=(N↑3+3*N↑2+2*N)/6
3 LET X9=X-INT(X/10)*10
4 LET X8=INT(X/10)-INT(X/100)*10
5 LET X7=INT(X/100)-INT(X/1000)*10
6 LET X6=INT(X/1000)-INT(X/10000)*10
7 LET X5=INT(X/10000)-INT(X/100000)*10
8 LET X4=INT(X/100000)-INT(X/1000000)*10
9 LET X3=INT(X/1000000)-INT(X/10000000)*10
10 LET X2=INT(X/10000000)-INT(X/100000000)*10
11 LET X1=INT(X/100000000)-INT(X/1000000000)*10
21 IF X1=X2 GOTO 58
22 IF X1=X3 GOTO 58
23 IF X1=X4 GOTO 58
24 IF X1=X5 GOTO 58
25 IF X1=X6 GOTO 58
26 IF X1=X7 GOTO 58
27 IF X1=X8 GOTO 58
28 IF X1=X9 GOTO 58
29 IF X2=X3 GOTO 58
30 IF X2=X4 GOTO 58
31 IF X2=X5 GOTO 58
32 IF X2=X6 GOTO 58
33 IF X2=X7 GOTO 58
34 IF X2=X8 GOTO 58
35 IF X2=X9 GOTO 58
36 IF X3=X4 GOTO 58
37 IF X3=X5 GOTO 58
38 IF X3-X6 GOTO 58
39 IF X3=X7 GOTO 58
40 IF X3=X8 GOTO 58
41 IF X3=X9 GOTO 58
42 IF X4=X5 GOTO 58
43 IF X4=X6 GOTO 58
44 IF X4=X7 GOTO 58
45 IF X4=X8 GOTO 58
46 IF X4=X9 GOTO 58
47 IF X5=X6 GOTO 58
48 IF X5=X7 GOTO 58
49 IF X5=X8 GOTO 58
50 IF X5=X9 GOTO 58
51 IF X6=X7 GOTO 58
52 IF X6=X8 GOTO 58
53 IF X6=X9 GOTO 58
54 IF X7=X8 GOTO 58
55 IF X7=X9 GOTO 58
56 IF X8=X9 GOTO 58
57 PRINT N;X1;X2;X3;X4;X5;X6;X7;X8;X9
58 NEXT N
59 PRINT "THE END"
60 END

RUN
9 0 5 1 2 3 9 4 6 0 8 5
1026 1 8 0 5 3 4 2 7 6
1 1 8 5 2 7 8 0 3 6 9 4 5
1 4 6 2 5 2 1 8 9 3 0 6 4
THE END
```

Solution by Sid Spital, Hayward, California.

The eccentricities of conjugate hyperbolas are given by $\sqrt{a^2 + b^2}/a$ and $\sqrt{a^2 + b^2}/b$ $(a,b > 0)$. Their product, $(a/b) + (b/a)$, can therefore be no less than 2 by the inequality between arithmetic and geometric means.

Also solved by R. C. Gebhardt, *Hopatcong, N.J.;* Ruth Johnson, *Southeastern Community College, Whiteville, North Carolina;* Charles H. Lincoln, *Fayetteville, North Carolina;* Charles W. Trigg, *San Diego, California;* and *the proposer.*

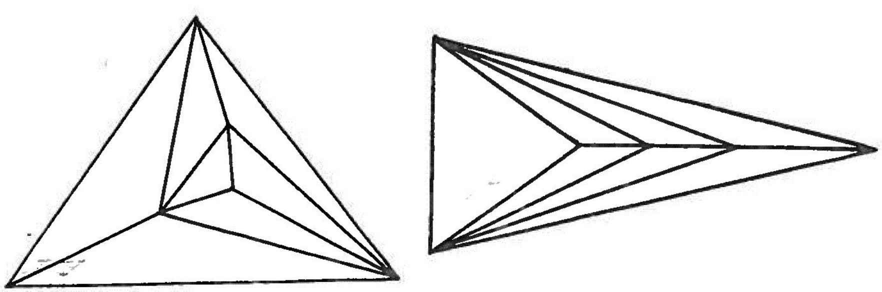
260. [Fall 1971] *Proposed by Paul Ehdoh, Budapest.*

Given n points in the plane, what is the maximum number of triangles you can form so that no two triangles have an overlap in area?

Solution by Charles W. Trigg, San Diego, California.

The number of non-overlapping triangles determined by n given points depends upon the distribution of these points. If the n points are collinear, no triangle will be determined.

Three points determine a triangle. Any point interior to the triangle removes that triangle from the tally and adds three more. Any exterior point, whose joins to two of the vertices and the join of those vertices form a triangle that includes the third vertex, adds two to the tally. Hence the maximum number of non-overlapping triangles determined by n points in the plane is $2(n - 3) + 1$, or $2n - 5$.



Also solved by Douglas L. Costa, *University of Kansas,* and *the Proposer.*

261. [Fall 1971] *Proposed by Solomon W. Golomb, California Institute of Technology and University of Southern California.*

Assume Goldbach's Conjecture in the form that every even integer > 6 can be written as the sum of two distinct primes. Use this to prove directly:

1) Bertrand's Postulate: For every integer $n > 1$, there is a prime between n and 2n.

2) There exist infinitely many sets of three primes in arithmetic progression, i.e., triples p, $p + a = q$, $p + 2a = r$, for some $a > 0$, and p, q, r all primes. (Different triples may use different values of a.)

Almost identical solutions by Douglas Costa, University of Kansas, Bob Prielipp, University of Wisconsin, Oshkosh, and the Proposer.

1) The assertion is clearly true for $n = 2$ and $n = 3$. For $n > 3$, we have $2n > 6$ and hence it follows from Goldbach's Conjecture that $2n = p + q$ where p, q are distinct primes. We cannot have $p < n$ and $q < n$ since $p + q = 2n$, so either $p \geq n$ or $q \geq n$. Equality in either case implies $p = q$, which is false. Hence $p > n$ or $q > n$. Clearly $p < 2n$ and $q < 2n$ always hold. Hence either $n < p < 2n$ or $n < q < 2n$.

2) Let q be any prime, $q > 3$. Then $2q = p + r$, where p, r are distinct primes, by Goldbach's Conjecture. As in proof above, we must have either $p > q$ or $r > q$. Without loss of generality assume $r > q$. Then $p < q$ and $q = p + (q - p)$, $r = p + 2(q - p)$ so that p, q, r is a triple of the required form. Since there are infinitely many primes available for q, there are infinitely many such triples.

The Proposer notes that we have actually shown somewhat more than the problem asked, viz, *for* every *prime* $q > 3$, *there is a set of 3 primes in arithmetic progression* with q in the middle.

262. [Fall 1971] *Proposed by Solomon W. Golomb, University of Southern California and California Institute of Technology.*

Ted: I have two numbers x and y, where $x + y = z$. The sum of the digits of x is 43 and the sum of the digits of y is 68. Can you tell me the sum of the digits of z?

Fred: I need more information. When you added x and y how many times did you have to carry?

Ted: Let's see .... It was five times.

Fred: Then the sum of the digits of z is 66.

Ted: That's right! How did you know?

Question: How did he know?

I. Solution by Robert C. Gebhardt, Hopatcong, N.J.

When adding digits, each carry counts as 1 instead of 10 (as it would in the ordinary addition of the numbers). Thus each carry will cause the sum of the digits to be nine less than the true sum of the numbers; five carries will cause the sum of the digits to be 45 less than the true sum. Thus, 43 + 68 - 45 = 66, the desired answer.

II. Solution by Charles W. Trigg, San Diego, California.

Each carry from the nth position to the $(n + 1)$th position reduces the digit sum in the nth position by b, the base of the system of numeration, and increases the sum in the $(n + 1)$th position by 1. Thus each carry reduces the sum of all the digits by $(b - 1)$. (Fred needed another bit of information, namely: the base of the system of numeration in which the computation was made. For example:

In base ten: $43 + 68 - 5\cdot 9 = 66$;

In base nine: $43 + 68 - 5\cdot 8 = 67$;

In base twelve: $43 + 68 - 5(E) = 64$.

The appearance of 8 in the digit sum of y establishes that $b \geq 9$. It could be argued that the spoken communication between Ted and Fred implied the decimal system.)

Also solved by Jeanette Bickley, *St. Louis, Missouri;* Marc Kaufman, *Mountain View, California;* Jim Metz, *Springfield, Illinois; and the Proposer.*

263. [Fall 1971] *Proposed by Gustave Solomon, TRW Systems, Los Angeles, California.*

Let $x^2 + bx + c = 0$ be a quadratic over a finite field of characteristic 2, $GF(2^k)$. Give necessary and sufficient conditions for solutions x_0 and $x_0 + b$ to lie in $GF(2^k)$, in terms of b and c for the case k odd. (Note: It is necessary to define a (new) discriminant, as the old one clearly does not work).

Solution by Leonard Carlitz, Duke University.

The equation $x^2 = c$ in $GF(2^k)$ has the unique solution $x = c^{2^{k-1}}$. We may accordingly assume that $b \neq 0$. Replacing x by bx, the equation $x^2 + bx + c = 0$ reduces to

$$(*) \qquad x^2 + x = a \qquad (a = c/b^2).$$

The equation $(*)$ is solvable in $GF(2^k)$ if and only if

$$(**) \qquad a = a^2 + a^{2^2} + \ldots + a^{2^{k-1}} = 0.$$

Proof. The necessity follows on raising both sides of $(*)$ to the $2^{j\text{-th}}$ power, $j = 0, 1, \ldots, k-1$ and adding the resulting equations. The sufficiency follows from the observation that the equation $(**)$ has 2^{k-1} solutions in $GF(2^k)$. This is a consequence of the identity

$$x^{2^k} - x = (x^{2^{k-1}} + \ldots + x^2 + x)(x^{2^{k-1}} + \ldots + x^2 + x + 1).$$

Note that nothing need be assumed about the parity of k. However, the equation $x^2 + x + 1 = 0$ is solvable in $GF(2^k)$ if and only if k is even.

Remark: The equation $x^q - x = a$ is solvable in $GF(q^k)$, where q is a prime power, if and only if $a + a^q + \ldots + a^{q^{k-1}} = 0$.

Also solved by the Proposer.

264. [Fall 1971] *Proposed by Bruce B. Olaf, Bethlehem, Pennsylvania.*

There are three prisoners: A, B, C. The prisoner with the highest degree of guilt will be executed. Prisoner A sees the warden and asks for any information he has. The warden says B will not be executed and that A's case has not yet been considered. Assuming no ties in the degree of guilt, what are A's chances that he will be executed?

Solution, by Jeanette Bickley, St. Louis, Missouri.

Given that A's case has not yet been considered and that B will not be executed merely implies that C is guiltier than B, an event that has no bearing on A's relative guilt. (A already knew that one or the other of B and C would not be executed, so the information supplied by the warden does not alter A's chances of being executed). Therefore the possible descending orders of guilt are ACB, CAB, or CBA. Of these three possibilities, there is only one in which A is the guiltiest. Hence the chance that A will be executed is 1 in 3.

Also solved by K. Burke, *Seton Hall University, South Orange, N.J.;* James C. Hickman, *University of Iowa;* Rick Johnson, *Southeastern Community College, Whiteville, N.C.;* Marc Kaufman, *Mountain View, California;* James Metz *and* Fr. Thomas Pisors, C. S. V. *(jointly), Springfield, Illinois; and* Charles W. Trigg, *San Diego, California.*

Editor's Note.

Marc Kaufman remarks that this kind of analysis extends to an n-person group. As long as A's case has not been evaluated, his chances stand at $1/n$.

Two of the solvers arrived at the result of 1/2 as the probability that A would be executed. This outcome could be considered correct if the initial sample space consisted only of A, whose case is the last to be evaluated, and, let us say, X, who has already been found to be guiltier than the other $(n - 2)$ of the original n number of prisoners. In our problem, however, the possible rankings of degrees of guilt consist of the six sets: ABC, ACB, BAC, BCA, CAB and CBA, with the first-named of each set considered the guiltiest. In this sample space, regardless of the relative ranking of B and C, only three possiblities remain in the reduced outcome space, either ABC, BAC, BCA (if B happens to be guiltier than C) or ACB, CAB, CBA (if C is guiltier than B). In either event A's chances of being executed are 1 in 3.

A further criticism of the model resulting in the probability of 1/2 is the following: Suppose it were C who asked the warden for information and were told that either A or B would not be executed, a fact C already knew. And suppose, further, that we arrived at the result 1/2 as C's probability of being executed. Now let us do the same with B and find that B's chances of being executed are 1 in 2. Then the combined probabilities of A, B and C being executed would add up to 3/2, an obvious absurdity.

A similar problem and its solution may be found in F. Mosteller *Fifty Challenging Problems in Probability*, Addison-Wesley, 1965, p. 28.

265. [Fall 1971] *Proposed by Lew Kowarski, Morgan State College, Baltimore.*

Prove that if $a \neq \pm 1$, $a^4 + 4$ is not a prime number.

I. Solution by C. Gendler, Clarion State College, Clarion, Pennsylvania.

By factoring $a^4 + 4$ over the complex field into linear factors we find

$$a^4 + 4 = (a - 1 + i)(a - 1 - i)(a + 1 + i)(a - 1 - i).$$

Combining these we obtain a factorization over R:

$$a^4 + 4 = [(a - 1)^2 + 1][(a + 1)^2 + 1] = pq$$

Hence $4 + a^4$ is composite except when one of the factors is equal to 1. This occurs only when

$$a - 1 = 0 \quad \text{or} \quad a + 1 = 0$$

(since all other values exceed 1).

Hence, $a^4 + 4$ is composite except for:

$$a = 1, \quad a^4 + 4 = 5$$
$$a = -1, \quad a^4 + 4 = 5.$$

II. Solution by Robert C. Gebhardt, Hopatcong, N.J.

If a is even, $a^4 + 4$ is even and thus a multiple of 2. If a ends in 1, 3, 7, or 9, then a^4 ends in 1 and $a^4 + 4$ ends in 5, and is a multiple of 5. (The exceptions are if $a = 1$ or $a = -1$, in each of which cases $a^4 + 4$ is 5.) If a ends in 5, then a is of the form $10n + 5$ $(n = \ldots, -2, -1, 0, 1, \ldots)$, and

$$a^4 + 4 = 10{,}000n^4 + 20{,}000n^3 + 15{,}000n^2 + 5{,}000n + 629$$

which can be factored as

$$a^4 + 4 = (100n^2 + 120n + 37)(100n^2 + 80n + 17),$$

and thus is not prime. There are no other cases.

Also solved by Jeanette Bickley, *St. Louis, Missouri;* K. Burke, *Seton Hall University, South Orange, N.J.;* Thomas J. Cato, Jr., *Adelphi University;* I. Kevin Colligan, *University of, Wisconsin, Madison;* Ray Haertel, *Central Oregon Community College, Bend, Oregon;* Mark Hunacek, *Brooklyn, N.Y.;* Francis Landolf, *Clarkson College of Technology, Potsdam, N.Y.;* Charles H. Lincoln, *Jerry Sanford Senior High, Fayetteville, N.C.;* Bob Prielipp, *University of, Wisconsin, Oshkosh;* Kenneth Rosen, *University of, Michigan, Ann Arbor;* Michael Shapiro, *University of Illinois, Champaign, Illinois;* Sid Spital, *Hayward, California;* Charles W. Trigg, *San Diego, California; and the Proposer.*

267 [Fall 1971] *Proposed by Charles W. Trigg, San Diego, California.*

Consecutive odd integers are equally spaced around a circle in order of magnitude. Under what conditions can a straight line be drawn through the circle dividing the integers into two groups with equal sums?

Solution by the Proposer

Being in arithmetic progression, the terns can be combined into pairs with equal sums— the largest and the smallest, the next largest and the next smallest, and so on. Then if the pairs are equally divided into two groups, the groups will have the same sum. Consequently, there must be 4k odd integers in the sequence, and a diameter drawn from between the kth and the $(k + 1)$th integers in the sequence will separate the integers as desired.

. *Also solved by* R. C. Gebhardt, *Hopatcong, N.J.; William A.* DePalo, *Polytechnic Institute of, Brooklyn, N.Y.; and* Charlie Carter, *University of Richmond, Virginia.*

268. [Fall 1971] *Proposed by Gregory Wulczyn, Bucknell University.*

List all the primitive roots of 3^n, where n is a positive integer.

Solution by Solomon W. Golomb, University of, Southern California.

For $n = 1$, the primitive root of 3^1 is 2. For $n = 2$, the primitive roots of 3^2 and 2 and 5. For $n > 2$, let $a_1, a_2, \ldots, a_{2\cdot 3^{n-3}}$ be the primitive roots mod 3^{n-1}. Then $\{a_i, a_i + 3^{n-1}, a_i + 2\cdot 3^{n-1}\}$, $i = 1, 2, \ldots, 2\cdot 3^{n-3}$ are the primitive roots mod 3^n.

For example, the primitive roots of 3^3 are 2, 5, 11, 14, 20, 23. If g is primitive mod p^n, then surely g is primitive mod p^{n-1}. The number of primitive roots mod p^n is $(p - 1)p^{n-2}$ for $n > 1$ and $p > 2$. Therefore the primitive roots mod p^2 generate all the primitive roots mod p^n for odd primes p and for $n > 2$. Hence, for $p = 3$, we observe that the primitive roots mod 9 are 2 and 5. Therefore, the primitive roots mod 3 are all numbers of the form $2 + 9k$, and $5 + 9k$.

Also solved by the Proposer.

269. [Fall 1971] *Proposed by the Problem Editor.*

If $A + B + C = 180°$, show that $\cos(A/2) + \cos(B/2) + \cos(C/2) \geq \sin A + \sin B + \sin C$.

I. Solution by Leonard Carlitz, Duke University, Durham, N.C.

Replace A, B, C by $180° - 2\alpha$, $180° - 2\beta$, $180° - 2\gamma$. It follows that α, β, γ are the angles of an acute triangle. The given inequality becomes

$$\sin\alpha + \sin\beta + \sin\gamma \geq \sin 2\alpha + \sin 2\beta + \sin 2\gamma .$$

This is a known inequality (see Bottema, Djordjević, Janić, Mitrinović and Vasić, *Geometric Inequalities*, Wolters-Noordhoff, Groningen, 1969, p. 18 no. 2.4).

II. Solution by the Problem Editor.

Consider the triangle ABC and its circumcircle, radius R. Let A' denote the midpoint of the arc BC containing A. It is known that

$A'B + A'C \geq AB + AC$, with equality* if and only if A and A' coincide. Now, $A'B = A'C = 2R\cos(A/2)$, and $AB + AC = 2R(\sin C + \sin B)$. So $2\cos(A/2) \geq \sin C + \sin B$. Similarly, $2\cos(B/2) \geq \sin A + \sin C$ and $2\cos(C/2) \geq \sin A + \sin B$. Hence $\Sigma_A \cos(A/2) \geq \Sigma_A \sin A$, with equality if and only if ABC is an equilateral triangle.

NEW OFFICERS ELECTED

The Pi Mu Epsilon Fraternity elected a new slate of officers during the past year, so we congratulate them and wish them well in their new office. For the benefit of the membership at large we introduce them below and include a brief background sketch for each officer.

President

Houston T. Karnes, Professor of Mathematics at Louisiana State University, holds a bachelor's degree from Vanderbilt University and received the Ph. D. from Peabody College in 1940. He has taught at Northwestern Junior College (Iowa), at Harding College, and in the Nashville school system. Coming to Louisiana State in 1938, he was Dean of Men 1945-46, and the Director of a Mathematics Institute in 1959. He went to Allahadbad, India as a visiting professor in 1965. He was a writer for the School Mathematics Study Group from 1958 to 1961, held the office of Secretary of the National Council of Teachers of Mathematics from 1954 to 1965, and was the Vice-president of Pi Mu Epsilon (national) from 1963 to 1972.

President-Elect

E. Allan Davis, Professor of Mathematics at the University of Utah, received his bachelor's and master's degree from the University of California at Berkeley and earned his Ph. D. in 1951 there also. He came to the University of Utah in 1955, and has taught at the Universities of California and Oregon. He was the Associate Program Director of the National Science Foundation Special Projects in Science Education in 1961-62 and was Program Director for the Student and Cooperative Program in Pre-College Education in Science from 1967 to 1970. He has also served as the faculty advisor of the Utah Alpha Chapter.

Councillors

Gloria C. Hewitt, Associate Professor of Mathematics at the University of Montana, received her bachelor's degree from Fisk University and in 1962 the Ph. D. from the University of Washington. She has served on various panels for the National Academy of Science and committees of MAA (the Mathematical Association of America) She has been a visiting lecturer for the MAA since 1964, and was one of the invited panel speakers at the 1971 summer meeting of the HAA.

Dale W. Lick, Vice-president for Academic Affairs and Dean of the Faculty at Russell Sage College (Troy, New York), won his bachelor's and master's degree from Michigan State University, and in 1963 the Ph. D. from the University of California at Riverside. Before assuming his new duties at Russell Sage College this year, he was Head of the Department of Mathematics at Drexel University, taught at the University of Redlands and the University of Tennessee, and was a consultant at Oak Ridge National Laboratories. He helped to organize the California Zeta Chapter in 1963 and reactivated Pennsylvania Theta while he was at Drexel.

Eileen L. Poiani, Assistant Professor at Saint Peter's College (Jersey City), received her bachelor's degree from Douglass College in 1965 and the Ph. D. from Rutgers University in 1971. She has taught at Saint Peter's College since 1967. Her activities in Pi Mu Epsilon include establishing the New Jersey Epsilon Chapter in 1968, accompanying two delegates to the 1971 summer meeting at University Park, Pennsylvania, and installing the New York Psi Chapter at Iona College.

Journal Editor

David C. Kay, Associate Professor of Mathematics at the University of Oklahoma, received his bachelor's degree from Otterbein College (Ohio), his master's degree form the University of Pittsburgh, and in 1963 the Ph. D. from Michigan State University. He taught at the University of Wyoming from 1963 to 1966 and came to Oklahoma in 1966. He is the author of a geometry textbook and several research articles on geometry and topology. He has served as faculty advisor of the Oklahoma Alpha Chapter.

We also welcome back *Richard V. Andree,* Professor of Mathematics at the University of Oklahoma, for another term as *Secretary-Treasurer.* He has served faithfully in that capacity for many years and has been a stalwart supporter of the fraternity. *J. C. Eaves,* Professor and Chairman of the Department of Mathematics at the University of West Virginia, steps down from the presidency to assume an active role as *Past-President.* He has been an eloquent spokesman for the fraternity and will continue to offer his sound advice. Finally, *E. Maurice Beesley,* Professor and Chairman of the Department of Mathematics at the University of Nevada, will be serving another term as a *Councillor.* He has been on the council since 1969, so his experience will be an asset to the organization during the coming years.

ANNOUNCEMENT

With this issue the listing of new initiates will be discontinued in compliance with action voted by the Council at its last meeting. It was felt that the list is of dubious value in view of the overall returns to the fraternity, recognition of local chapters, and its high cost. In the future more emphasis will be given to innovative programs by local chapters and to winners of annual awards and contests.

We therefore urge local chapters to inform us of their awards programs and to promptly send us names of the winners of those awards or contests they conduct during the year

INITIATES

ALABAMA ALPHA, University of Alabama

Gabriel C. Armijo
Helen H. Atkins
Barbara Bailey
Boyd L. Bailey, Jr.
Marvin W. Bassett
Cynthia L. Bathurst
Richard E. Broughton
Sarah E. Brown
Phyllis J. Burnett
Carl R. Canada
Debs D. Cokely
Rebecca A. Conway
Eugene P. Cooper
Alyce Y. Cox
Nancy S. Crawford
Curtis R. Croft
Louis Dale
Elizabeth T. Davis
Faye C. Deal
John G. Falls
Thomas M. Fortner
Ann S. Fralish
Peggy A. Gilbert
Bedford K. Goodwin, III
Terry A. Green
Malcolm S. Harris
Deborah E. Head
Thomas A. Henry
Bernice K. Hodge
Sandra L. Hodges
Jess H. Howton, III
Barry S. Johnston
Catherine D. Jordan
John C. Kagan
Nanette J. Lathan
Alan R. Lee
James R. Light
Thomas D. Loftin
William C. Massey
Ronald E. May
John D. McCamy
Elena M. Medina
Lawrence D. Miller
Joseph E. Morris
William S. Morrow
Dwight W. Moss
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