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WORMHOLES AND THE COSMOLOGICAL CONSTANT*

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ABSTRACT

We review Coleman's wormhole mechanism for the vanishing of the cosmological constant. We show that in a minisuperspace model wormhole-connected universes dominate the path integral. We also provide evidence that the Euclidean path integral over geometries with spherical topology is unstable with respect to formation of infinitely many wormhole-connected 4-spheres. Consistency is restored by summing over all topologies, which leads to Coleman's result. Coleman's argument for determination of other parameters is reviewed and applied to the mass of the pion. A discouraging result is found that the pion mass is driven to zero. We also consider qualitatively the implications of the wormhole theory for cosmology. We argue that a small number of universes containing matter and energy may exist in contact with infinitely many cold and empty universes. Contact with the cold universes insures that the cosmological constant in the warm ones is zero.

1. Introduction

The cosmological constant plays two roles in physics. The first role is that of a coupling constant, similar to other mass and coupling parameters in microscopic physics. Its origin is likely to include short distance physics including wavelengths down to the Planck scale. The other role, as its name suggests, is that of a macroscopic parameter controlling the large scale behaviour of the universe. From the microscopic point of view we have no explanation of why the cosmological constant vanishes. From the cosmic viewpoint it vanishes so that the universe can be big and flat, as observed. Thus, it seems a miracle that microscopic physics should be fine tuned with practically infinite precision just so that the large scale structure of space-time can look as it does. What seems to be needed, as emphasized by Linde,[1] is a direct connection between the cosmic scale physics and the microscopic machinery which creates coupling constants. Wormholes provide just such a large distance – small distance connection. As far as we know, early speculations about wormholes date back to John Wheeler. Hawking and others[2] have emphasized that unusual and surprising effects can be associated with them.[3] In particular, he speculated that wormholes may play an important role in shifting the cosmological constant to zero.[4,5]

A wormhole is a microscopic connection between two otherwise smooth and large regions of space-time. For example, in figure 1 a wormhole is shown connecting two flat two-dimensional sheets. The two sheets may actually be portions of the same sheet, as in figure 2, or may be parts of otherwise disconnected universes, as in figure 3. The important thing about wormholes is that they are small and cost little action but can connect arbitrarily distant regions of space-time. Evidently, there is a potential connection between the very large and the very small.

Recently, Coleman[6] and Giddings and Strominger[7] have considered the effects of wormholes in the Euclidean path integral of quantum gravity. Similar ideas have been explored in ref. [9,10]. Remarkably, it was shown that the entire effect of wormholes is to modify coupling constants and to provide a probability distribution for them. Even more remarkable is Coleman's claim[11] that the probabilty for a given value of the cosmological constant is overwhelmingly concentrated at zero.[12] One purpose of this paper is to review Coleman's arguments and discuss some subtle points about the Euclidean path integral and the wave function of the universe. A second purpose is to clarify the implications of Coleman's theory for other parameters as well as for physics of the early universe.

In sec. 2 we review Coleman's arguments and rederive his results using a somewhat different method. In sec. 3 we point to some of the subtleties in defining the Euclidean path integral for gravity using a minisuperspace model as an example. In sec. 4 we argue that some of these subtleties are actually clarified once the wormholes are taken into account. Sec. 5 is an attempt to use Coleman's approach to fix other fundamental parameters. The results of a naive treatment turn out to be quite discouraging: wormholes shift the pion mass to zero and the neutrino mass away from zero. We speculate on how one might avoid these unphysical conclusions. Finally, in sec. 6 we address the issue of whether generation of heat in the early universe is consistent with the mechanism that shifts the cosmological constant to zero.

2. Coleman's Mechanism

In this section we will derive Coleman's results in a way that some people have found more transparent than Coleman's original arguments. Consider the Euclidean path integral version of quantum gravity. We integrate over all compact topologies of space-time; in particular, we focus on geometries which consist of some number of large universes connected by tiny wormholes. To begin with, we will assume that the wormholes can be treated as dilute so that their emissions are independent. This means that their average space-time separation is much greater than their size, which we take to be of the order of the Planck scale. For definiteness, we take the large universes to have spherical topology. Let us first focus on a single universe with no wormholes. The Euclidean path integral for the expectation value of some observable M is

$$\langle M\rangle_\lambda = \frac{\int dg e^{-I(g,\lambda)} M}{\int dg e^{-I(g,\lambda)}} \tag{2.1}$$

where the symbols have the following meaning. The parameters, such as couplings, masses and the cosmological constant, are collectively indicated by λ. The expression $\langle M\rangle_\lambda$ denotes the expectation value of M in a theory with only a single large universe without wormholes and with parameters λ. The integration $\int dg$ indicates a sum over metrics and other local fields and $I(g,\lambda)$ is the action functional.

Now consider the effects of wormholes connecting distant regions of a single large universe. In particular, suppose that the two points connected by the wormhole are x and x'. Let $\phi_i(x)$ be a basis for the local operators at x. We assume that the effect of a wormhole is to insert the expression

$$\frac{1}{2}\sum_{ij} C_{ij}\phi_i(x)\phi_j(x') \tag{2.2}$$

into the integrand of the path integral, where $C_{ij} \sim \exp(-S_w)$ and S_w is the wormhole action. Thus, for example, the numerator of eq. (2.1) would be replaced by

$$\int dg M e^{-I(g,\lambda)} \int dx dx' \sum_{ij} \frac{1}{2} C_{ij}\phi_i(x)\phi_j(x') \tag{2.3}$$

The process in eq. (2.3) can be represented by a figure in which a line connects x and x' (see figure 4). It is important to distinguish such processes from ordinary propagators connecting x and x'. The ordinary processes propagate through a large region of space-time and depend on space-time separation between x and x'. Instead, wormholes 'short circuit' space-time. Therefore, the coefficients C_{ij} do not depend on x and x', at least when the two points are distant. This lack of dependence on space-time separation makes the wormhole amplitudes very different from amplitudes for the ordinary processes. In fact, if the wormholes are sufficiently dilute, the amplitude for each wormhole will typically scale like the square of the space-time volume instead of the volume.

Next consider the sum over any number of wormholes, as in figure 5. It is easy to see that the sum exponentiates to yield

$$\int dgMe^{-I(g,\lambda)}\exp(\frac{1}{2}\int dxdx'\sum_{ij}C_{ij}\phi_i(x)\phi_j(x')) \tag{2.4}$$

Let us write this in a different form through the use of the identity

$$\exp(\frac{1}{2}C_{ij}V_iV_j) \sim \int\prod_k d\alpha_k \exp(-\frac{1}{2}D_{ij}\alpha_i\alpha_j)\exp(-\alpha_l V_l) \tag{2.5}$$

where D_{ij} is the inverse of C_{ij}. The matrix element in question becomes

$$\int\prod_k d\alpha_k\int dgM\exp(-\frac{1}{2}D_{ij}\alpha_i\alpha_j)\exp(-I(g,\lambda)-\alpha_l\int dx\phi_l(x)) \tag{2.6}$$

If λ_i are the coefficients of $\int dx\phi_i$ in the lagrangian, then eq. (2.6) takes the form

$$\int\prod_k d\alpha_k\int dgM\exp(-\frac{1}{2}D_{ij}\alpha_i\alpha_j)\exp(-I(g,\lambda+\alpha)) \tag{2.7}$$

In a similar manner we can take into account processes involving additional closed universes. Each additional universe gives a factor $\int dg\exp(-I(g,\lambda+\alpha))$ in the α-integrand. The combinatorics again exponentiate giving

$$\langle M\rangle = \frac{1}{N}\int d\alpha dgM\exp(-\frac{1}{2}D_{ij}\alpha_i\alpha_j)e^{-I(g,\lambda+\alpha)}\exp\Big(\int dg'e^{-I(g',\lambda+\alpha)}\Big) \tag{2.8}$$

where N is a normalization factor. Let us compare eq. (2.8) and eq. (2.1). We see that $\langle M\rangle$ can be written in the form

$$\langle M\rangle = \int d\alpha\rho(\alpha)\langle M\rangle_{\lambda+\alpha} \tag{2.9}$$

where

$$\rho(\alpha) = \frac{1}{N}\exp(-\frac{1}{2}D_{ij}\alpha_i\alpha_j)\int dge^{-I(g,\lambda+\alpha)}\exp\Big(\int dg'e^{-I(g',\lambda+\alpha)}\Big) \tag{2.10}$$

Eq. (2.9) has a remarkable form. It says that any expectation value computed in our universe is a weighted average over expectation values in universes without wormholes but with couplings $\lambda + \alpha$. This is precisely the formula for an ensemble of worlds with a statistical distribution of coupling constants. In other words, if God created a large number of big smooth worlds, each with couplings $\lambda + \alpha$, drawn from a statistical distribution with weight $\rho(\alpha)$, exactly the same formula would result. Needless to say, an observer in one of the members of the ensemble would have no way to deduce the existence of the others.

One might wonder if two experiments at different locations of space-time would agree on the values of the couplings. The answer is yes because the integration variables α_i are not functions of position. There is a single overall integral over α_i. Thus, one of the effects of the wormholes is to equalize the couplings in all regions of space-time, even in large universes which would otherwise be disconnected. Furthermore, there is no restriction that the operator M must only involve a single region of space-time. In fact, M could be a product of observables in our region and some vastly different place and time. The formula states that, in that case too, an integration over a single set of α_i defines the expectation value. Therefore, there can be no disparity between the values of the coupling constants in distant regions of space-time. This completes the first part of Coleman's argument.

The second part involves the computation of the probability function ρ. Let us define

$$X(\alpha) = \int dg e^{-I(g,\lambda+\alpha)} \tag{2.11}$$

where the integration is over geometries with spherical topology and no wormholes. There are geometries of spherical topology which can be described as several large spheres connected by wormholes (see figure 6). We do not wish to include these in the definition X. We will return to this point later. For now, let us assume that this separation can be made. Then the probabilty function is

$$\rho(\alpha) = X e^{X} e^{-\frac{1}{2} D_{ij} \alpha_i \alpha_j}. \tag{2.12}$$

Coleman suggests that the leading approximation to X is the contribution from the classical stationary point associated with Euclidean de Sitter space. The

Euclidean de Sitter space is a 4-sphere whose radius is controlled by the physical cosmological constant Λ. Consider a large smooth universe of spherical topology with metric g_{ij}. Let us compute the effective action for gravity by integrating over all fluctuations including matter and gauge fields. The result can be expanded in powers of the curvature tensor and its derivatives[⋆]

$$S_{eff} = \int d^4x\sqrt{g}(\Lambda - \frac{1}{16\pi G}R + aR_{abcd}R^{abcd} + bR_{ab}R^{ab} + cR^2 + \ldots) \qquad (2.13)$$

where Λ, G, a, etc., are functions of the wormhole-shifted fundamental parameters $\lambda + \alpha$. If we approximate S_{eff} by Einstein gravity, then the variational equation is

$$R_{ij} = 8\pi G\Lambda g_{ij} \qquad (2.14)$$

The maximum volume solution of this equation is the 4-sphere whose radius becomes large as $\Lambda \to 0$. Therefore, let us in general restrict our attention to large 4-spheres of radius r. Then

$$R_{abcd} = \frac{1}{r^2}(g_{ac}g_{bd} - g_{ad}g_{bc}) \qquad (2.15)$$

Substituting this into eq. (2.13), we find

$$S_{eff}(r) = \frac{8\pi^2}{3}(\Lambda r^4 - \frac{3}{4\pi G}r^2 + A_1 + \frac{A_2}{r^2} + \ldots) \qquad (2.16)$$

A dominant contribution to the Euclidean path integral comes from the stationary point of $S_{eff}(r)$. For large r (small Λ), this occurs at $r^2 \approx 3/8\pi G\Lambda$. Plugging this into eq. (2.16) gives

$$S_{eff} = -\frac{3}{8G^2\Lambda} \qquad (2.17)$$

Approximating X by such a saddle point, we find

$$\rho \sim \exp(-\frac{1}{2}D_{ij}\alpha_i\alpha_j)\exp(\frac{3}{8G^2\Lambda})e^{\exp(\frac{3}{8G^2\Lambda})} \qquad (2.18)$$

where Λ is the physical value of the cosmological constant which, in general, depends in a complicated way on many wormhole parameters α_i. However, Λ has a

⋆ We thank Steve Weinberg for emphasizing this to us.

simple linear dependence on the α which shifts the 'bare value' of the cosmological constant. Analogous statements apply to the physical value of Newton's constant $G(\alpha)$. Obviously, the function ρ is infinitely peaked at $G^2(\alpha)\Lambda(\alpha) = 0$. This is the basis for Coleman's claim that wormholes provide a mechanism for setting the cosmological constant to zero.

How much does the above argument depend on the details of physics at and below Planck scale? It seems to us that the answer is: very little. In particular, it does not depend on the existence of classical wormhole solutions of Planck size. To see this, consider a geometry consisting of two smooth asymptotically flat regions connected by a wormhole of size a much larger than the Planck scale (see figure 7). Certainly, including such configurations does not depend on peculiar small distance effects. How does the action depend on a? As a approaches zero, it varies like a^2, as long as a is large enough to ignore the effects of terms of order R^3 in the effective lagrangian (2.13). When a becomes of order Planck scale, we do not know how the action varies. In fact, we do not know whether the notion of a smooth metric or even a 4-dimensional manifold describes the wormhole adequately. The only important assumption is that the integral over a makes sense and gives rise to an effective description in terms of bilocal operators, as in eq. (2.2). For example, in Einstein gravity, the contribution of a wormhole with positive action is maximized at the endpoint $a = 0$. There, in place of a saddle point associated with a classical solution, the path integral is dominated by the endpoint contribution.

Let us now consider what happens if the dilute gas approximation breaks down. This is likely to occur if the shift of the cosmological constant is of the order M_p^4. Under these circumstances we must introduce interactions among wormholes. For example, a process in which two wormholes are absorbed close to one another may have to be taken into account. We can always represent this by the two wormholes coalescing to form a third one, which is then absorbed. This can be accounted for by adding a cubic term in the α's to $D_{ij}\alpha_i\alpha_j$ in eq. (2.8). More generally, this term should be replaced by some unknown function of α_i. It may be that the natural variables are not α_i but some θ_i which are non-linear functions of the α_i. In fact, in this regime, the space of α_i may be compact so that the shifts of couplings are bounded.

3. Euclidean Path Integrals in Minisuperspace

In this section we will review the Hartle-Hawking definition of the Euclidean path integral. There are two main components involved in this definition. One is the idea of a sum over compact Euclidean geometries. This idea is basic to our entire discussion. The other component is a method of definition of divergent integrals by continuation to complex values of the conformal factor. We will find that this second component is not consistent with Coleman's treatment.* Rather than discuss the full path integral for quantum gravity, we will restrict our attention to a minisuperspace model. The main points can be easily understood in this context. Let us consider the Euclidean geometries with metric

$$ds^2 = \frac{2G}{3\pi}(dt^2 + a^2(t)d\Omega_3^2) \tag{3.1}$$

where $d\Omega_3^2$ is the metric of a unit three-sphere. The dynamical variable is the scale factor $a(t)$. We would like to integrate over the compact Euclidean geometries, i.e., the trajectories which begin with $a = 0$ at $t = 0$ and end with $a = 0$ at some final time $t = T$. The Euclidean action of such a geometry is

$$I = -\frac{1}{2}\int_0^T dt(a\dot{a}^2 + a - h^2a^3) \tag{3.2}$$

where $h = 4G\sqrt{\Lambda}/3$. This action defines a hamiltonian

$$H = \frac{1}{2}(-\frac{p^2}{a} - a + h^2a^3) \tag{3.3}$$

The total Euclidean path integral contains a functional integral over $a(t)$ and an

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⋆ This was also mentioned in a footnote in the second reference of [10].

integration over T:† [13]

$$X = \int_0^\infty dT \int da(t) e^{-I(a(t))} \tag{3.4}$$

The integral is only a formal expression because the action in eq. (3.2) is not bounded from below. First of all, it can become large and negative because of rapid oscillations of a which cause $\dot{a}$ to be large. This ultraviolet instability can be cured by adding, for example, an R^2 term to the gravitational action. Perhaps, of greater interest are configurations of very long duration. First, consider the classical de Sitter solution which is a stationary point of eq. (3.2):

$$a(t) = \frac{1}{h} \sin(ht) \tag{3.5}$$

with duration $T = \pi/h$. This is the configuration Coleman uses to compute the probability function $\rho(\alpha)$. It describes a 4-sphere. The action of this solution is $I_1 = -\frac{3}{8G^2\Lambda}$. Next consider the configuration with duration slightly less than $2\pi/h$. The new configuration is made by joining two 4-spheres as in figure 8. The neck where the two spheres join has the minimum value of the scale factor a_{min}. This is the simplest wormhole configuration included in the minisuperspace model. The action of such a configuration is

$$I_2 \approx -\frac{6}{8G^2\Lambda} + \frac{1}{2} a_{min}^2. \tag{3.6}$$

Similarly, N spheres can be joined by narrow necks with the Euclidean action

$$I_N \approx -\frac{3N}{8G^2\Lambda} + \frac{1}{2}(N-1) a_{min}^2. \tag{3.7}$$

These configurations are what remains in the minisuperspace of the wormhole-connected universes of Coleman. If Coleman is right, we expect them to dominate the Euclidean path integral in the minisuperspace.

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† The range of integration over T is not *a priori* fixed. We have made the choice which is natural in the Euclidean formalism.

Hartle and Hawking have proposed a definition of the Euclidean path integral which somehow regulates the divergences associated with the unbounded action.[14] When applied to the minisuperspace model considered in this paper, their prescription amounts to the following. The contour of integration over a is rotated to ia and $t \to it$. The resulting path integral is given by eq. (3.4), with I replaced by

$$\mathcal{I} = \frac{1}{2}\int_0^T dt(a\dot{a}^2 + a + h^2a^3) \tag{3.8}$$

Eq. (3.8) is the action for a quantum mechanical problem with the hamiltonian

$$\mathcal{H} = \frac{1}{2}(\frac{p^2}{a} + a + h^2a^3) \tag{3.9}$$

Note the difference between H and $\mathcal{H}$. All the terms in $\mathcal{H}$ are positive and in the limit $\Lambda \to 0$ nothing special happens. The quantum mechanics problem with $\Lambda = 0$ is completely stable. Therefore, it is not possible that the resulting path integral is of order $\exp(\frac{3}{8G^2\Lambda})$. This property of the Hartle-Hawking definition of the Euclidean path integral continues to be true if we integrate over geometries more general than in the minisuperspace model. Consider, for example, the path integral over conformally flat geometries of spherical topology: $g_{ij} = \phi^2\delta_{ij}$. As explained in the next section, this set includes networks of wormhole-connected spherical universes which constitute the tree approximation to the path integral considered by Coleman. The Euclidean path integral for Einstein gravity reduces to

$$X = \int[d\phi]\exp\Big(\int d^4x(\frac{3}{8\pi G}(\partial\phi)^2 - \Lambda\phi^4)\Big) \tag{3.10}$$

Clearly, this expression is formal due to the unconventional sign of the kinetic term for ϕ. With the Hartle-Hawking continuation $\phi \to i\phi$, it is defined to be

$$X = \int[d\phi]\exp\Big(-\int d^4x(\frac{3}{8\pi G}(\partial\phi)^2 + \Lambda\phi^4)\Big) \tag{3.11}$$

This is just the Euclidean path integral for the stable ϕ^4 theory.* Therefore, it

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$\star$ This is not a conventional theory since it must be regulated in a conformally invariant way.

cannot develop an exponential singularity as $\Lambda \to 0$.

The non-appearance of singular terms in the minisuperspace Euclidean path integral as $\Lambda \to 0$ is related to properties of the Hartle-Hawking wave function of the universe. This wave function can be defined in the following way. Let $\tilde{\psi}(a)$ be the Euclidean path integral for the analytically continued problem (3.8) over all trajectories which begin at $a = 0$ and end at a.

$$\tilde{\psi}(a) = \int_0^\infty dT \int da(t) \exp[-\mathcal{I}(a(t))]\delta(a(T) - a) \tag{3.12}$$

Another representation of this formula is

$$\tilde{\psi}(a) =< a| \int_0^\infty dT e^{-\mathcal{H}T} |a = 0 >=< a|\frac{1}{\mathcal{H}}|a = 0 > \tag{3.13}$$

A subsequent continuation $a \to -ia$ in the argument of $\tilde{\psi}$ gives a Wheeler-De Witt wave function for the original problem defined by eq. (3.2). It can be written formally as

$$\psi(a) =< a| \int_0^\infty dT e^{-HT} |a = 0 >=< a|\frac{1}{H}|a = 0 > \tag{3.14}$$

In terms of the original problem, specified by the hamiltonian H, there are two stationary paths that can contribute to the wave function significantly. They correspond to the two classical trajectories which begin at $a = 0$ and end at a. The first trajectory terminates at a while $\dot{a} > 0$, as in figure 9. Its Euclidean action is $S_1(a) \approx -\frac{1}{4}a^2$ for $a \ll h^{-1}$. In the same limit the second trajectory, shown in figure 10, has action

$$S_2(a) \approx -\frac{3}{8G^2\Lambda} + \frac{1}{4}a^2 \tag{3.15}$$

Thus, in the semiclassical approximation, the two wave functions are

$$\psi_1 \sim e^{-S_1} = \exp(\frac{1}{4}a^2); \qquad \psi_2 \sim e^{-S_2} = \exp(\frac{3}{8G^2\Lambda} - \frac{1}{4}a^2) \tag{3.16}$$

From the definition of the wave function in eq. (3.12) it is clear that the total path integral over all closed compact geometries is just $\psi(0)$. Coleman's saddle point estimate obviously comes from $\psi_2(0)$. However, as emphasized in ref. [14], rotating the contours of a and t integrations entirely eliminates ψ_2 and picks out ψ_1 as the Hartle-Hawking wave function! Can this prescription be consistent with Coleman's theory? Hartle and Hawking claim a different connection between $\psi(a)$ and the total Euclidean path integral. They argue that the Euclidean path integral is the norm of the Hartle-Hawking wave function

$$X = \int_0^\infty \psi_1^\star(a)\psi_1(a)da \tag{3.17}$$

Indeed, this quantity is of the order $\exp(\frac{3}{8G^2\Lambda})$. To see this, let us study the behaviour of the Hartle-Hawking wave function ψ_1. For small a, ψ_1 grows exponentially. It is easy to see that for $a > h^{-1}$ we enter the classically allowed region where ψ_1 oscillates with the amplitude $\sim \exp(3/16G^2\Lambda)$. Therefore, the norm in eq. (3.17) seemingly gives the answer required for Coleman's theory. Unfortunately, we see no basis for the claim that the Euclidean path integral in eq. (3.4) is the conventional norm of the Wheeler-De Witt wave function.[15] To show that the two are different we make use of eq. (3.14):

$$\begin{aligned} \int_0^\infty \psi^\star(a)\psi(a)da &= \int_0^\infty da < a = 0|\frac{1}{H}|a >< a|\frac{1}{H}|a = 0 > \\ &=< a = 0|\frac{1}{H^2}|a = 0 > \end{aligned} \tag{3.18}$$

Obviously, this is not the same as

$$X = \psi(0) =< a = 0|\frac{1}{H}|a = 0 > \tag{3.19}$$

Perhaps, one should simply conclude that Coleman is just wrong and the path integral does not exhibit the $\exp(\frac{3}{8G^2\Lambda})$ required for his argument. On the other hand, maybe the Hartle-Hawking prescription is not the only possible way to make sense of the Euclidean path integral. Below we propose a different regularization of the Euclidean path integral in the minisuperspace, which leads to consistency with Coleman's sum over wormhole-connected universes.

If the Wheeler-De Witt Hamiltonian was bounded from below, the wave function could be obtained in two steps. First, we must solve the imaginary time Schroedinger equation

$$H\psi(a,T) = -\frac{\partial\psi(a,T)}{\partial T} \tag{3.20}$$

with initial condition

$$\psi(a,0) = \delta(a) \tag{3.21}$$

Integrating $\psi(a,T)$ over T gives the wave function $\psi(a)$

$$\psi(a) = \int_0^\infty \psi(a,T)dT \tag{3.22}$$

These steps can be formally expressed by eq. (3.14). However, the above procedure for defining $\psi(a)$ is not meaningful due to the unconventional sign of the kinetic term in the hamiltonian of eqn. (3.20). We propose to define the wave function formally by real time (Minkowskian) path integration for the quantum mechanics problem associated with the hamiltonian $-H$:

$$\psi(a) = -i < a|\frac{1}{-iH}|a=0> = -i < a|\int_0^\infty dT e^{iHT}|a=0> \tag{3.23}$$

The potential energy contained in $-H$ is plotted in figure 11.* The wave function defined by eq. (3.23) satisfies[13]

$$H\psi(a) = \delta(a), \tag{3.24}$$

i.e. it satisfies the Wheeler-De Witt equation everywhere but the origin. In fact,

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⋆ Note that the mass of the 'particle' is also position dependent: $m = a$.

the precise meaning of the Wheeler-De Witt equation at $a = 0$ is obscure for several reasons. First of all, since no meaning is attached to $a < 0$, the derivatives in H are ambiguous. Secondly, the boundary conditions at $a = 0$ involve sub-Planckian physics. However, the path integral in eq. (3.23) does imply eq. (3.24). Calculation of this path integral reveals two interesting results. The first conclusion is that, as with the Hartle-Hawking prescription, $\psi(0)$ does not become large or small in the limit $\Lambda \to 0$. To see this we note that, as $\Lambda \to 0$, the hamiltonian becomes

$$-H = \frac{1}{2}(\frac{p^2}{a} + a) \tag{3.25}$$

which is once again the hamiltonian of a stable quantum mechanics problem.

The second conclusion is that, unlike the Hartle-Hawking wave function, $\psi(a)$ is complex and corresponds to an outgoing wave at $a \gg \frac{1}{h}$. This should not come as a surprise since the physical problem we have set up corresponds to the quantum mechanics of a particle starting at $a = 0$ at $t = 0$. Therefore, our wave function describes a particle tunneling from under the barrier, which extends from $a = 0$ to $a = 1/h$, to large values of a.[16] One might object to our regularization, which is essentially substituting a Minkowskian path integral for a Euclidean in the minisuperspace context. Then, a 'manifestly real' Euclidean path integral gives rise to a complex wave function. Actually, this is a standard phenomenon in problems which involve an instability, such as the quantum mechanics of minisuperspace.

With our alternate regularization we once again find no evidence for $\exp(\frac{3}{8G^2\Lambda})$ in the normalization of the wave function. This is despite the fact that both ψ_1 and ψ_2 are present in the wave function:

$$\psi(a) \approx -\exp(-3/8G^2\Lambda)(\psi_2(a) + i\psi_1(a)) \tag{3.26}$$

The naive saddle point approximation to $\psi(a)$ would have given

$$\psi(a) \sim \psi_2 + i\psi_1 \tag{3.27}$$

What is the origin of the normalization factor in eq. (3.27) and how does it affect Coleman's arguments? The answer is given in the next section.

4. Wormholes in Minisuperspace and Beyond

In this section we will show that a consistent treatment of wormholes in minisuperspace, analogous to Coleman's, explains the normalization factor of the wave function given by the path integral. As we have emphasized, the Euclidean path integral is divergent due to unboundedness of the action. A 4-sphere of radius $r = \sqrt{3/8\pi G\Lambda}$ is a stationary point where the action has the value $-\frac{3}{8G^2\Lambda}$. We can, however, easily generate configurations with much larger negative action. As described in sec. 3, N almost complete 4-spheres glued together in a sequence have the action $\approx -\frac{3N}{8G^2\Lambda}$. Each of these configurations is also a stationary point of the action, and there is no consistent reason to ignore them. But do we even want to discard contributions such as those in figure 12? Obviously not, since they are just the wormhole-connected universes of Coleman, or what is left of them in minisuperspace. What we want to do is to apply Coleman's reasoning and sum them up. In the saddle point approximation the Euclidean path integral becomes

$$\psi(0+) = i\sum_{N=0}^{\infty} \exp(\frac{3N}{8G^2\Lambda}) + \sum_{N=1}^{\infty} \exp(\frac{3N}{8G^2\Lambda}) \tag{4.1}$$

The first term sums up the processes in which any number of bounces precede a termination of a trajectory at a point where $\dot{a} > 0$ and $a = 0+$. The second series involves trajectories terminating with $\dot{a} < 0$. The reason for the factor of i in the first term is the presence of an extra negative mode in the fluctuations about the trajectories which end with $\dot{a} > 0$. Although divergent, the series in eq. (4.1) can be formally summed up to

$$\psi(0+) \approx \frac{\exp(\frac{3}{8G^2\Lambda}) + i}{1 - \exp(\frac{3}{8G^2\Lambda})}. \tag{4.2}$$

This manipulation can be made more convincing by introducing an R^2 stabilizer term into the action. Then, for Λ greater than some critical value, the geometric series which sums up the multiple bounces converges. Analytic continuation to small Λ essentially reproduces eq. (4.2).

Note that (4.2) approaches -1 as $\Lambda \to 0$. Thus, in this limit, only if we sum over the wormholes do we get the correct saddle point approximation to the wave function. This is not to say that tunneling amplitudes cannot be understood without multiple bounces. In particular, if the usual Euclidean path integral is applied to a hamiltonian with the conventional sign of the kinetic term, such as $-H$, where H is given in eq. (3.3), the multiple bounce has the amplitude $\sim \exp(-\frac{3N}{8G^2\Lambda})$. Thus, the successive bounces are strongly suppressed. The reader can verify that the path integral carried out this way leads to the same wave function. Our point is that, if we insist on using the standard sign for the gravitational kinetic energy and the usual saddle point definition of the Euclidean path integral, the sum over the geometric series generated by the multiple bounces is necessary. This supports the view that the wormholes provide important contributions to the Euclidean path integral. We also see that, if Λ is small, no serious error is made by ignoring wormhole-connected universes since their only effect is to change normalization of the wave function. In Coleman's case, the analogue of summing up the geometric series is just the calculation of sec. 2. The effects of the wormhole summation and the exponentially large contribution from each 4-sphere result in a finite prescription: quantum gravity with zero cosmological constant.

As far as we can tell, similar arguments do not apply to the Hartle-Hawking prescription, where it appears that trajectories with multiple bounces do not dominate the Euclidean path integral. The sum over multiple bounces in figure 12 a) results in normalization $\sim \exp(-\frac{3}{8G^2\Lambda})$ for $\psi_1(0)$. Since the Hartle-Hawking method of evaluating the Euclidean path integral gives $\psi_1(0)$ of order 1, it appears that it is not consistent with the idea that the wormhole-connected universes dominate the Euclidean path integral.

Another interesting question is what is the role of higher 4-topologies in Coleman's argument. Let us consider the Euclidean path integral over all geometries with a spherical topology. An important subset of these is formed by the conformally flat geometries discussed in the previous section. Among these geometries there are approximate saddle points, which are generalizations of the wormhole-connected series of 4-spheres encountered in the minisuperspace. These are the tree diagrams of the 'universal field theory'. Typical examples are shown in figure 6. In ordinary field theory the sum over such diagrams is found by a saddle point approximation to the Euclidean path integral. Let us therefore construct a simple path integral which sums up the effects of wormhole-connected universes. Define a 'field' $B(V)$ which describes large spherical universes of volume V and a field α for wormholes. The emission of wormholes by a universe will be represented by a graph like that shown in figure 13. The path integral which reproduces the wormhole sum is*

$$\int d\alpha dB(V) \exp\Big(-\frac{1}{2}D\alpha^2 + \int_0^\infty \frac{dV}{V}(B(V)e^{\frac{1}{2}(\frac{1}{G}\sqrt{\frac{3V}{2}}-\Lambda V+\alpha V)} - \frac{1}{2}B^2(V))\Big) \quad (4.3)$$

The reader can check that the expansion in powers of $1/D$ generates the wormhole summation including loops. Integrating out $B(V)$ with some particular integration measure† reduces eq. (4.3) to

$$\int d\alpha \exp(-\frac{1}{2}D\alpha^2 + \frac{1}{G}\int_0^\infty \frac{dV}{\sqrt{V}} e^{\frac{1}{G}\sqrt{\frac{3V}{2}}-\Lambda V+\alpha V}) \quad (4.4)$$

For $\alpha < \Lambda$ the volume integration in the exponent converges to

$$\sqrt{\frac{\pi}{G^2(\Lambda-\alpha)}} \exp(\frac{3}{8G^2(\Lambda-\alpha)}) \quad (4.5)$$

Ordinarily, the sum of connected tree graphs would be given by the extremum of

$$-\frac{D}{2}\alpha^2 + \sqrt{\frac{\pi}{G^2(\Lambda-\alpha)}} \exp(\frac{3}{8G^2(\Lambda-\alpha)}) \quad (4.6)$$

In our case this sum diverges for any D due to the singularity of (4.6) at $\Lambda = \alpha$.[10] We observe, however, that, for a sufficiently large Λ, there exists a local maximum

$\star$ A similar path integral appears in the work of Giddings and Strominger (ref.[10]).

† Our conclusions do not depend sensitively on the choice of measure.

of (4.6) with

$$\alpha \sim \frac{1}{GD\Lambda^{3/2}}, \tag{4.7}$$

We can regard the value of (4.6) at this extremum as the definiton of the sum of connected tree graphs. With this definition, the cosmological constant is only weakly shifted by wormholes. As Λ decreases, the local maximum in eq. (4.7) disappears. This happens well before the wormhole-shifted cosmological constant vanishes. Thus, the sum of tree diagrams ceases to make sense below some value of the physical cosmological constant. It should be noted that all the tree diagrams are spheres topologically, while metrically they are very far from spherical. This raises an interesting possibility. It suggests that the Euclidean path integral over geometries of spherical topology is unstable with respect to break up into infinitely many wormhole-connected 4-spheres. For Λ less than some critical value, this integral is uncontrollably divergent.

On the other hand, the result of integration over α, which sums up the loop diagrams in eq. (4.4), is well-defined in the sense that all the physical amplitudes are dominated by the singularity at $\Lambda - \alpha = 0$. In the region where $\Lambda - \alpha < 0$ the volume integral in eq. (4.4) is problematic. However, we can formally continue from the region where $\Lambda - \alpha > 0$ to obtain the same answer as in eq. (4.5). This procedure seems to indicate that there is no preference for a negative physical cosmological constant.

Let us consider what the Hartle-Hawking prescription says about the Euclidean path integral over geometries of spherical topology. An important class of these geometries, which includes the wormhole-connected spheres, is the conformally flat geometries $g_{ij} = \phi^2 \delta_{ij}$ considered in the previous section. The theory specified by eq. (3.10) has an instanton

$$\phi(r) = \left(1 + \frac{2\pi G\Lambda r^2}{3}\right)^{-1} \tag{4.8}$$

This is simply the 4-sphere of radius $\sqrt{3/8\pi G\Lambda}$ which constitutes the Euclidean de Sitter space. Multi-instanton configurations are the wormhole-connected 4-spheres.

Coleman's treatment assumes that the Euclidean path integral is saturated by the instanton sum. As we have seen above, this sum diverges. Even if defined by the extremum of the action in eq. (4.6), it becomes singular at some critical value of Λ, below which it does not make sense. On the other hand, the Hartle-Hawking prescription leads to the Euclidean path integral in eq. (3.11) which defines a stable ϕ^4 theory and is presumably well-behaved as $\Lambda \to 0$. This once again suggests that this prescription eliminates the instability which leads to Coleman's wonderful effect. This is not meant to imply that the Hartle-Hawking prescription is incorrect. We believe, however, that it cannot coexist with Coleman's results. Some other prescription must be found to justify them.

5. Fixing Other Coupling Constants

Once the cosmological constant has been set to zero, it is natural to ask if Coleman's theory predicts some or all of the additional parameters, such as the mass and coupling constants. Coleman's answer is yes. Let us review his argument. We imagine carrying out the path integral over all fields in a background Euclidean de Sitter space of radius r. Explicit examples indicate that the result has the form $\exp(-S_{eff}(r))$ with $S_{eff}(r)$ given by the eq. (2.16). As in sec. 2, we eliminate the radius by solving

$$\frac{\partial S_{eff}}{\partial r} = 0 \tag{5.1}$$

This gives $r^2 = 3/8\pi G\Lambda + O(G^3\Lambda)$. S_{eff} can then be written in terms of G, Λ, and a set of parameters A_i which depend on the wormhole-shifted fundamental constants.

$$S_{eff} = -\frac{3}{8G^2\Lambda} + \frac{8\pi^2}{3}A_1 + O(G^2\Lambda)A_2 + \dots \tag{5.2}$$

The probability for the wormhole-shifted couplings and masses is proportional to exp(exp($-S_{eff}$)), which for small $G^2\Lambda$ reduces to

$$\rho(\alpha) \sim \exp\big(\exp(\frac{3}{8G^2\Lambda} - \frac{8\pi^2}{3}A_1)\big) \tag{5.3}$$

The absolute maximum of this function will generally occur at $G^2\Lambda(\alpha) = 0$. This defines some subspace in the space of wormhole-shifted constants. On this surface the probability varies, being infinitely sharply peaked at the place where $A_1(\alpha)$ achieves its minimum. If this occurs at a point in the space α_i, then all the parameters are determined. If it occurs on some higher-dimensional surface, then only some relations between couplings are fixed. In this case the process can be continued by minimizing A_2 and so on. Whether the process determines all the interesting couplings is not known.*

It is important to know whether symmetries restrict the allowable couplings which can be generated by wormhole effects. For example, suppose that the fundamental lagrangian is invariant under some global symmetry, such as the chiral symmetry or baryon number conservation. Wormholes can then break this symmetry. The mechanism involves a wormhole through which the conserved current flows. For example, a unit of baryon number can pass through a wormhole. This induces a bilocal operator

$$\int dx dx' O^\dagger(x) O(x') \tag{5.4}$$

where x and x' are the ends of the wormhole and O is a baryon number violating operator. The arguments in sec. 2 indicate that the phenomenology of a single large universe will require the operator $\alpha_o(O + O^\dagger)$ in the lagrangian. Of course, it may happen that the probability is maximized at $\alpha_o = 0$. However, this is not a priori implied by the symmetry of the theory without the wormholes.

$\star$ This procedure requires that we introduce a lower cut-off on $G^2\Lambda$, and take it to zero at the end of the calculation. Grinstein and Wise[17] have suggested that the proper quantity to be cut off is $G_0^2\Lambda$, where G_0 is the 'bare' Newton's constant, which does not depend on the α's. With this regulator the probability is maximized at $G/G_0 = 0$. Unfortunately, this seems to imply that wormhole effects make gravity a free theory. However, the full consequences of this approach have not yet been worked out.

The obvious question is to what extent the A_i can be computed from a knowledge of low-energy physics alone. Power counting indicates that A_1 depends on the short-distance physics. Since it is dimensionless, it will generally be logarithmically divergent in the ultraviolet when expressed in terms of integrals over wave numbers. Therefore, it is sensitive to physics at arbitrarily short distances. Nevertheless, it appears that, when applied to masses of spin-0 and spin-$\frac{1}{2}$ particles, Coleman's procedure leads to some discouraging conclusions.

Consider a light pseudoscalar particle, such as the pion. We wish to study the Euclidean path integral on a 4-sphere of radius r as a function of the pion mass. Let us assume that m_π is much smaller than the QCD scale f_π. The low energy interactions of pions are well described by the $SU(2) \times SU(2)$ non-linear sigma model with the cut-off set around f_π:

$$\mathcal{L} = \frac{f_\pi^2}{4} tr(\nabla_i U \nabla^i U^\dagger) + tr(MU + U^\dagger M^\dagger), \tag{5.5}$$

where the $SU(2)$ variable U is related to $\vec{\pi}$ by

$$U = \exp(\frac{i\vec{\sigma} \cdot \vec{\pi}}{f_\pi}) \tag{5.6}$$

The portion of the path integral with momenta $\leq f_\pi$ will be dealt with using this model. The rest of the Euclidean path integral should make use of a more fundamental theory involving quarks, gluons, Z's, W's, etc.

Now consider the limit of large f_π in which the pion becomes a free minimally coupled point particle with lagrangian

$$\mathcal{L} = \frac{1}{2}((\nabla\vec{\pi})^2 + m_\pi^2 \vec{\pi}^2) \tag{5.7}$$

In this case the path integral reduces to evaluation of the determinant of $\nabla^2 + m_\pi^2$. Methods and results of such a calculation in arbitrary curved backgrounds can be found in ref. [18]. Recently, the calculation was performed directly on a 4-sphere by Grinstein and Wise[17] with the purpose of application to Coleman's theory. They isolated the coefficient A_1 in eq. (5.3)

$$A_1 = \frac{1}{32\pi^2}(2 - \frac{1}{15}) \log \frac{m_\pi^2}{f_\pi^2} + O(\frac{m_\pi^2}{f_\pi^2}) \tag{5.8}$$

The logarithmic dependence on m_π originates in the infrared modes and is totally independent of the ultraviolet cut-off. Thus, corrections to eq. (5.8), coming from the interaction terms in eq. (5.5), depend on the strength of these perturbations among the infrared modes. We have checked by an explicit calculation that, to lowest order, the interactions do not generate additional infrared logarithms. The reason is that Goldstone bosons decouple at low momenta. Although we have no general proof of this, we expect the coefficient of $\log m_\pi$ in (5.8) to be universal. Thus, it appears that the probability for the pion mass is infinitely peaked at $m_\pi = 0$. Unless a way around this conclusion is found, the theory of wormholes is in trouble.

It is also interesting to do a similar calculation for a free massive fermion. Using the knowledge of eigenvalues and degeneracies of the Dirac operator on a 4-sphere,[19] we find

$$\log det(i\gamma^\mu D_\mu + m) \sim \sum_{l=0}^{\infty} \frac{(l+3)!}{l!} \exp(-\frac{l+2}{rM}) \log(\frac{(l+2)^2}{r^2 M^2} + \frac{m^2}{M^2}) \tag{5.9}$$

where M is the cut-off mass. We would like to calculate the sign of the coefficient of $\log(m/M)$ in A_1. Since this term originates in the infrared modes, the coefficient is cut-off independent. In contrast with the result for a free scalar, we find that the sign is negative. Therefore, wormholes drive the free fermion mass toward the cut-off scale.

Let us apply this to neutrino physics. Since wormholes break chiral symmetry, ν acquires Majorana mass through operators like

$$\phi^\dagger \psi_L \gamma_2 \phi^\dagger \psi_L + h.c. \tag{5.10}$$

where ϕ is the Higgs field. At low momenta, neutrinos are almost free and the above calculation should be applicable. Our calculation indicates that wormholes drive the neutrino mass away from zero.

A possible way out of this predicament involves assuming that the dilute wormhole approximation is bad. If the shift of the cosmological constant produced by wormholes is of order M_p^4, then wormholes must develop a significant density. As we have mentioned, under these circumstances the dynamics of α's becomes non-linear. In fact, the α's may be poor global coordinates for the space of wormhole fields. This space may be better described by some coordinates $\theta(\alpha)$. The manifold may even be compact. In general, the parameters α_i which multiply the specific operators are highly non-linear functions of θ_i. Now consider the surface $\Lambda(\theta) = 0$. This will be some curved surface in θ-space, as shown in figure 14. Similar remarks apply to the surface $m_\pi(\theta) = 0$. Since we know very little about the θ-space, we see no general reason for the two surfaces to intersect. This might happen if the required shift of the cosmological constant is fairly large in Planck units and the wormholes required for shifting Λ become dense. Furthermore, the wormholes required for shifting m_π carry different quantum numbers from those that shift Λ, since the former must carry off chiral charge while the latter should be chiral singlets. A high density of neutral wormholes could leave very little space for the ones that shift m_π. On the other hand, the probability distribution near $m_\pi = 0$ is

$$\rho \sim \exp\{(\frac{f_\pi}{m_\pi})^{0.32} \exp(\frac{3}{8G^2\Lambda})\} \tag{5.11}$$

Evidently, as $\Lambda \to 0$, the driving force on the chiral wormholes diverges and it is not clear that they can resist increasing their density.

V. Kaplunovsky suggested an even more disturbing possibility that larger scale wormholes may be forced to occur if the Planck scale wormholes are not sufficient to shift m_π to zero. Once we have integrated out the effects of wormholes and other fluctuations above a given scale, there is no reason why a new round of wormholes cannot be important at larger scale. One might object that the factor $\exp(-S_w)$ in the amplitude for each wormhole decreases rapidly with the wormhole size. This causes the coefficient D in eq. (2.8) to be of order $\exp(S_w)$. Unfortunately, the factor in eq. (5.3) is so strong as $\Lambda \to 0$ that it easily overwhelms any finite value of D, no matter how large. This suggests that, if m_π is not driven to zero by microscopically small wormholes, then wormholes should occur with maximum possible density at every scale up to f_π. This seems unphysical since it would almost certainly adversely affect predictions of the standard model. At the moment, we have no answer to this puzzle.

6. Wormholes and Cosmology

An important question about Coleman's theory is whether it is consistent with a reasonable cosmology. It will be disappointing if the theory truly predicts nothing rather than something: namely, a cold universe devoid of matter and energy. We must hope that there is at least a finite number of universes which have undergone an interesting cosmological development. Obviously, the relevant issue is the absolute number of such universes and not the fractional number. We will argue that a possible outcome of the wormhole theory is that the number of warm universes is finite while the number of cold ones diverges. As a result, the expectation values of all observables will be dominated by cold empty universes. Under these circumstances the quantities of physical interest are conditional probabilities given that one is in a warm universe. Let us begin with extending the minisuperspace model by including a scalar variable $\phi(t)$ which represents the state of all matter fields in the universe. The Euclidean action becomes

$$I = \int_0^T dt \Big(-\frac{1}{2} a \dot{a}^2 - \frac{1}{2} a + a^3 \big(\frac{1}{2} \dot{\phi}^2 + V(\phi) \big) \Big) \tag{6.1}$$

For our purposes we will restrict the shape of $V(\phi)$ to be the one in figure 15. A warm universe corresponds to a Euclidean trajectory which emerges in the classically allowed region of a with ϕ in the vicinity of ϕ_b. Subsequently, it can tunnel to the lower well around ϕ_a dumping the energy difference into heat. We are making no attempt here to model a realistic cosmology. We prefer $V(\phi)$ because this potential makes it easy to separate the important saddle point trajectories. The Euclidean equations of motion are

$$\begin{aligned} \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} &= \frac{dV}{d\phi} \\ (\frac{\dot{a}}{a})^2 &= \frac{1}{a^2} + \dot{\phi}^2 - 2V(\phi) \end{aligned} \tag{6.2}$$

The two important saddle points correspond to classical trajectories which start at $a = 0$ with $\dot{\phi} = 0$ and $\phi = \phi_a$ or ϕ_b. We observe that, if a trajectory starts at the bottom of the well and has $\dot{\phi} = 0$ initially, it never acquires any non-zero $\dot{\phi}$. Therefore, the solutions are

$$\phi = \phi_a; \qquad a(t) = \frac{1}{h_a}\sin(h_a t) \tag{6.3}$$

and

$$\phi = \phi_b; \qquad a(t) = \frac{1}{h_b}\sin(h_b t) \tag{6.4}$$

where $h_a^2 = 2V(\phi_a)$ and similarly for b. Ignoring wormholes, these two trajectories approximately saturate the Euclidean path integral for the Wheeler-De Witt wave function in the classically forbidden region $a^2 < 1/2V(\phi)$ (neglecting $\dot{\phi}$). To extrapolate to large values of a, the Euclidean trajectories must be matched on to the solutions of the Minkowski equations of motion for $a^2 > 1/2V(\phi)$. Eventually, within the classically allowed region tunneling to the lower well, accompanied by heat generation, takes place. We assume that the tunneling rate between the two wells is much slower than the rate for a to tunnel from $a = 0$ into the classically allowed region.⋆ Under these circumstances it is clear that a warm universe can be recognized in the Euclidean path integral as the extremum in eq. (6.4), and a cold universe – as the extremum in eq. (6.3).

⋆ This is true if the wells are separated by a high barrier.

Let us now consider the effect of wormholes in the Euclidean path integral. In analogy with the discussion of sec. 4, a wormhole is just a reflection off $a = 0$. There are three types of wormholes. The two diagonal processes take place entirely at ϕ_a or ϕ_b. In addition, transitions between ϕ_a and ϕ_b are possible. The reason appreciable transitions exist, even if the barrier is high, is that the ϕ-motion at $a = 0$ costs no action. If the wormholes are taken into account, the Euclidean path integral is saturated by series of large and small bubbles which correspond to cold and warm universes. When we depart from minisuperspace, the path integral will contain arbitrarily connected bubbles of type a (cold) and of type b (warm), as in figure 16. The probability in eq. (5.3) generalizes to a function of $\Lambda_a = 9V(\phi_a)/8G^2$ and $\Lambda_b = 9V(\phi_b)/8G^2$, as well as other constants. Typically, it will have the form

$$\rho \sim \exp\big(F_a \exp(\frac{3}{8G^2\Lambda_a}) + F_b \exp(\frac{3}{8G^2\Lambda_b})\big) \tag{6.5}$$

where F_a and F_b are the prefactors which depend on the wormhole-shifted fundamental constants of the theory.

An important question is whether both Λ_a and Λ_b are shifted to zero by wormholes. If this is the case, we will be forced to conclude that both a and b become cold and uninteresting universes. However, it is quite possible that the prefactors in eq. (6.5) are such that this does not occur. Recall that both F and Λ depend on the fundamental constants $\lambda + \alpha$. Therefore, the F's have an implicit dependence on the Λ's. Let us imagine a simplified model where the prefactors are functions of just Λ_a and Λ_b:

$$F_a = F_a(\Lambda_a, \Lambda_b); \qquad F_b = F_b(\Lambda_a, \Lambda_b). \tag{6.6}$$

Suppose that the prefactors satisfy

$$F_a(0,0) + F_b(0,0) < F_a(0,x), \tag{6.7}$$

where $F_a(0,x)$ is the maximum of $F_a(0,\Lambda_b)$. Then the probability is infinitely larger at $\Lambda_a = 0$ and $\Lambda_b = x$ than at $\Lambda_a = 0$ and $\Lambda_b = 0$. The property (6.7) holds for a broad variety of smooth functions F. Therefore, we do not consider this to be a 'fine-tuned' possibility.

It is interesting to determine the mean numbers of universes of type a and b in a connected diagram. The result is

$$N_a \sim \exp(\frac{3}{8G^2\Lambda_a}), \qquad N_b \sim \exp(\frac{3}{8G^2\Lambda_b}) \tag{6.8}$$

Thus, if (6.7) holds, the number of cold universes is driven to infinity, while the warm ones stay finite in number. However, the cosmological constant in these warm universes is driven to zero by contact with the infinity of cold universes. This follows from the fact that $\Lambda_a = 0$ and that eventually the universe tunnels to the well at ϕ_a.

The details of this scenario may vary depending on the specific mechanism for inflation. However, the idea that the cosmological constant in our warm universe is driven to zero by contact with an infinity of cold universes can be quite general. It suggests that the reason why the cosmological constant is zero lies outside our own universe.

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FIGURE CAPTIONS

1) A wormhole is a microscopic configuration connecting two asymptotically flat regions of space-time.

2) The two asymptotically flat regions can actually be connected even in the absence of a wormhole.

3) A wormhole between two otherwise disconnected universes.

4) A large universe with one wormhole.

5) A large universe with multiple wormholes.

6) Geometries of spherical topology which consist of many wormhole-connected spheres.

7) A more detailed picture of a wormhole of size a.

8) Two 4-spheres connected by a wormhole.

9) The minisuperspace trajectory which corresponds to the smaller part of the Euclidean sphere.

10) The minisuperspace trajectory which corresponds to the bigger part of the Euclidean sphere.

11) The potential energy in the hamiltonian $-H$. Note that the mass of the 'particle' is also position dependent: $m = a$.

12) Minisuperspace trajectories which involve several reflections off the barrier at $a = 0$.

13) A representation of emission of baby universes in terms of a Feynman graph.

14) Schematic drawing of $\Lambda = 0$ and $m_\pi = 0$ surfaces in the space of wormhole variables $\theta(\alpha)$.

15) The convenient potential for the scalar variable ϕ.

16) Important contributions to the Euclidean path integral in a theory with two types of large universes.

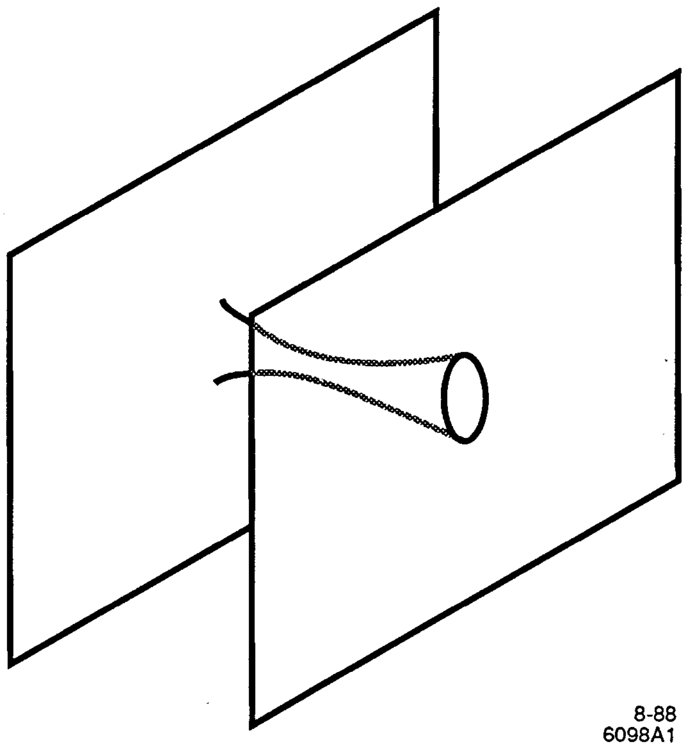

Fig. 1

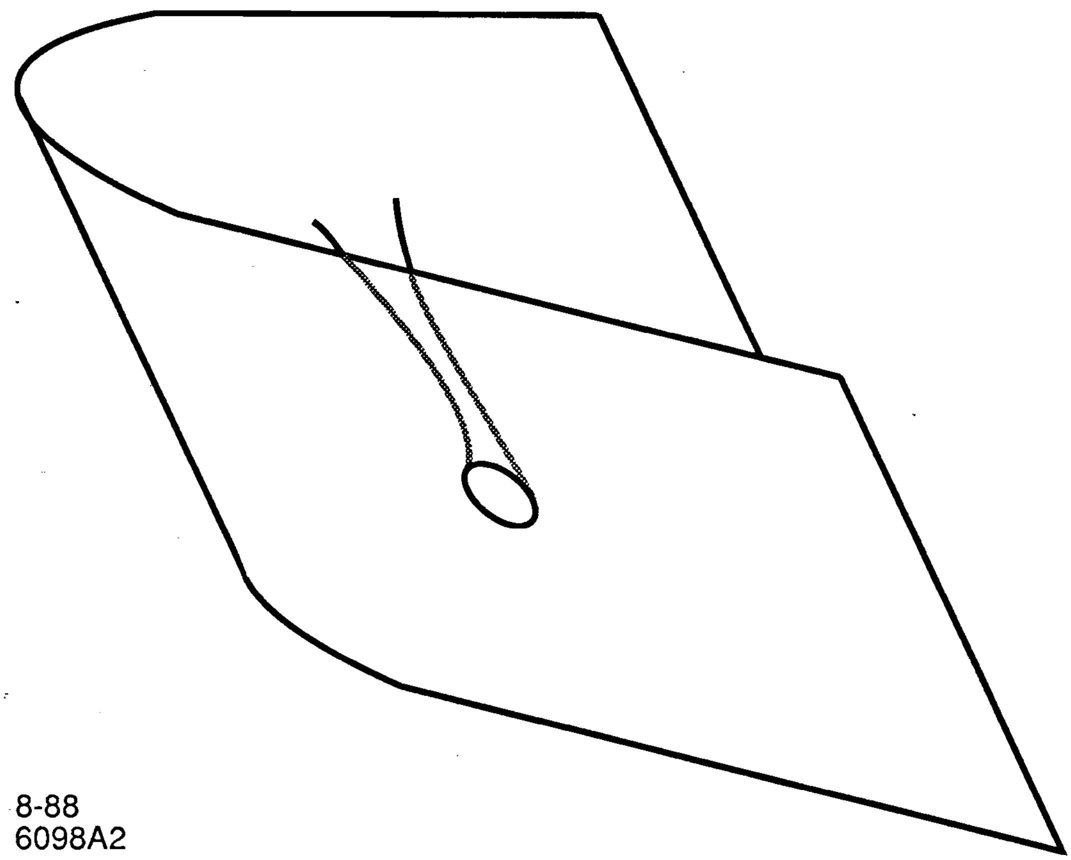

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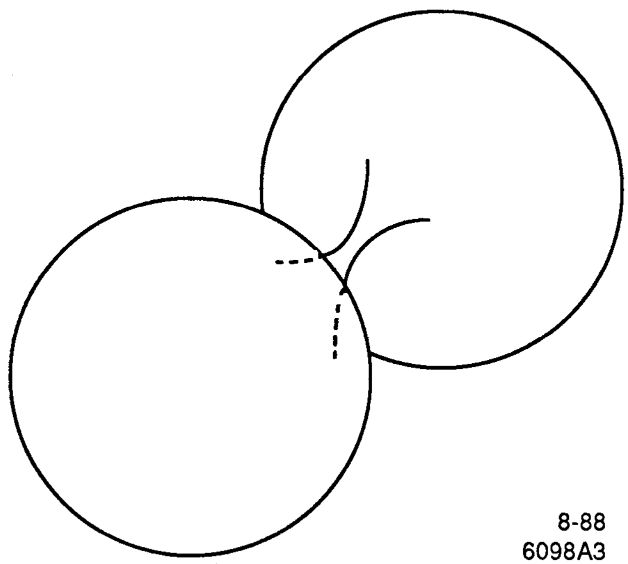
Fig. 2



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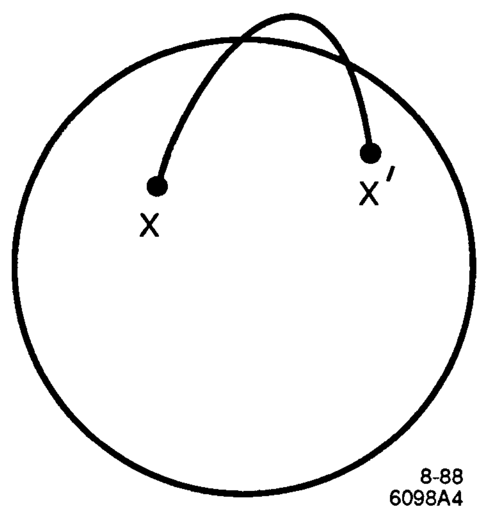

Fig. 4

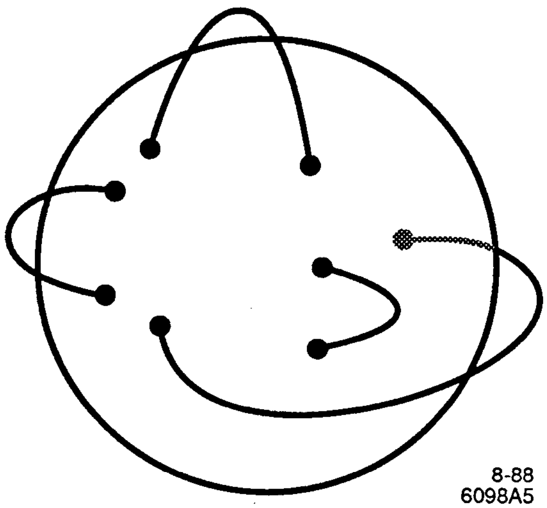

Fig. 5

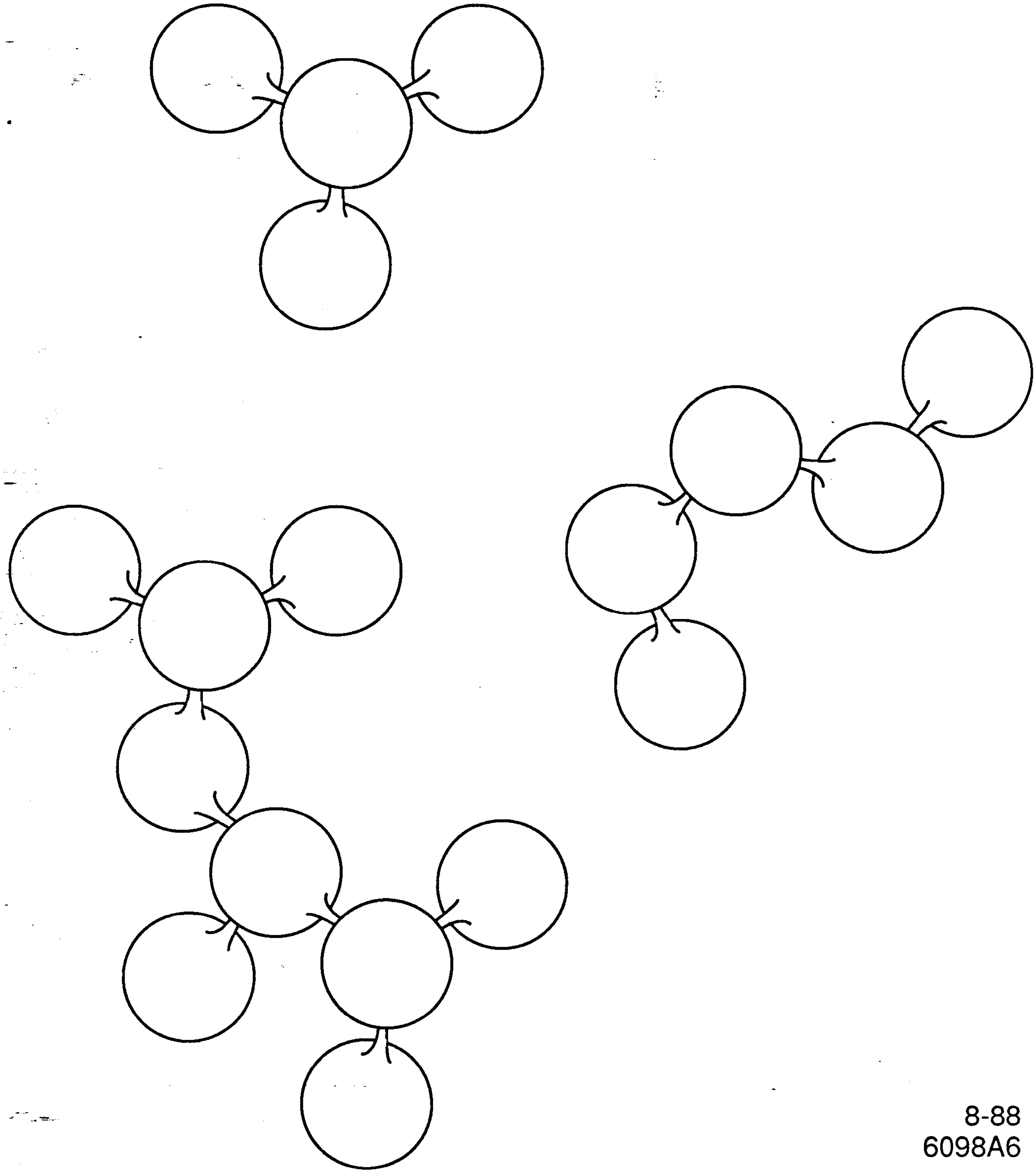

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Fig. 6

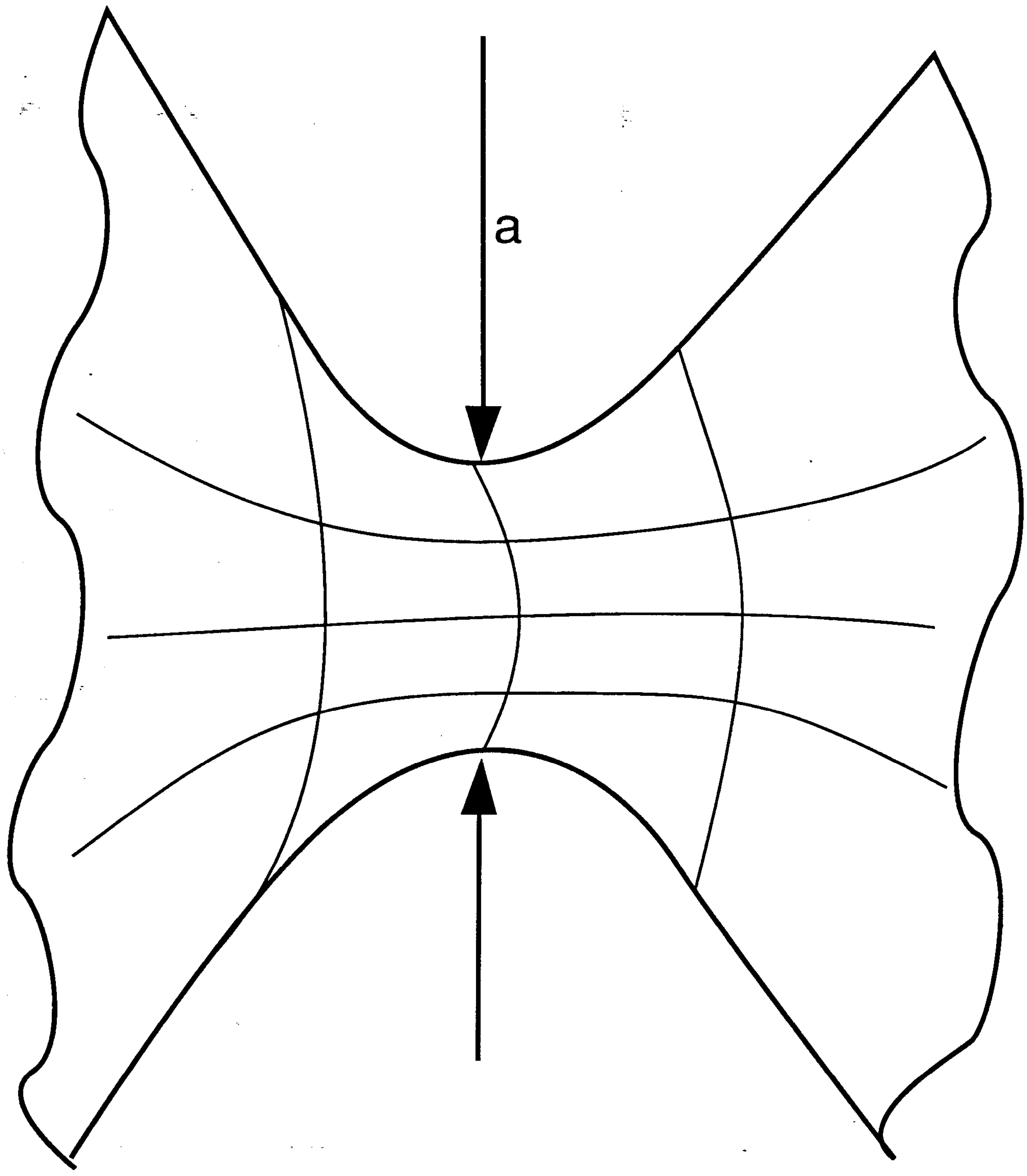

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Fig. 7

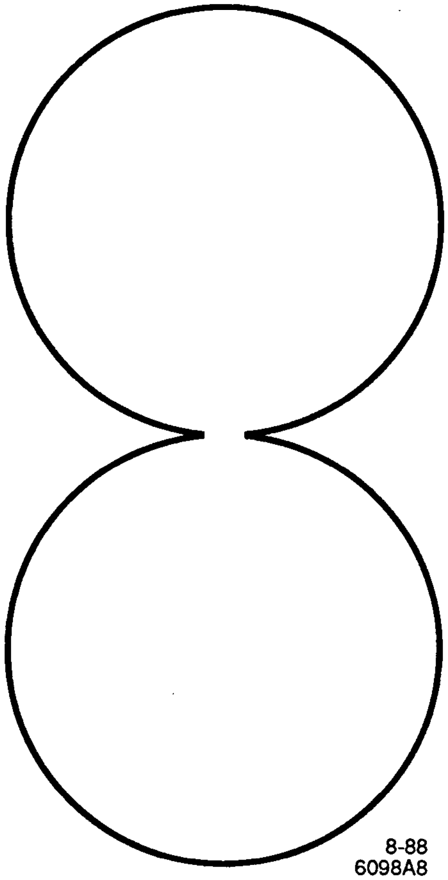

Fig. 8

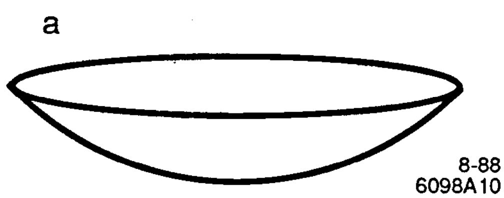

Fig. 9

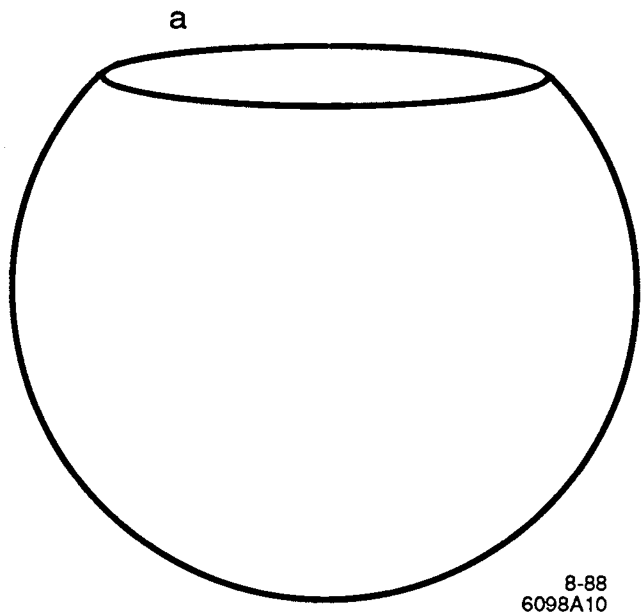

Fig. 10

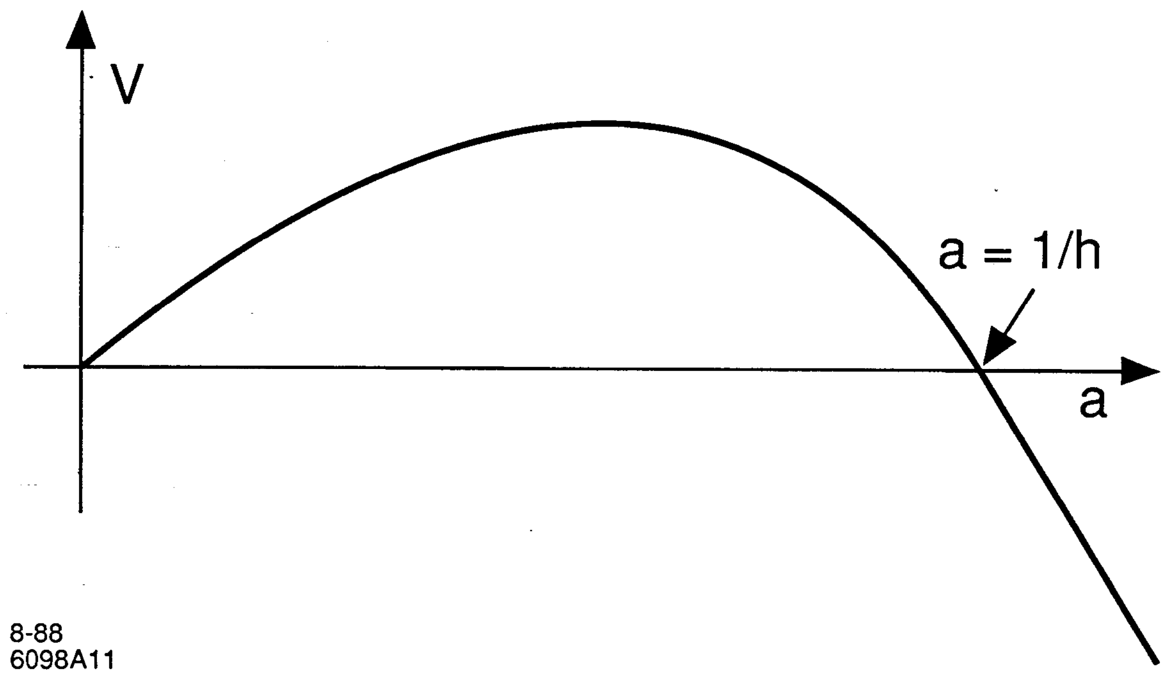

Fig. 11

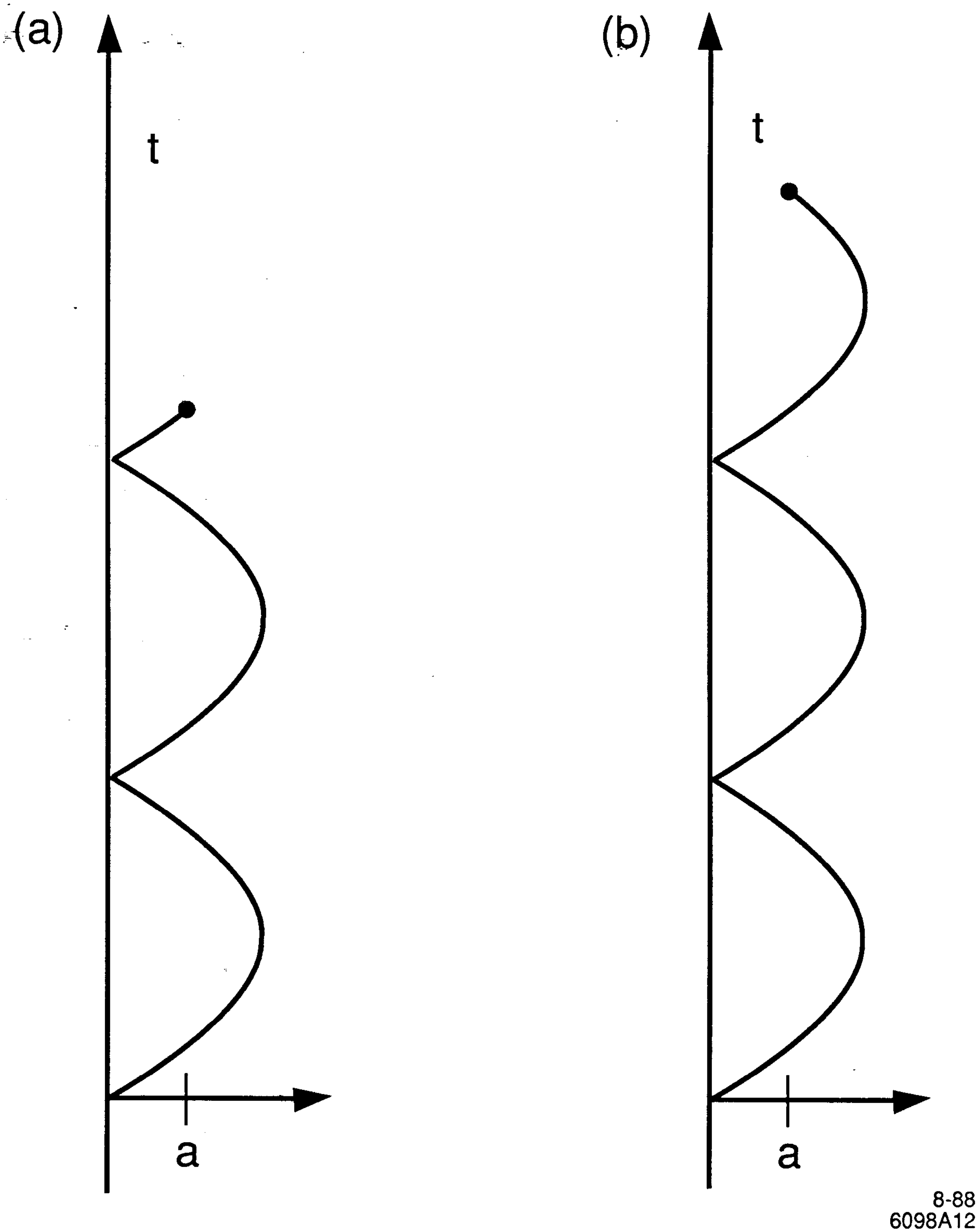

Fig. 12

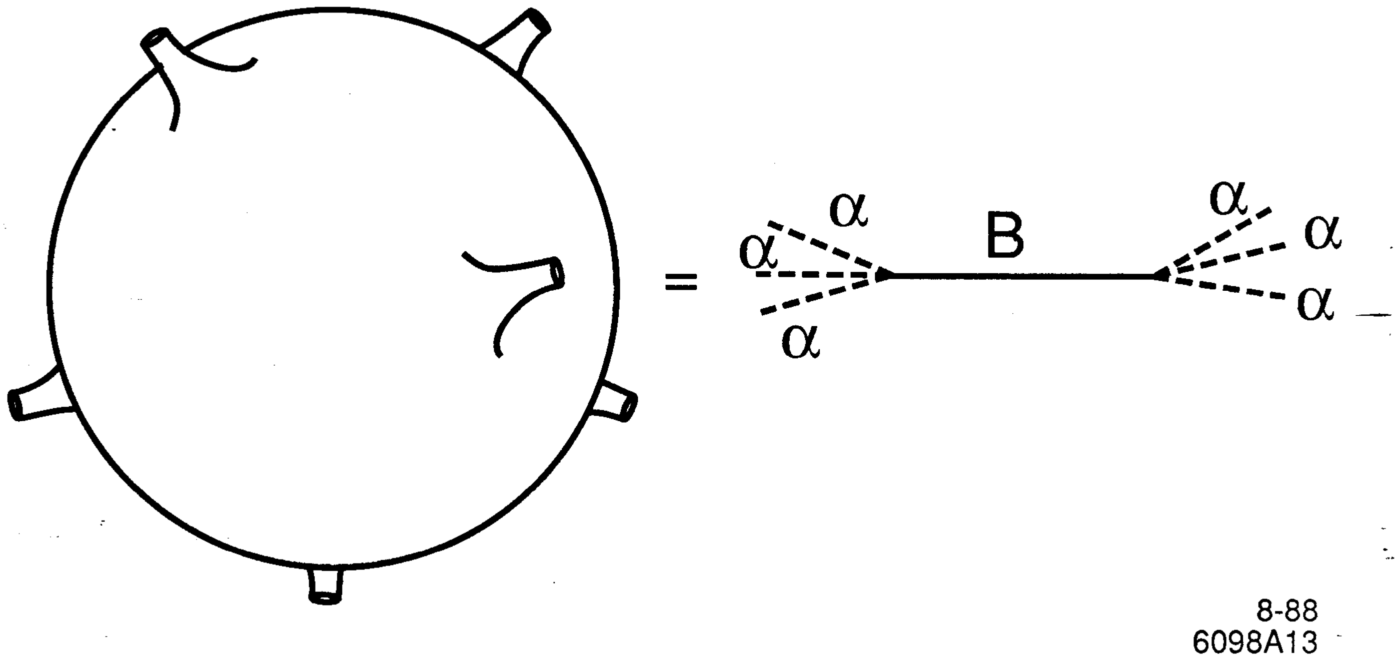

Fig. 13

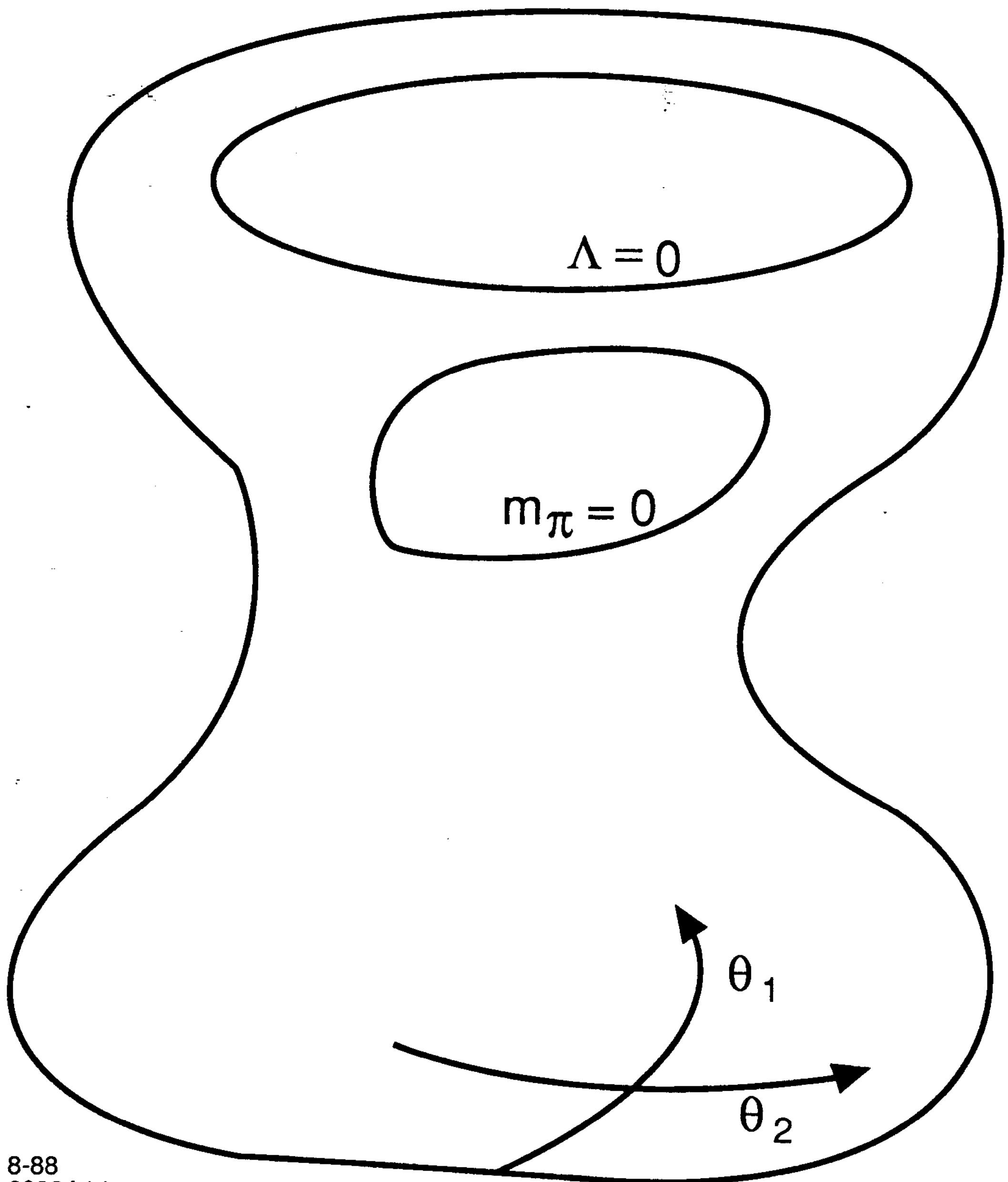

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Fig. 14

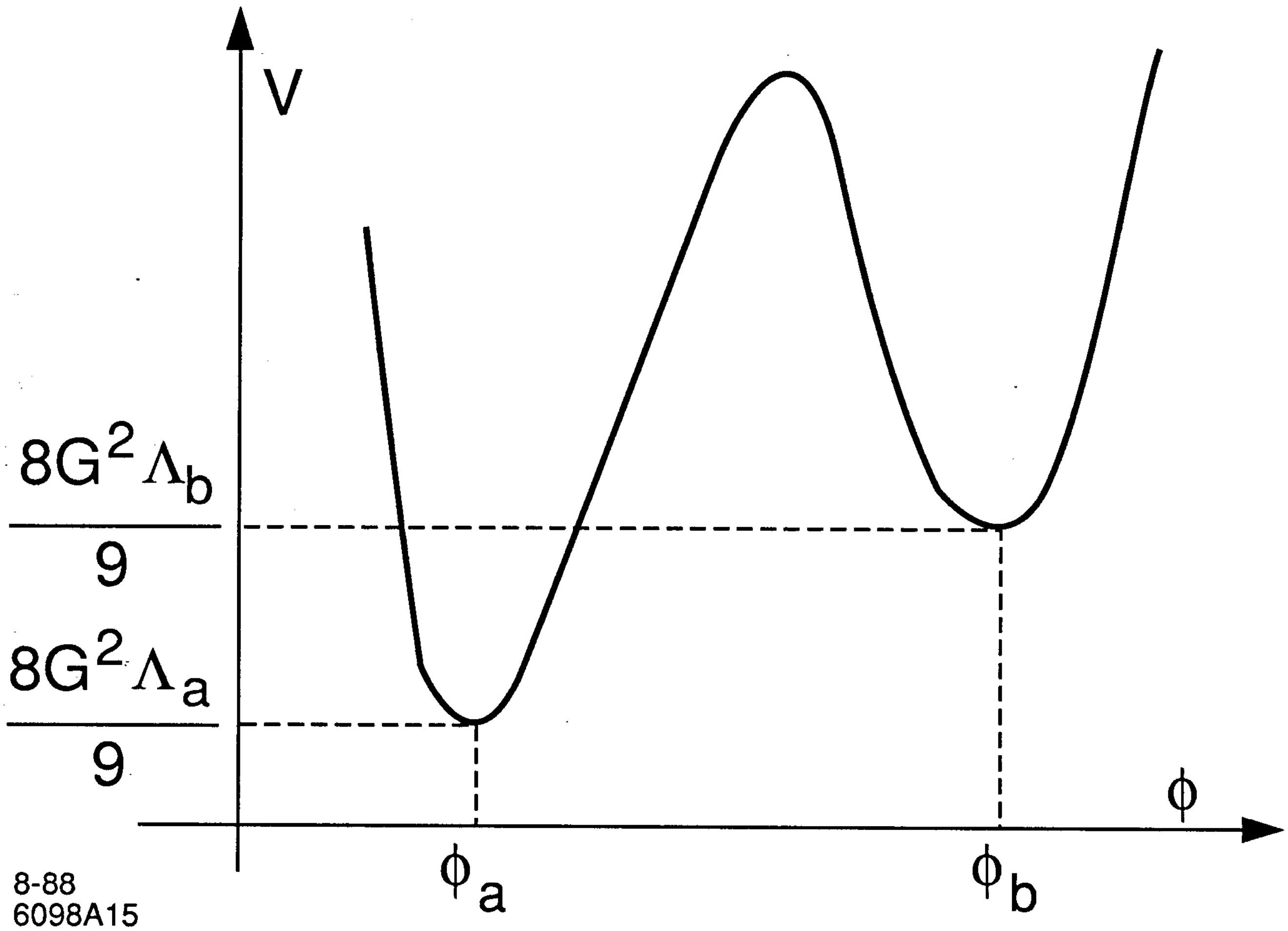

Fig. 15

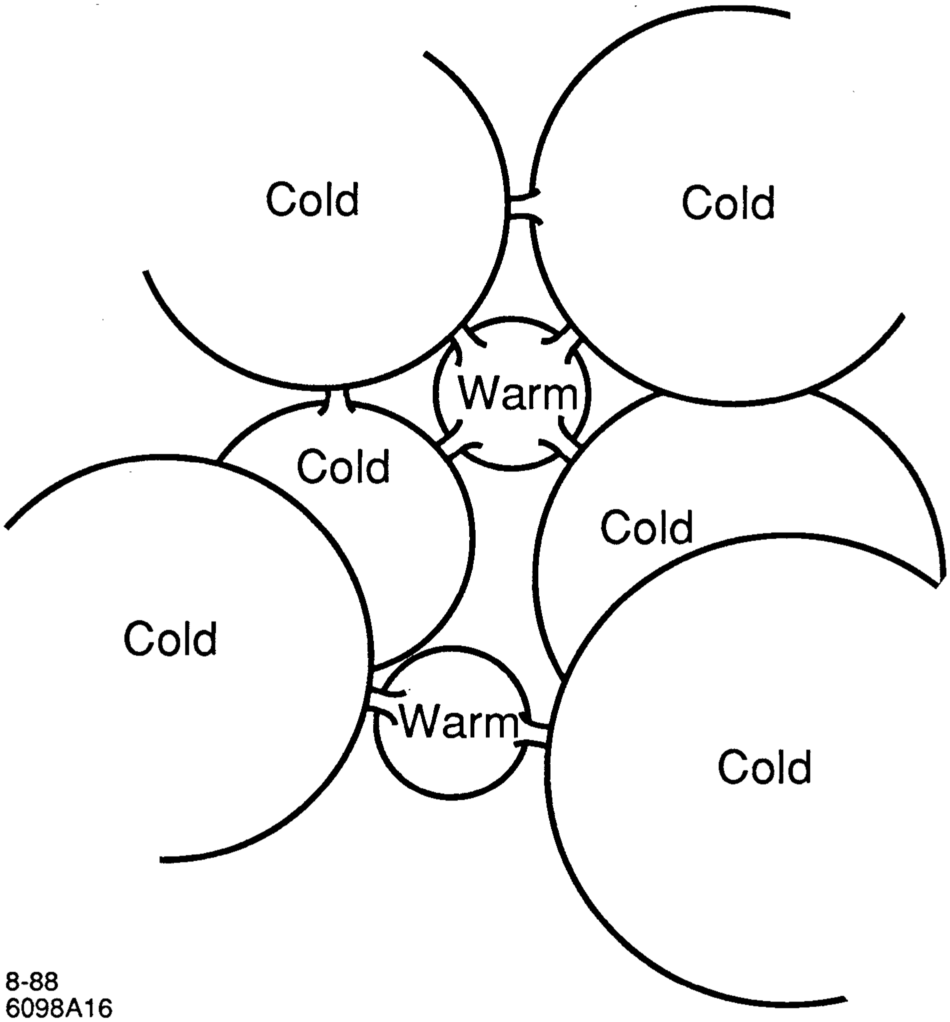

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Fig. 16