Ca' Foscari University of Venice

Department of Economics

Working Paper

Monica Billio, Roberto Casarin, Michele Costola, and Andrea Pasqualini

An entropy-based early warning indicator for systemic risk

ISSN: 1827-3580
No. 09/WP/2015

Working Papers
Department of Economics
Ca' Foscari University of Venice
No. 09/WP/2015
ISSN 1827-3580

An entropy-based early warning indicator for systemic risk

Monica Billio
University of Venice at Ca' Foscari
Roberto Casarin
University of Venice at Ca' Foscari
Michele Costola
University of Venice at Ca' Foscari
Andrea Pasqualini
University of Venice at Ca' Foscari

Abstract
The purpose of this paper is the construction of an early warning indicator for systemic risk using entropy measures. The analysis is based on the cross-sectional distribution of marginal systemic risk measures such as Marginal Expected Shortfall, Delta CoVaR and network connectedness. These measures are conceived at a single institution for the financial industry in the Euro area. We estimate entropy on these measures by considering different definitions (Shannon, Tsallis and Renyi). Finally, we test if these entropy indicators show forecasting abilities in predicting banking crises. In this regard, we use the variable presented in Babeckỳ et al. (2012) and Alessi and Detken (2011) from European Central Bank. Entropy indicators show promising forecast abilities to predict financial and banking crisis. The proposed early warning signals reveal to be effective in forecasting financial distress conditions.

Keywords
Entropy, systemic risk measures, early warning indicators, aggregation.
JEL Codes
C10, C11, G12, G29.

Address for correspondence:
Professor Monica Billio,
Ca' Foscari University of Venice,
San Giobbe 873/b
30121 Venezia, Italy
Email: billio@unive.it.

The Working Paper Series
is available only on line
(http://www.unive.it/nqcontent.cfm?a_id=86302)
For editorial correspondence, please contact:
wp.dse@unive.it

Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

An entropy-based early warning indicator for systemic risk*

Monica Billio[‡] Roberto Casarin[‡]
Michele Costola[‡] Andrea Pasqualini[‡]

[‡]University Ca' Foscari of Venice

May 7, 2015

Abstract

The purpose of this paper is the construction of an early warning indicator for systemic risk using entropy measures. The analysis is based on the cross-sectional distribution of marginal systemic risk measures such as Marginal Expected Shortfall, Delta CoVaR and network connectedness. These measures are conceived at a single institution for the financial industry in the Euro area. We estimate entropy on these measures by considering different definitions (Shannon, Tsallis and Renyi). Finally, we test if these entropy indicators show forecasting abilities in predicting banking crises. In this regard, we use the variable presented in Babeckỳ et al. (2012) and Alessi and Detken (2011) from European Central Bank. Entropy indicators show promising forecast abilities to predict financial and banking crisis. The proposed early warning signals reveal to be effective in forecasting financial distress conditions.

1 Introduction

Given the relevance of latest financial crisis, much attention has been reserved to modeling systemic events, which represent a potential threats to financial stability in an interconnected economic and financial system. As stated in (Billio et al., 2012), these linkages are parts of a complex and strongly interrelated system where the interconnectedness among financial

*Authors' research is supported by funding from the European Union, Seventh Framework Programme FP7/2007-2013 under grant agreement SYRTO-SSH-2012-320270, by the Institut Europlace of Finance, "Systemic Risk grant", the Global Risk Institute in Financial Services, the Louis Bachelier Institute, "Systemic Risk Research Initiative", and by the Italian Ministry of Education, University and Research (MIUR) PRIN 2010-11 grant MISURA. This research used the SCSCF multiprocessor cluster system at University Ca' Foscari of Venice.

institutions in period of financial distress may result in a rapid propagation of illiquidity, insolvency, and losses through the system. Given the endogenous nature of systemic risk, its measurement represents a complex task which involves different financial and macroeconomic aspects. In fact, the implications of systemic risk is relevant both in the macro and micro perspectives. At macro level, the aim of policy makers such as European Central Bank (ECB), European Systemic Risk Board (ESRB) and Federal Reserve (FED) is to guarantee the stability of the banking system (Rochet and Tirole, 1996; Freixas et al., 2000) while at micro level, systemic risk reduces the gains of diversification in an investors perspective (Das and Uppal, 2004). In this regard, different measures have been proposed in the literature to exploit the variety of aspects expressed by relevant economic and financial variables. Part of this literature think at the economic system as many interconnected subjects (consumers, firms, banks, ...), where systemic risk lies on the basis of such connections. In this line, studies investigate network linkages among financial institutions, where the purpose of the analysis has been the study of the transmission and propagation mechanism in term of connectedness. Billio et al. (2012) developed a pairwise Granger causality to detect significant linkages among financial institutions to describe which ones are systemically important and how those relate to the rest of the financial sectors.

Other measures are cross-sectional based, meaning that the analysis is on the co-dependence of financial institutions to measure their marginal contribution to systemic risk. Among these, the Δ-CoVaR, defined as the difference between the VaR of the financial system conditional on the institution being under distress and the VaR of the financial system conditional on state of that institution. Another relevant measure is the marginal expected shortfall (MES) proposed by Acharya et al. (2010), defined as the average return of the return of a financial institution during the 5% worst day for the overall market return. These measures consider the financial system as a “portfolio” of institutions, where the dynamic of the market price may impact the others, and where financial regulators are interested in the overall portfolio, rather than how its components behave individually and interact among themselves. Bisias et al. (2012) presents an excellent survey on systemic risk measures in literature. It is worth noting, that network representation allows to infer the contribution (connection) of each institution (node) to the overall system, and thus also network measures can be related to the cross-sectional type.

The motivation of our study relies on the capability to detect and predict likeness of systemic events defined as financial distress condition. In this regard, we propose a new approach to capture the structural changes of the system through systemic risk measures. The idea is that systemic risk is associated to the joint variability into a whole financial system, where linkages among institutions, e.g. banks, insurances, financial dealers, are the condi-

tions for observing loss cascade effects. That is, if an institution experiences a distress condition then other institutions, linked to the former, may experience distress. Our aggregation relies on the use of entropy applied to a feature distribution estimated on the market such as the cross-sectional systemic risk measures at a given point in time. In fact, movements of entropy built on these measures may reveal first signs of changes on systemic risk.
Therefore, the dynamic of this entropy could be used as a quasi real time early warning indicator for financial distress condition. Example of a quasi real time early warning indicator related to our study can be found in Alessi and Detken (2011) where they used this type of indicators to predict asset price booms that may have a great impact in real economy.

Entropy is used in a variety of fields to characterize the complexity of a system. In this regard, entropy measures of the information flow through a computer network and the entropy of the network behavioural features have been used in computer science (e.g., see Nychis et al. (2008)) to detect anomalies in computer networks. Lee and Xiang (2001) suggest to use several information-theoretic measures entropy, conditional entropy, relative conditional entropy, information gain, and information cost for anomaly detection.

Entropy has been involved also in finance. Zhou et al. (2013) provides an up-to-date review of the concepts and principles of entropy with applications to finance. Studies in our direction are in Gao and Hu (2013) and Alvarez-Ramirez et al. (2012). Gao and Hu (2013) study the income structures of different sectors of an economy separately and provide an early warning indicator based on entropy. They show that the losses, measured in term of quarterly negative incomes, in exposure networks can be modelled by a two-parameter Omori-law-like distribution for earthquake aftershocks. Our analysis is based at daily frequency and it considers risk measures as a proxy of the daily losses on stock prices. Alvarez-Ramirez et al. (2012) apply approximate entropy measures to study the time evolution of the market efficiency from an informational view point. They find evidence of changes in the efficiency structure for the US market. Their approach is at univariate level considering financial returns on a multi-scale analysis.

We consider instead multiple series and focus on the cross-section dimension at each point in time. To our knowledge we are the first to apply entropy to cross sectional risk measures.

Davis and Karim (2008) propose a comparison using a multinomial-logit and signal extraction as early warning for banking crises. The authors highlight the importance of the use of early warning system for policy makers to prevent crises. Their analysis is based mainly on macroeconomic, fiscal and financial variables at yearly frequency. In this regard, they suggest logit models are the most appropriate choice for these indicators for a global equally warning system. Following this approach, we propose entropy based on cross sectional measures as an early warning system for banking crisis using the variable proposed in Alessi and Detken (2014) for European countries. We focus on the Euro area who has experienced recently a sovereign debt crisis where the prominent role has been played by a frail financial and banking system (Lane, 2012). The remainder of this paper is organized as follows. Section 2 presents the entropy measures used in this paper and the estimation method. Section 3 presents the entropy estimates for the European stock market. Section 4 provides evidence of the nowcasting ability of the entropy for the financial crisis. Finally, Section 5 concludes.

2 System Entropy

Entropy measures are widely used in finance. Jiang et al. (2014) provide an entropy measure for asymmetrical dependency in asset returns. Their findings show that asymmetry is much more pervasive than previously thought, and that stocks which have greater asymmetric movements with the market earn higher average returns. Chabi-Yo and Colacito (2013) propose a new entropy-based correlation measure (co-entropy) to evaluate the performance of international asset pricing models. Co-entropy captures the codependence of two random variables beyond normality. They document that the coentropy of international stochastic discount factors (SDFs) can be decomposed into a series of entropy-based correlations of permanent and transitory components of the SDFs. A large cross-section of countries is employed to obtain model-free estimates of all the components of co-entropy at various horizons. They compare several models and find that they cannot account for the composition of codependence at all horizons. Bera and Park (2008) propose to use cross-entropy measure as the objective function with side conditions coming from the mean and variance–covariance matrix of the resampled asset returns. This automatically captures the degree of imprecision of input estimates. It can be viewed as shrinkage estimation of portfolio weights (probabilities) which are shrunk towards the predetermined portfolio. The novel feature of our application in finance is to apply entropy on systemic risk. Intuitively, in the proximity of a systemic event, the financial institutions that are those that cause the event, since they are the systemic relevant or frail, may be the first to react and thus to provoke a structural change in the cross-sectional distribution. In this regard, entropy can detect these changes in the (cross-sectional) distribution of these measures.

2.1 Entropy measures

In many applications the object of interest is a function of the probability distribution which summarizes the information content of the distribution. One of the most used probability functional is the entropy. Let $\boldsymbol{\pi}_t = (\pi_{1t}, \ldots, \pi_{mt})$, $t = 1, \ldots, T$, be a sequence of vector of probabilities associated to the cross-section distribution of a given feature of the financial assets measured over time t on the market. In this paper we apply entropy to $\boldsymbol{\pi}_t$. There exists many alternative definitions of entropy.

The Shannon entropy (Shannon, 1948), also known as Gibbs-Boltzman-Shannon, is defined as

$$H_S(\boldsymbol{\pi}_t) = -\sum_{j=1}^{m} \pi_{jt} \log \pi_{jt} \tag{1}$$

where $m < \infty$.

Two measures of entropy which have been widely used in the literature are the Tsallis (see Tsallis (1988)) entropy and the Renyi (Rényi, 1960). They allow for power tail behaviour and that are defined as

$$H_T(\boldsymbol{\pi}_t) = \frac{1}{\alpha - 1}\left(1 - \sum_{i=1}^{m} \pi_{it}^{\alpha}\right) \tag{2}$$

$$H_R(\boldsymbol{\pi}_t) = \frac{1}{1 - \alpha}\log\left(\sum_{i=1}^{m} \pi_{it}^{\alpha}\right) \tag{3}$$

and $m < \infty$. Alternative entropy measures have been proposed in the literature, such as the κ-entropy (Wada and Suyari, 2013) and the fuzzy entropy (e.g., see De Luca and Termini (1972)), that will be not considered in this paper.

As discussed in Maszczyk and Duch (2008), the entropy in Shannon (1948) is a special case of the other two formulations. In particular, according to the value of α, the measures in Equations 3 and 2 assign more or less weight to the tails of the distribution. Especially the the Tsallis entropy has been used in the analysis of systems involving long-range interactions, in a wide range of fields such as physics, chemistry, astronomy, engineering and economics (see, e.g. Tsallis (2001),Tsallis et al. (2002), Beck (2000), Reynolds (2003), Niven (2006)).

In order to show the relationship between tail probability and entropy, we consider $\boldsymbol{\pi}_t = (\pi_{1t}, \pi_{2t})$ with $\pi_{2t} = 1 - \pi_{1t}$ and report in Fig. 1. Compared to the entropy index in Shannon (1948), and depending on the value of the parameter α, the entropy in Rényi (1960) penalizes the mid-way between the uniform and the impulse distributions, while the entropy in Tsallis (1988) assigns less importance to randomness, that is it penalizes uniformity in the distribution. Therefore, for the entropy in Rényi (1960), the higher the parameter α and the less the entropy for distributions far from the uniform, i.e., the tails of the distribution are penalized. In contrast, for the entropy in Tsallis (1988), the higher the parameter α (see dashed-dotted line in panel (c), and the less the entropy for distributions close to the uniform. Note that the farther a distribution is from the uniform, the thinner its tails are (see the log-kurtosis in panel (a)). Thus for large α the Tsallis entropy is less

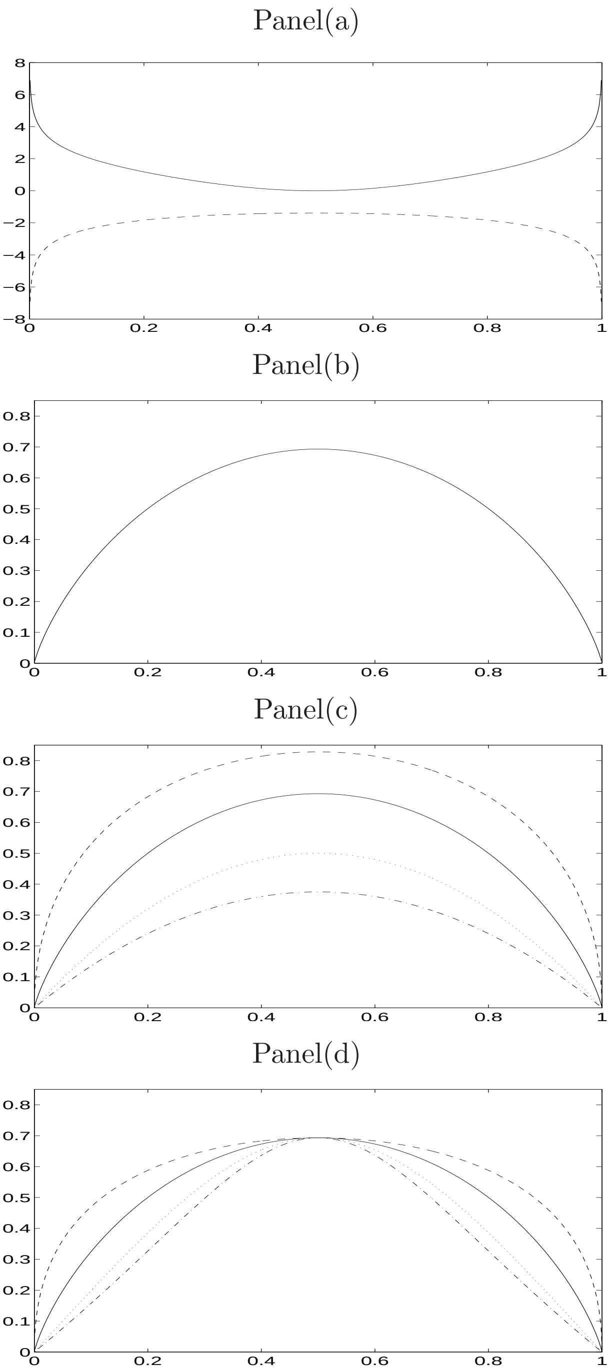

Figure 1: Logarithmic kurtosis (solid line) and variance (dashed line) (panel (a)). Shannon (panel (b)), Tsallis (panel (c)) and Renyi (panel (d)) entropies for $\boldsymbol{\pi}_t = (\pi_{1t}, \pi_{2t})$ and $\alpha = 0.5$ (dashed line), $\alpha = 1$ (solid line), $\alpha = 2$ (dotted line), and $\alpha = 3$ (dashed-dotted line).

sensitive to changes in the probabilities associated to common events (Bentes and Menezes, 2012; Tsallis et al., 2003) and more sensitive to changes in the tail probabilities, when there is a large change in the kurtosis (see panel (a)). We conclude that as regards to the Tsallis entropy α may be regarded as a biasing parameter which privileges common events when $\alpha > 1$ (dotted and dashed-dotted line in panel (c)) and rare events when $\alpha < 1$ (dashed line in panel (c)). Comparing the results in panel (c) and (d) one can see that the parameter α has opposite effects in the two definitions of entropy.

2.2 Early warning signal

Early warning system in financial system has been widely discuss and analyzed in literature (Davis and Karim, 2008; Bussière and Fratzscher, 2006; Edison, 2003; Martin, 1977). Most of these studies and more recently as in Davis and Karim (2008); Alessi and Detken (2011) have used macroeconomic variables. Demirgüc-Kunt and Detragiache (1999) define an early warning system, a system that will issue a signal in case the likelihood of a crisis crosses a specified threshold. Duca and Peltonen (2013) detect systemic risks in a unified framework resulting from domestic and global macro-financial vulnerabilities. Their results show the relevance in considering jointly indicators in a multivariate setting. Our analysis implements entropy on systemic risk measures as an early warning indicator to signal banking crisis.

2.3 Entropy estimation

We follow a Bayesian approach to inference for the distribution $\boldsymbol{\pi}_t$ and the entropy (e.g., see Minka (2003) and Mazzuchi et al. (2008)). Bayesian estimation of entropy is closely related to the expected information in Bayesian analysis (Bernardo, 1979; Zellner, 1991) and to the average entropy in the communication theory (Campbell, 1995), and has been proved to be consistent and to exhibit comparatively low bias on finite data sets, thus outperforming alternative procedures. Let $\mathbf{x}_t = (x_{1t}, \ldots, x_{n_t t})'$ be a vector of observations for a variable of interest over n_t different assets traded on the market at time t, with $t = 1, \ldots, T$, and let B_i, $i = 1, \ldots, m$ a sequence of intervals such that $B_i \cup B_j = \emptyset$, $i \neq j$. Define the sequence of discrete distributions

$$p(x|\boldsymbol{\pi}_t) = \sum_{j=1}^{m} \pi_{jt} \mathbb{I}_{B_j}(x) \tag{4}$$

$t = 1, \ldots, T$, with $m < \infty$, $\sum_{j=1}^{m} \pi_j = 1$, and $\mathbb{I}_A(x)$ is the indicator function, which take value 1 if $x \in A$ and 0 otherwise. The likelihood of the data at time t is a product of multinomial distribution and can be written as

$$p(\mathbf{x}_t|\boldsymbol{\pi}_t) = \prod_{i=1}^{n_t}\prod_{j=1}^{m}\pi_{jt}^{\mathbb{I}_{B_j}(x_{it})} = \prod_{j=1}^{m}\pi_{jt}^{n_jt} \tag{5}$$

where $\mathbf{x}_t = (x_{1t}, \ldots, x_{n_t t})$, and $n_{it} = \sum_{j=1}^{n_t} \mathbb{I}_{B_i}(x_{jt})$ is the count for bin B_i.

We assume a conjugate Dirichlet prior distribution (see Robert (2001), pp. 116-117) for the probability vector, $\boldsymbol{\pi}_t \sim \mathcal{D}ir(\phi\nu_1, \ldots, \phi\nu_m)$, with density function

$$f(\boldsymbol{\pi}_t) = \Gamma(\phi)\prod_{i=1}^{m}\frac{1}{\Gamma(\phi\nu_j)}\pi_{it}^{\nu_j}\mathbb{I}_{\Delta_{[0,1]^m}}(\boldsymbol{\pi}) \tag{6}$$

with $\nu_1 + \ldots + \nu_m = 1$, $\nu_i > 0$, $\phi > 0$ and $\Delta_{[0,1]^m}$ denoting the m-dimensional standard simplex. The posterior distribution is a Dirichlet $\boldsymbol{\pi}_t|\mathbf{x}_t \sim \mathcal{D}ir(\phi\nu_1 + n_{1t}, \ldots, \phi\nu_m + n_{mt})$, and the Bayesian estimator $\hat{\boldsymbol{\pi}}_t$ of $\boldsymbol{\pi}_t$ is a probability vector with elements $\hat{\pi}_{jt} = (\phi\nu_j + n_{jt})/(\phi + n_t)$, $j = 1, \ldots, m$. Finally we define the following Bayesian estimator of entropy

$$\hat{H}_k = \int H_k(\boldsymbol{\pi}_t)p(\boldsymbol{\pi}_t|\mathbf{x}_t)d\boldsymbol{\pi}_t \tag{7}$$

with $k = S, R, T$, The integral can be easily approximated by using Monte Carlo samples generated from the posterior, that is

$$\hat{H}_k \approx \frac{1}{N}\sum_{i=1}^{N} H_k(\boldsymbol{\pi}_t^{(i)}) \tag{8}$$

where $\boldsymbol{\pi}_t^{(i)} \sim \mathcal{D}ir(\phi\nu_1 + n_{1t}, \ldots, \phi\nu_m + n_{mt})$ i.i.d. $i = 1, \ldots, N$.

3 Features of financial market participants

As stated in Billio et al. (2012), we define systemic risk as "a collection of interconnected financial institutions that have mutually beneficial business relationships". In particular, illiquidity, insolvency, and losses quickly propagate during periods of distress through the financial system.

Our variables of interest x_{it} used in the entropy calculation are the Marginal Expected Shortfall (MES), the Δ-CoVaR and the network connectedness.

As regards the MES we follow Acharya et al. (2010) and starting from a series of asset returns r_{it}, $t = 1, \ldots, T$, where i denotes the asset, MES_{it} is defined as the expected value of r_{it} when a reference asset (or a reference market) is in its "worst state" and is experiencing losses. This state is identified when the return of the reference asset r_{mt} is below a given quantile q_k. That is, for $k = 0.05$,

$$MES_{it} = \mathbb{E}\{r_{it}|r_{mt} < q_{5\%}\}. \tag{9}$$

The authors, in their original formulation, put a minus in front of the expectation in order to meet consistency with the definition of "shortfall," as the expected returns in case of a tail event are intuitively thought to be negative. Moreover, Acharya et al. (2010) consider MES as a measure of systemic risk, which assesses the expected losses in case the market faces a tail event. The intuition behind MES is that, if institution i is linked to a systemic event, the conditional returns should highlight it. The authors propose and analyze MES properties at a firm-level risk management point of view. In particular, they analyse its predictive power. However, as shown in Löffler and Raupach (2013), MES is successful in capturing systemic relations if calculated on assets' returns, but it does not perform sufficiently well for other financial instruments, like bonds and derivatives. As it turns out, MES filters data in order to pick loss cascades during market downturns, thus allowing for a specific analysis of tail events.

The Δ-CoVar proposed by Adrian and Brunnermeier (2011) represents the value at risk (VaR) of the financial system conditional on institutions being under distress. Let us define the VaR and CoVar as follows

$$\begin{aligned} &\mathbb{P}(r_{it} \leq VaR_{it,q}) = q, \\ &\mathbb{P}(r_{jt} \leq CoVaR_{jit,q} | r_{it} = VaR_{it,q}) = q \end{aligned} \tag{10}$$

then the authors define a contribution of a given institution to systemic risk as the difference between the CoVaR conditional on the institution being under distress and the CoVaR in the median of the institution (Δ-CoVaR), that is

$$\Delta CoVaR_{mit,q} = CoVaR_{mit,q} - CoVaR_{mit,0.5}, \tag{11}$$

where r_{it} is the asset return value of the institution i and r_{mt} represents the system. $CoVaR_{mit,0.5}$ represents the VaR of the system at time t when returns of asset i are at 50^{th} percentile. Like European Systemic Risk Board (ESRB)[1], we use stock returns rather than asset returns as in (Adrian and Brunnermeier, 2011).

Following Billio et al. (2012) first we extrapolate a network from the asset returns and then focus on some feature of this network. Billio et al. (2012) focused on the total degree of the network and proposed a connectedness measure. In this paper we focus on the in-out degree of each node, IO_{it}, and its distribution on the network. A network is defined as a set of nodes $V_t = \{1, 2, \ldots, n_t\}$ and directed arcs (edges) between nodes. The network can be represented through an n_t-dimensional adjacency matrix A_t, with the element $a_{ijt} = 1$ if there is an edge from i directed to j with $i, j \in V_t$ and 0 otherwise. The matrix A_t is estimated by using a pairwise Granger causality approach to detect the direction and propagation of the relationships between the institutions. In order to test the causality direction the

[1] The Risk Dashboard pubblications are available at http://www.esrb.europa.eu/pub/rd/html/index.en.html.

following model is estimated

$$\begin{aligned} r_{it} &= \sum_{l=1}^{m} b_{11l} r_{it-l} + \sum_{l=1}^{m} b_{12l} r_{jt-l} + \epsilon_{it} \\ r_{jt} &= \sum_{l=1}^{m} b_{21l} r_{it-l} + \sum_{l=1}^{m} b_{22l} r_{jt-l} + \epsilon_{jt} \end{aligned} \tag{12}$$

$i \neq j$, $\forall i, j = 1, \ldots, n_t$, where m is the max lag (selected according a BIC criteria) and ϵ_{it} and ϵ_{jt} are uncorrelated white noise processes. The definition of causality implies,

- if $b_{12l} \neq 0$ and $b_{21l} = 0$, r_{jt} causes r_{it} and $a_{jit} = 1$.
- if $b_{12l} = 0$ and $b_{21l} \neq 0$, r_{it} causes r_{jt} and $a_{ijt} = 1$.
- if $b_{12l} \neq 0$ and $b_{21l} \neq 0$, there is a feedback relationship among r_{it} and r_{jt} and $a_{ijt} = a_{jit} = 1$.

The in-out degree measure is then defined as

$$IO_{it} = \sum_{j=1}^{n_t} a_{ijt} + \sum_{j=1}^{n_t} a_{jit} \tag{13}$$

$t = 1, \ldots, T$. As a reference measure we also consider the dynamic causality index (DCI), proposed by (Billio et al., 2012), which is defined as

$$DCI_t = \binom{n_t}{2}^{-1} \sum_{i=1}^{n_t} \sum_{j=1}^{n_t} a_{ijt} \tag{14}$$

$t = 1, \ldots, T$, when $(DCI_t - DCI_{t-1}) > 0$, there is an increase of system interconnectedness.

4 Effectiveness of the early warning indicators

The aim of this section is to provide empirical evidence of the effectiveness of the entropy indicators in nowcasting financial instability.

4.1 The European financial sector

We consider the daily closing price series for the European firms of the financial sector from from 1st January 1985 to 12th May 2014. See Appendix A for a detailed description of the dataset. As stated above, we focus on the Eurozone, who has experienced recently a sovereign debt crisis. Among others, **?** study the nature of systemic sovereign credit risk using CDS for US and Europe. They find that US sovereigns contain less systemic risk

with respect to Euro counterpart. However, in both area systemic sovereign risk is strongly related to financial market variables. **?** defined systemic risk as the propensity of a financial institution to be under-capitalized when the financial system is under-capitalized. The authors analyse the largest European financial firms, findings show that for certain countries, the cost for the taxpayer to rescue the riskiest domestic banks is so high that some banks might be considered too big to be saved.
In our study, looking at the European area, we considered a total of 437 financial institutions of the Industrial Classification Benchmark (ICB) class. We consider the MSCI Europe index a proxy for the European market, which provides a comprehensive overview of 15 countries in Europe, where the considered institutions are based.
To estimate systemic risk measures, we use a rolling window approach (e.g., see Zivot and J. (2003), Billio et al. (2012), Diebold and Yilmaz (2014)) with a window size of 252 daily observations, which corresponds approximately to a year of daily observations.[2]

Figure 2 shows the estimation results in terms of inter-quantile range at the 95% (gray are) and the mean (solid line) of the cross-sectional distribution of MES, ΔCoVaR and In-Out network degree over time.

The entropy estimates for the different indicators are reported in Figure (3).

4.2 Crisis indicators

Different indicators have been presented in the literature to detect economic and financial crises. Among others, there are Reinhart and Rogoff (2008), Reinhart and Rogoff (2010a), Reinhart and Rogoff (2010b), Valencia and Laeven (2008) and Valencia and Laeven (2012). One relevant aspect concerns the definition of "crisis" with respect to the aim of the analysis. As in Reinhart and Rogoff (2008), the idea of crisis concerns several perspectives such as sovereign debt crisis, banking crisis or currency crisis. Davis and Karim (2008) show the importance of the banking crisis in the design of an equally warning system. Therefore, we focus on European banking crisis as presented in Babeckỳ et al. (2012) and Alessi and Detken (2014). This indicator represents also one of the target variables monitored by European Systemic Risk Board (ESRB).

We use this indicator in the implementation of the early warning system. The variable is defined as,

$$C_t = \begin{cases} 1 & \text{if more than one country is in crisis at time } t \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

[2]The sequential estimation have been implemented in Matlab and takes approximately 15 hours on a cluster multiprocessor system which consists of 4 nodes; each comprises four Xeon E5-4610 v2 2.3GHz CPUs, with 8 cores, 256GB ECC PC3-12800R RAM, Ethernet 10Gbit, 20TB hard disk system with Linux.

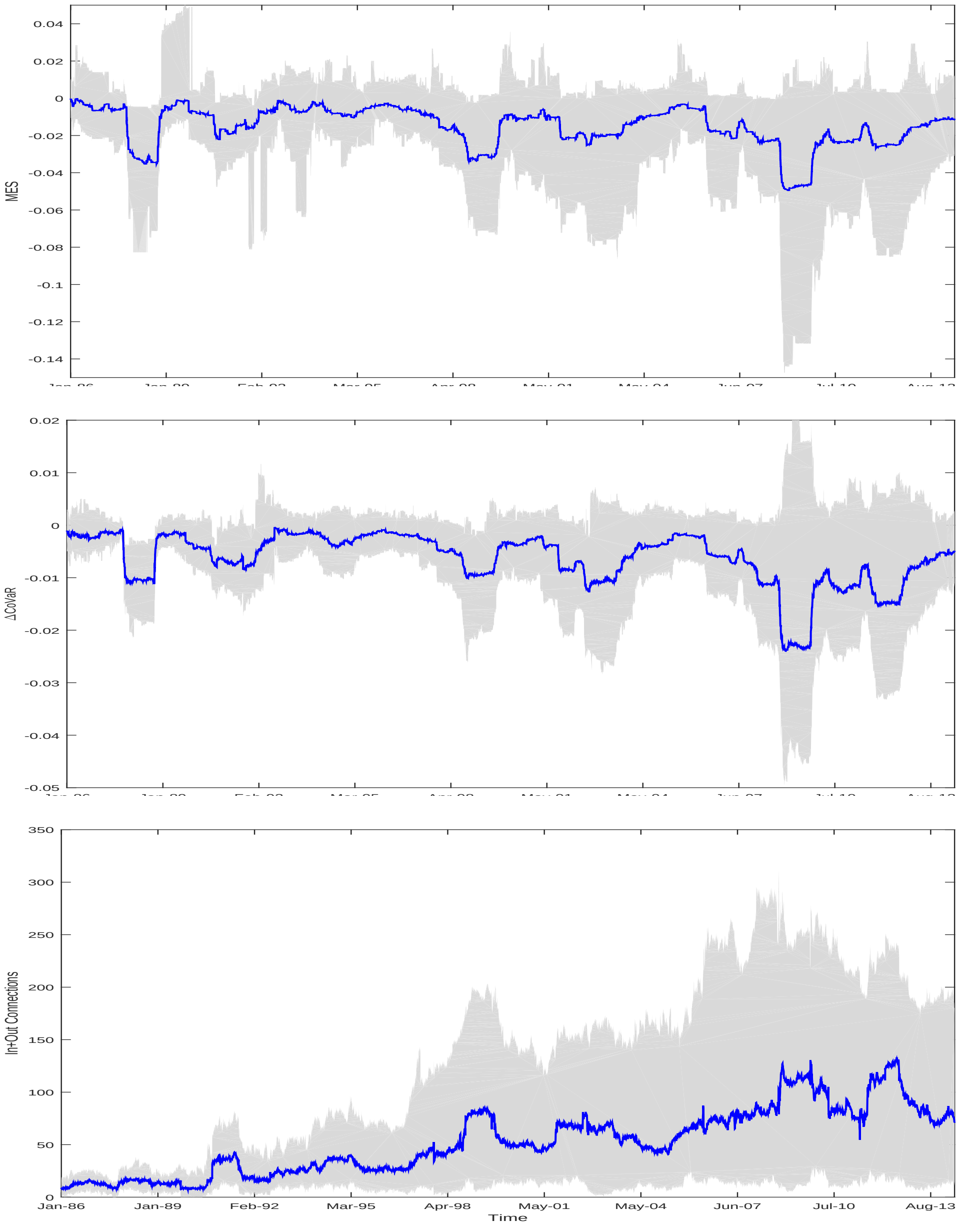

Figure 2: Distribution of MES (first panel), Δ-CoVaR (second panel) and In–Out network degrees (third panel) for the European financial sector over time.

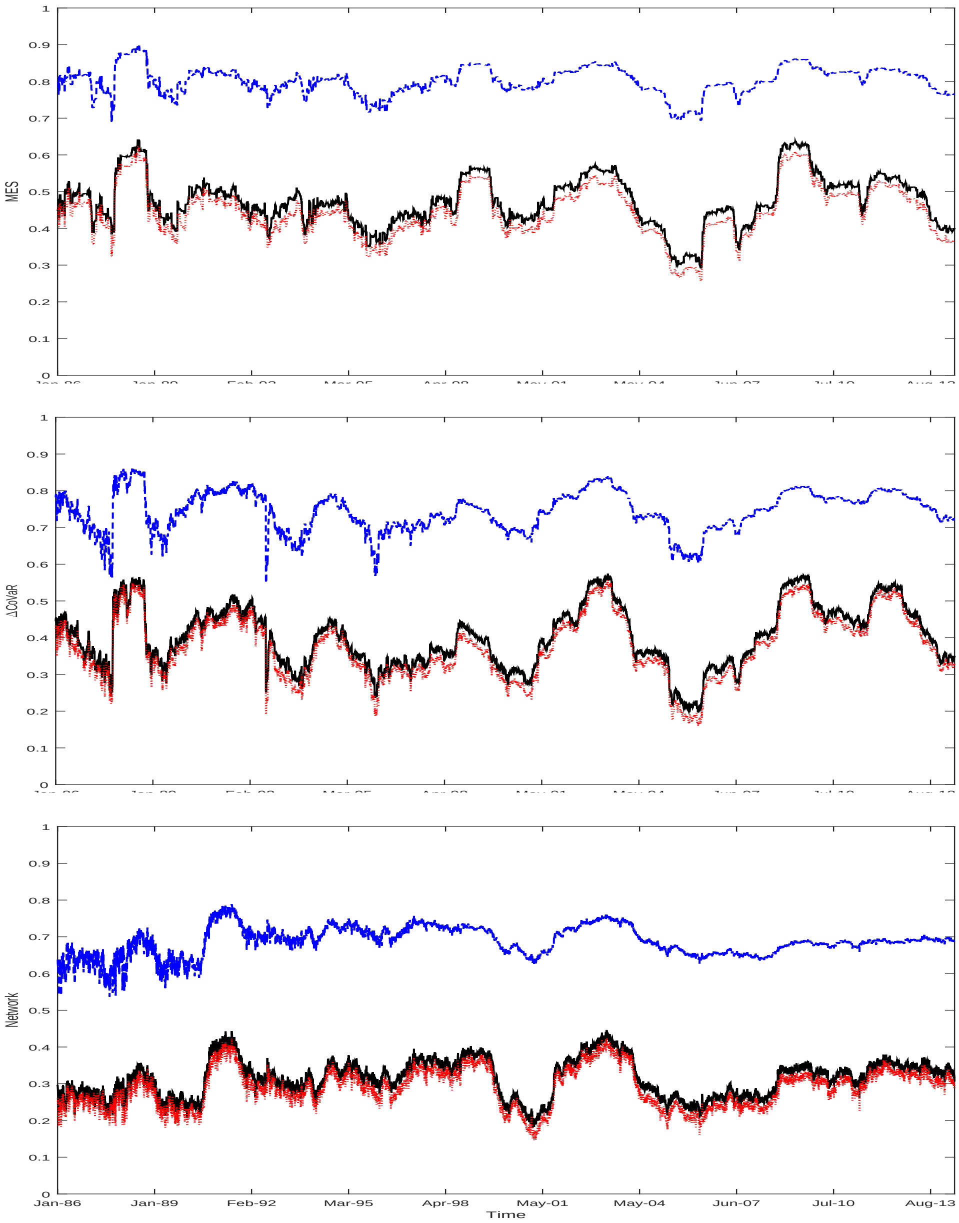

Figure 3: Shannon (solid line), Tsallis (dashed line) and Renyi (dotted line) Entropy measures of MES (first panel), Δ-CoVaR (second panel) and In–Out network degrees (third panel) for the European financial sector over time.

Given that the banking crisis indicator in Alessi and Detken (2014) has its last record in December 2012, we will focus on the period from January 1986 to December 2012. The crisis indicator is given on a per-country basis. Since the crisis indicator are at quarterly frequency and the returns are at daily frequency, we assume that the crisis indicator will equal 1 for all days in a given quarter, if the indicator equals 1 for that quarter. As Robustness Check, in Appendix C, we formulate two alternative banking crisis variables by changing the number of countries to be required to define a banking crisis in all the European area.

4.3 Forecasting results

In order to study the effectiveness of the entropy-based indicators in detecting conditions of financial distress we build a forecasting model for the crisis indicator variables given in Alessi and Detken (2014). We set a logistic model with entropy indicators for MES, $\Delta CoVaR$ and In-Out network degree as covariates.

We denote E_t the entropy index at time t and with C_t is the crisis indicator at time t, from the Alessi and Detken (2014) database, then the specified logistic model is

$$P(C_t = 1|E_t) = G(\beta_0 + \beta_1 E_t). \tag{16}$$

The estimation results from the logit specification are presented in Table 1. All entropies are significant at 1% confidence-level. The best explanatory variable is the entropy based on $\Delta CoVaR$.
We report in the paper the estimation for Shannon entropy. In Appendix B are also reported the estimation for Tsallis and Renyi entropy which confirm the results.

For evaluating the goodness of the models, many approaches can be considered. See, for example, Greene (2008) for a list of most common goodness-of-fit measures. Given the problem, we employ the *percent of correctly predicted* indicators. With reference to the dependent variable, the crisis indicator, we can define a threshold as the percent of times where it is equal to one.

Namely,

$$\text{threshold} = \frac{\sum_{t=1}^{T} C_t}{T}, \tag{17}$$

where C_t is the crisis indicator defined in Equation 15. For the time between January 1986 and December 2012, such threshold is equal to 47.25%.

If $\hat{C}_t$ is the predicted probability of crisis returned by the logit model, we can define a binary variable $\tilde{C}_t$ such that:

$$\tilde{C}_t = \begin{cases} 1 & \text{if } \hat{C}_t \geq 0.4725 \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

Table 1: Logit specification where the dependent variable is the banking crisis from Alessi and Detken (2014) and the explanatory variables are Shannon entropy indicator based on cross-sectional systemic risk measures of european financial institutions. The considered measures are MES, ΔCoVaR and In-Out network degrees. Significance level: 1% (***).

	Crisis Indicator		
(Intercept)	-5.3340***	-6.3911***	-6.4137***
	(0.1996)	(0.1641)	(0.1851)
$H_S(MES)$	10.9669***		
	(0.4151)		
$H_S(\Delta CoVaR)$		15.5536***	
		(0.4001)	
$H_S(InOut)$			20.0670***
			(0.5817)
R-squared	0.1140	0.2905	0.1952
Adjusted-R-squared	0.1139	0.2905	0.1951
LogLikelihood	-4455.92	-3790.80	-4107.88
LLR	0.0854	0.2219	0.1568
AIC	8915.84	7585.61	8219.77
BIC	8929.56	7599.33	8233.49
Sample	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12
Obs	7044	7044	7044

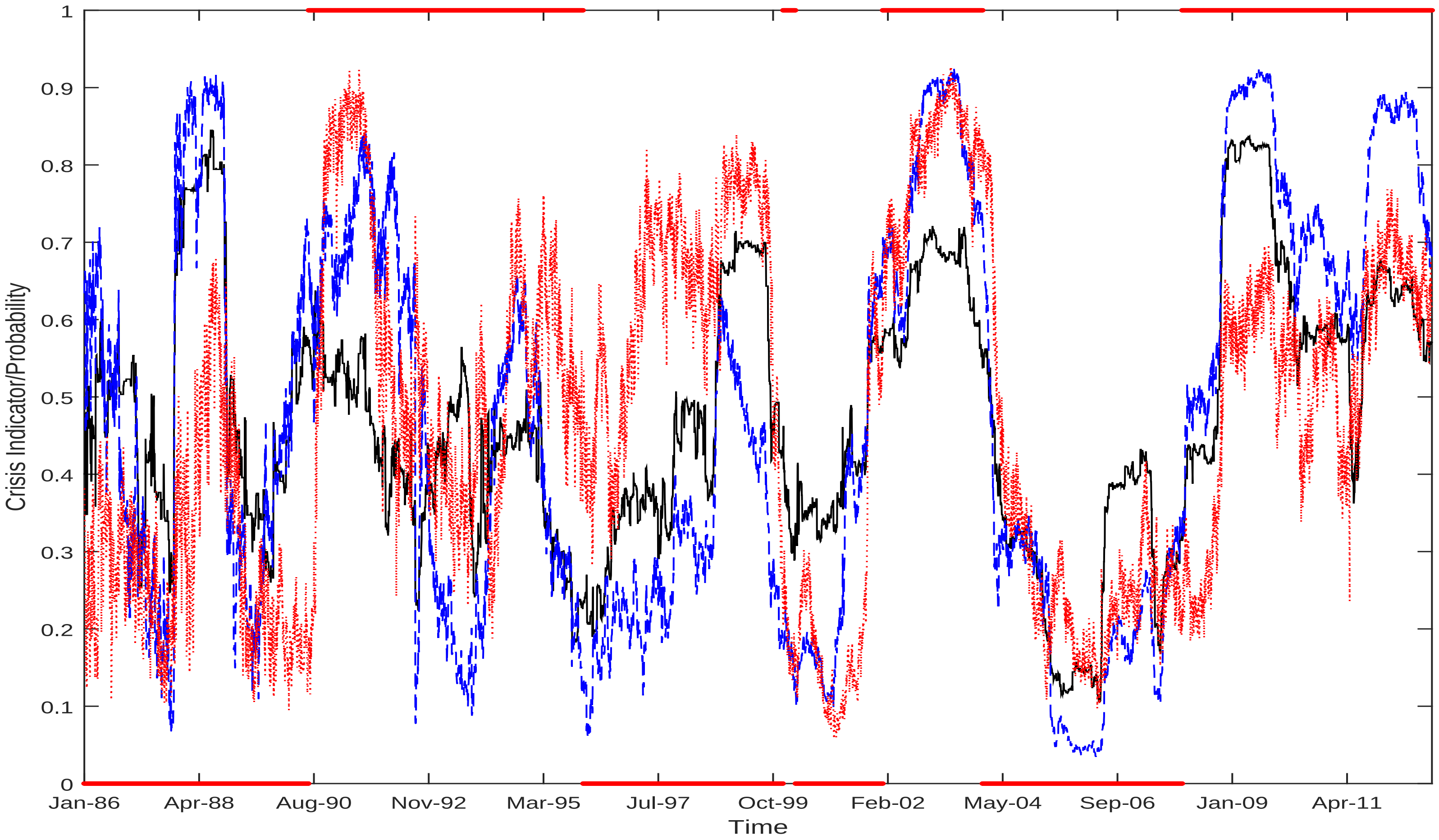

Figure 4: Overview of actual and estimated response variable over time of MES (solid line), Δ-CoVaR (dashed line) and In–Out network degrees (dotted panel).

This way, we have T pairs of values $(C_t, \tilde{C}_t)$ which, at any t, can form four possible combinations: either they are both equal to 1 or 0, or they are different. The percent of correctly predicted indicators is the number of times where $C_t = \tilde{C}_t$ relative to T. Table 2 reports the value for the estimated logit models. The entropy based on $\Delta CoVar$ confirms the superior ability in predicting banking crisis. It is worth nothing from Figure **??** that entropy indicators detect 1987 market crash and the consequent turmoil which is not marked as a banking crisis.

Table 2: Percent of correctly predicted banking crisis on the logit models using MES, ΔCoVaR and In-Out network degree entropy.

	MES	$\Delta CoVar$	In-Out degree
% corrected predicted	65.32%	77.51%	69.79%

Moreover, as Robustness check, we reported in Appendix C logit estimations with systemic risk indicator as DCI from (Billio et al., 2012) and alternative specifications for cross-sectional systemic risk measures such as mean and volatility. Entropy measures show their superior ability.

5 Conclusion

As shown in the latest two crises, systemic events represent a potential threats to financial and economic stability. Policy makers such as European Central Bank and Federal Reserve (FED) are interested in monitoring this risk using different target variables and econometrics techniques.
In this research line, this paper focused the attention on the construction of an early warning indicator for systemic risk using entropy measures. We based the analysis on the cross-sectional distribution of marginal systemic risk measures (measures conceived at institution-level) such as Marginal Expected Shortfall (MES), ΔCoVaR and network connectedness. Entropy measures are estimated considering alternative entropy definitions such as Shannon (1948),Tsallis et al. (2002) and Rényi (1960). In the empirical application, we estimated a logit model using as dependent variable the banking crisis presented in Babeckỳ et al. (2012) and Alessi and Detken (2011). Findings highlight the goodness of entropy indicators in forecasting and predicting banking crises. In future research, this approach could easily be extend to other areas and industries. Moreover, in an early warning system perspective, further investigation should be performed using other risk measures and target variables.

References

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. P. (2010). Measuring systemic risk. Technical report.

Adrian, T. and Brunnermeier, M. K. (2011). Covar. Technical report, National Bureau of Economic Research.

Alessi, L. and Detken, C. (2011). Quasi real time early warning indicators for costly asset price boom/bust cycles: A role for global liquidity. *European Journal of Political Economy*, 27(3):520–533.

Alessi, L. and Detken, C. (2014). Identifying excessive credit growth and leverage.

Alvarez-Ramirez, J., Rodriguez, E., and Alvarez, J. (2012). A multiscale entropy approach for market efficiency. *International Review of Financial Analysis*, 21:64–69.

Babeckỳ, J., Havranek, T., Mateju, J., Rusnák, M., Smidkova, K., and Vasicek, B. (2012). Banking, debt and currency crises: early warning indicators for developed countries.

Beck, C. (2000). Application of generalized thermostatistics to fully developed turbulence. *Physica A: Statistical Mechanics and its Applications*, 277(1):115–123.

Bentes, S. R. and Menezes, R. (2012). Entropy: A new measure of stock market volatility? *Journal of Physics: Conference Series*, 394.

Bera, A. K. and Park, S. Y. (2008). Optimal portfolio diversification using the maximum entropy principle. *Econometric Reviews*, 27(4-6):484–512.

Bernardo, J. M. (1979). Expected information as expected utility. *The Annals of Statistics*, 7:686–690.

Billio, M., Getmansky, M., Lo, A. W., and Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535 – 559.

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012). A survey of systemic risk analytics. *Annu. Rev. Financ. Econ.*, 4(1):255–296.

Bussière, M. and Fratzscher, M. (2006). Towards a new early warning system of financial crises. *journal of International Money and Finance*, 25(6):953–973.

Campbell, L. L. (1995). Averaging entropy. *IEEE Transactions on Information Theory IT*, 41:338–339.

Chabi-Yo, F. and Colacito, R. (2013). The term structure of co-entropy in international financial markets. Technical report, Charles A. Dice Center Working Paper No. 2013-17.

Das, S. R. and Uppal, R. (2004). Systemic risk and international portfolio choice. *The Journal of Finance*, 59(6):2809–2834.

Davis, E. P. and Karim, D. (2008). Comparing early warning systems for banking crises. *Journal of Financial stability*, 4(2):89–120.

De Luca, A. and Termini, S. (1972). A definition of non-probabilistic entropy in setting of fuzzy set theory. *Information and Control*, 20(301-312).

Demirgüc-Kunt, A. and Detragiache, E. (1999). Monitoring banking sector fragility: a multivariate logit approach with an application to the 1996-97 banking crises. *World Bank Policy Research Working Paper*, (2085).

Diebold, F. and Yilmaz, K. (2014). On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms. *Journal of Econometrics*, 182(1):119–134.

Duca, M. L. and Peltonen, T. A. (2013). Assessing systemic risks and predicting systemic events. *Journal of Banking & Finance*, 37(7):2183–2195.

Edison, H. J. (2003). Do indicators of financial crises work? an evaluation of an early warning system. *International Journal of Finance & Economics*, 8(1):11–53.

Freixas, X., Parigi, B. M., and Rochet, J.-C. (2000). Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of money, credit and banking*, pages 611–638.

Gao, J. and Hu, J. (2013). Financial crisis, omori's law, and negative entropy flow. In *Social Computing, Behavioral-Cultural Modeling and Prediction Lecture Notes in Computer Science*, volume 7812, pages 449–457.

Greene, W. H. (2008). *Econometric analysis*. Granite Hill Publishers.

Jiang, L., Wu, K., and Zhou, G. (2014). Asymmetry in stock returns: An entropy measure. Technical report.

Lane, P. R. (2012). The european sovereign debt crisis. *The Journal of Economic Perspectives*, 26(3):49–67.

Lee, W. and Xiang, D. (2001). Information-theoretic measures for anomaly detection. In *Security and Privacy, 2001. S&P 2001. Proceedings. 2001 IEEE Symposium on*, pages 130–143. IEEE.

Löffler, G. and Raupach, P. (2013). Robustness and informativeness of systemic risk measures. *Available at SSRN 2264179*.

Martin, D. (1977). Early warning of bank failure: A logit regression approach. *Journal of banking & finance*, 1(3):249–276.

Maszczyk, T. and Duch, W. (2008). Comparison of Shannon, Renyi and Tsallis entropy used in decision trees. In *Artificial Intelligence and Soft Computing–ICAISC 2008*, pages 643–651. Springer.

Mazzuchi, T. A., Soofi, E. S., and Soyer, R. (2008). Bayes estimate and inference for entropy and information index of fit. *Econometric Reviews*, 27(4-6):428–456.

Minka, T. (2003). Bayesian inference, entropy, and the multinomial distribution. Technical report, Microsoft, Available: http://www.stat.cmu.edu/ minka/papers/multinomial.html.

Niven, R. (2006). Exponential structure of arbitrary-order reaction kinetics. *Chemical Engineering Science*, 61(11):3785–3790.

Nychis, G., Sekar, V., Andersen, D. G., Kim, H., and Zhang, H. (2008). An empirical evaluation of entropy-based traffic anomaly detection. In *Proceedings of the 8th ACM SIGCOMM conference on Internet measurement*, pages 151–156. ACM.

Reinhart, C. M. and Rogoff, K. S. (2008). This time is different: A panoramic view of eight centuries of financial crises. Technical report.

Reinhart, C. M. and Rogoff, K. S. (2010a). From financial crash to debt crisis.

Reinhart, C. M. and Rogoff, K. S. (2010b). Growth in a time of debt.

Rényi, A. (1960). On measures of information and entropy. In *Proceedings of the 4th Berkeley Symposium on Mathematics, Statistics and Probability*, pages 547–561.

Reynolds, A. (2003). Superstatistical mechanics of tracer-particle motions in turbulence. *Physical review letters*, 91(8):084503.

Robert, C. P. (2001). *The Bayesian Choice*. Springer.

Rochet, J.-C. and Tirole, J. (1996). Interbank lending and systemic risk. *Journal of Money, credit and Banking*, pages 733–762.

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, 27(3):379–423.

Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics*, 52(1-2):479–487.

Tsallis, C. (2001). *Nonextensive Statistical Mechanics and its Applications*. Springer, Berlin.

Tsallis, C., Anteneodo, C., Borland, L., and Ososrio, R. (2003). Nonextensive statistical mechanics and economics. *Physica A*, 324(1):89–100.

Tsallis, C., Borges, E. P., and Baldovin, F. (2002). Mixing and equilibration: protagonists in the scene of nonextensive statistical mechanics. *Physica A: Statistical Mechanics and its Applications*, 305(1):1–18.

Valencia, F. and Laeven, L. (2008). Systemic banking crises: a new database. Technical report.

Valencia, F. and Laeven, M. L. (2012). Systemic banking crises database: An update (epub).

Wada, T. and Suyari, H. (2013). The κ-generalizations of stirling approximation and multinominal coefficients. *Entropy*, 15:5144–5143.

Zellner, A. (1991). Bayesian methods and entropy in economics and econometrics. In *Maximum Entropy and Bayesian Methods*, pages 17–31. Netherlands: Kluwer.

Zhou, R., Cai, R., and Tong, G. (2013). Applications of entropy in finance: A review. *Entropy*, 15(4909-4931).

Zivot, E. and J., W. (2003). *Modelling Financial Time Series with S-Plus®*. Springer.

A Data description

The dataset constitutes of the European firms which are classified under the ICB code class 8000. This is the class for financial firms. A description of this class of assets is in Table 1.

	Supersector		Sector
8330	Banks	8350	Banks
8500	Insurance	8530	Nonlife Insurance
		8570	Life Insurance
8600	Real Estate	8630	Real Estate Investment & Services
		8670	Real Estate Investment Trust
8700	Financial Services	8770	Financial Services
		8980	Equity Investment Instruments

Table 1: Description of the ICB 8000 *financials* asset class.

The dataset includes closing prices (source, DataStream ®) from 1st January 1985 to 12th May 2014 at a daily frequency. Table 2 shows the list of the 20 financial markets (countries) considered with the corrisponding number of assets.

Austria	43	Belgium	73	Denmark	179
Finland	30	France	285	Germany	344
Greece	82	Hungary	16	Ireland	30
Italy	139	Latvia	1	Lithuania	5
Luxembourg	40	The Netherlands	87	Norway	78
Portugal	29	Spain	84	Sweden	113
Switzerland	149	United Kingdom	1310		

Table 2: List of financial markets and no. of assets collected.

The overall EU market, that is aggregating all the data in a unique array, is summarized in Figures 1 and 2. Red line reprsents the cross-section average, and the green lines the 0.95% and 5% quantiles

We reported also in Figure 4 and Figure 5 the network representation for 2005 and 2012. It can be clearly seen that the number of connections increase during banking crisis.

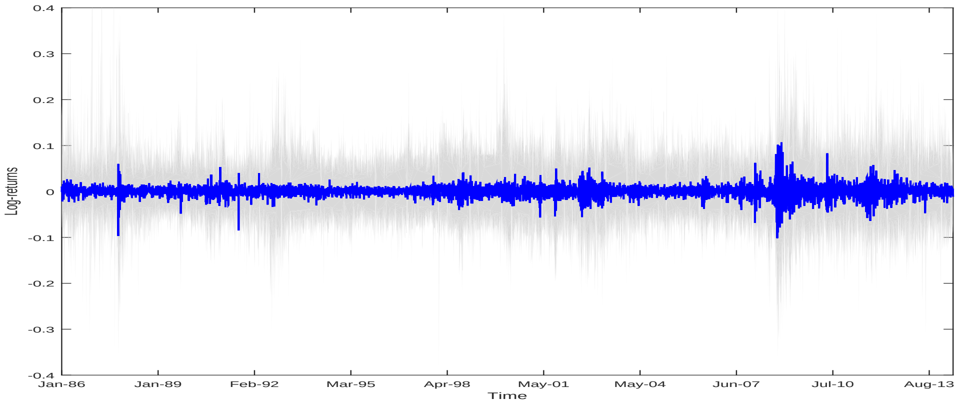

Figure 1: Distribution of returns in Europe over time.

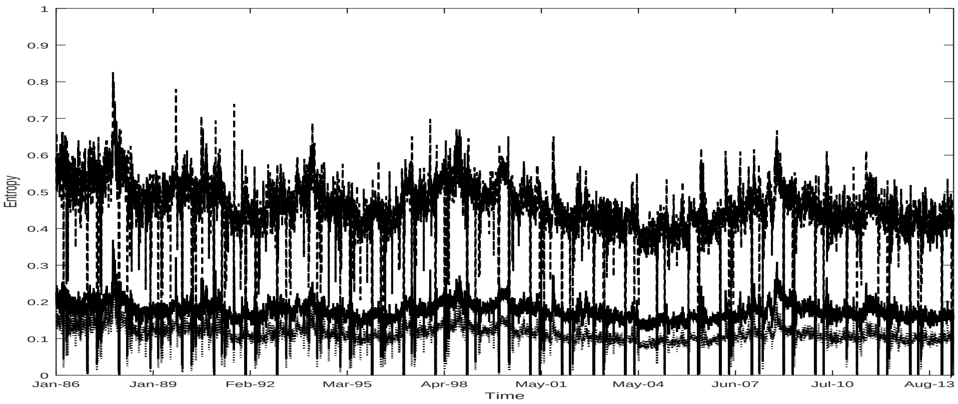

Figure 2: Shannon (solid line), Tsallis (dashed line) and Renyi (dotted line) entropy of returns in Europe over time.

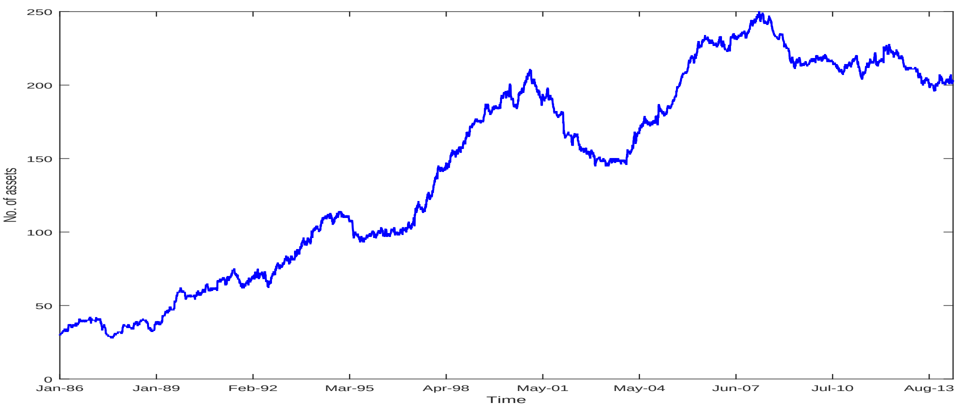

Figure 3: Sample size of returns in Europe over time.

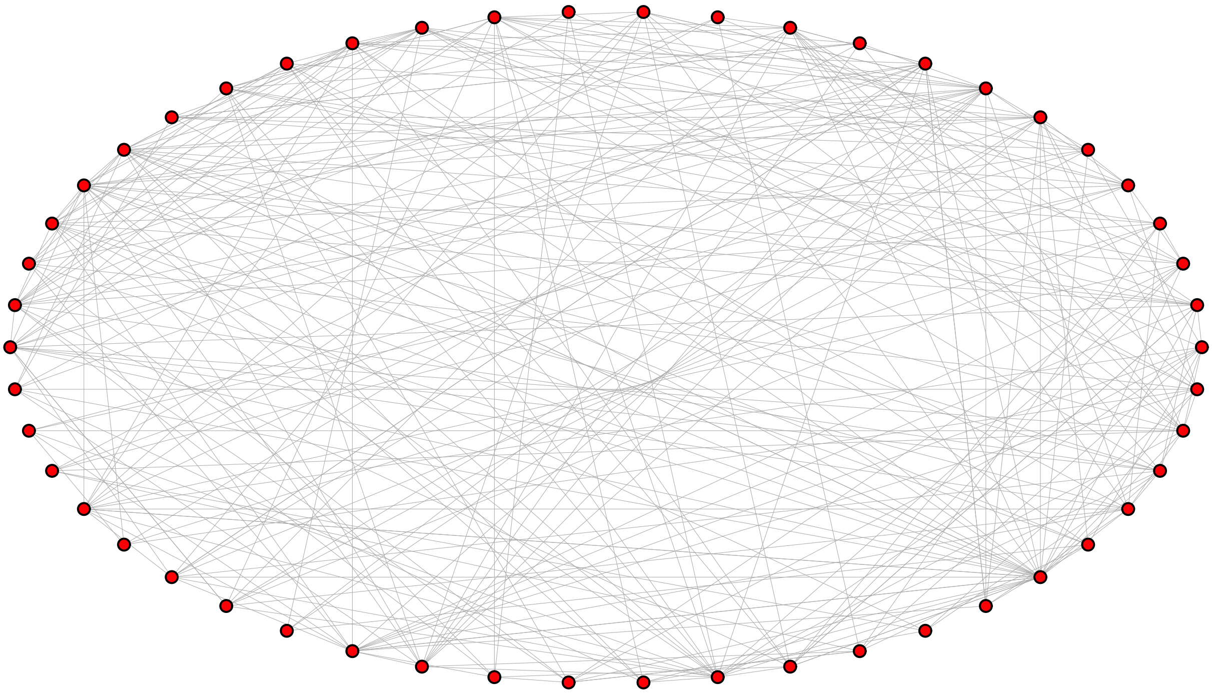

Figure 4: Network diagram of linear Granger-causality relationships that are statistically significant at the 5% level among the daily returns in April 2005.

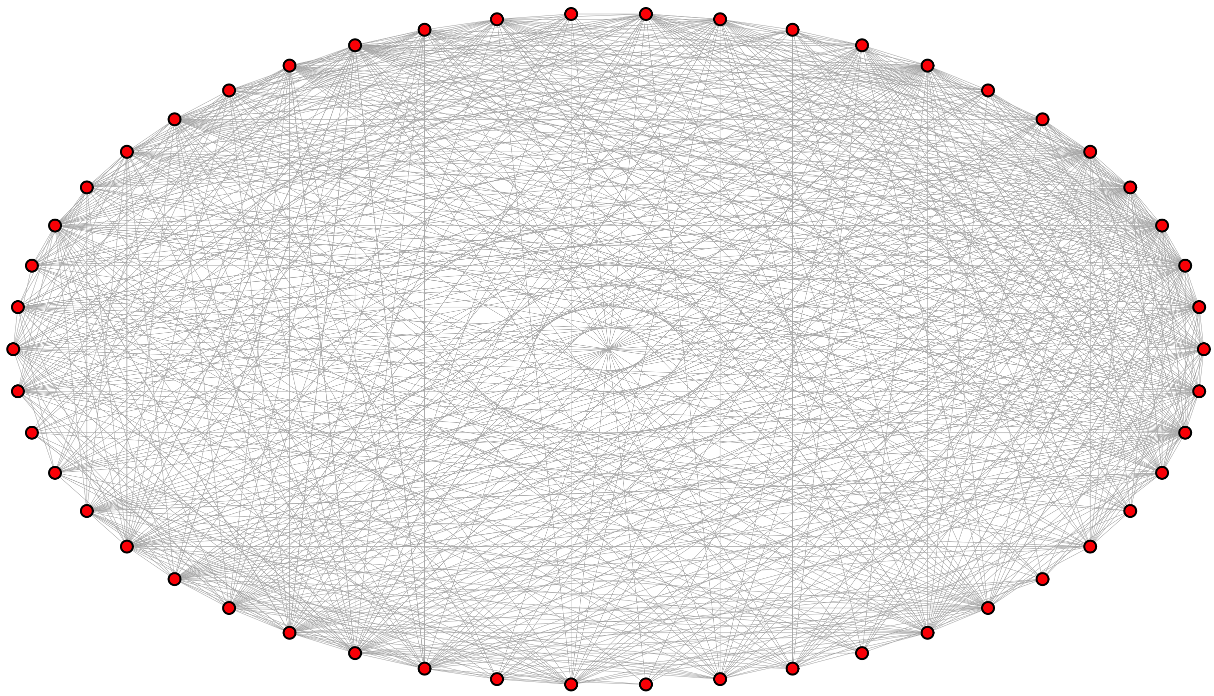

Figure 5: Network diagram of linear Granger-causality relationships that are statistically significant at the 5% level among the daily returns in June 2012.

B Entropy calibration

More attention can be paid to the entropy indexes in Rényi (1960) and Tsallis (1988). As they embed the parameter α, they allow the researcher to fine-tune the models.

Economically, it means identifying how much the tails of the distributions of MES are relevant to the crises prediction. Intuitively, the more spread the distribution, and the "fatter" its tails. As already discussed, the parameter α in the entropy definitions helps understanding how relevant are the tails. One may further develop the argument stating that such parameter assigns more or less weight to the degree of uncertainty of the scenarios.

Statistically, that means minimizing a loss function. Such function is identified, in the paper, with the AIC criterion, namely

$$\mathrm{AIC} = 2k - 2\ln(\mathrm{L}), \tag{1}$$

where L is the likelihood returned by the logit in Equation 16.

For the entropies in Rényi (1960) and Tsallis (1988), L is a function of α, as it derives from the logistic regression run on those indexes. We can therefore want to optimize the *AIC* according to that parameter. That translates to solving

$$\min_{\alpha} \mathrm{AIC}(\alpha). \tag{2}$$

Figure 1-3 show the behavior of the AIC criteria as a function of α for both Renyi and Tsallis entropy indexes. The implication of such behavior is twofold: asymptotically, the Tsallis entropy has to be preferred to the Renyi one. The second implication has to do with the optimal value of α which is greater than one. This suggests that changes in the tail probabilities of the variable of interest play a crucial role in measuring systemic risk and in detecting changes in the risk level.
To avoid an ad-hoc selection, the tuning of the parameter α is performed in the first part of the sample.

Table (1) and (2) report the results for the estimation with Tsallis and Renyi entropy, respectively.
Shannon entropy provides better estimates in terms of AIC and BIC criteria with respect to the Tsallis and Renyi.

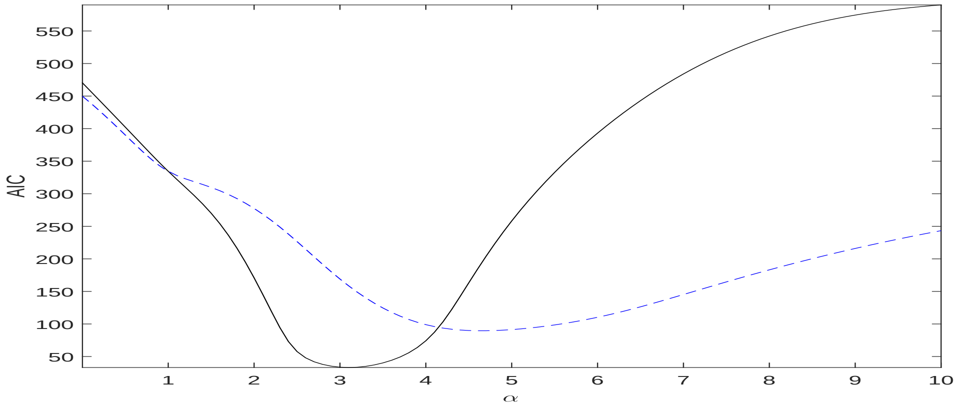

Figure 1: AIC as function of α for Tsallis (solid) and Renyi (dashed) entropies in the logit model (MES).

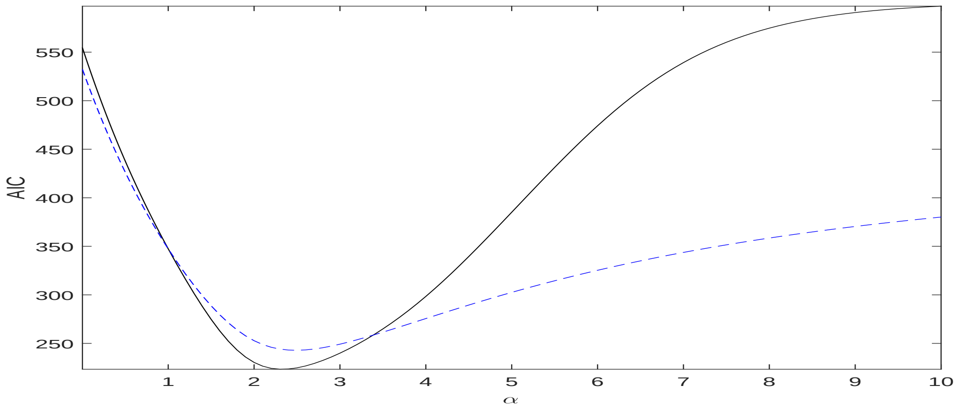

Figure 2: AIC as function of α for Tsallis (solid) and Renyi (dashed) entropies in the logit model (ΔCoVaR).

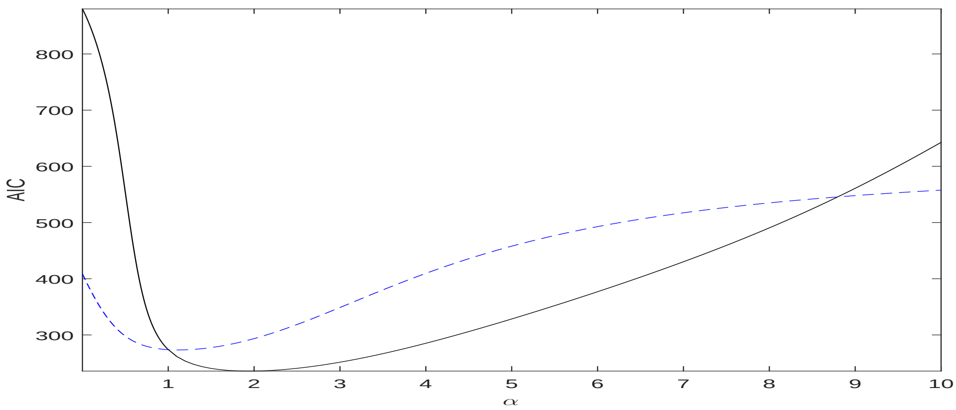

Figure 3: AIC as function of α for Tsallis (solid) and Renyi (dashed) entropies in the logit model (In-Out network degree).

Table 1: Logit specification where the dependent variable is the banking crisis from Alessi and Detken (2014) and the explanatory variables are Tsallis entropy indicator based on cross-sectional systemic risk measures of european financial institutions. The considered measures are MES, ΔCoVaR and In-Out network degrees. Tsallis entropy with the optimal α according to the AIC. Significance level: 1% (***).

	Crisis Indicator		
(Intercept)	-85.4113***	-26.2500***	-19.2738***
	(5.0148)	(0.9337)	(0.6490)
$H_S(MES)$	86.7045***		
	(5.0961)		
$H_S(\Delta CoVaR)$		29.0981***	
		(1.0367)	
$H_S(InOut)$			25.0981***
			(0.8479)
R-squared	0.0457	0.1585	0.1352
Adjusted-R-squared	0.0456	0.1583	0.1351
LogLikelihood	-4706.41	-4362.64	-4325.85
LLR	0.0340	0.1045	0.1121
AIC	9416.83	8729.28	8655.71
BIC	9416.83	8743.00	8669.43
Sample	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12
Obs	7044	7044	7044

Table 2: Logit specification where the dependent variable is the banking crisis from Alessi and Detken (2014) and the explanatory variables are Renyi entropy indicator based on cross-sectional systemic risk measures of european financial institutions. The considered measures are MES, ΔCoVaR and In-Out network degrees. Renyi entropy with the optimal α according to the AIC. Significance level: 1% (***).

	Crisis Indicator		
(Intercept)	-4.4896***	-5.3212***	-6.3234***
	(0.1647)	(0.1391)	(0.1822)
$H_S(MES)$	11.2202***		
	(0.4159)		
$H_S(\Delta CoVaR)$		14.5302***	
		(0.3791)	
$H_S(InOut)$			20.0556***
			(0.5803)
R-squared	0.1196	0.2887	0.1965
Adjusted-R-squared	0.1195	0.2886	0.1963
LogLikelihood	-4438.51	-3828.46	-4103.57
LLR	0.0889	0.2142	0.1577
AIC	8881.03	7660.92	8211.14
BIC	8894.75	7674.64	8224.86
Sample	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12
Obs	7044	7044	7044

C Robustness Checks

As Robustness check in our analysis, we consider the Dynamic Causality Index as in Billio et al. (2012), cross-sectional mean and standard deviation for systemic risk measures. In terms of AIC and BIC criteria, Shannon entropy provides better estimates with respect to the mean of the cross-sectional risk measures. Similar for Δ-CoVaR and In-Out connection degrees entropy returns better estimates with respect to the standard deviation except for the case of MES.

Finally, the last robustness check is performed on the dependent variable by changing the number of countries to be on crisis to have an european crisis. In this regards, we require more than two countries to be on crisis on the first check while more than three countries on the second check.
All entropy measures based on MES, Δ-CoVaR and in-out network degree are strongly significant in both the logit estimations. Table (2) and report the estimation for the banking crisis variable obtained when at least three and four European countries are on crisis, respectively.

Table (4) reports the percent of correctly predictid banking crisis for the two different constructed dependent variables. The values drop substantially as it could have been expected.

Table 1: Logit specification where the dependent variable is the banking crisis from Alessi and Detken (2014) and the explanatory variables are mean and standard deviation based on cross-sectional systemic risk measures of european financial institutions. The considered measures are MES, ΔCoVaR and In-Out network degrees. Significance level: 1% (***).

	Crisis Indicator						
(Intercept)	-3.3815***	-0.8583***	-1.4844***	-1.2704***	-1.8362***	-2.5419***	-1.0187***
	(0.1108)	(0.0444)	(0.0483)	(0.0505)	(0.0689)	(0.0701)	(0.0486)
DCI	18.5333***						
	(0.6148)						
$\mu(MES)$		-49.9143***					
		(2.5072)					
$\mu(\Delta CoVaR)$			-227.2496***				
			(7.0982)				
$\mu(InOut)$				0.0227***			
				(0.0009)			
$\sigma(MES)$					167.26876***		
					(6.3684)		
$\sigma(\Delta CoVaR)$						745.1144***	
						(21.3651)	
$\sigma(InOut)$							0.0398***
							(0.0018)
R-squared	0.1466	0.0605	0.1843	0.1004	0.1124	0.2579	0.0676
Adjusted-R-squared	0.1465	0.0603	0.1842	0.1003	0.1123	0.2578	0.0674
LogLikelihood	-4307.77	-4650.66	-4150.46	-4479.90	-4439.83	-3798.74	-4619.54
LLR	0.1158	0.0454	0.1481	0.0805	0.0887	0.2203	0.0518
AIC	8619.54	9305.32	8304.93	8963.79	8883.67	7601.47	9243.08
BIC	8633.26	9319.04	8318.65	8977.51	8897.39	7615.19	9256.79
Sample	jan-86	jan-86	jan-86	jan-86	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12	dec-12	dec-12	dec-12	dec-12
Obs	7044	7044	7044	7044	7044	7044	7044

Table 2: Logit specification where the dependent variable is the banking crisis defined as more than two countries on crisis obtained from Alessi and Detken (2014) and the explanatory variables are Shannon entropies based on cross-sectional systemic risk measures of European financial institutions. The considered measures are MES, ΔCoVaR and In-Out network degrees. Significance level: 1% (***).

Crisis Indicator			
(Intercept)	-5.8689***	-5.7522***	-3.9969***
	0.2125	0.1617	0.1699
$H_S(MES)$\$	10.4722***		
	0.4298		
$H_S(\Delta CoVaR)$		11.8787***	
		0.3714	
$H_S(InOut)$			10.0374***
			0.5199
R-squared	0.0838	0.1773	0.0429
Adjusted-R-squared	0.0837	0.1771	0.0427
LogLikelihood	-3972.91	-3679.20	-4113.54
LLR	0.0788	0.1469	0.0462
AIC	7949.83	7362.40	8231.08
BIC	7963.53	7376.10	8244.78
Sample	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12
Obs	7044	7044	7044

Table 3: Logit specification where the dependent variable is the banking crisis defined as more than three countries on crisis obtained from Alessi and Detken (2014). The explanatory variables are Shannon entropies based on cross-sectional systemic risk measures of European financial institutions. The considered measures are MES, ΔCoVaR and In-Out network degrees. Significance level: 1% (***).

	Crisis Indicator		
(Intercept)	-10.4567***	-7.8415***	-3.7623***
	(0.3045)	(0.2216)	(0.1983)
$H_S(MES)$	17.9179***		
	(0.5871)		
$H_S(\Delta CoVaR)$		14.5885***	
		(0.4779)	
$H_S(InOut)$			6.9981***
			(0.6021)
R-squared	0.1684	0.1962	0.0081
Adjusted-R-squared	0.1683	0.1960	0.0080
LogLikelihood	-2661.13	-2650.60	-3198.72
LLR	0.1857	0.1889	0.0212
AIC	5326.27	5305.20	6401.45
BIC	5339.97	5318.90	6415.15
Sample	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12
Obs	7044	7044	7044

Table 4: Percent of correctly predicted banking crisis on the logit models using MES, ΔCoVaR and In-Out network degree entropy. Banking crisis 1 (Banking crisis 2) is defined as more than two (three) countries on crisis which have been obtained from Alessi and Detken (2014).

% corrected predicted	MES	ΔCoVaR	In-Out degree
banking crisis 1	19.30%	24.00%	19.42%
banking crisis 2	17.33%	15.91%	17.80%