MISCELLANEOUS

TRACTS

ON

Some curious, and very interesting Subjects

IN

MECHANICS, PHYSICAL-ASTRONOMY, and SPECULA-
TIVE MATHEMATICS;

WHEREIN

The Precession of the EQUINOX, the Nutation of the
EARTH'S AXIS, and the Motion of the MOON in her
ORBIT are determined.

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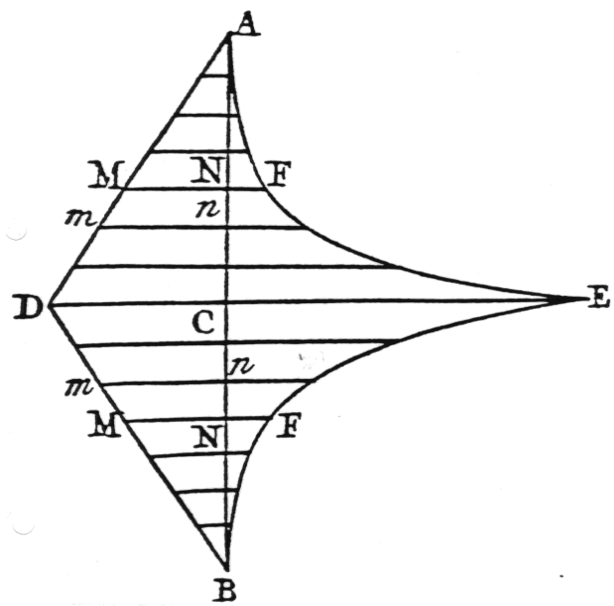

[*This extract consists of a paper on pages 64 to 75 of* Miscellaneous Tracts ..., *which is a slightly expanded version of the paper published in* Philosophical Transactions of the Royal Society of London **49** (1755), 82–93. *The original pagination is shown in square brackets. The notation is generally the same as in the original, except that for typographical simplicity a bar over an expression is replaced by braces ..., and a bar over an expression joined to a following vertical line is replaced by braces followed by a vertical line not joined to them, as ...—.*]

[64]

An ATTEMPT to shew the Advantage, arising by Taking the Mean of a Number of Observations, in practical Astronomy

ALTHOUGH the method practiced by Astronomers. in order to diminish the errors arising from the imperfections of instruments and of the organ of sense, by taking the mean of several observations, is of very great utility, and almost universally followed, yet it has not, that I know of, been hitherto subjected to any kind of demonstration.

In this Essay, some light is attempted to be thrown on the subject, from mathematical principles: in order to the application of which, it seemed necessary to lay down the following suppositions.

1. That there is nothing in the construction, or position of the instrument, whereby the errors are constantly made to tend the same way, but that the respective chances for their happening in excess, and in deficit, are either accurately, or nearly, the same.
2. That there are certain assignable limits between which all these errors may be supposed to fall; which limits depend on the goodness of the instrument and the skill of the observer.

These particulars being premised, I shall deliver what I have to offer on the subject, in the following Proposition.

PROPOSITION I.

Supposing that the several chances for the different errors that any single observation can admit are expressed by the terms of the series $r^{-v}, \ldots\ r^{-3}, r^{-2}, r^{-1}, r^0, r^1, r^2, r^3, \ldots\ r^v$ *where the exponents denote the quantities and qualities of the respective errors, and the terms themselves, the respective chances for their happening; it is proposed to determine the probability, or [65] odds, that the error, by taking the mean of a given number (n) of observations,* exceeds not a given quantity $\left(\frac{m}{n}\right)$.

It is evident from the laws of chance that if the given series $r^{-v} \cdots + r^{-3} + r^{-2} + r^{-1} + r^0 + r^1 + r^2 + r^3 \cdots + r^v$, expressing all the chances in one observation, be raised to the nth power, the terms of the series thereby arising will duly exhibit all the proposed (n) observations. But in order to raise this power, with the greatest facility, our given series may be reduced to $r^{-v} \times \frac{1-r^{2v+1}}{1-r}$ (by the known rule for summing up the terms of a geometrical progression); whereof the nth power (making $w = 2v+1$) will be $r^{-nv} \times \{1-rv\}|^n \times (1-r)|^{-n}$; which expanded, becomes $r^{-nv} - nr^{w-nv} + \frac{n}{1}.\frac{n+1}{2}r^{2w-nv} + \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}r^{3w-nv} +$&c.$\times$ into $1 + nr + \frac{n}{1}.\frac{n+1}{2}.r^2 + \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}r^3 + \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}r^4 +$ &c.

Now, to find from hence the sum of all the chances, whereby the excess of the positive errors above the negative ones, can amount to a given number m precisely, it will be sufficient (instead of multiplying the former series by the whole of the latter) to multiply by such terms of the latter only, as are necessary to the production of the given exponent m, in question.

Thus the first term (r^{-nv}) of the former series, is to be multiplied by that term of the second whose exponent is $nv+m$, in order that the power of r, in the product, may be r^m: but this term (putting $nv+m=q$) will be $\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q)$, q being the number of factors; and consequently, that the product under consideration will be $\frac{n}{1}\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q) \times r^m$.

Again, the second term of the former series being $-nr^{w-nv}$, the exponent of the corresponding term of the latter must therefore be $-w+nv+m(=q-w)$, and the term it-[66]self, $\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}(q-w) \times r^{q-w}$; which drawn into $-nr^{w-nv}$, gives $\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}(q-w) \times nr^m$ for the second term required.

In the like manner the third term of the product whose whole exponent is m, will be found $\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}(q-2w) \times \frac{n}{1}.\frac{n-1}{r}^m$. And the sum of all the terms, having the same, given exponent, will consequently be

$$+\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q) \times r^m$$

$$
\begin{aligned}
&- \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q-w) \times nr^m \\
&+ \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q-2w) \times \frac{n}{1}.\frac{n-1}{2}r^m \\
&- \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q-3w) \times \frac{n}{1}.\frac{n-1}{2}.\frac{n-2}{3}.r^m \\
&\qquad \text{\&c.} \qquad \text{\&c.}
\end{aligned}
$$

From which general expression, by expounding m by 0, $+1$, -1, $+2$, -2 &c. successively, the sum of the several chances whereby the difference of the positive and negative errors can fall within the proposed limits $(+m, -m)$ will be found: which divided by the total of all the chances, or $r^{-nv} \times \{1-rv\}|^n \times \{1-r\}|^{-n}$, will be the true measure of the probability sought. From whence the advantage, by taking the mean of several observations, might be made to appear: but this will be shewn more properly in the next Proposition; which is better adapted, and to which this is premised as a Lemma.

REMARK

If r be taken $+1$, or the chances for the positive, and the negative errors be supposed accurately the same; then our expression, by expunging the powers of r. will be the very same with that shewing the chances for throwing $n+q$ points precisely, with n dice, each die having as many faces (w) as the result of any single observation can come out different ways. Which may be made to appear, independent of any [67] kind of calculation, from the bare consideration, that the chances of throwing, precisely, the number m, with n dice, whereof the faces of each, are numbered $-v, \ldots\ -3, -2, -1, -0, +1, +2, +3\ \ldots\ +v$, must be the very same as the chances by which the positive errors can exceed the negative ones by that precise number: but the former are, evidently, the same as the number $\{v+1\} \times n + m$ (or $n+q$) with the same n dice, when they are numbered in the common way, with the terms of the natural progression 1, 2, 3, 4, 5, and so on; because the number on each face being, *here*, increased by $v+1$, the whole increase upon all the (n) faces will be expressed by $\{v+1\} \times n$; so that there will be, now, the very same chances for the number $\{v+1\} \times n + m$, as there was before for the number m; since the chances for throwing any faces assigned will continue the same, however the faces are numbered.

PROPOSITION II.

Supposing the respective chances for the different errors, which any single observation can admit of, to be expressed by the terms of the series $r^{-v} + 2r^{1-v} + 3r^{3-v} \cdots + \{v+1\}.r^0 \ldots 3r^{v-2} + 2r^{v-1} + r^v$ *(whereof the coefficients, from the middle one* $(v+1)$ *decrease both ways, according to the terms of an arithmetical progression); it is proposed to find the probability, or odds, that the error, by taking the mean of a given number* (t) *of observations exceeds not a given quantity* $\left(\frac{m}{n}\right)$

Following the method laid down in the preceding proposition, the sum, or value of the series here proposed will appear to be $\frac{r^{-v} \times \{1-r^{v+1}\}|^2}{(1-r)^2}$ (being the same with the square of the geometrical progression $r^{\frac{1}{2}v} \times \{1+r+r^2+r^3 ..... + r^v\}$). And the power thereof whose exponent is t (by making $n = 2t$, and $w = v + 1$) will therefore be $r^{-tv} \times \{1 - r^w\}|^n \times \{1 - r\}|^{-n} = r^{-w} - nr^{w-tv} + \frac{n}{1}.\frac{n-1}{2}r^{2w-tv} -$ &c. into $1 + nr + \frac{n}{1}.$[68]$\frac{n+1}{2}r^2 + \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}r^3 +$ &c. Which two series being the same with those in the preceding Problem (excepting only, that the exponents in the former of them are expressed in terms of t, instead of n), it is plain, that, if q be here put $= tv + m$ (instead of $nv + m$) the conclusion here brought out will answer equally here; and consequently that the sum of all the chances, whereby the excess of positive errors, above the negative errors, can amount to the number m, precisely, will here, also, be truly defined by

$$\begin{aligned}
&+ \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q) \times r^m \\
&- \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q-w) \times nr^m \\
&+ \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q-2w) \times \frac{n}{1}.\frac{n-1}{2}r^m \\
&- \frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}.\frac{n+3}{4}(q-3w) \times \frac{n}{1}.\frac{n-1}{2}.\frac{n-2}{3}.r^m \\
&\qquad \text{\&c.} \qquad \text{\&c.}
\end{aligned}$$

But this general expression, as several of the factors in the numerators and denominators mutually destroy each other, may be transformed to another more commodious.

Thus the quantity $\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}(q)$, in the first line, by breaking the numerator and denominator, will become

$$\frac{n.(n+1).(n+2).(n+3)\ldots q.(q+1).(q+2).(q+3)\ldots.(q+n-1)}{1.\quad 2\quad .\quad 3\quad .\quad 4\quad \ldots.n.(n+1).(n+2).(n+3)\ldots q}$$

which, by equal division, is reduced to

$$\frac{\{q+n-1\}.\{q+n-2\}.\{q+n-3\}....q+1}{1 \qquad\qquad 2 \qquad\qquad 3} = \frac{p-1}{2}.\frac{p-2}{2}.\frac{p-3}{3}(n-1);$$

supposing $p = q + n = tv + m + n$.

In the very same manner, by making $q' = q - w$, and $p' = q' + n$ $(= p - w)$ it appears that $\frac{n}{1}.\frac{n+1}{2}.\frac{n+2}{3}(q-w)$ [69] $= \frac{p'-1}{1}.\frac{p'-2}{2}.\frac{p'-3}{3}(n-1)$ &c. Consequently our whole given expression (making $p'' = p - 2w$, $p''' - 3w$, &c.) will be transformed to

$$\begin{aligned}
&+ \frac{p-1}{1}.\frac{p-2}{2}.\frac{p-3}{3}(n-1) \times r^m \\
&+ \frac{p'-1}{1}.\frac{p'-2}{2}.\frac{p'-3}{3}(n-1) \times nr^m \\
&+ \frac{p''-1}{1}.\frac{p''-2}{2}.\frac{p''-3}{3}(n-1) \times \frac{n}{1}.\frac{n-1}{2}r^m \\
&+ \frac{p'''-1}{1}.\frac{p'''-2}{2}.\frac{p'''-3}{3}(n-1) \times \frac{n}{1}.\frac{n-1}{2}.\frac{n-2}{3}r^m \\
&\qquad \text{\&c.} \qquad \text{\&c.}
\end{aligned}$$

which expression is to be continued until some of the factors become nothing or negative; and which, with $r = 1$, is the very same with that exhibiting the number of chances for p points, precisely, on n dice, each having w faces.

And, in this case, where the chances for the errors in excess and in defect are the same, the solution is the most simple it can be; since, from the chances determined, answering to the number p precisely, the sum of the chances for all the inferior numbers to p, may be readily obtained, being given (from the method of increments equal to

$$\begin{aligned}
&+ \frac{p-1}{1}.\frac{p-2}{2}.\frac{p-3}{3}(n) - \frac{p'-1}{1}.\frac{p'-2}{2}.\frac{p'-3}{3}(n-1) \times (n) \times n \\
&+ \frac{p''-1}{1}.\frac{p''-2}{2}.\frac{p''-3}{3}(n) \times \frac{n}{1}.\frac{n-1}{2} \\
&- \frac{p'''-1}{1}.\frac{p'''-2}{2}.\frac{p'''-3}{3}(n) \times \frac{n}{1}.\frac{n-1}{2}.\frac{n-2}{3} + \text{\&c.}
\end{aligned}$$

The difference between which and half (w^n) the sum of all the chances (which difference I shall denote by D) will consequently be the true number of the chances whereby the errors in excess (or in defect) can fall within the given limit (m); so that $\frac{D}{\frac{1}{2}w^n}$ will be the true measure of the required probability, that the error, by taking the mean of t observations, exceeds not the quantity $\frac{m}{t}$ proposed.

[70] But now, to illustrate this by example, from whence the utility of the method in practice may clearly appear, it will be necessary, in the first place, to assign some number for v, expressing the limits of the errors to which any observation is subject. These limits indeed (as has been observed) depend on the goodness of the instrument, and the skill of the observer: but I shall here suppose, that every observation may be relied on, to five seconds; and that the chances, for the several errors $-5''$, $-4''$, $-3''$, $-2''$, $-1''$, $0''$, $+1''$, $+2''$, $+3''$, $+4''$, $+5''$ included within the limits thus assigned, are respectively proportional to the terms of the series 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1. Which series is much better adapted, than if all the terms were to be equal; since it is highly reasonable to suppose, that the chances for the respective errors decrease, in proportion as the errors themselves increase.

These particulars being premised, let it be now required to find what probability, or chance for an error of 1, 2, 3, 4, or 5 seconds will be, when, instead of relying on one, the mean of the observations is taken.

Here v being $= 5$, and $t = 6$, we shall have $n\ (= 2t) = 12$, $w\ (= v + 1) = 6$, and $p\ (= tv + n + m) = 42 + m$; but the value of m, if we first seek the chances whereby the error exceed not one second, will be found from the equation $\frac{m}{t} = \pm 1$; where either sign may be used (the chances being the same) but the negative one is commodious: from whence we have $m\ (= -t) = -6$; and therefore $p = 36$, $p' = 30$, $p'' = 24$, &c.

Which values being substituted in the general expression above determined, it will become $\frac{35}{1}.\frac{34}{2}.\frac{33}{3}.(12) - \frac{29}{1}.\frac{28}{2}.\frac{27}{3}(12) \times 12 + \frac{23}{1}.\frac{22}{2}.\frac{21}{3}(12) \times 66 - \frac{17}{1}.\frac{16}{2}.\frac{15}{3}(12) \times 220 = 299576368$: and this subtracted from 1088391168 ($= \frac{1}{2} \times 6^{12}$) leaves 78881480, for the value of D corresponding: therefore the required probability that the error, by taking the mean of the six observations, exceeds not a single *second*, will be truly measured by the fraction $\frac{78881480}{1088391168}$; and consequently the odds [71] will be as 78881480 to

299576368 or near as $2\frac{1}{2}$ to 1. But the odds, or proportion, when one single observation is taken, is only as 16 to 20, or as $\frac{8}{10}$ to 1.

To determine, now, the probability that the result comes within two seconds of the truth, let $\frac{m}{t}$ be made $= -2$; so shall $m\ (= -2t) = -12$; therefore $p = 30$, $p' = 24$, $p'' = 18$, &c. and our general expression will here come out = 36079407; whence $D = 1052311761$. Consequently $\frac{1052311761}{1088391168}$ will be the true measure of the probability sought: and the odds, or proportion of the chances, will therefore be that of 1052311761 to 36079407, or as 29 to 1, nearly. But the proportion, or odds, when a single observation is taken, is only as 2 to 1: so that the chance for an error exceeding two seconds, is not $\frac{1}{10}$th part so great, from the mean of six, as from one single observation. And it will be found in the same manner, that the chance for an error exceeding three seconds is not here $\frac{1}{1000}$th part so great as it will be from one observation only. Upon the whole of which it appears, that, the taking of the mean of a number of observations greatly diminishes the chances for all the smaller errors, and cuts off almost all possibility of any large ones: which last consideration alone is sufficient to recommend the use of the method, not only to *Astronomers*, but to all *Others* concerned in making experiments, or observations of any kind, which will allow of being repeated under the same circumstances.

In the preceding calculations, the different errors to which any observation is supposed subject, are restrained to whole quantities, or a certain, precise, number of seconds; it being impossible, from the most exact instruments, to take off the quantity of an angle to a geometrical exactness. But I shall now show how the chances may be computed, when the error admits of any value whatever, whole or broken, within the proposed limits, or when the result of each observation is supposed to be exactly known. [72]['Fig. 20' in the margin at this point] Let, then, the line AB represent the whole extent of the given interval, within which all the observations are supposed to fall; and conceive the same to be divided into an exceeding great number of very small, equal particles, by perpendiculars terminating in the sides AD, BD of an isoceles triangle ABD formed by the base AB: and let the probability or chance whereby the result of any observation tends to fall within any of these very small intervals Nn, be proportional to the corresponding area NMmn, or to the perpendicular NM; then, since these chances (or areas) reckoning from the extremes A and B. increase according to the terms of the arithmetical progression 1, 2, 3, 4, &c. it is evident that the case is here the same with that in the latter part of Prop. II; only, as the number v (expressing the particles in AC or BC) is indefinitely great, all (finite) quantities joined to v, or its multiples, with the signs of addition or subtraction, will here vanish, as being nothing in comparison to v. By which means the general expression D $\left(\frac{p-1}{1}.\frac{p-2}{2}.\frac{p-3}{3}(n) - \frac{p'-1}{1}.\frac{p'-2}{2}.\frac{p'-3}{3}(n) \times n + \frac{p''-1}{1}.\frac{p''-2}{2}.\frac{p''-3}{3}(n) \times n.\frac{n-1}{2}, \&c.\right)$ *there* determined, will *here* become $\frac{p}{1}.\frac{p}{2}.\frac{p}{3}(n) - \frac{p'}{1}.\frac{p'}{2}.\frac{3}{p}'(n) \times n$, &c. $= \frac{1}{1.2.3.4(n)} \times$ $\{p^n - np'^n + n.\frac{n-1}{2}p''^n - n.\frac{n-1}{2}.\frac{n-2}{3}'''^n$, &c. (wherein $p = tv \mp m$, $p' = p - m$, $p'' = p - 2v$, $p''' = p - 3v$, &c.) and therefore, the value of D in the present case being $\frac{1}{2}v^n - \frac{1}{1.2.3(n)} \times \{p^n - n.\{p-v\}|^n + n.\frac{n-1}{2}.\{p-2v\}|^n$, &c.$\}$ it is evident that the probability $\left(\frac{D}{\frac{1}{2}v^n}\right)$ of the error's not exceeding the quantity $\frac{m}{t}$ (in taking the mean of t observations) will truly be deformed by $1 - \frac{2}{1.2.3(n)} \times \left\{\frac{p}{v} - 1\right\}\Big|^n - n \times \left\{n \times \left\{\frac{p}{v} - 1\right\}\right\}\Big\}\,|^n +$

$\frac{n}{1}.\frac{n-1}{2}\times\left\{\frac{p}{v}-2\right\}\Big|^{n}$, &c. which may which may be represented by the curvilinear area CNFE, cor-[73]responding to the given value or abscissa CN ($=\frac{n}{t}$). Now, though the number v, p, and m are, all of them, here supposed to be indefinitely great, yet they may be exterminated, and the value of the expression determined, from their known relation to one another. For if the given ratio of $\frac{m}{t}$ to v of CN to CA, be expressed as that of x to 1, or, which is the same, if the error in question be supposed the x part of the greatest error; then m being $= tvx$, p $(= tv\mp m)$ will be $= tv\mp tvx$, and therefore $\frac{p}{v} = t\times\{1\mp x\}$; which let be denoted by y: then, by substitution, our last expression will become $1-\frac{2}{1.2.3(n)}\times\{y^n - n\{y-1\}|^n + \frac{n}{1}.\frac{n-1}{2}.\{y-2\}|^n - \frac{n}{1}.\frac{n-1}{2}.\frac{n-2}{3}.\{y-3\}|^n$, &c.$\}$ which series is to be continued till the quantities y, $y-1$, $y-2$, &c. become negative.

As an example of what is above delivered, let it be now required to find the probability, or odds that the error, by taking the mean of six observations, exceeds not a single second; supposing (as in the former example) that the greatest error, that any observation can admit of, is limited to five seconds.

Here t being $=6$, n $(=2t)=12$, and $x=\frac{1}{5}$, we have y $(=t\times\{1-x\})=4,8$; and therefore the probability will be equal to $1-\frac{2}{1.2.3(12)}\times\{\{4,8\}|^{12}-\{12\times 3,8\}|^{12}+\{66\times 2,8\}|^{12}-\{22\times 1,8\}|^{12}+495\times 0,8\}|^{12}\} = 0,7668$, nearly; so that the odds, that the error exceeds not a single second, will be as 0,7768 to 0,2332; which is more than three to one. But the proportion, when one single observation is relied on, is only as 36 to 64, or as 9 to 19. In the same manner, taking $x=\frac{2}{5}$, it will be found, that [74] the odds, of the error's not exceeding two seconds, when the mean of six observations is taken, will be as 0,985 to 0,015, nearly, or as 65 to 1; whereas the odds on one single observation, is only as 6 4 to 36, or as $\frac{17}{9}$ to 1: so that the chance for an error of two seconds is not $\frac{1}{20}$th part so great, from the mean of six, as from one single observation. And it will farther appear, by making $x=3$, that the probability of an error of three seconds, here, is not $\frac{1}{1400}$th part so great as from one single observation: so that in this, as well as in the former hypothesis, almost all probabilities of any large error is cut off. And the case will be found the same, whatever the hypothesis is assumed to express the chances for the errors to which any single observation is subject.

From the same general expression by which the foregoing properties are derived, it will be easy to determine the odds, that the mean of a given number of observations is nearer to the truth than one single observation, taken indifferently. For, if z be put $(=1-x)\ =\frac{y}{t}$, and $s=\frac{1}{t}$, then, y being $=tz$, the quantity

$$\frac{2}{1.2.3(n)}\times\left\{y^n - n.\{y-1\}|^n + n.\frac{n-1}{2}.\{y-2\}|^n, \&\text{c.}\right\}$$

(expressing the probability that the result falls within the distance z of the greatest limit) will here, by substitution, become

$$\frac{2t^n}{1.2.3(n)}\times\left\{z^n - n.\{z-s\}|^n + n.\frac{n-1}{2}.\{z-2s\}|^n, \&\text{c.}\right\}$$

which, in the case of one single observation (when $t=1$, and $n=2$) is barely z^2, and its fluxion $2z\dot{z}$: therefore, if we now multiply by $2z\dot{z}$, the product

$$\frac{2t^n}{1.2.3(n)}\times\left\{z^{n+1}\dot{z} - n.\{z-s\}|^n.z\dot{z} + n.\frac{n-1}{2}.\{z-2s\}|^n z\dot{z}, \&\text{c.}\right\}$$

will give the fluxion of the probability that the result of t observations is farther from the truth, or nearer to the limits, than one single observation taken indifferently. And consequently the fluent thereof, which is $\frac{4t^n}{1.2.3(n)}$ into $\frac{z^{n+2}}{n+2}$ [75] $-\frac{n}{1}\times\left\{\frac{s.\{z-s\}|^{n+1}}{n+1}+\frac{\{z-s\}|^{n+2}}{n+2}\right\}+\frac{n}{1}.\frac{n-1}{2}\times\left\{\frac{2s.\{z-2s\}|^{n+1}}{n+1}+\frac{\{z-2s\}|^{n+2}}{n+2}\right\}$ &c. will, when $z = 1$, be the true measure of the probability itself which in the case above proposed, where $t = 6$ and $n = 12$, will be found $= 0, 245$, and consequently, the odds that the mean of six, is nearer to the truth than one single observation, as 755 to 245, or as 151 to 49.