Review of *Introduction to Algebraic and Constructive Quantum Field Theory*, by J.C. Baez, I.E. Segal, and Z. Zhou.

For *Physics Today*, by Mark A. Kon

Mathematical quantum field theory (in its many current forms) has become an established and diverse area of physics as well as of mathematics. Work in the area began in in earnest in the 1950's, after the realization that elaborate and apparently ad-hoc mathematical constructs were required for the construction of a quantum field theory with phenomenological value. As in the case of Feynman's path integral approach to quantum mechanics (for which there is in fact no underlying measure space in the classical sense), there are in quantum field theory strong and almost overwhelming suggestions of an underlying mathematically cohesive and relatively simple description of the phenomenology, though serious difficulties with divergences arise when such canonical-looking theories are implemented in the infinite dimensional home of quantum field theory. Dealing with such problems in a mathematically rigorous and understandable way was the essential purpose of the mathematical approach.

The primary workers in this field have included, among others, James Glimm, Arthur Jaffe, Oscar Lanford, Edward Nelson, Barry Simon, and Arthur Wightman. Irving Segal, a major contributor since the area's inception, was one of the first to introduce a probabilistic approach to solving the problems of this area, and this approach has become widely used in the course of the field's development. His book with John Baez and Zhengfang Zhou approaches the construction of quantum fields in two space-time dimensions through a very general algebraically oriented approach, which is then specialized to the construction of two space-time dimensional fields.

This book emphasizes a methodology (algebraic quantization) for quantizing arbitrary linear systems (i.e., transforming classical linear systems to quantum ones). It gives a description of Fock space in its particle representation, and describes the quantization of single particle operators, i.e., their extension to many particle operators on Fock space. Here Weyl systems, the exponentiated versions of the canonical commutation relations, are introduced rigorously. Functional integral representations of Fock space are presented as well; these, in their two forms, have the advantage of diagonalizing momentum or creation operators. The book also introduces the analogs of such systems for fermions (in addition to the boson structures studied initially), in implementations of the canonical anti-commutation relations. Quantizations of general symplectic and orthogonal systems are also introduced in this context.

The issue of unitary implementability of canonical transformations is discussed throughout the book, and a remedy to the problem of implementability of infinite dimensional canonical transformations is considered in a study of the C^*-algebraic approach to quantum field theory.

Specializations of the algebraic approach are given to quantizations of linear differential equations (such as the Schrödinger, Klein-Gordon, and Dirac equations), and then the process of constructing a nonlinear field theory begins. Renormalized products of quantum field operators ϕ(x,t) are defined, and finally are used to complete the rigorous construction of a continuum quantum field in two space-time dimensions.

A benefit of the abstract algebraic approach is that it describes in a general context results which sometimes seem quite specialized. The book presents the quantization process for the two space-time dimensional theory elegantly, in operations on a (single particle) Hilbert space with a distinguished family of physical operators. This differs from other approaches, which specify physical wave equations and their related Hilbert spaces earlier on, and use analytic techniques to quantize the equations. The process of quantizing hyperbolic equations is clarified in the book, and, in the end, there is a renormalized quantum field theory in two space-time dimensions, with the overview of a general mathematical context to draw on. The book reveals how this quantization applies, for example, to the quantized Schrödinger equation (for which there is an initial complex Hilbert space) and also other physical systems for which a real Hilbert space is natural, and which need to be complexified before the general *Ansatz* of quantization can be accomplished.

The book's intended audience consists of mathematicians and mathematically inclined physicists. The level of the book presumes a knowledge of functional analysis, though there is a novel glossary which explicates a large portion of the background required. The glossary explains terms in the book, and includes much of the functional analytic background information necessary. It is a very useful tool.

The book is quite well put together, with few typographical errors and an index, along with the above-mentioned glossary. In any text with mathematical notation a list of symbols is a great convenience, and this book has one. To the present reviewer, the book provides an important overview and a useful synthesis of some significant contributions to the mathematical understanding of quantum field theory.

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