2001 Veblen Prize

Jeff Cheeger

Yakov Eliashberg

Michael J. Hopkins

Oswald Veblen (1880–1960), who served as president of the AMS in 1923 and 1924, was well known for his work in geometry and topology. In 1961 the trustees of the Society established a fund in memory of Veblen, contributed originally by former students and colleagues and later doubled by his widow. Since 1964 the fund has been used for the award of the Oswald Veblen Prize in Geometry. Subsequent awards were made at five-year intervals. The current amount of the prize is $4,000 (in case of multiple recipients, the amount is divided equally).

At the Joint Mathematics Meetings in New Orleans in January 2001, the 2001 Veblen Prize was presented to JEFF CHEEGER, YAKOV ELIASHBERG, and MICHAEL J. HOPKINS.

Previous recipients of the Veblen Prize are: Christos D. Papakyriakopolous and Raoul H. Bott (1964); Stephen Smale, Morton Brown, and Barry Mazur (1966); Robion C. Kirby and Dennis P. Sullivan (1971); William P. Thurston and James Simons (1976); Mikhael Gromov and Shing-Tung Yau (1981); Michael H. Freedman (1986); Andrew Casson and Clifford H. Taubes (1991); and Richard Hamilton and Gang Tian (1996).

The Veblen Prize is awarded by the AMS Council, acting through a selection committee whose members at the time of this award were Mikhael Gromov (chair), Richard S. Hamilton, Robion C. Kirby, and Gang Tian. The text that follows contains, for each of the three prize recipients, the prize citation, a biographical sketch, and the response of the recipient upon receiving the prize.

Jeff Cheeger

Citation

The 2001 Veblen Prize in Geometry is awarded to Jeff Cheeger for his work in differential geometry. In particular the prize is awarded for:

1. His works on the space of Riemannian metrics with Ricci curvature bounded from below, such as his rigidity theorems for manifolds of nonnegative Ricci curvature and his joint efforts with Colding on the structure of the space of metrics with Ricci curvature bounded from below. These works led to the resolution of various conjectures in Riemannian geometry and provided significant understanding of how singularities form in the degeneration of Einstein metrics on Riemannian manifolds.

J. Cheeger and T. H. Colding, "Lower bounds on the Ricci curvature and the almost rigidity of warped products", *Ann. of Math.* **144** (1996), 189–237.

J. Cheeger and T. H. Colding, "On the structure of spaces with Ricci curvature bounded below. I", *J. Differential Geom.* **45** (1997), 1–75.

2. His works on eta invariants and index theory, e.g.,

J. M. Bismut and J. Cheeger, "The index theorem for families of Dirac operators on manifolds with boundary; superconnections and cones; I, II",

J. Funct. Anal. **89**, no. 2 (1990), 313–363; and **90**, no. 2 (1990), 306–354.

Biographical Sketch

Jeff Cheeger was born on December 1, 1943, in Brooklyn, New York. He graduated from Harvard University in 1964 and received his Ph.D. from Princeton University in 1967. After one-year stays at the University of California, Berkeley, and the University of Michigan, he joined the mathematics department at the State University of New York, Stony Brook. Since 1989 he has been a member of the Courant Institute of Mathematical Sciences, New York University.

Cheeger has been the recipient of National Science Foundation Postdoctoral, Sloan, and Guggenheim Fellowships, as well as of a Max Planck Research Award from the Alexander von Humboldt Society. He gave the Marston Morse Lectures at the Institute for Advanced Study in Princeton in 1992, and 45-minute invited addresses at the International Congress of Mathematicians in 1974 and in 1986. He is a member of the National Academy of Sciences and a foreign member of the Finnish Academy of Science and Letters.

Response

I am very honored to be named, along with Yasha Eliashberg and Michael Hopkins, as a recipient of the Veblen Prize. It is especially gratifying to be recognized for work on subjects that have been central to my research throughout my career. My work on these subjects includes joint efforts with a number of wonderful collaborators: Mike Anderson, Jean-Michel Bismut, Toby Colding, and Detlef Gromoll.

A unifying theme is the key role played by spaces possessing a simple canonical geometric structure, spaces which split off lines as isometric factors and metric cones.

Rigidity theorems for Ricci curvature assert that geometric conditions that cannot be realized if the Ricci curvature has a particular strict lower bound are realized only in the presence of special structure if "strict" is relaxed to "weak". In almost rigidity theorems both the hypotheses and conclusions hold up to a specified error. If the Ricci curvature has a definite lower bound, then on a small but definite scale, after rescaling, the hypotheses of certain almost rigidity theorems for nonnegative Ricci curvature—the splitting theorem and volume cone imply the metric cone theorem—are satisfied "generically". So these theorems govern the small-scale structure of such manifolds and, in limiting cases, the structure of the singular set.

Gromoll and I proved the splitting theorem for nonnegative Ricci curvature in 1970. Colding's and my almost rigid version took another twenty-five years. Precursors of our work include Bochner's formula (1946), Toponogov's splitting theorem for nonnegative sectional curvature (1959), Bishop's inequality (1963), Cheng-Yau's gradient estimate (1975), Gromov's compactness theorem for the Gromov-Hausdorff distance (1981), Anderson's convergence theorems (1989), Abresch-Gromoll's inequality (1990), and the new techniques introduced by Colding (1994) in proving conjectures of Anderson's and mine.

A point that came up in my study of analytic torsion suggested that one could do index theory L_2-cohomology on singular spaces (circa 1975). The most basic case was that in which a metric cone was attached to the boundary of a manifold with boundary. The resulting index formula for the signature operator was identical to that of Atiyah-Patodi-Singer, but rather than being associated to the boundary, the η-invariant term arose from the singularity. Much later this turned out to be of technical importance in Bismut's and my proof of the families index theorem for manifolds with boundary. There the Bismut superconnection was also crucial.

I want to express my gratitude to my father, Thomas Cheeger, who first aroused my passion for mathematics; to my teachers, Salomon Bochner, Raoul Bott, Jim Simons, and Shlomo Sternberg; to S. S. Chern and Is Singer; and to my collaborators. All have helped me immeasurably.

Yakov Eliashberg

Citation

The 2001 Veblen Prize in Geometry is awarded to Yakov Eliashberg for his work in symplectic and contact topology. In particular the prize is awarded for:

1. His proof of the symplectic rigidity, presented in his ICM talk:

"Combinatorial methods in symplectic geometry", *Proceedings of the International Congress of Mathematicians*, Vols. 1, 2 (Berkeley, California, 1986), Amer. Math. Soc., Providence, RI, 1987, pp. 531–539.

2. The development of 3-dimensional contact topology, presented in the papers:

"Classification of overtwisted contact structures on 3-manifolds", *Invent. Math.* **98**, no. 3 (1989), 623–637; and "Invariants in contact topology", *Proceedings of the International Congress of Mathematicians*, Vol. II (Berlin, 1998).

Biographical Sketch

Yakov Eliashberg was born in December 1946 in Leningrad, USSR. He received his Ph.D. from Leningrad University in 1972 under the direction of V. A. Rokhlin. From 1972 to 1979 he taught at the Syktyvkar University of Komi Republic of Russia and from 1980 to 1987 worked in industry as the head of a computer software group. In 1988 Eliashberg moved to the United States, and since 1989 he has been a professor of mathematics at Stanford University. Eliashberg received the Leningrad Mathematical Society Prize in 1972. In 1986 and in 1998 he was an invited speaker at the

International Congress of Mathematicians. He delivered the Porter Lectures at Rice University (1992), the Rademacher Lectures at the University of Pennsylvania (1996), the Marston Morse Lectures at the Institute for Advanced Study (1996), the Frontiers in Mathematics Lectures at Texas A&M University (1997), and the Marker Lectures at Pennsylvania State University (2000). In 1995 Eliashberg was a recipient of the Guggenheim Fellowship.

Response

I am greatly honored to be a corecipient of the Oswald Veblen Prize of the AMS along with such outstanding mathematicians as Jeff Cheeger and Mike Hopkins. Symplectic geometry and topology has flourished during the last two decades, and I am happy that I was able to contribute to its success. I want first to thank my wife, Ada, for her lifelong support. I am also grateful to N. M. Mitrofanova, who converted me to mathematics from music when I was in school, and to Professor V. A. Rokhlin, who shared with me his topological insights when I was a student in Leningrad University. I was greatly influenced by my friend and colleague Misha Gromov. From him I learned about symplectic structures; then I struggled to find a balance between the apparent flexibility and hidden rigidity of the symplectic world. I am grateful to Misha for sharing his vision with me. My gratitude also goes to V. I. Arnold for many stimulating and critical discussions, and to D. B. Fuchs, who invested a lot of time to help me clarify the proof of my first result in symplectic geometry, the Arnold conjecture for surfaces. I owe a lot to all my coauthors: R. Brooks, M. Fraser, A. Givental, M. Gromov, H. Hofer, C. McMullen, N. M. Mishachev, L. Polterovich, T. Ratiu, D. Salamon, and W. P. Thurston. Finally, I thank my colleagues and students at the Stanford mathematics department for creating a stimulating environment for geometric research.

Michael J. Hopkins

Citation

The 2001 Veblen Prize in Geometry is awarded to Michael J. Hopkins for his work in homotopy theory. In particular the prize is awarded for:

1. His work on nilpotence and periodicity, beginning with his work with coauthors Ethan Devinatz and Jeff Smith, "Nilpotence and stable homotopy theory, I and II", *Ann. of Math.* **128** (1988), 207–241; and **148** (1998), 1–49.

2. His work on rigid analytic geometry and its application to homotopy theory, represented in his papers with Dick Gross, "Equivariant vector bundles on the Lubin-Tate moduli space", *Contemp. Math.* **158** (1994), 23–88; and "The rigid analytic period mapping, Lubin-Tate space, and stable homotopy theory", *Bull. Amer. Math. Soc. (N.S.)* **30** (1994), 76–86.

3. His work on elliptic spectra, represented in part in the paper with Matthew Ando and Neil Strickland, "Elliptic spectra, the Witten genus, and the theorem of the cube".

Biographical Sketch

Michael Hopkins was born on April 18, 1958, in Alexandria, Virginia. He received his Ph.D. from Northwestern University in 1984 under the direction of Mark Mahowald. In 1984 he also received his D.Phil. from the University of Oxford under the supervision of Ioan James.

Hopkins has held the position of professor of mathematics at the Massachusetts Institute of Technology since 1990 after a few years of teaching at Princeton University, a one-year position with the University of Chicago, and a visiting lecturer position at Lehigh University. He gave invited addresses at the 1990 Winter Meeting of the American Mathematical Society in Louisville, Kentucky, and at the the 1994 International Congress of Mathematicians in Zurich. He presented the 1994 Everett Pitcher Lectures at Lehigh University, the 2000 Namboodiri Lectures at the University of Chicago, and the 2000 Marston Morse Memorial Lectures at the Institute for Advanced Study, Princeton. At the 2001 Joint Mathematics Meetings, where he received the Veblen Prize, he presented an AMS Invited Address. Hopkins received postdoctoral fellowships from the National Science Foundation and the Sloan Foundation, a Rhodes Scholarship, and a Presidential Young Investigator award.

Response

It is a great pleasure to receive the Oswald Veblen Prize for my work in homotopy theory. The mathematical world would be very different had it not been for the efforts of Oswald Veblen, and I feel both humbled and honored to be given the prize that bears his name.

One of the things that enchants me the most about mathematics is its capacity for sudden and profound transformations of context. I have been lucky to take part in this many times. Homotopy theory meets geometry in both its geometric and function theoretic aspects, cannily straddling our most fundamental metaphor. It has taken me on a remarkable journey, and I am excited by the prospects for its future.

I am indebted to many teachers, friends, and collaborators, but I would especially like to thank Mark Mahowald, Ib Madsen, and Is Singer for all that they have given me. I would also like to thank the selection committee. It is an honor to receive this award and to share it with Jeff Cheeger and Yakov Eliashberg, both of whom have my admiration.