2002 Steele Prizes

The 2002 Leroy P. Steele Prizes were awarded at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–06, and Birkhoff served in that capacity during 1925–26. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Mathematical Exposition: for a book or substantial survey or expository-research paper; (2) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research; and (3) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students. Each Steele Prize carries a cash award of $5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2002 prizes, the members of the selection committee were: M. S. Baouendi, Sun-Yung A. Chang, Michael G. Crandall, Constantine M. Dafermos, Daniel J. Kleitman, Hugh L. Montgomery, Barry Simon, S. R. S. Varadhan (chair), and Herbert S. Wilf.

The list of previous recipients of the Steele Prize may be found in the November 2001 issue of the *Notices*, pages 1216–20, or on the AMS website at `http://www.ams.org/prizes-awards/`.

The 2002 Steele Prizes were awarded to Yitzhak Katznelson for Mathematical Exposition, to Mark Goresky and Robert MacPherson for a Seminal Contribution to Research, and to Michael Artin and Elias Stein for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Mathematical Exposition: Yitzhak Katznelson

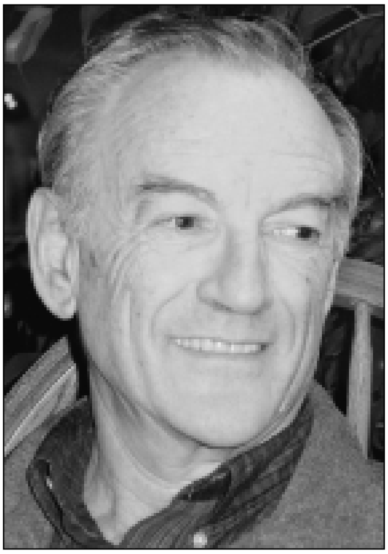
Citation

Although the subject of harmonic analysis has gone through great advances since the sixties, Fourier analysis is still its heart and soul. Yitzhak Katznelson's book on harmonic analysis has withstood the test of time. Written in the sixties and revised later in the seventies, it is one of those "classic" Dover paperbacks that has made the subject of harmonic analysis accessible to generations of mathematicians at all levels.

The book strikes the right balance between the concrete and the abstract, and the author has wisely chosen the most appropriate topics for inclusion. The clear and concise exposition and the presence of a large number of exercises make it an ideal source for anyone who wants to learn the basics of the subject.

Biographical Sketch

Yitzhak Katznelson was born in Jerusalem in 1934. He graduated from the Hebrew University with a



Yitzhak Katznelson

Mark Goresky

Robert MacPherson

master's degree in 1956 and obtained the Dr. és Sci. degree from the University of Paris in 1959.

After a year as a lecturer at the University of California, Berkeley, and a few more at the Hebrew University, Yale University, and Stanford University, he settled in Jerusalem in 1966. Until 1988 he taught at the Hebrew University, while making extended visits to Stanford and Paris. He is now a professor of mathematics at Stanford University.

Katznelson's mathematical interests include harmonic analysis, ergodic theory (and in particular its applications to combinatorics), and differentiable dynamics.

Response

What a pleasant surprise!

I am especially gratified by the committee's approval of "the balance between the concrete and the abstract," which was one of my main concerns while teaching the course and while developing the notes into a book.

How should one look at things, and in what generality? If a statement and its proof apply equally in an abstract setup, should it be introduced in the most general or the most familiar terms?

When I came to Paris in 1956 I heard a rumor that the old way of doing mathematics was being replaced by a new, "abstract" fashion which was the only proper way of doing things. The rumor was spread mostly by younger students—typically hugging a freshly-purchased volume of Bourbaki—but seemed confirmed also by the way some courses were taught.

As late as 1962, Kahane and Salem found the need to apologize (undoubtedly tongue-in-cheek) in the preface to their exquisite book *Ensembles Parfaits et Séries Trigonométriques* for dealing with subject matter that might be considered too concrete.

The balance I tried to strike in the book—and I believe that I was strongly influenced by Kahane and Salem—was to set up the subject matter in the most concrete terms and allow as much generality and abstraction as needed for development, methods, and solutions.

Seminal Contribution to Research: Mark Goresky and Robert MacPherson

Citation

In two closely related papers, "Intersection homology theory", *Topology* 19 (1980), no. 2, 135–62 (IH1) and "Intersection homology. II", *Invent. Math.* 72 (1983), no. 1, 77–129 (IH2), Mark Goresky and Robert MacPherson made a great breakthrough by discovering how Poincaré duality, which had been regarded as a quintessentially manifold phenomenon, could be effectively extended to many singular spaces. Viewed topologically, the key difficulty had been that Poincaré duality reflects the transversality property that holds within a manifold but which fails in more general spaces. IH1 introduced "intersection chain complexes", which are the subcomplexes of usual chain complexes consisting of those chains which satisfy a transversality condition with respect to the natural strata of a space. More precisely, by introducing a kind of measure, called a "perversity", of the amount of variation from transversality a chain would be allowed, Goresky and MacPherson actually introduced a parametrized family of intersection chain complexes. Each of these yielded a corresponding sequence of intersection homology groups, and these theories intermediated between homology and cohomology. Starting with methods of local piecewise-linear transversality that had been developed by investigations of M. Cohen, E. Akin, D. Stone, and C. McCrory, IH1 showed that its intersection homology theories were related to each other by a version of Poincaré duality; in particular, the intersection homology theory which was positioned midway between homology and cohomology satisfied, when defined, a self-duality, as was familiar for manifolds. This immediately yielded a signature invariant for many singular varieties, and that, in turn, was used in IH1 to yield, in analogy with the Thom-Milnor treatment of piecewise linear manifolds, rational characteristic classes for many triangulated singular varieties. However, these characteristic classes of singular varieties naturally were elements in homology rather than cohomology groups, a distinction which for singular varieties was significant.

The continuation paper, IH2, reformulated this theory in a natural and powerful sheaf language. This language, suggested by Deligne, gave local formulations of a version of Poincaré duality for singular spaces in terms of a Verdier duality of sheaves. Furthermore, IH2 presented beautiful axiomatic characterizations of its intersection chain sheaves. These were all the more valuable as the achievement of duality for nonsingular spaces came at the cost of giving up the familiar functorial and homotopy properties that characterized usual homology theories; in particular, intersection homology theory is not a "homology theory" in the sense of homotopy theory.

IH1 and IH2 made possible investigations across a great spectrum of mathematics which further extended key classical manifold phenomena and methods to singular varieties and used these to solve well-known problems. While it is impossible to list all of these, a few important ones in 1) differential geometry, 2) algebraic geometry and representation theory, 3) geometrical topology, and 4) geometrical combinatorics will be indicated.

1) An immediate question was the relation of intersection homology theory to an analytic theory of L^2 differential forms and L^2 cohomology on suitable singular varieties with metrics that J. Cheeger had concurrently developed. In fact, for many metrics the resulting groups were seen to be isomorphic by a generalization of the classical de Rham isomorphism of manifold theory. Questions about when and how this can be generalized to various natural metrics have since occupied many investigators.

2) The work of IH2 led to the discovery of the important category $P(X)$ of perverse sheaves on an algebraic variety X. In the case when X is a smooth algebraic variety over a field of characteristic zero the [generalized] Riemann-Hilbert theorem says that the category $P(X)$ is equivalent to the category of D-modules on X. This equivalence made possible the applications of Grothendieck's yoga to the theory of D-modules and, in particular, to the formulation and proof of the Kazhdan-Lusztig conjecture, which gives a formula for characters of reducible representations of Lie groups in terms of intersection homology of the closures of Schubert cells. In the case when X is an algebraic variety over a finite field F, $P(X)$ is used in investigating "good" functions on the points $X(F)$ of X over F. This is the basic ingredient in the geometrization of representation theory which has had remarkable successes in recent years.

3) Paul Segal used the methods of Goresky and MacPherson and a cobordism theory of singular varieties to show that their rational characteristic classes could in many cases be lifted, after inverting 2, to a KO-homology class. Intersection chain sheaves were extensively used in various collaborations of Cappell, Shaneson, and Weinberger which extended results of classical Browder-Novikov-Sullivan-Wall surgery theory of manifolds to yield topological classifications of many singular varieties, which developed new invariants for singular varieties and their transformation groups, which gave methods of computing the characteristic classes of singular varieties, and which related these to knot invariants.

4) In investigations of the geometrical combinatorics of convex polytopes, the intersection homology groups of their associated toric varieties have become a fundamental tool. This began with R. Stanley's investigations of the face vectors of polytopes. A calculation of the Goresky-MacPherson characteristic classes of toric varieties was used by Cappell and Shaneson in obtaining an Euler-MacLaurin formula with remainder for lattice sums in polytopes. Recent works of MacPherson and T. Braden on flags of faces of polytopes used results on the intersection chain sheaves of toric varieties. The already astonishing range of research areas influenced by this seminal work continues to grow.

Biographical Sketch: Mark Goresky

Mark Goresky received his B.Sc. from the University of British Columbia in 1971 and attended graduate school at Brown University. He spent the 1974–75 academic year at the Institut des Hautes Études Scientifiques and received his Ph.D. in 1976. He was a C. L. E. Moore Instructor at the Massachusetts Institute of Technology (1976-78) and an assistant professor at UBC (1978-81). In 1981 he moved to Northeastern University, where he eventually attained the rank of professor with a joint appointment in mathematics and computer science. Since 1995 he has lived in Princeton, New Jersey, where he is currently a member at the Institute for Advanced Study. He has held other visiting positions at the University of Chicago, the Max-Planck-Institut für Mathematik, the IHÉS, and the University of Rome.

Goresky received a Sloan Fellowship in 1981. He is a fellow of the Royal Society of Canada, and he received the Coxeter-James Award (1984) and the Jeffrey-Williams Prize (1996) from the Canadian Mathematical Society.

Biographical Sketch: Robert MacPherson

Robert MacPherson received a B.A. from Swarthmore College and a Ph.D. from Harvard University.

He held faculty positions at Brown University from 1970 to 1987, at MIT from 1987 to 1994, and at the Institute for Advanced Study since then. Over the years, he has held visiting positions at the Institut des Hautes Études Scientifiques in Paris, Université de Paris VII, Steklov Institute in Moscow, IAS in Princeton, Università di Roma I, University of Chicago, Max-Planck-Institut für Mathematik in Bonn, and Universiteit Utrecht. He received the National Academy of Sciences Award in Mathematics and honorary doctorates from Brown University and Université de Lille. He served as chair of the National Research Council's Board on Mathematical Sciences from 1997 to 2000. He is a member of the American Academy of Arts and Sciences, the National Academy of Sciences, and the American Philosophical Society.

Response

We are very grateful to the American Mathematical Society for awarding us the Steele Prize. We are particularly pleased to receive a joint prize for our joint research. We know of no other mathematical prize that is awarded jointly to the participants of a collaboration. Given the increasing role of collaborative research in mathematics, this policy on the part of the AMS seems particularly enlightened to us.

In September 1974 we began a year at the Institut des Hautes Études Scientifiques with a pact to try to understand what intersection theory should mean for singular spaces. We thought the question might have importance for several areas of mathematics, given the ubiquity with which singular spaces naturally arise. By late autumn, we had found intersection homology and Poincaré duality. Jeff Cheeger, Pierre Deligne, Clint McCrory, John Morgan, and Dennis Sullivan played significant roles in the early stages of this research.

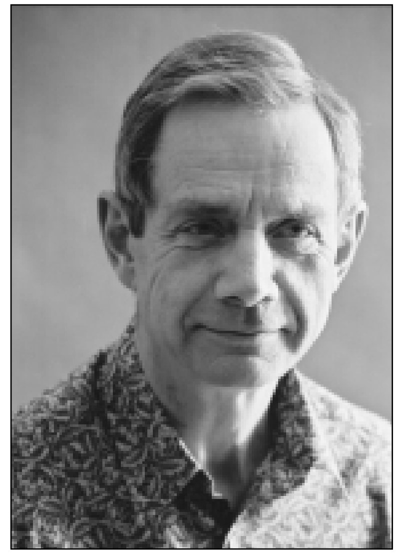
Starting around 1980, an explosion of activity surrounding intersection homology occurred. Our dream that the subject would find applications suddenly became true. Many mathematicians contributed a remarkable collection of ideas to this activity, and our collaboration was swept along with this flow into new fields such as combinatorics and automorphic forms.

Today, extensions and applications of the theory are pursued by a new generation of highly talented mathematicians, some of whom have already received mathematical awards in Europe (where prizes for younger mathematicians are more common). It is gratifying to see that these ideas, in whose discovery we participated, are now in such capable hands.

Lifetime Achievement: Michael Artin

Citation

Michael Artin has helped to weave the fabric of modern algebraic geometry. His notion of an algebraic space extends Grothendieck's notion of scheme. The point of the extension is that Artin's theorem on approximating formal power series solutions allows one to show that many moduli spaces are actually algebraic spaces and so can be studied by the methods of algebraic geometry. He showed also how to apply the same ideas to the algebraic stacks of Deligne and Mumford. Algebraic stacks and algebraic spaces appear everywhere in modern algebraic geometry, and Artin's methods are used constantly in studying them.



Michael Artin

He has contributed spectacular results in classical algebraic geometry, such as his resolution (with Swinnerton-Dyer in 1973) of the Shafarevich-Tate conjecture for elliptic $K3$ surfaces. With Mazur, he applied ideas from algebraic geometry (and the Nash approximation theorem) to the study of diffeomorphisms of compact manifolds having periodic points of a specified behavior.

For the last twenty years he has worked to create and define the new field of noncommutative algebraic geometry.

Artin has supervised thirty doctoral students and influenced a great many more. His undergraduate algebra course was for many years one of the special features of an MIT education; now some of that insight is available to the rest of the world through his textbook.

Biographical Sketch

I have departed from the usual format here to write a bit about my early life and the origins of my interest in mathematics.

When I was nearly forty years old I had a revelation: A recurring dream that I'd had since age twelve was an allegory of my birth! In the dream, I was stuck in a secret passage in our house but eventually worked my way out and emerged into a sunlit cupola. After my revelation, the dream went away.

My mother says that I was a big baby and it was a difficult birth, although I don't know what I weighed. The conversion from German to English pounds adds ten percent, and I suspect that my mother added another ten percent every few years. She denies this, of course. Anyway, I'm convinced that a birth injury caused my left-handedness and some seizures, which, fortunately, are under control.

The name Artin comes from my great-grandfather, an Armenian rug merchant who moved to Vienna in the nineteenth century.

Armenians were declared "Aryan" by the Nazis, but one side of my mother's Russian family was Jewish, and because of this, my father Emil was fired from the university in Hamburg. We came to America in 1937, when I was three years old.

My father loved teaching as much as I do, and he taught me many things: sometimes mathematics, but also the names of wild flowers. We played music and examined pond water. If there was a direction in which he pointed me, it was toward chemistry. He never suggested that I should follow in his footsteps, and I never made a conscious decision to become a mathematician.

I had decided to study science when I began college, but fields such as chemistry and physics gradually fell away, until biology and mathematics were the only ones left. I loved them both, but decided to major in mathematics. I told myself that changing out of mathematics might be easier, since it was at the theoretical end of the science spectrum, and I planned to switch to biology at age thirty when, as everyone knew, mathematicians were washed up. By then I was too involved with algebraic geometry. My adviser Oscar Zariski had seen to that.

Response

I thank the AMS and the prize committee for choosing to award me the Steele Prize for Lifetime Achievement, and I congratulate my fellow recipient Eli Stein. This award gives me great pleasure.

I also want to thank the many inspiring people who have surrounded me throughout my career. It has been a privilege to teach at MIT, where the students are gifted and motivated, and where my colleagues are as deserving of an award for lifetime achievement as I am. My thesis students there have been a constant source of inspiration. The financial support provided by the National Science Foundation for my work has been invaluable.

Alexander Grothendieck, Barry Mazur, John Tate, and of course my thesis adviser Oscar Zariski, are among the people who influenced me the most during the 1950s and 1960s. Those were exciting times for algebraic geometry. The crowning achievement of the Italian school, the classification of algebraic surfaces, was just entering the mainstream of mathematics. The sheaf theoretic methods introduced by Jean-Pierre Serre were being absorbed, and Grothendieck's language of schemes was being developed. Zariski's dynamic personality, and the explosion of activity in the field, persuaded me to work there. I became his student along with Peter Falb, Heisuke Hironaka, and David Mumford. Later, in the 1960s, I visited the Institut des Hautes Études Scientifiques several times to work with Grothendieck and Jean-Louis Verdier.

My interest in noncommutative algebra began with a talk by Shimshon Amitsur and a visit to Chicago, where I met Claudio Procesi and Lance Small. They prompted my first foray into ring theory, and in subsequent years noncommutative algebra gradually attracted more of my attention. I changed fields for good in the mid-1980s, when Bill Schelter and I did experimental work on quantum planes using his algebra package, Affine.

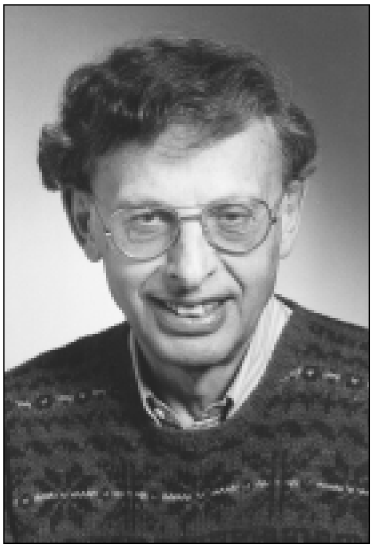
My early training has led me to concentrate on dimension two, or noncommutative surfaces. They display many interesting phenomena which remain to be explained, and I've come to understand that two is a critical dimension. Thanks to recent work of people such as Johan de Jong, Toby Stafford, and Michel Van den Bergh, the methods of algebraic geometry are playing a central role in this area too, and I hope to see it absorbed into the mainstream in the near future.

Lifetime Achievement: Elias Stein

Citation

During a scientific career that spans nearly half a century, Eli Stein has made fundamental contributions to different branches of analysis.

In harmonic analysis, his Interpolation Theorem is a ubiquitous tool. His result about the relation between the Fourier transform and curvature revealed a deep and unsuspected property and has far reaching consequences. His work on Hardy spaces has transformed the subject. He has made important contributions to the representation theory of Lie groups as well.



Elias Stein

His work on several complex variables is equally striking. His explicit approximate solutions for the $\bar{\partial}$-problems made it possible to prove sharp regularity results for solutions in strongly pseudoconvex domains. In this connection he also obtained subelliptic estimates which sharpened and quantified Hörmander's hypoellipticity theorem for second order operators.

Besides his contributions through his own research and excellent monographs, Stein has worked with and influenced many students, who have gone on to make profound contributions of their own.

Biographical Sketch

Elias M. Stein was born in Belgium in 1931 and came to the U.S. at the age of ten. He received his Ph.D. from the University of Chicago in 1955. Since 1963 he has taught at Princeton University, where he has served twice as chair of the mathematics department (1968–71 and 1985–87).

Stein's many fellowships and awards include a National Science Foundation Postdoctoral Fellowship (1955–56), an Alfred P. Sloan Foundation Fellowship (1961–63), Guggenheim Fellowships (1976–77 and 1984–85), membership in the National Academy of Sciences (1974) and the American Academy of Arts and Sciences (1982), the von Humboldt Award (1989–90), the Schock Prize from the Swedish Academy of Sciences (1993), and the Wolf Prize (1999). He was awarded the AMS Steele Prize in 1984 for his book *Singular Integrals and the Differentiability Properties of Functions* published in 1970 by Princeton University Press. Stein received an honorary Ph.D. from Peking University and an honorary D.Sc. from the University of Chicago.

Response

I want to express my deep appreciation to the American Mathematical Society for the honor represented by this award. At this occasion I am mindful of the great debt I owe others for my present good fortune. Beginning with my teachers and mentors and continuing with my peers, colleagues, and students, I have had the advantage of their warm support and encouragement and the indispensable benefit of their inspiration and help. To all of them I am very grateful.

I would like also to say something about the area of mathematics of which I am a representative. For more than a century there has been a significant and fruitful interaction between Fourier analysis, complex function theory, partial differential equations, real analysis, as well as ideas from other disciplines such as geometry and analytic number theory, etc. That this is the case has become increasingly clear, and the efforts and developments involved have, if anything, accelerated in the last twenty or thirty years. Having reached this stage, we can be confident that we are far from the end of this enterprise and that many exciting and wonderful theorems still await our discovery.