Edward Saff, Vanderbilt University, USA
"*Minimal Discrete Energy and Maximal Polarization*"

This talk concerns minimal energy point configurations as well maximal polarization (Chebyshev) point configurations on manifolds, which are optimization problems that are asymptotically related to best-packing and best-covering. In particular, we discuss how to generate N points on a d-dimensional manifold that have the desirable local properties of well-separation and optimal order covering radius, while asymptotically having a uniform distribution (as N grows large). Even for certain small numbers of points like $N = 5$, optimal arrangements with regard to energy and polarization can be challenging problems. Connections to the very recent major breakthrough on best-packing results in R^8 and R^{24} will also be described.

Dave Hewett, UCL, UK
"*Homogenized boundary conditions and resonance effects in Faraday cages*"

We consider two-dimensional electrostatic and electromagnetic shielding by a cage of conducting wires (the so-called 'Faraday cage effect'). In the limit as the number of wires in the cage tends to infinity we use the asymptotic method of multiple scales to derive continuum models for the shielding, which involve homogenized boundary conditions on an idealised cage boundary. We investigate how the resulting models depend on the key cage parameters such as the size and shape of the wires, and in the electromagnetic case the frequency and polarisation of the incident field. We find in the electromagnetic case that there are resonance effects, whereby at frequencies close to the natural frequencies of the equivalent solid shell, the presence of the cage actually amplifies the incident field, rather than shielding it. By appropriately modifying our continuum model we are able to calculate to high precision the modified resonant frequencies, and their associated peak amplitudes. We discuss applications to radiation containment in microwave ovens and acoustic scattering by perforated membranes.

Bruno Carneiro da Cunha, UFPE, Recife, Brazil
"*The isomonodromy method and applications to physics* "

Many problems and physics can be phrased in terms of the connection problem of the solutions of (ordinary) differential equations. This is, in turn, related to the Riemann-Hilbert problem, which displays an interesting set of symmetries encoded by the Schlesinger equations and is intimately related to the theory of Painlev transcendents. I will review the recent efforts by myself and collaborators to apply these techniques to extract analytical solutions to interesting physical problems, ranging from laminar flow to black hole scattering.

Dimitris Askitis, University of Copenhagen, Denmark
"*Complete monotonicity of ratios of products of entire functions*"

In their recent paper [1], Karp and Prilepkina investigate conditions for logarithmic complete monotonicity of ratios of products of weighted gamma functions on $(0, +\infty)$, i.e. products of the form $\frac{\prod_j \Gamma(A_j x + a_j)}{\prod_j \Gamma(B_j x + b_j)}$ where the argument of each gamma function has different scaling factor. The proof there is based on the classical integral representation of the gamma function $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} \mathrm{d}t$. Noting that the reciprocal of Γ is an entire function of order 1 with negative zeros, we show that an analogue of their result holds for more general entire functions of arbitrary order ρ with negative zeros of divergent class, i.e. where the following sum diverges: $\sum_0^\infty \frac{1}{\lambda_k^{\rho_0}} = \infty$, where $\{-\lambda_k\}_{k=0}^\infty$ is the sequence of zeros and $\rho_0 = \inf\{\sigma \geq 0 | \sum_0^\infty \frac{1}{\lambda_k^\sigma} < \infty\}$ is its convergence exponent.

[1] D. B. Karp and E. G. Prilepkina, Completely monotonic gamma ratio and infinitely divisible H-function of Fox. *Computational Methods and Function Theory*, **16**(1):135–153, (2016).

Takashi Sakajo and Yuuki Shimizu, Kyoto University
"*Point Vortex Dynamics on a toroidal surface*"

Interactions of vortex structures play an important role in the understanding of complex evolutions of fluid flows. Incompressible and inviscid flows with point-wise vorticity distributions in two-dimensional space, called point vortices, have been used as a theoretical model to describe such vortex interactions. The motion of point vortices has been investigated well in unbounded planes with boundaries as well as on a sphere owing to their physical relevance. On the other hand, it is of a theoretical interest to investigate how geometric nature of curved surfaces and the number of holes gives rise to different vortex interactions that are not observed in vortex dynamics in the plane and on the sphere. In the preceding studies, point-vortex interactions on surfaces of revolution have been investigated. In this presentation, we consider the dynamics of point vortices on a toroidal surface, which is a compact, orientable 2D Riemannian manifold with a non-constant curvature with one handle. Deriving the equation of motion of point vortices, we obtain some stationary point-vortex configurations and describe the interactions of two point vortices in order to cultivate an insight into vortex interactions on this manifold.

[1] T. Sakajo and Y. Shimizu, Point vortex interactions on a toroidal surface, Proceedings of Royal Society A, vol. 472 20160271 (2016) (doi:10.1098/rspa.2016.0271)

Takaaki Nara, Tetsuya Furuichi, and Motofumi Fushimi, Tokyo University
"*Generalized Cauchy formula for magnetic resonance electrical property tomography*"

Recently, magnetic resonance electrical property tomography (MREPT) has attracted attention as an imaging modality that reconstructs the electrical conductivity and permittivity from radio-frequency (RF) magnetic fields measured by a magnetic resonance imaging (MRI) scanner. It can provide important diagnostic information, since electrical properties of cancerous tissues are different from those of normal tissues [1]. In this talk, we show that the time-harmonic Maxwell equations for the electric and magnetic fields inside the body can be reduced to a Dbar problem. Then, by using the generalized Cauchy formula, we obtain an explicit reconstruction formula which expresses the electrical conductivity and permittivity in terms of the measured magnetic field and the boundary condition.

Denote the magnetic and electric field inside the body generated by an MRI scanner by $\boldsymbol{H} = (H_x, H_y, H_z)^T$ and $\boldsymbol{E} = (E_x, E_y, E_z)^T$, respectively, where the z-axis is the body axis. Let Ω be an arbitrary 2D region-of-interest (ROI) in the xy-plane and $\Gamma \equiv \partial\Omega$. Let

$$\partial \equiv \frac{1}{2}(\partial_x - i\partial_y), \quad \bar{\partial} \equiv \frac{1}{2}(\partial_x + i\partial_y), \quad H^+ \equiv \frac{1}{2}(H_x + iH_y), \quad E^+ \equiv \frac{1}{2}(E_x + iE_y).$$

When the magnetic field is generated by the so-called birdcage coil and the electric properties is homogeneous with respect to the z-axis, we can assume that $H_z = 0$ and $\partial_z H^+ = 0$ [2]. Under these assumptions, the time harmonic Maxwell equations are written as

$$4\partial H^+ = i\gamma E_z, \quad \bar{\partial} E_z = \omega\mu_0 H^+, \tag{1}$$

where $\gamma = \sigma + i\omega\epsilon$ is the admittivity to be reconstructed, with the electrical conductivity σ and the permittivity ϵ, respectively, μ_0 is the permeability inside the body and is the same as that in the free space, and ω is the Larmor frequency. MREPT inverse problem is to reconstruct γ from H^+ that can be measured by using the MRI scanner.

Since E_z satisfies the second equation in (1), that is a Dbar equation, it holds from the generalized Cauchy formula [3] that

$$E_z(w, \bar{w}) = \frac{1}{2\pi i}\int_\Gamma \frac{E_z(\zeta, \bar{\zeta})}{\zeta - w} d\zeta - \frac{\omega\mu_0}{\pi}\int\int_\Omega \frac{H^+(\zeta, \bar{\zeta})}{\zeta - w} d\xi d\eta, \quad w \in \Omega. \tag{2}$$

Substituting this into the first equation in (1) (Ampere's law), we obtain

$$\gamma(w, \bar{w}) = \frac{4\pi i \partial H^+(w, \bar{w})}{\displaystyle\int_\Gamma \frac{\frac{2}{\gamma(\zeta,\bar{\zeta})}\partial H^+(\zeta, \bar{\zeta})}{\zeta - w} d\zeta + \omega\mu_0 \int\int_\Omega \frac{H^+(\zeta, \bar{\zeta})}{\zeta - w} d\xi d\eta}, \quad w \in \Omega. \tag{3}$$

Eq. (3) is our explicit reconstruction formula in which the admittivity γ at an arbitrary point in Ω can be reconstructed from the measured H^+ and the boundary value of γ.

Verification with numerical simulations as well as phantom experiments will be shown in the presentation.

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Tomoki Uda, Kyoto University
"*Shape derivative of the contour integral type and its application to vortex patch equilibria*"

We propose a new shape derivative formula for singular contour integrals with logarithmic kernels which yields a numerical scheme to compute vortex patch equilibria. Owing to its simplicity, any steady configuration of point vortices can be extended to that of vortex patches. As a test problem, a doubly periodic array of vortex patches is considered to show the efficiency of the new formula. Non-trivial families of stationary vortex patch lattices are found and presented.

In a two-dimensional ideal flow, the finite area region on which the uniform vorticity distribution is supported is called a vortex patch. In the planar flow domain $\mathbf{C}$, a vortex patch $D \subset \mathbf{C}$ of vorticity $\omega \in \mathbf{R}$ induces the velocity field of the form

$$u - iv = \frac{\omega}{2\pi i} \iint_D \frac{\mathrm{d}w_1\,\mathrm{d}w_2}{z-w} = -\frac{\omega}{4\pi} \oint_{\partial D} \log(z-w)\,\mathrm{d}\overline{w}. \tag{4}$$

Elcrat and Protas [1] have applied the shape calculus to (4), whereafter the linear stability of vortex patch equilibria is considered. In order to apply the shape derivative formula of the boundary integral type, one needs to deal with the singularity of the integrand in (4). In general, this gives rise to difficulties in dealing with a logarithmic kernel coming from geometry of an arbitrary flow domain. We thus derive an alternative formula of the singular contour integral type which is applicable to contour integrals with any logarithmic kernels.

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Saleh Tanveer, Ohio State University, USA
"*Proof of existence of a steadily translating oppositely rotating vortex patch pair*"

A canonical problem in 2-D vortex dynamics is a translating pair of oppositely rotating vortex patches. These have been calculated numerically by Pierrehumbert in the early eighties. The theoretical mathematical problem of showing existence of solution is limited to near circular shapes when the translating pairs are far apart. However, the vortices are quite distorted when the distance between the centroids is smaller.

We adapt a quasi-solution method, where strongly nonlinear problems can be analyzed through a weakly nonlinear analysis, to the nonlocal integro-differential equation arising in this problem. We use an analytical expression of an approximate solution, obtained through numerics, and prove that there is an actual solution in the neighborhood of this solution. This requires use of a good space of functions for which non-local, non-linear terms can be controlled. There are no theoretical restrictions on how distorted the shapes are in this approach, and the approach can be generalized to other vortex configurations.
(Work with T.E. Kim)

Samuel Brzezicki, Imperial College
"*A theoretical study of low-Reynolds-number swimming near corners*"

An analytical determination of the dynamical system governing the motion of an idealized two-dimensional microorganism in a corner of arbitrary angle is given. A novel solution method capable of fully resolving the complicated singularity structure typically associated with biharmonic boundary value problems in corners is described. The microorganism studied is modelled using the point swimmer introduced by Crowdy & Or [*Phys. Rev. E*, **81**, (2010)]. Such swimmers are non-self-propelling in free space but are capable of both steady and unsteady translation along a straight wall. Swimmers approaching corners of sufficiently small angle are found to be liable to trapping in these wedge regions. Those swimmers approaching corners with opening angles greater than π generally scatter from the corner point. [Joint work with D. Crowdy]

Koya Sakakibara, Tokyo University
"*Method of fundamental solutions for biharmonic equation based on Almansi-type decomposition*"

In this talk, we consider the boundary value problem for the biharmonic equation. Namely, let Ω be a bounded region in the plane with smooth boundary, and consider the following problem.

$$\begin{cases} \triangle^2 u = 0 & \text{in } \Omega, \\ u = f & \text{on } \partial\Omega, \\ \dfrac{\partial u}{\partial \nu} = g & \text{on } \partial\Omega, \end{cases}$$

where $\triangle^2 = \dfrac{\partial^4}{\partial x^4} + 2\dfrac{\partial^4}{\partial x^2 \partial y^2} + \dfrac{\partial^4}{\partial y^4}$ is the biharmonic operator in the plane, $\dfrac{\partial u}{\partial \nu}$ denotes the derivative of u along outward normal direction. The conventional scheme for the method of fundamental solutions (MFS) offers an approximate solution for the above problem as a linear combination of the fundamental solutions of the biharmonic operator and ones of the Laplace operator. Namely, $u^{(N)}$ is of the form

$$u^{(N)}(x) = \sum_{k=1}^{N} \left(Q_k^{(1)} E(x - y_k) + Q_k^{(2)} F(x - y_k) \right),$$

where $E(x) = \dfrac{1}{2\pi} \log |x|$ and $F(x) = \dfrac{1}{8\pi} |x|^2 \log |x|$ are the fundamental solutions for the Laplace operator and the biharmonic operator, respectively, and $\{y_k\}_{k=1}^N$ are the singular points taken from the exterior of Ω. Coefficients $\{Q_k\}_{k=1}^N$ are determined by the collocation method, that is, take the collocation points $\{x_j\}_{j=1}^N$ on $\partial\Omega$, and impose the following approximate boundary conditions.

$$u^{(N)}(x_j) = f(x_j), \quad \frac{\partial u^{(N)}}{\partial \nu}(x_j) = g(x_j), \quad j = 1, 2, \ldots, N.$$

Although the above is the conventional scheme for MFS applied to biharmonic equation, in this talk, we consider the another scheme for MFS based on Almansi-type decomposition of biharmonic function. Namely, we seek an approximate solution for the above problem having the following form:

$$u^{(N)}(x) = \sum_{k=1}^{N} \left(Q_k^p + Q_k^q |x|^2 \right) E(x - y_k).$$

Since there are no mathematical result for MFS applied to biharmonic equation, we consider the case where Ω is a disk as a first step to establish mathematical theory, and then we prove that an approximate solution actually exists uniquely and that an approximation error decays exponentially with respect to N. We also present results of numerical experiments, which verify that our error estimate is almost optimal.

Elliott Ginder, University of Hokkaido, Research Institute for Electronic Science, Department of Mathematical Modeling

"*Multiphase optimization in phononic crystal design*"

This research approaches the design and imaging of phononic crystals (PnC) through means of experimentation, mathematics, and computation. We will present surface wave imaging results of composite elastic materials where we are aiming at the development of techniques for preforming noninvasive CT imaging. Finite element methods for approximating the solution to the model equations are then used to investigate the control of band-gaps through related eigenvalue problems. We will also remark about our technique for expressing the multiphase nature of PnC and its role in formulating shape and density gradient optimization problems.

Michael Chen, University of Oxford

"*Pressurisation in microstructured optical fibre drawing*"

A series of experiments where cylindrical glass preforms (diameter 10-30 mm) with air channels running along their length are heated and stretched (or drawn) to produce microstructured optical fibres (diameter 160 microns) are compared to a model of this fabrication process. The softened glass is modelled as a 3D Stokes flow, with the shape of the air channels determined by solving a free boundary problem in a multiply-connected domain. Although there is excellent agreement between the model and most experiments, there are marked discrepancies with others. One possible (or at least partial) explanation is that an overpressure is induced in each air channel as the fibre is drawn, and modelling the air flow inside the channels confirms that, under some conditions, there is indeed significant pressure. The magnitude of this pressure varies along the direction of the fibre axis and depends on a number of factors, including the cross-sectional shape of the channel.

Khadija Al-Amoudi, UCL

"*Using singularity structure to find special solutions of differential equations"*"

In this talk I will explain how to use singularity structure combined with global methods to identify special exact solutions of a differential equation, even if it is not integrable.

Rhodri Nelson, Imperial College

"*Outer boundary effects in a petroleum reservoir*"

A new toolkit for potential theory based on the Schottky-Klein prime function is first introduced. This potential theory toolkit is then applied to study fluid flow structures in bounded 2D petroleum reservoirs. In the model, reservoirs are assumed to be heterogeneous and isotropic porous media and can thus be modeled using Darcys equation. First, computations of flow contours are carried out on some test domains and benchmarked against results from the ECLIPSE reservoir simulator. Following this, a case study of the Quitman oil field in Texas is presented. [Joint work with D. Crowdy, R. Weijermars, L. Zuo]

Relaxations of discrete gradients for differential equations

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1 Introduction

As you know, discrete gradients play essential role to design some structure-preserving schemes for ordinary/partial differential equations. For a set U with an inner product $<,>$ and a map $V : U \to K$, typical discrete gradients $\overline{\nabla V} : U^2 \to U$ are defined [1, 3, 5, 6, 7] to satisfy the following two conditions:

$$\begin{cases} V(\boldsymbol{x}') - V(\boldsymbol{x}) &= \left\langle \overline{\nabla V}(\boldsymbol{x}', \boldsymbol{x}), (\boldsymbol{x}' - \boldsymbol{x}) \right\rangle, \\ \overline{\nabla V}(\boldsymbol{x}, \boldsymbol{x}) &= \nabla V(\boldsymbol{x}), \end{cases} \quad \text{for any } \boldsymbol{x}, \boldsymbol{x}' \in U. \tag{1}$$

These conditions are symmetric and rigorous, however, we are able to relax these requirements to design some structure-preserving schemes "superior" in performance to conventional ones. Here, we would like to introduce those relaxed discrete gradients and applications, i.e., structure-preserving schemes for ordinary/partial differential equations.

With some appropriate boundary conditions and a definition of the inner product like $< f, g > \stackrel{\text{def}}{=} \sum_k f_k g_k \Delta x$, we are able to treat discrete variational derivatives as discrete gradients. This means that the conventional discrete variational derivative method (conventional DVDM) [3] are one of those structure-preserving methods mentioned above and we have a hope to develop some relaxed or extended DVDM schemes based on relaxed discrete gradients.

2 Extended DVDM and relaxed discrete gradients

To date, we have developed three main families of extended DVDM schemes with relaxed discrete variational derivative, i.e., relaxed discrete gradients for PDEs. The first one is "(symmetric) linearized DVDM" [3] which is based on a straightforward extension of discrete gradients. This extension applies to only polynomial problems, and the obtained schemes are unstable frequently. The second is "asymmetric linearized DVDM" similar to the first. However, the extensions are asymmetric, and sometimes we are able to expect the obtained schemes are superior in performance. The last "asymmetric non-linearized DVDM" is based on most flexible relaxations of discrete gradients, and we obtain some excellent schemes fairly infrequently via this idea. In this talk, we will describe them in detail and show relationships between their relaxed discrete gradients and those DVDM schemes.

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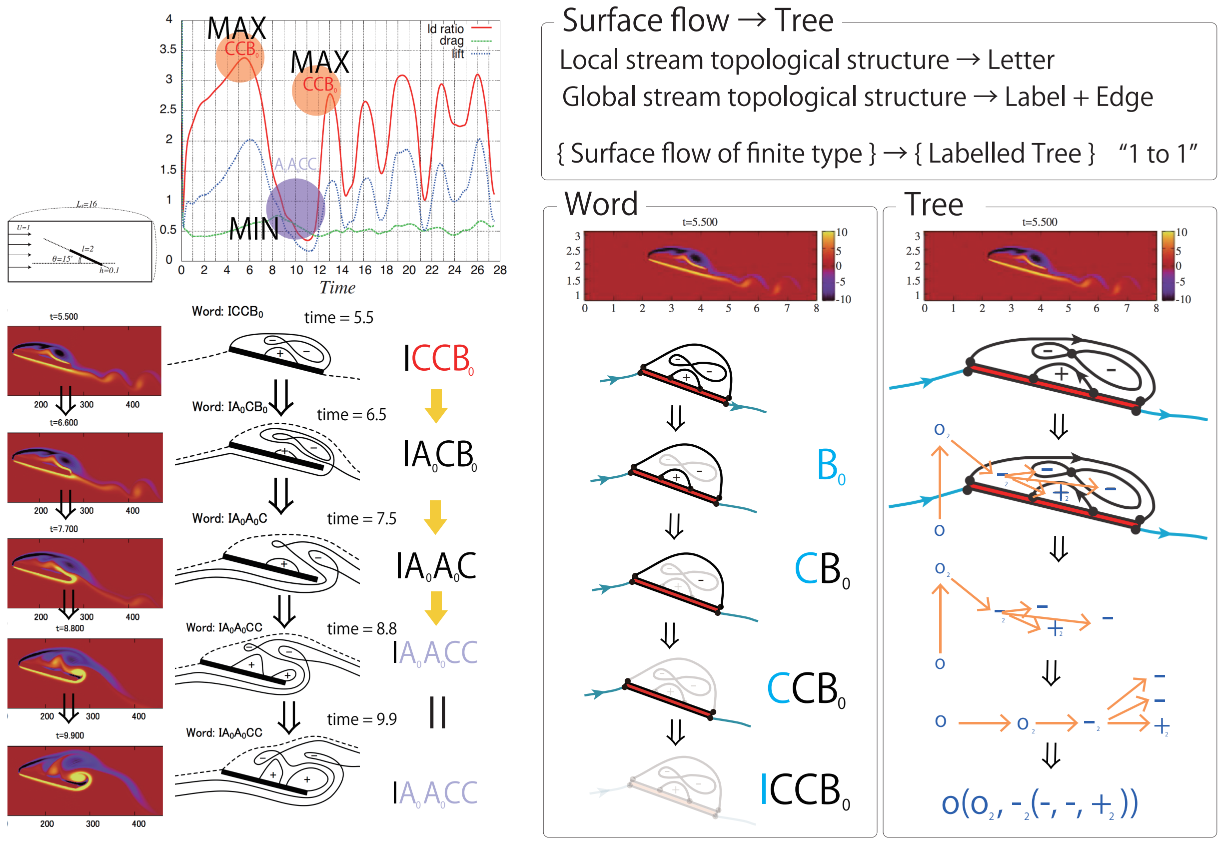
TOPOLOGICAL PROPERTIES OF SURFACES FLOWS

TOMOO YOKOYAMA

Abstract. In this talk, we introduce that generic topological structures of global streamline patterns generated by the complex velocity potentials of uniform flows and point vortices are uniquely represented by labelled trees. Moreover, we show that topological structures of generic surface flows can be represented by finite combinational structures. Finally, we discuss the relations between topological structures and data structures.



Surface flows and Data structures

Topology of a flow	$\Longrightarrow$	Regular tree grammar + Cyclic order + Label		
Resolution	$\Longrightarrow$	Depth of nodes		
Good data structure	$\Longrightarrow$	Persistent	$\Longrightarrow$	Easy to implement
Bad data structure	$\Longrightarrow$	Sensitive to error	$\Longrightarrow$	Hard to implement

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