Model for organizing cargo transportation with an initial station of departure and a final station of cargo distribution[1]

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Abstract

A model for organizing cargo transportation between two node stations connected by a railway line which contains a certain number of intermediate stations is considered. The movement of cargo is in one direction. Such a situation may occur, for example, if one of the node stations is located in a region which produce raw material for manufacturing industry located in another region, and there is another node station. The organization of freight traffic is performed by means of a number of technologies. These technologies determine the rules for taking on cargo at the initial node station, the rules of interaction between neighboring stations, as well as the rule of distribution of cargo to the final node stations. The process of cargo transportation is followed by the set rule of control. For such a model, one must determine possible modes of cargo transportation and describe their properties.

This model is described by a finite-dimensional system of differential equations with nonlocal linear restrictions. The class of the solution satisfying nonlocal linear restrictions is extremely narrow. It results in the need for the "correct" extension of solutions of a system of differential equations to a class of quasi-solutions having the distinctive feature of gaps in a countable number of points. It was possible numerically using the Runge–Kutta method of the fourth order to build these quasi-solutions and determine their rate of growth. Let us note that in the technical plan the main complexity consisted in obtaining quasi-solutions satisfying the nonlocal linear restrictions. Furthermore, we investigated the dependence of quasi-solutions and, in particular, sizes of gaps (jumps) of solutions on a number of parameters of the model characterizing a rule of control, technologies for transportation of cargo and intensity of giving of cargo on a node station.

Key words: organizing cargo transportation, dynamic model, differential equations, solution of the traveling wave type, numerical realization.

Citation: Khachatryan N.K., Akopov A.S. (2017) Model for organizing cargo transportation with an initial station of departure and a final station of cargo distribution. *Business Informatics*, no. 1 (39), pp. 25–35.
DOI: 10.17323/1998-0663.2017.1.25.35.

[1] This work was supported by the Russian Foundation for Basic Research (project No. 16-01-00110)

Introduction

Research has been is devoted to the problem of organizing cargo transportation; in particular, it is considered in works [1–7]. In [8–10] a model devoted to studying the process of organizing cargo transportation realized through a number of technologies is investigated. The distinctive feature of this model is studying the modes of cargo transportation satisfying the set "simple" control system. Several variants of the model are considered.

The first version describes transnational transportation, i.e. transportation without a selected initial station of departure and a final station of distribution of cargo. This version of the model is given by a countable system of differential equations with nonlocal linear restrictions:

$$\dot{z}_i(t) = \alpha z_{i-1} - 2\alpha z_i + \alpha z_{i+1} + \varphi(z_i),\ i \in Z,\ t \in [0, +\infty), \quad (1)$$

$$z_i(t) = z_{i+1}(t+\tau),\ i \in Z,\ t \in [0, +\infty), \quad (2)$$

where $z_i(t)$ – the number of the involved nodes at the station with number i in timepoint t;

τ – the characteristic of control.

Definition 1. The family of absolutely continuous functions $\{z_i(.)\}_{i \in Z}$, defined on $[0, +\infty)$, It called the decision of the system of differential equations (1), if almost all $t \in [0, +\infty)$ functions $z_i(.)$ satisfy this system.

Solutions of the system of differential equations (1), satisfying the condition (2) are called solutions of the traveling wave type. The theorem of existence and uniqueness of solutions of traveling wave type is proven. From the substantive point of view, this theorem describes a possible mode of cargo transportation at the quantity of the involved nodes recorded in the initial time point on a randomly selected station.

The second version describes transportation with a selected initial station of departure of the cargo. This version of the model is given by the countable system of differential equations with nonlocal linear constraints:

$$\dot{z}_0(t) = \psi(t) - \alpha z_0 + \alpha z_1 + \varphi_0(z_0),\ t \in [0, +\infty), \quad (3)$$

$$\dot{z}_i(t) = \alpha z_{i-1} - 2\alpha z_i + \alpha z_{i+1} + \varphi(z_i),\ i = 1, 2, \ldots,\ t \in [0, +\infty), \quad (4)$$

$$z_i(t) = z_{i+1}(t+\tau),\ i = 0, 1, 2, \ldots,\ t \in [0, +\infty). \quad (5)$$

Apparently, unlike system (1) – (2), the first equation is allocated here. The function participating in this equation describes the intensity of handover of cargo on the initial station. As it appeared, the class of solutions of system (3) – (5) is extremely narrow. This leads to the need to "correct" the extension of the class of solutions of the traveling wave type to the class "quasi-travelling" wave type.

Definition 2. The family of piecewise absolutely continuous functions $\{z_i(.)\}_0^{+\infty}$, defined on $[0, +\infty)$, it called a quasi-solution of thetraveling wave type with characteristic $\tau > 0$ for the system (3) – (5), if almost all $t \in [0, +\infty)$ functions $z_i(.)$ satisfy the system, and gaps are located at the points $k\tau,\ k = 1, 2, \ldots$.

It should be noted that another possible way to expand the class of traveling wave type solutions to the class of "quasi-traveling" wave type solutions making waves is the weakening of the nonlocal linear restrictions (5) (assuming implementation of these restrictions with some error). However, for these restrictions to define the control system at cargo transportation in this type of expansion is unacceptable as the control system (5) is the simplest and therefore realized.

The theorem of existence and uniqueness of the traveling wave type quasi-solution is proven. As follows from definition 2, quasi-solution components have gaps in points multiple τ. This theorem also describes the possible mode of cargo transportation at the quantity of the involved nodes recorded in initial the time point at an arbitrary fixed station. However, unlike the previous version of the model, this mode of transportation involves a sharp change in the number of involved nodes (jumps) in time points multiple to the characteristic of the control system. Considering the small equipment of intermediate stations, a change of number of the involved nodes cannot be too big. This leads to the formulation of the problem of minimizing the magnitude of the jumps, depending on the parameters of the system, which is also solved.

The third version describes transportation between a dedicated initial station of departure and final station of distribution of cargo. This version of the model describes the cargo on the long section of the route between the two node stations. Unlike the first and second versions, this version of the model is described by a finite number of differential equations with nonlocal linear constraints. The class of solutions of this system is also very narrow, so, here, as well as for system (3) – (5), there is an expansion of a class of solutions of traveling-wave type to class of solutions of "quasi-traveling" wave type. As in the previous case, considering the small equipment of intermediate stations, a change of number of the involved nodes cannot be too big. This leads to the formulation of the problem of minimizing the magnitude of the jumps, depending on the parameters and the data of system, which is also solved. From the practical point of view, the numerical realization of this version of the model is very important. This work is devoted to this task. The numerical realization will allow us to investigate dependence of traveling-wave type quasi-solutions, and in particular the sizes of gaps (jumps) of solutions, from a number of parameters of model.

Before passing on to the description of the results of numerical realization, we will provide a description of the model and theoretical base.

1. Description of the model

We will consider a model of transportation with an initial station of departure of cargo $i = 0$, intermediate stations $i = 1, 2, ..., m$ and final station of distribution of cargo $i = m + 1$. It is supposed that between two neighboring stations there is an interexchange railway track where part of cargo can temporarily be stored in special storage areas. We consider the capacity of such storage areas unlimited. The movement of cargo happens in one direction. On any intermediate station with number i cargo can arrive as from the previous station with number $(i - 1)$ and the storage area located between them. Similarly, from any intermediate station with number i cargo can be sent or to the following station with number $(i + 1)$ or on the storage area located between them. Work of all stations consists of receiving, processing and shipping cargo, and stations have a set capacity. Capacity is understood as the maximum volume of cargo which can pass through the intermediate station for a single interval of time. Processing of cargo happens in nodes of stations. At any time, the number of involved nodes at n-th station is denoted by $z_n(t)$. In each node during a unit of time, a single volume of cargo is processed. It is obvious that the quantity of the involved nodes for processing freight at trouble-free operation of the whole chain of transportation is limited. The maximum number of such nodes designated through Δ determines the capacity of stations.

The organization of similar freight traffic depends on technologies for receiving, processing and dispatch of freight. We will describe these technologies.

The first technology for intermediate stations is based on the established standard rules of interaction of the neighboring stations. For each station with number i there are rules of interaction from previous $(i - 1)$-th station and the subsequent $(i + 1)$-th station. According to the rule of interaction with the previous station, the station with number i, depending on the quantity of the involved nodes on $(i - 1)$-th station, has to increase or reduce the quantity of the involved nodes with a speed $\alpha(z_{i-1} - z_i)$ (i.e. to accept freight from the previous station if the quantity of the involved nodes on $(i - 1)$-th station is more than on i-th station, or to send to a storage area if the quantity of the involved nodes on $(i - 1)$-th station is less than on i-th station). According to the rule of interaction with the subsequent station, the station with number i, depending on the quantity of the involved nodes on $(i + 1)$-th station, has to reduce or increase the quantity of the involved nodes with a speed $\alpha(z_i - z_{i+1})$ (i.e. to send to the following station if the quantity of the involved nodes on i-th station is more than on $(i + 1)$-th station, or to accept from a storage area if the quantity of the involved nodes on i-th station is less than on $(i + 1)$-th station).

At the initial station $(i = 0)$, the first technology is determined by means of the rule of interaction with the subsequent station and rules of handover of freight on it, determined by the function $\psi_1(t)$, depending on time variable $t \geq 0$. We assume that the function $\psi_1(.)$ is piecewise infinitely differentiable.

The first technology does not account for conditions of limited throughput capacity of stations. Furthermore, it does not allow us to use the full potential of stations. Therefore, along with the first technology, other technology is also used.

The second technology for intermediate stations allows us to increase the number of the involved nodes (if it does not exceed Δ) or decrease them (if it exceeds Δ). In this case, the freight is accepted from a storage area or goes to a storage area. It follows from determination of the second technology that the function setting the speed of change of the number of involved processing nodes within this technology has the following properties: on a half-line $(-\infty, 0]$ identically equal 0, on an interval $(0, z_{opt})$ is increasing, in a point z_{opt} accepts the maximum value, on a half-line $(z_{opt}, +\infty)$ is decreasing, in a point Δ accepts zero value, and on a half-line is linear. The schedule of this function is represented in *Figure 1.*

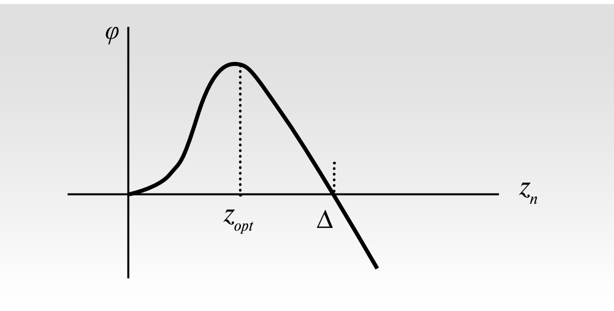


Fig. 1. Speed of change of the number of involved processing nodes at the intermediate station within the second technology

We assume that function $\varphi(.)$ is twice continuously differentiable, with regularly limited first and second derivatives. It is obvious that such a function and its derivative meet Lipschitz's condition with constants L_0 and L_1, respectively. We will designate $c = -\dot{\varphi}(\Delta)$. Parameter $c > 0$ determines the intensity of sending freight from intermediate stations to storage areas.

For the initial station ($i = 0$), the second technology is used only for unloading. Therefore, the speed of change of the number of involved handling nodes at the initial station within the second technology is described by the function $\varphi_0(.)$, depending on the quantity of the involved nodes at the initial station which schedule is represented in *Figure 2.*

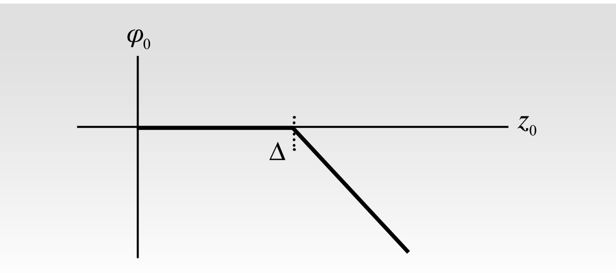


Fig. 2. Speed of change of the number of involved processing nodes at the initial station within the second technology

It is obvious that in case the amount of freight on 0-th station is not exceeding Δ, only the first technology is used. We will designate $c_0 = -\dot{\varphi}_0(\Delta)$. Parameter $c_0 > 0$ determines the intensity of sending freight from the initial station to the storage area.

At the final station ($i = m + 1$), the first technology is determined by means of the rule of interaction with the previous station and the rule for apportionment of freight from it, described by function $\psi_2(t)$, $t \geq 0$. We assume that function $\psi_2(.)$ is piecewise continuous. The second technology for a final station is the same as for intermediate stations.

For cargo transportation, it is necessary to have an efficient and simple control system. The amounts of the processed freight for any planned interval of time at all stations shall match a certain log of time, single for all stations. Such a condition can be described in the following way: there is a number $\tau > 0$, which is not depending on t and i, it that at all $i = 0, 1, 2, ..., m$ and $t \in [0, +\infty)$ equality is carried out:

$$z_i(t) = z_{i+1}(t + \tau).$$

Thus, taking into account the work of the first and second technologies, and also the control systems, acceptance and sending freight will be described by the following system of differential equations:

$$\dot{z}_0(t) = \psi_1(t) - \alpha z_0 + \alpha z_1 + \varphi_0(z_0),\ t \in [0, +\infty), \quad (6)$$

$$\dot{z}_i(t) = \alpha z_{i-1} - 2\alpha z_i + \alpha z_{i+1} + \varphi(z_i),$$
$$i = 1, 2, ..., m,\ t \in [0, +\infty) \quad (7)$$

$$\dot{z}_{m+1}(t) = \alpha z_m - \alpha z_{m+1} - \psi_2(t) + \varphi(z_{m+1}), t \in [0, +\infty), \quad (8)$$

$$z_i(t) = z_{i+1}(t + \tau),\ i = 0, 1, 2, ..., m,\ t \in [0, +\infty). \quad (9)$$

The solution of the system of differential equations (6) — (8), satisfying the condition (9) is called a solution of traveling-wave type.

2. Theoretical basis of research

In works [8—10] the system (6) — (9) has been investigated. We will give substantial aspects of this research.

By means of replacement of time $t \rightarrow \tau t$ we will rewrite the system (6) — (9) in the form:

$$\dot{x}_0(t) = \tau[\bar{\psi}_1(t) - \alpha x_0 + \alpha x_1 + \varphi_0(x_0)],\ t \in [0, +\infty), \quad (10)$$

$$\dot{x}_i(t) = \tau[\alpha x_{i-1} - 2\alpha x_i + \alpha x_{i+1} + \varphi(x_i)],$$
$$i = 1, 2, ..., m,\ t \in [0, +\infty), \quad (11)$$

$$\dot{x}_{m+1}(t) = \tau[\alpha x_m - \alpha x_{m+1} - \bar{\psi}_2(t) + \varphi(x_{m+1})],\ t \in [0, +\infty), \quad (12)$$

$$x_i(t) = x_{i+1}(t + 1),\ \ i = 0, 1, 2, ..., m,\ t \in [0, +\infty) \quad (13)$$

where

$$x_i(t) = z_i\left(\frac{t}{\tau}\right),\quad \bar{\psi}_1(t) = \psi_1\left(\frac{t}{\tau}\right),\quad \bar{\psi}_2(t) = \psi_2\left(\frac{t}{\tau}\right),$$
$$i = 0, 1, 2, ..., m, m + 1,\quad t \in [0, +\infty).$$

At the initial stage we considered narrowing of system (10) — (13) on the segment [0,1], i.e. system:

$$\dot{x}_0(t) = \tau[\bar{\psi}_1(t) - \alpha x_0 + \alpha x_1 + \varphi_0(x_0)],\ t \in [0, 1] \quad (14)$$

$$\dot{x}_i(t) = \tau[\alpha x_{i-1} - 2\alpha x_i + \alpha x_{i+1} + \varphi(x_i)],$$
$$i = 1, 2, ..., m,\ t \in [0, 1] \quad (15)$$

$$\dot{x}_{m+1}(t) = \tau[\alpha x_m - \alpha x_{m+1} - \bar{\psi}_2(t) + \varphi(x_{m+1})],\ t \in [0, 1] \quad (16)$$

$$x_i(0) = x_{i+1}(1),\quad i = 0, 1, 2, ..., m. \quad (17)$$

The theorem of existence and uniqueness of the solution of system (14) — (17) has been proven. According to the solution $\{\tilde{x}_i(.)\}_{i=0,1,2,...,m+1}$ of system (14) — (17) functions $\overline{\psi}_1(.)$, $\overline{\psi}_2(.)$ and $\{x_i(.)\}_{i=0,1,2,...,m+1}$ on a half-line $[0,+\infty)$ have been constructed:

$$\begin{cases} x_i(t)=\tilde{x}_i(t),\ i=0,1,2,...,m+1,\ t\in[0,1]; \\ x_i(t)=x_{i-1}(t-1),\ i=1,2,...,m+1, \\ t\in(k,k+1),\ k=1,2,...; \\ x_0(t)=x_0(t-1)+\dfrac{\overline{\psi}_1(t-1)}{\alpha}+\dfrac{\varphi_0(x_0(t-1))-\varphi(x_0(t-1))}{\alpha}, \\ t\in(k,k+1),\quad k=1,2,...; \\ \overline{\psi}_1(t)=\dfrac{1}{\alpha\tau}\dot{\overline{\psi}}_1(t-1)+\dfrac{1}{\alpha\tau}[\dot{\varphi}_0(x_0(t-1))-\dot{\varphi}(x_0(t-1))]\dot{x}_0(t-1)+ \\ +\dfrac{\dot{x}_0(t-1)}{\tau}+\overline{\psi}_1(t-1)+\varphi_0(x_0(t-1))-\varphi(x_0(t-1))- \\ -\varphi_0\left(x_0(t-1)+\dfrac{\overline{\psi}_1(t-1)}{\alpha}+\dfrac{\varphi_0(x_0(t-1))-\varphi(x_0(t-1))}{\alpha}\right), \\ t\in(k,k+1),\quad k=1,2,...; \\ \overline{\psi}_2(t)=\alpha x_m(t-1)-\alpha x_{m+1}(t-1),\ t\in(k,k+1),\ k=1,2,.... \end{cases} \tag{18}$$

Definition 3. The quasi-solution of system (10)-(13) is called the set of piecewise strongly absolutely continuous functions $\{x_i(.)\}_{i=0,1,2,...,m+1}$ with gaps only in points k = 1, 2, ... and almost everywhere satisfying to this system.

The lemma is proven according to which the solution of the boundary value problem (14) – (17), extended to the half $[0,+\infty)$ in the relations (18), is the quasi-solution of system (10) – (13).

3. Results of numerical experiments

As was noted, this work is devoted to numerical implementation of the given model for organizing cargo transportation. We will give the results of research into the numerical solution of the system (10) – (13) describing this model. In numerical experiments, the number of stations was equal to 10: the initial station of positioning freight (i = 0), intermediate stations (i = 1, 2, ..., 8) and final station of distribution of freight (i = 9). According to the results given in the previous paragraph, numerical implementation of system (10) – (13) consists of two stages. At the first stage, the solution of the system which is restriction of the initial system (10) – (13) on the segment [0, 1], i.e. the solution of the following system is found:

$$\dot{x}_0(t)=\tau[\overline{\psi}_1(t)-\alpha x_0+\alpha x_1+\varphi_0(x_0)],\ t\in[0,1] \tag{19}$$

$$\dot{x}_i(t)=\tau[\alpha x_{i-1}-2\alpha x_i+\alpha x_{i+1}+\varphi(x_i)],\quad i=1,2,...,8,\ t\in[0,1] \tag{20}$$

$$\dot{x}_9(t)=\tau[\alpha x_8-\alpha x_9-\overline{\psi}_2(t)+\varphi(x_9)],\ t\in[0,1] \tag{21}$$

$$x_i(0)=x_{i+1}(1),\quad i=0,1,2,...,8. \tag{22}$$

Before passing to the numerical solution of this system, it is necessary to define functions $\varphi(.)$, $\overline{\psi}_1(t)$, $\overline{\psi}_2(t)$. Function $\varphi(.)$ on the segment [0, Δ] is set by means of a parabola $y=-ax^2+bx$, where $a>0$, $y(\Delta)=0$ (see *Figure 1*). Thus, coefficients of a parabola are connected by the ratio $b=a\Delta$. It is obvious that the greater is a, the greater the intensity of cargo received on the second technology. We will pass to the choice of functions $\overline{\psi}_1(.)$ and $\overline{\psi}_2(.)$. We remind the reader that these functions determine, respectively, intensity of shipping freight at the initial station and intensity of distribution of freight from the final station. It is obvious that the intensity of shipping freight at the initial station is subject to seasonality. Moreover, in order to avoid jams at stations, the cargo receiving period at the initial station shall be replaced by the period of more intensive freight shipment with initial to the following station. Similar reasoning is fair also for the final station (the distribution period of freight from a final station is replaced by the period of more intensive receiving of cargo from the previous station). Owing to the above, in quality $\overline{\psi}_1(.)$ and $\overline{\psi}_2(.)$ periodic functions $\overline{\psi}_1(t)=\overline{\psi}_2(t)=\gamma\cos(\omega t)$ are used, while amplitude and the period of functions are model parameters. After definition of functions $\varphi(.)$, $\overline{\psi}_1(t)$, $\overline{\psi}_2(t)$ we proceed to the solution of the boundary value problem (19) – (22). The solution algorithm is as follows.

1. We find the solution of equations (19) – (21) with initial conditions

$$x_0(0)=\Delta,\ x_1(0)=\Delta,\ ...,\ x_9(0)=\Delta.$$

For the solution found $\{x_i(.)\}_0^9$ we calculate the following expression:

$$Q=(x_0(0)-x_1(1))^2+(x_1(0)-x_2(1))^2+...+(x_8(0)-x_9(1))^2.$$

2. For a previously set small value $\varepsilon>0$ by means of a gradient method, we find the solution of the system of differential equations (19) – (21) with initial conditions for which the condition $Q<\varepsilon$ is satisfied. On each iteration of a gradient method, the solution of the system of equations (19) – (21) is found by means of the method of Runge–Kutta, fourth order. Thus, we receive the solution of the system (19) – (21) for which the condition (22) is satisfied with a certain accuracy. We will call such decisions solutions of the system (19) – (22). At the second stage, the solution of the boundary value problem (19) – (22) continues in $(1,+\infty)$ accordance with the re-

lations (18). As we know from the previous section, as a result we get the quasi-solution system (10) – (13), i.e. functions $x_i(.)$, satisfying this system and having gaps at the points $k = 1, 2, ...$.

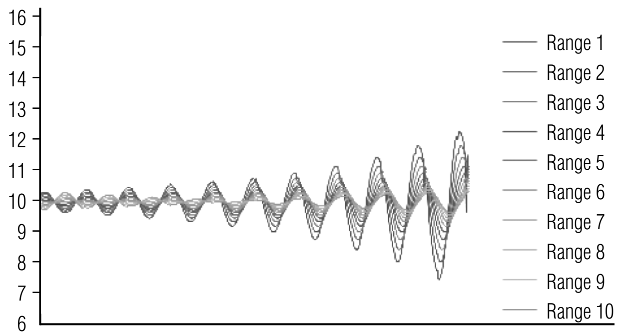
The main purpose of the study – to determine the form and dynamics of the quasi-solution of the system (10) – (13) and also to study their dependence on model parameters $\alpha, a, \gamma, \omega, c_0, c$ and τ. Note that all these parameters are positive. The results of the numerical experiments are presented in the following two propositions.

Proposition 1. Quasi-solutions of system (10) – (13) satisfy the following restriction

$$\Delta_1 - e^{\beta_1 t} \le x_i(t) \le \Delta_2 + e^{\beta_2 t},\ i = 0, 1, ...9, \text{ where}$$
$$\Delta_1 < \Delta + 1, \Delta_2 > \Delta - 1, \beta_1 > 0, \beta_2 > 0. \quad (23)$$

Thus, according to proposition 1, quasi-solutions of the system (10) – (13) both from above, and from below are majorized by exponential functions. For example, the schedule of one of quasi-solutions of system (10) – (13) is provided on *Figure 3*.

This quasi-solution is received at $\Delta = 10$ and the following values of parameters:



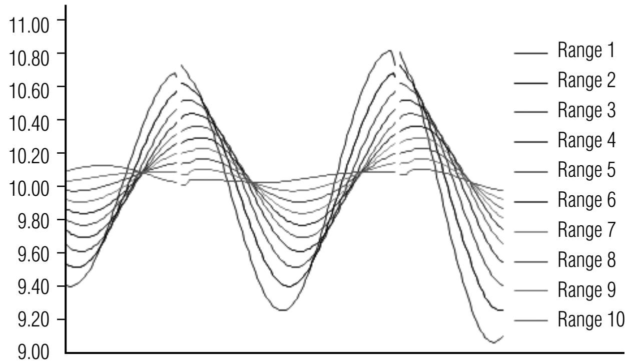
Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

Fig. 3. Quasi-solution schedule of the system of differential equations (basic case)

$$\alpha = 60, a = 0.1, \gamma = 5, \omega = 2\pi, c_0 = 0.1, c = 0.1, \tau = 1. \quad (24)$$

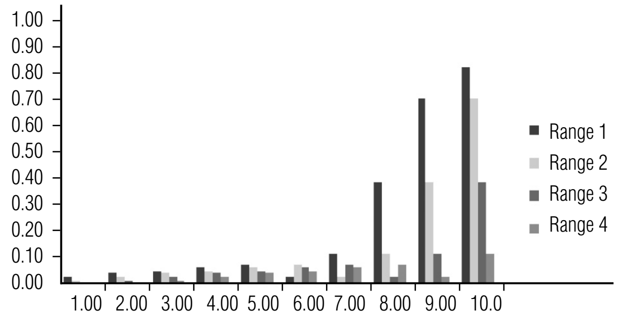
To see ruptures of functions on schedules, we will give a small fragment of *Figure 3* (a segment [3.5; 5.5], gaps in points 4 and 5).

As can be seen from *Figure 4*, the biggest gap at these points has a function $x_0(.)$. Further, with increasing numbers coordinates quasi-solution breaks are reduced. This trend remains also in the subsequent integer points. For comparison, in *Figure 5* we will provide histograms of ruptures of functions $x_0(.)$, $x_1(.)$, $x_2(.)$, $x_3(.)$ (histograms of other functions aren't provided to avoid encumbering the figure).



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

Fig. 4. Fragment of the quasi-solution schedule of system of the differential equations (basic case)



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$,
Range 3 – $x_2(.)$, Range 4 – $x_3(.)$

Fig. 5. Histograms of gaps (basic case)

For these values of the parameters of inequality (23) takes the following form:

$$10.75 - e^{0.09t} \le x_i(t) \le 9.25 + e^{0.08t},\ i = 0, 1, ... 9.$$

Parameters β_1, β_2 of the functions majorizing quasi-solutions of the system (10) – (13) depend on the parameters of this system; therefore, we will designate them:

$$\beta_1 = \beta_1(\alpha, a, \gamma, \omega, c_0, c, \tau),\ \beta_2 = \beta_2(\alpha, a, \gamma, \omega, c_0, c, \tau).$$

As a result of numerous experiments, it has been revealed that function $\beta_1(.)$ is monotone in all parameters except for the parameter *c* concerning which it is invariable. Function $\beta_2(.)$ also is monotone in all parameters except for the parameter *a*, concerning which it is invariable. We will provide more detailed formulation of this result in the following proposition.

Proposition 2. Functions $\beta_1(.)$ and $\beta_2(.)$ have the following properties:

1) $\dfrac{\partial\beta_1(.)}{\partial\alpha}<0,\ \dfrac{\partial\beta_2(.)}{\partial\alpha}<0;$ 2) $\dfrac{\partial\beta_1(.)}{\partial a}>0,\ \dfrac{\partial\beta_2(.)}{\partial a}=0;$

3) $\dfrac{\partial\beta_1(.)}{\partial\gamma}>0,\ \dfrac{\partial\beta_2(.)}{\partial\gamma}>0;$ 4) $\dfrac{\partial\beta_1(.)}{\partial\omega}>0,\ \dfrac{\partial\beta_2(.)}{\partial\omega}>0;$

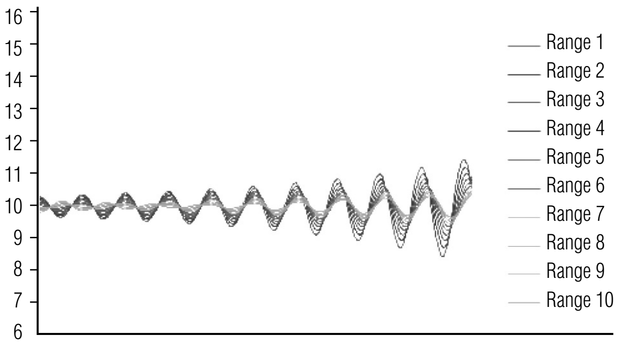
5) $\dfrac{\partial\beta_1(.)}{\partial c_0}>0,\ \dfrac{\partial\beta_2(.)}{\partial c_0}<0;$ 6) $\dfrac{\partial\beta_1(.)}{\partial c}=0,\ \dfrac{\partial\beta_2(.)}{\partial c}>0,$

7) $\dfrac{\partial\beta_1(.)}{\partial\tau}<0,\ \dfrac{\partial\beta_2(.)}{\partial\tau}<0.$

We will give below schedules of quasi-solutions of system (10) – (13) in which alternately value of one of parameters differs from the value given in (24) at invariable values of other parameters. At the same time value Δ is also invariable and equal to 10. Besides, for these quasi-solutions we will receive estimate (23).

The schedule of the quasi-solution of system (10) – (13) with the following values of parameters is shown on *Figure 6*:

$$\alpha=85, a=0.1, \gamma=5, \omega=2\pi, c_0=0.1, c=0.1, \tau=1.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

Fig. 6. Quasi-solution schedule of the system of differential equations with the increased value of parameter α

We will notice that in comparison with (24), value α is increased. For these values of the parameters, inequality (23) takes the following form:

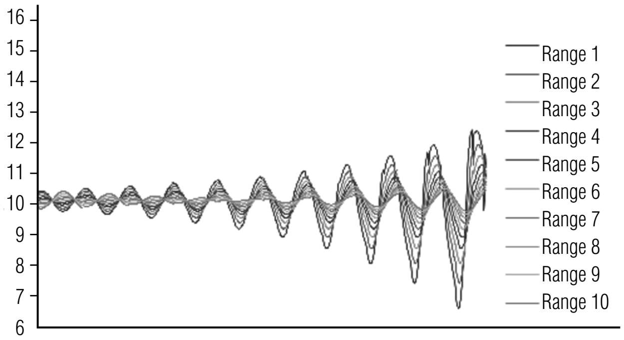
$$10.86-e^{0.07t}\le x_i(t)\le 9.29+e^{0.06t},\ i=0,1,...9.$$

The schedule of the quasi-solution of system (10) – (13) with the following values of parameters is shown on *Figure 7*:

$$\alpha=60, a=0.4, \gamma=5, \omega=2\pi, c_0=0.1, c=0.1, \tau=1.$$

In comparison with (24), value a is increased. For these values of the parameters inequality (23) takes the following form:

$$10.88-e^{0.12t}\le x_i(t)\le 9.39+e^{0.08t},\ i=0,1,...9.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

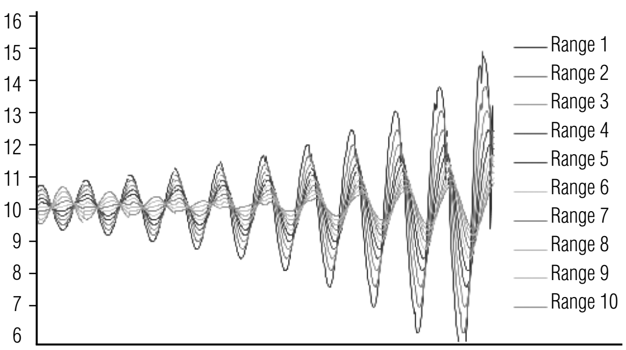
Fig. 7. Quasi-solution schedule of system of the differential equations with the increased value of parameter a

The schedule of the quasi-solution of system (10) – (13) with the following values of parameters is shown on *Figure 8*:

$$\alpha=60, a=0.1, \gamma=10, \omega=2\pi, c_0=0.1, c=0.1, \tau=1.$$

In comparison with (24), value γ is increased. For these values of the parameters, inequality (23) takes the following form:

$$10.61-e^{0.15t}\le x_i(t)\le 9.63+e^{0.13t},\ i=0,1,...9.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

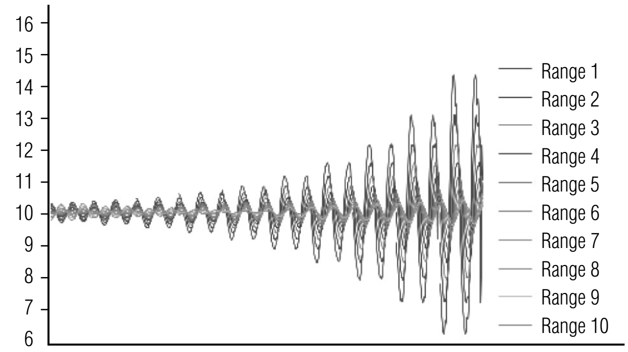
Fig. 8. Quasi-solution schedule of the system of differential equations with the increased value of parameter γ

The schedule of the quasi-solution of system (10) – (13) with the following values of parameters is shown on *Figure 9:*

$$\alpha=60, a=0.1, \gamma=5, \omega=4\pi, c_0=0.1, c=0.1, \tau=1.$$

In comparison with (24), value ω is increased. For these values of the parameters, inequality (23) takes the following form:

$$10.86-e^{0.12t}\le x_i(t)\le 9.2+e^{0.1t},\ i=0,1,...9.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

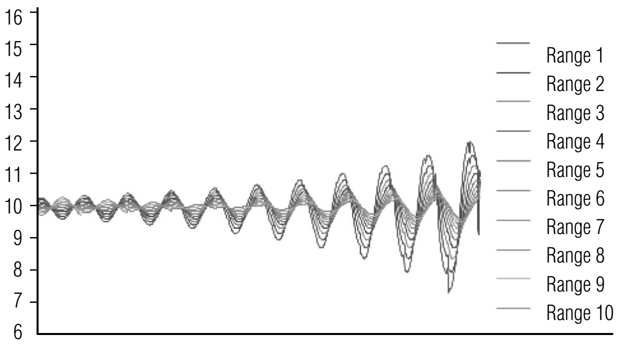
Fig. 9. Quasi-solution schedule of the system of differential equations with the increased value of parameter ω

The schedule of the quasi-solution of system (10) – (13) with the following values of parameters is shown on *Figure 10*:

$$\alpha = 60, a = 0.1, \gamma = 5, \omega = 2\pi, c_0 = 2, c = 0.1, \tau = 1.$$

In comparison with (24), value c_0 is increased. For these values of the parameters, inequality (23) takes the following form:

$$10.92 - e^{0.11t} \le x_i(t) \le 9.26 + e^{0.07t}, \; i = 0,1,...9.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

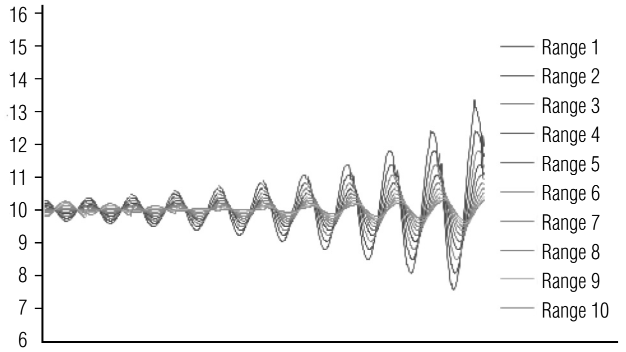
Fig. 10. Quasi-solution schedule of the system of differential equations with the increased value of parameter c_0

The schedule of the quasi-solution of system (10) – (13) with the following values of parameters is shown on *Figure 11*:

$$\alpha = 60, a = 0.1, \gamma = 5, \omega = 2\pi, c_0 = 0.1, c = 2, \tau = 1.$$

In comparison with (24), value c is increased. For these values of the parameters, inequality (23) takes the following form:

$$10.72 - e^{0.09t} \le x_i(t) \le 9.2 + e^{0.1t}, \; i = 0,1,...9.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

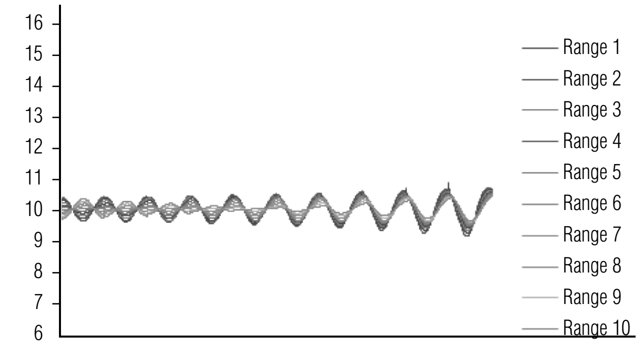
Fig. 11. Quasi-solution schedule of the system of differential equations with the increased value of parameter c

Lastly, the schedule of the quasi-solution of the system (10) – (13) with the following values of parameters is shown on *Figure 12*:

$$\alpha = 60, \gamma = 5, \omega = 2\pi, a = 0.1, c_0 = 0.1, c = 0.1, \tau = 4.$$

In comparison with (24), value τ is increased. For these values of the parameters, inequality (23) takes the following form:

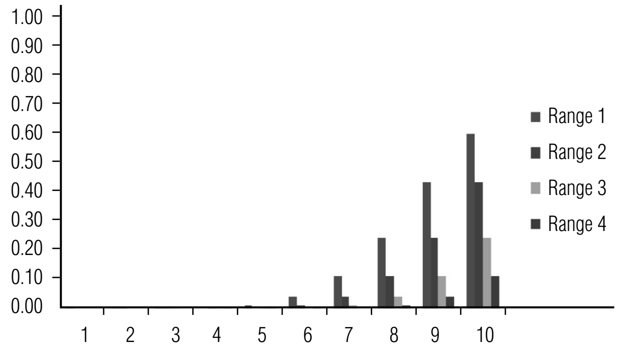
$$10.79 - e^{0.04t} \le x_i(t) \le 9.42 + e^{0.03t}, \; i = 0,1,...9.$$



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$, ..., Range 10 – $x_9(.)$

Fig. 12. Quasi-solution schedule of the system of differential equations with the increased value of parameter τ

In conclusion, we will pass to the analysis of results of the following from proposition 2 and having practical value. It follows from proposition 2 that growth of quasi-solutions of the system (10) – (13) decreases with an increase of parameters α and τ, and also with reduction of parameters γ and ω. Numerical experiments have shown that similarly conduct to themselves and ruptures of quasi-solutions of the system (10) – (13), i.e. they decrease with an increase of parameters α and τ and reduction of parameters γ and ω. For example, for comparison with *Figure 5*, we will provide histograms of ruptures of functions $x_0(.)$, $x_1(.)$, $x_2(.)$, $x_3(.)$, which are components of quasi-solutions of the system (10) – (13) with the increased value of parameter α (with 60 to 85) and at invariable values of other parameters (*Figure 13*).



Range 1 – $x_0(.)$, Range 2 – $x_1(.)$,
Range 3 – $x_2(.)$, Range 4 – $x_3(.)$

Fig. 13. Histograms of gaps with an increased value of parameter α

We remind the reader that parameters γ and ω are characteristics of intensity of shipping freight on the initial station and cannot be operated by the organizer of cargo transportation unlike parameters α and τ. Thus, the organizer of cargo transportation can effectively reduce the load of stations, increasing intensity of the movement of freight traffic (parameter α) and the characteristic of the control system (parameter τ). However, here it must be kept in mind that for great values τ the control system loses relevance. Thus, the choice of parameter α depends only on the technical capabilities of the infrastructure of cargo transportation, and the choice of parameter τ has to be reached at the expense of a compromise between technical capabilities of the infrastructure of cargo transportation and the relevance of the control system.

Conclusion

This article is devoted to numerical realization of a model for organizing cargo transportation between two node stations with a set rule of control. Such a model is described by a finite-dimensional system of differential equations with nonlocal linear restrictions (the rule of control). The class of the solution satisfying nonlocal linear restrictions is extremely narrow. It results in the need for the "correct" extension of solutions of the system of differential equations to a class of quasi-solutions. Based on the theoretical basis presented in [8–10], it was possible numerically using the Runge–Kutta method to build these quasi-solutions and determine their rate of growth. Furthermore, dependence of the quasi-solutions and, in particular, the sizes of gaps (jumps) on solutions from a number of the parameters of the model characterizing a rule of control, and technologies for transportation of cargo and intensity of shipping cargo on a node station has been researched. ■

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Модель организации грузоперевозок с начальной станцией отправления и конечной станцией распределения грузов[2]

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Аннотация

В статье рассматривается модель организации грузоперевозок между двумя узловыми станциями, соединенными железнодорожной линией, которая содержит определенное количество промежуточных станций. Движение грузопотока осуществляется в одном направлении. Такая ситуация может иметь место, например, в случае, если одна из узловых станций расположена в регионе добычи сырья для предприятия, находящегося в другом регионе и располагающего другой узловой станцией. Организация грузопотока осуществляется с помощью ряда технологий. Эти технологии определяют правило подачи грузов на начальную узловую станцию, правила взаимодействия соседних станций, а также правило распределения грузов с конечной узловой станции. Процесс грузоперевозок сопровождается заданным правилом контроля. Для такой модели требуется определить возможные режимы грузоперевозок и описать их свойства.

Данная модель описывается конечномерной системой дифференциальных уравнений с нелокальными линейными ограничениями. Класс решений, удовлетворяющих нелокальным линейным ограничениям, оказывается чрезвычайно узким. Это приводит к необходимости «правильного» расширения решений системы дифференциальных уравнений до класса квазирешений, отличительной особенностью которых является наличие разрывов в счетном числе точек. С помощью метода Рунге–Кутта четвертого порядка удалось численно построить указанные квазирешения и определить скорость их роста. Отметим, что в техническом плане основная сложность заключалась именно в получении квазирешений, удовлетворяющих нелокальным ограничениям. Кроме того, исследована зависимость квазирешений и, в частности, величин разрывов (скачков) решений от ряда параметров модели, характеризующих правило контроля, технологии перевозки грузов и интенсивность подачи грузов на узловую станцию.

Ключевые слова: организация грузоперевозок, динамическая модель, дифференциальные уравнения, решения типа бегущей волны, численная реализация.

Цитирование: Khachatryan N.K., Akopov A.S. Model for organizing cargo transportation with an initial station of departure and a final station of cargo distribution // Business Informatics. No. 1 (39). P. 25–35. DOI: 10.17323/1998-0663.2017.1.25.35.

Работа выполнена при поддержке Российского фонда фундаментальных исследований (проект № 16-01-00110)

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