

## 1. Introduction

**Sea level (SL)** is usually reconstructed by combining the long but spatially sparse tide gauge data set with the shorter but complete global coverage of satellite altimetry [1][2]. The most widely used technique is the reduced space optimal interpolation (RSOI) [3]. The technique involves the calculation of empirical orthogonal functions (EOFs) from satellite altimetry observations, which are then fitted in a weighted least square sense to a set of tide gauge records. To account for uniform sea level changes, it is common practice to add a spatially uniform EOF (often referred to as **EOF0**) to the basis functions [1][2].

Here we investigate the **skill** of the RSOI to reconstruct the global mean sea level (GMSL) and assess the impact of adding the EOF0 to the basis functions. We first explore the analytical solution of the method and then perform a series of numerical experiments using modeled data. In addition we present a **new GMSL reconstruction** computed both with and without the EOF0.

## 2. Data

**Tide gauge records** are obtained from the data archive of the PSMSL. Selected records are corrected for both GIA and the IB effect.

**Satellite altimetry data** were obtained from the CSIRO website. These data were produced using a combination of T/P, Jason-1, and Jason-2/OSTM and are available on a 1° x 1° global grid from January 1993 to November 2012.

**Model data** are from the Simple Ocean Data Assimilation (SODA) reanalysis [4]. The model output is provided on a 0.5° x 0.5° global grid and covers the period 1871-2008. Long-term SL changes caused by the exchange of water between oceans and land are incorporated into the model by adding the SL fingerprint of continental water and ice mass change of [5].

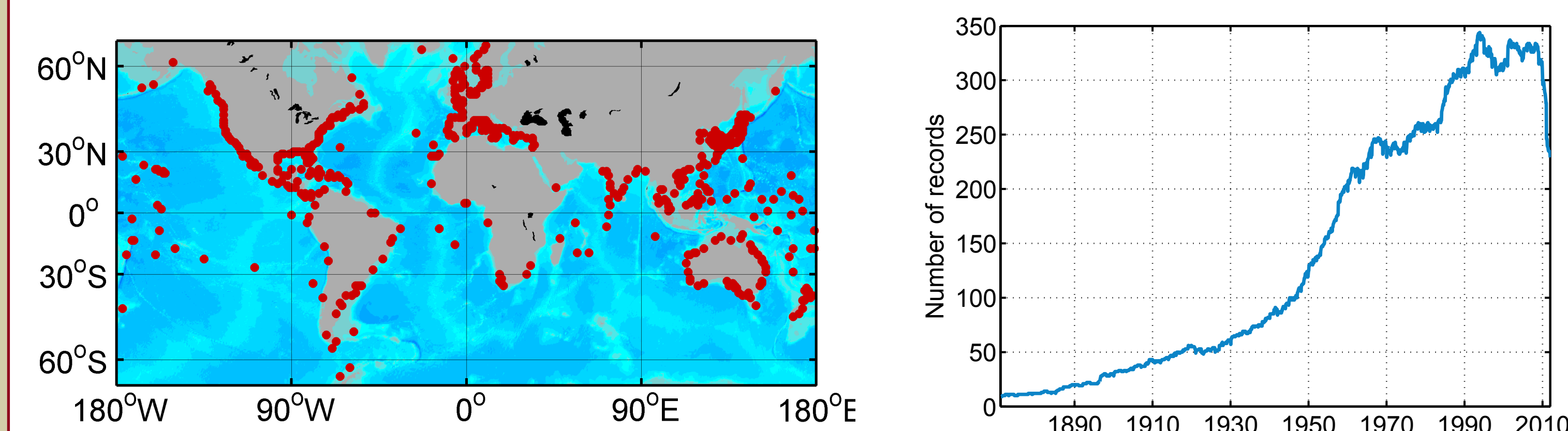


FIG 1a. Location of tide gauges used in the reconstruction

FIG 1b. Number of available tide gauges over time.

There are significant **discrepancies** between tide gauges and nearby altimetry

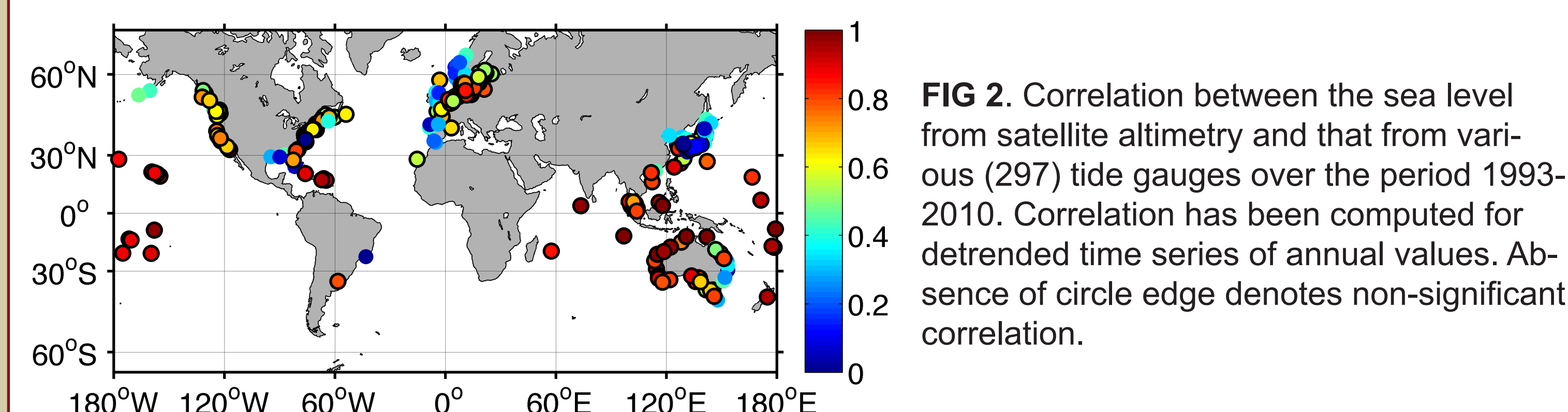


FIG 2. Correlation between the sea level from satellite altimetry and that from various (297) tide gauges over the period 1993-2010. Correlation has been computed for detrended time series of annual values. Absence of circle edge denotes non-significant correlation.

## 3. Methodology and theoretical analysis

In the **RSOI method**, the temporal amplitudes of a truncated set of EOFs are estimated, for each month, by minimizing the cost function [3]:

$$S[\alpha] = (\mathbf{H}\mathbf{E}\alpha - \mathbf{T}^\circ)^\top (\mathbf{R}_\circ + \mathbf{R}_\tau)^{-1} (\mathbf{H}\mathbf{E}\alpha - \mathbf{T}^\circ) + \alpha^\top \mathbf{\Lambda}^{-1} \alpha$$

$\alpha$  : temporal amplitudes       $\mathbf{R}_\circ$  : observational error  
 $\mathbf{H}$  : sampling operator       $\mathbf{R}_\tau$  : truncation error  
 $\mathbf{E}$  : retained EOFs       $\mathbf{\Lambda}$  : retained eigenvalues  
 $\mathbf{T}^\circ$  : tide gauge observations

The GMSL is reconstructed with (**OIH**) and without (**OI**) the inclusion of the EOF0.

**Surrogate SL fields** are generated by applying a phase-randomized Fourier-transform algorithm to the SODA sea level fields.

Once the temporal amplitudes have been estimated, the GMSL can be computed by

$$\text{GMSL} = \sum_{i=0}^L \bar{\mathbf{E}}_i \alpha_i \quad (1), \text{ which, when using the EOF0, reduces to } \text{GMSL} = \bar{\mathbf{E}}_0 \alpha_0 \quad (2)$$

After some algebra, Equation (2) can be written as [2]  $\text{GMSL} = \frac{\mathbf{1}^\top \mathbf{\Sigma}^{-1} \mathbf{t}^\circ}{\mathbf{1}^\top \mathbf{\Sigma}^{-1} \mathbf{1}} \quad (3)$

where  $\mathbf{\Sigma}$  is the **covariance matrix** of altimetry records only **at tide gauge locations**. The GMSL takes the form of a weighted mean, where the weights associated with each tide gauge are given by the solution of the generalized weighted mean problem of altimetry records at tide gauge locations. Note that with the EOF0, the estimated GMSL contains **no global information**. Note also that because tide gauges and coastal altimetry can differ significantly, the weights in (3) will not, in general, minimize the variance of the mean.

## 4. Numerical experiments

The **interannual variability** is better captured without the EOF0 (FIG 3 and FIG 4A). To capture the **long-term trend**, however, the EOF0 is needed (FIG 4B).

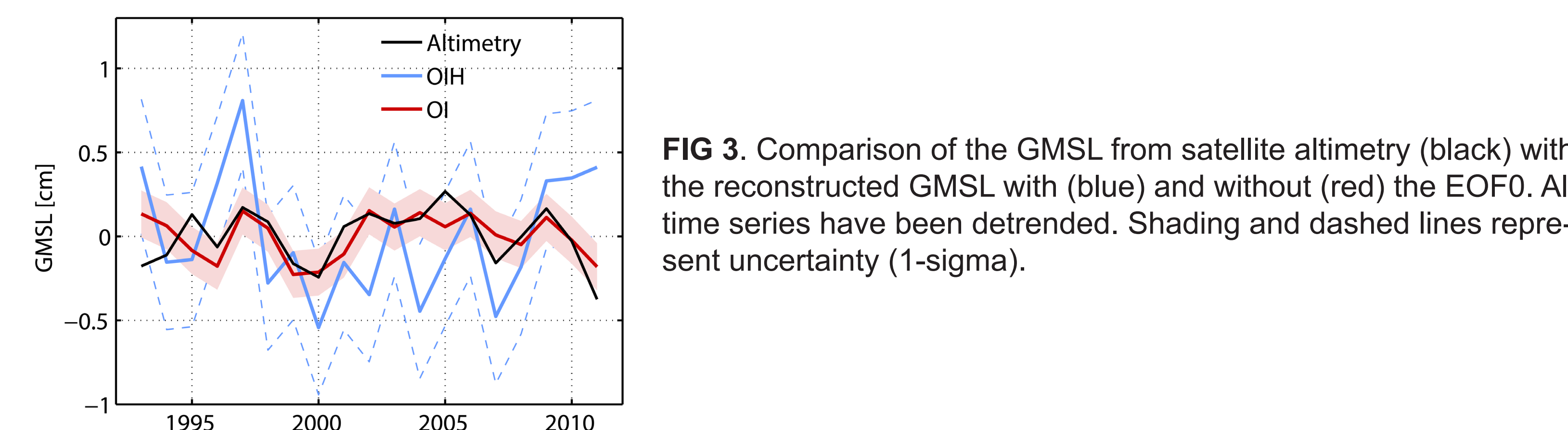


FIG 3. Comparison of the GMSL from satellite altimetry (black) with the reconstructed GMSL with (blue) and without (red) the EOF0. All time series have been detrended. Shading and dashed lines represent uncertainty (1-sigma).

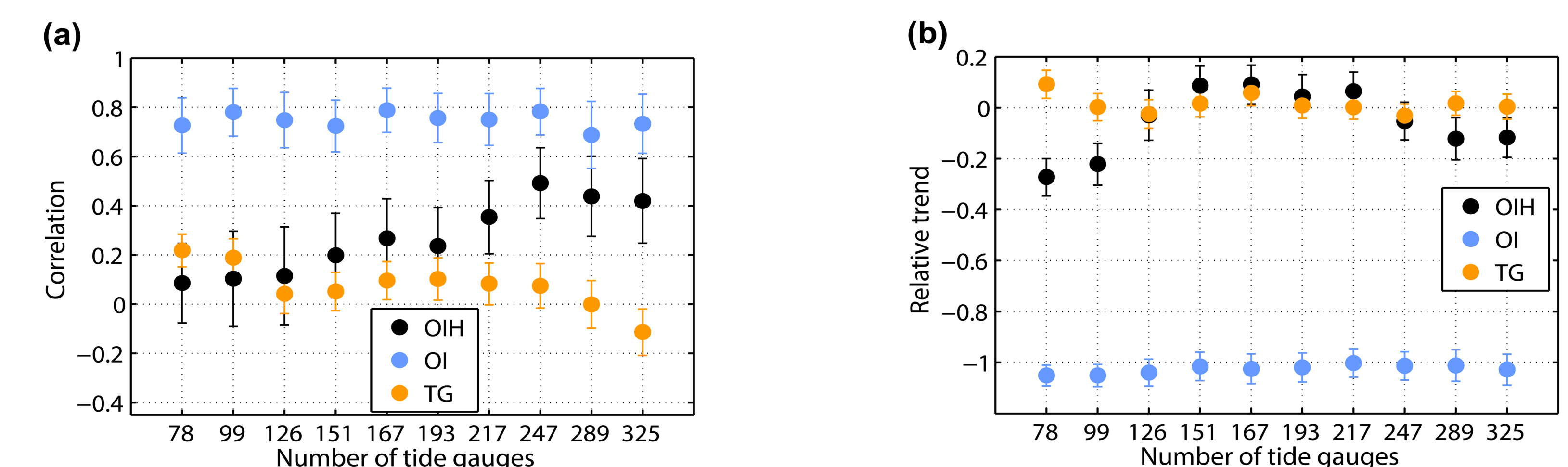


FIG 4. (a) Correlation, and (b) relative trend difference for different tide gauge distributions, and for the cases with (OIH) and without (OI) the EOF0 and a simple average of tide gauges (TG). Results are from the ensemble of surrogate SL fields based on SODA. The dots represent the ensemble mean while the error bars denote 1-sigma.

## 5. GMSL reconstruction for the period 1900-2011

Here we present a new reconstruction of the GMSL for the period 1900-2011 with and without the EOF0 and using all available tide gauges.

- Both the GMSL from altimetry and that from the reconstruction **without the EOF0 agree well with land hydrology** changes at interannual and decadal timescales.
- The GMSL with EOF0 has no correlation with land hydrology changes and shows **much larger variability**.

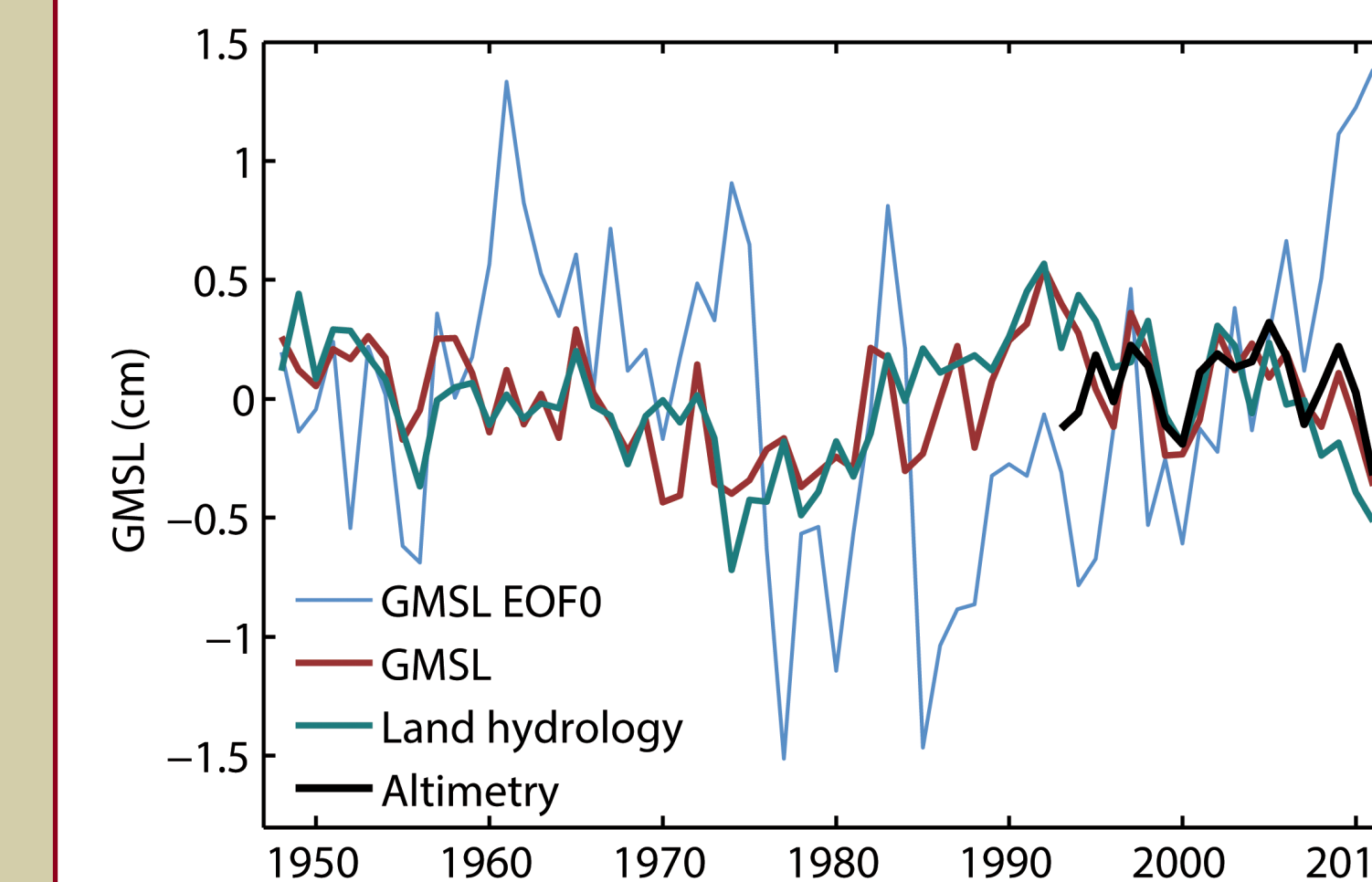


FIG 5. Comparison of land hydrology changes (green) with the GMSL derived from the EOF reconstruction based on altimetry and tide gauge data with (blue) and without (red) the use of a spatially uniform EOF. The GMSL from satellite altimetry is also shown (black). All time series have been detrended.

- The reconstructed GMSL without the EOF0 is correlated with both the **ENSO index** at interannual timescales and with the **AMO index** at decadal time scales (FIG 6)

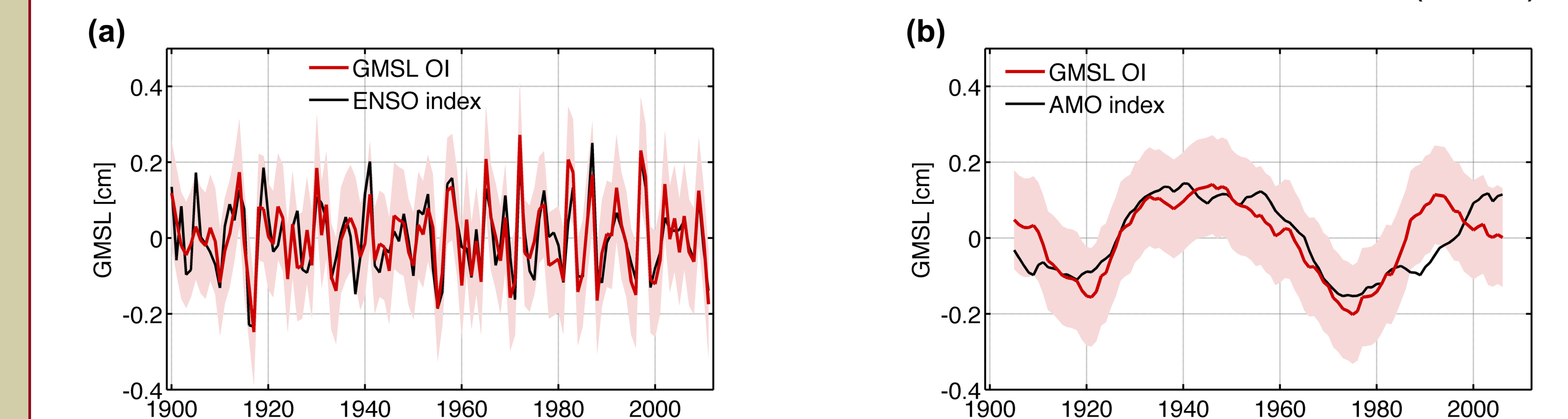


FIG 6. (a) Comparison of the 10-year high-pass filtered reconstructed GMSL without the EOF0 (red) with the ENSO index (black). (b) Comparison of the 10-year low-pass filtered reconstructed GMSL without the EOF0 (red) with the AMO index (black). Shading represent uncertainty (1-sigma).

## 6. Take-home messages

- **The method without the EOF0 uses global information**, which leads to a better reconstruction of the variability. The trend, however, is not captured.
- **The method with the EOF0 reduces to a weighted mean of tide gauges**, with the weights being the solution of the **generalized weighted mean** problem of altimetry data **at tide gauges locations**. Hence the method does not use global information.
- **The trend is better captured with the EOF0**, however using an interannual covariance matrix to determine the trend is dubious because it erroneously assume that the interannual variability and the trend and driven by the same mechanisms.
- **Improving coastal altimetry is critical** as the method is based on a covariance matrix that assumes a perfect agreement between tide gauges and nearby altimetry.

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### References

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