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Regular black hole remnants in de Sitter space

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ABSTRACT

We address the question of thermodynamical evolution of regular spherically symmetric cosmological black holes with de Sitter center. Space–time is asymptotically de Sitter as $r \to 0$ and as $r \to \infty$. A source term in the Einstein equations connects smoothly two de Sitter vacua with different values of cosmological constant and corresponds to anisotropic vacuum dark fluid defined by symmetry of its stress–energy tensor. In the range of masses $M_{cr1} \leqslant M \leqslant M_{cr2}$ it describes a regular cosmological black hole with three horizons, an internal horizon r_a, a black hole horizon $r_b > r_a$, and a cosmological horizon $r_c > r_b$. Thermodynamical preference for a final product of evaporation is a double-horizon ($r_a = r_b$) black hole remnant with the positive specific heat.

1. Introduction

The Hawking radiation from a black hole horizon [1] is the most remarkable example of a successful marriage between quantum mechanics and general relativity which gave the birth to thermodynamics of black holes [2].

Black hole remnants are considered as a source of dark matter for more than two decades [3–7] (for a review [8,9]), and planned to be searched at CERN LHC [10].

Gibbons and Hawking found that also cosmological horizon can radiate [11], and this gave rise to thermodynamics of horizons [12–16] (for a recent review [17]).

Astronomical observations testify that our universe is dominated at above 73% of its density by a dark energy responsible for its accelerated expansion due to negative pressure, $p = w\rho$, $w < -1/3$ [18], with the best fit $w = -1$ [19] corresponding to a cosmological constant λ. This motivates study of black hole remnants in de Sitter space.

Theoretical developments suggest the existence of a holographic duality between quantum gravity on de Sitter space and a certain Euclidean conformal field theory on its spacelike boundary, dS/CFT correspondence [20].

Studies on black hole thermodynamics in the de Sitter space are thus important in the context of dark matter physics and of the quantum theory of gravity.

A loop quantum gravity [21] provides arguments in favor of a regular black hole. Analyzing a Schwarzschild interior in frame of a minisuperspace model, Modesto found that the curvature invariant and the inverse volume operator have a finite spectrum inside a horizon [22].

The Einstein equations admit the class of regular spherically symmetric solutions asymptotically de Sitter as $r \to 0$ [23,24]. The idea of replacing a Schwarzschild singularity with a de Sitter vacuum, goes back to 1965 papers by Sakharov [25] who considered $p = -\rho$ as the equation of state for superdense matter and by Gliner who interpreted $p = -\rho$ as a vacuum with a non-zero density [26], and to 1988 paper by Poisson and Israel [27] who studied Schwarzschild–de Sitter transition as $r \to 0$.

A spherically symmetric space–time with de Sitter center ($T_i^k \to \rho_0 \delta_i^k = (8\pi G)^{-1} \Lambda \delta_i^k$ as $r \to 0$) is described by the Einstein equations with a source term satisfying [28]

$$T_t^t = T_r^r; \qquad T_\theta^\theta = T_\phi^\phi \tag{1}$$

The equation of state reads [27–29]

$$p_r = -\rho; \qquad p_\perp = -\rho - \frac{r}{2}\rho' \tag{2}$$

where $\rho(r) = T_t^t$ is the energy density, $p_r(r) = -T_r^r$ is the radial pressure, and $p_\perp(r) = -T_\theta^\theta = -T_\phi^\phi$ is the tangential pressure for anisotropic perfect fluid.

Spherically symmetric solutions specified by (1) belong to the Kerr–Schild class [30], so that extension to the Kerr family is straightforward [31] (for a recent review [32]).

A stress–energy tensor (1) represents a spherically symmetric anisotropic vacuum fluid [28,23,24,33,34] whose symmetry is re-

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duced as compared with maximally symmetric de Sitter vacuum $T_i^k = \rho_0 \delta_i^k$.[1]

Vacuum with a reduced symmetry (for a review [36–41]) provides a unified description of dark ingredients in the Universe by a vacuum dark fluid [34], which represents both distributed vacuum dark energy by a time evolving and spatially inhomogeneous cosmological term [23], and gravitational vacuum solitons, G-lumps, as dark matter candidates [34] which are regular gravitationally bound vacuum structures without horizons (dark particles or dark stars, dependently on a mass) [29,24,42]. Remnants of regular black holes with de Sitter center are naturally incorporated into this picture.

Spherically symmetric regular solutions satisfying (1), belong to the class of metrics [23]

$$ds^2 = g(r)\,dt^2 - \frac{dr^2}{g(r)} - r^2\,d\Omega^2 \tag{3}$$

asymptotically de Sitter as $r \to 0$ if the weak energy condition is satisfied [24]. In the case of two vacuum scales, at the center and at infinity, a space–time can be described by a metric function [43]

$$g(r) = 1 - \frac{2G\mathcal{M}(r)}{r} - \frac{\lambda r^2}{3}; \quad \mathcal{M}(r) = 4\pi \int_0^r \rho(x) x^2\,dx \tag{4}$$

whose asymptotics are the de Sitter metrics with λ as $r \to \infty$ and with $(\Lambda + \lambda)$, $\Lambda = 8\pi G \rho_0$, as $r \to 0$.

An asymptotically flat ($\lambda = 0$), de Sitter–Schwarzschild regular black hole [28] has in general two horizons [24], which coincide for a certain value M_{cr} of the mass parameter $M = \mathcal{M}(r \to \infty)$. It evaporates from both horizons, and generic asymptotic behavior of the metric function $g(r)$ defines generic dynamic of evaporation: It involves a phase transition where a specific heat is broken and changes its sign; a mass decreases during evaporation, temperature vanishes at a double horizon and evaporation stops leaving a regular double-horizon remnant with $M = M_{cr}$ [29,24]. A phase transition of this kind was found [44] also in the case of *a minimal model* [45] of a regular black hole.

Thermodynamics of two horizons has been studied in the literature for the case of the Schwarzschild–de Sitter black hole described by the metric function

$$g(r) = 1 - \frac{2GM}{r} - \frac{r^2}{l^2} \tag{5}$$

where M is the mass parameter and l is related to the background cosmological constant λ by $l^2 = 3/\lambda$. It emits the Hawking radiation from both horizons which are not in thermal equilibrium [11]. A global temperature can be defined only when the relation of surface gravities on horizons is a rational number [46].

Following Teitelboim who considered the Euclidean Schwarzschild–de Sitter geometry as an extremum of two different action principles [15,16], dynamical evolution is studied for two different thermodynamical systems [47]: a cosmological horizon with a black hole as a boundary, and a black hole horizon with a cosmological horizon as a boundary. Teitelboim identified a black hole mass M as a thermodynamical energy (without taking into account a pressure), and found the tendency of growing the mass M in the course of evaporation so that a black hole would evolve to the Nariai state [15]. Applying the second law of thermodynamics for a manifold between the horizons, Aros found an opposite tendency

[1] (1) is invariant under radial Lorentz boosts which makes impossible to single out a preferred comoving reference frame and thus fix the velocity with respect to a medium specified by (1) – which is the intrinsic property of a vacuum [35].

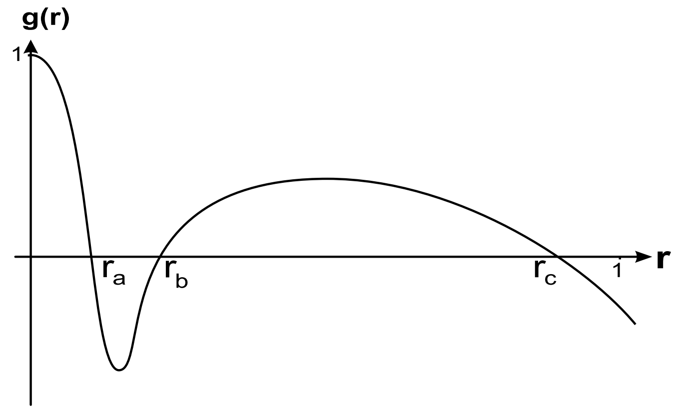

Fig. 1. Metric function $g(r)$ for a regular cosmological black hole with the de Sitter center.

– complete evaporation of a black hole [17]. An open question noted by Aros concerns the causal structure of space–time, namely the fate of energy radiated once the black hole disappears leaving behind the de Sitter space in which there is nothing beyond the cosmological horizon but the de Sitter space itself, and thus, "*roughly speaking* the energy cannot be hidden there" [17].

Complete evaporation of a singular black hole would involve also serious changes in space–time symmetry – from a singularity in a distinguished center to a maximally symmetric de Sitter or Minkowski space.

An asymptotically flat regular black hole leaves behind a double-horizon remnant [29,24,44]. We will show in this Letter that a regular black hole in de Sitter space does not evaporate completely too. In both cases space–time symmetry leaves unchanged.

A regular black hole in de Sitter space described by (4), represents a nonsingular modification of two-horizon Schwarzschild–de Sitter black hole. A singularity is replaced with de Sitter vacuum (with $\Lambda > \lambda$), and space–time involves two scales of vacuum density. The number of horizons is related to the number of extrema of the metric function $g(r)$ which in turn is related to the number of zeros of a tangential pressure $p_\perp$ [24], since in an extremum of the metric function $g(r)$, the Einstein equation with the $T_\theta^\theta = T_\phi^\phi$ in the right-hand side reads $g'' = 16\pi G p_\perp$. Zeros of $p_\perp$ are surfaces of zero-gravity at which the strong energy condition, $\rho + \sum p_k \geqslant 0$, is violated. For the class of metrics (3) corresponding to the equation of state (2), it reduces to $p_\perp \geqslant 0$. In the case of two vacuum scales it is violated twice, $p_\perp$ has two zero points, and space–time can have not more than three horizons [48], a black hole horizon r_b, a cosmological horizon r_c, and an internal horizon r_a which is the cosmological horizon for an observer in the region $0 \leqslant r < r_a$. Typical behavior of a metric function is shown in Fig. 1.

An extreme state with $r_a = r_b$ puts a lower limit M_{cr1}, and the extreme state $r_b = r_c$ puts an upper limit M_{cr2} on a black hole mass [43]. Two extreme states and two one-horizon configurations are shown in Fig. 2.

Uncertainties in predictions concerning an endpoint of evaporation of the Schwarzschild–de Sitter black hole, are related to the problem of definition of thermodynamical variables in a multi-horizon case with non-zero pressure.

In this Letter we study thermodynamical evolution in the case of three horizons by applying the Padmanabhan approach which takes into account that a pressure is not zero in de Sitter space [13]. Padmanabhan deduced the thermodynamical identity directly from the Einstein equations on a horizon for the class of solutions described by (3), and independently by consideration of a canoni-

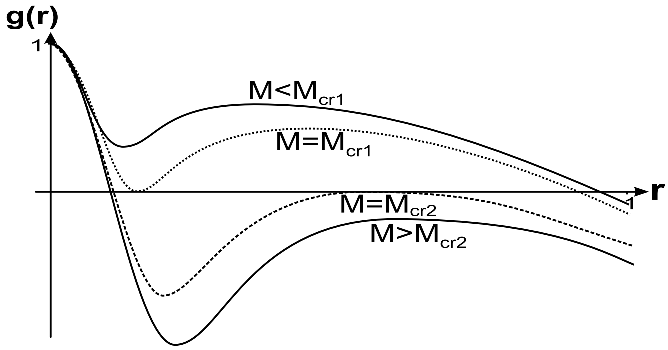

Fig. 2. Metric function $g(r)$ for one-horizon and double-horizon regular configurations in de Sitter space.

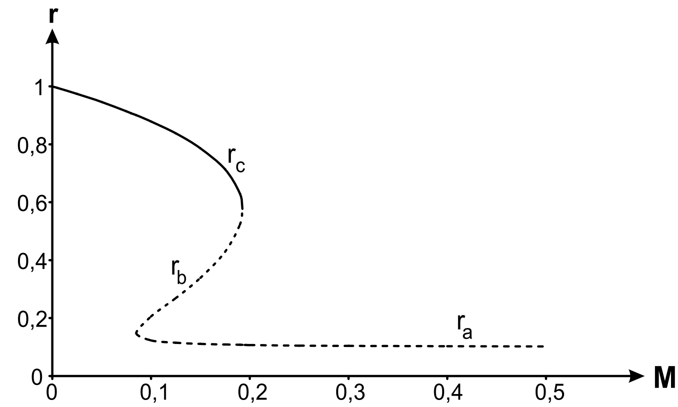

Fig. 3. Horizons for the case $q = 25$.

cal ensemble of metrics belonging to the class (3) at the constant temperature of the horizon determined by the periodicity of the Euclidean time in the Euclidean continuation of the Einstein action [13].

We keep fixed an internal Λ (as corresponding to a certain fundamental symmetry scale [42,23]). We keep also fixed (following [15,17]) a background λ (the question of λ-decay [49] is out of scope of this Letter), so that the results are applicable to the case when a time scale of evaporation is less than a time scale of changing λ.

Thermodynamics of a regular black hole with de Sitter interior is dictated by the typical behavior of the metric function $g(r)$, generic for the considered class of space–times specified by symmetry (1) of a stress–energy tensor satisfying the weak energy condition. A particular form of the density profile $\rho(r)$ affects only numerical values of thermodynamical parameters but not their dynamical behavior.

In Section 2 we outline the basic features of space–time. In Section 3 we study thermodynamics of horizons, and show that in the case of three horizons there exists a certain range of parameters for which there can exist a global temperature. In Section 4 we study evolution during evaporation and find a thermodynamically stable double-horizon remnant ($M = M_{cr1}$ in Fig. 2) with a positive specific heat. In Section 5 we summarize and discuss the results.

2. Metric of space–time

The stress–energy tensor responsible for geometry (4) connects two de Sitter vacua: $T_{ik} = (8\pi G)^{-1}(\Lambda + \lambda)g_{ik}$ at the center, and $T_{ik} = (8\pi G)^{-1}\lambda g_{ik}$ at infinity.

A density component of T_i^k is given by

$$T_t^t(r) = \rho(r) + (8\pi G)^{-1}\lambda; \quad \rho(r \to 0) \to (8\pi G)^{-1}\Lambda \tag{6}$$

It includes a background vacuum density $\rho_\lambda = (8\pi G)^{-1}\lambda$ and the dynamical density ρ vanishing as $r \to \infty$ quickly enough to ensure the finiteness of the total mass

$$M = 4\pi \int_0^\infty \rho(r) r^2\, dr \tag{7}$$

Geometry defined by (4) has three characteristic lengths

$$r_g = 2GM; \qquad r_0 = \sqrt{\frac{3}{\Lambda}} = \sqrt{\frac{3}{8\pi G\rho_0}}; \qquad l = \sqrt{\frac{3}{\lambda}}$$

where l is related to the background vacuum density $\rho_\lambda = (8\pi G)^{-1}\lambda$, and r_0 to the de Sitter vacuum in the origin $\rho_0 = (8\pi G)^{-1}\Lambda$. The characteristic parameter relating two vacuum scales, Λ and λ, is given by

$$q = \frac{l}{r_0} = \sqrt{\frac{\Lambda}{\lambda}} \tag{8}$$

For $r \gg r_*$ where

$$r_* = \left(r_0^2 r_g\right)^{1/3} \tag{9}$$

is the characteristic scale of space–time with the de Sitter interior [27,28], the metrics (4) are asymptotically Schwarzschild–de Sitter, Eq. (5), or asymptotically Schwarzschild in the case $\lambda = 0$. For $r \ll r_*$ the metrics are asymptotically de Sitter with $\Lambda + \lambda$.

For numerical calculations needed to produce pictures illustrating typical behavior, we adopt the density profile

$$\rho(r) = \rho_0 e^{-r^3/r_0^2 r_g} \tag{10}$$

which corresponds to replacing Schwarzschild singularity with de Sitter vacuum in a simple semiclassical model for vacuum polarization in the spherically symmetric gravitational field [28,29]. The mass function is given by

$$\mathcal{M}(r) = M\left(1 - e^{-r^3/r_0^2 r_g}\right) \tag{11}$$

In all pictures below we use the normalization $r \to r/l$ so that the mass parameter M is normalized to l/G.

A mass (7) of a regular cosmological black hole is confined within a certain range $M_{cr1} < M < M_{cr2}$ which depends on the parameter q [43]. The critical values M_{cr1} and M_{cr2} correspond to the two double-horizon configurations, $r_a = r_b$ and $r_b = r_c$. Horizon-mass diagram is plotted in Fig. 3.

In geometries with de Sitter center there exist zero gravity surfaces defined by $p_\perp(r) = 0$ [29,23], beyond which the strong energy condition is violated and gravitational attraction becomes gravitational repulsion.

In geometries satisfying the weak energy condition and not satisfying the dominant energy condition (which requires $p_k \leqslant \rho$), there exist also surfaces at which 4-curvature scalar and 3-curvature scalar

$$\mathcal{R} = 16\pi G(\rho - p_\perp); \qquad \mathcal{P} = 8\pi G(2\rho - p_\perp) \tag{12}$$

vanish, which can be essential for evaporation dynamics [42,24]. For geometries satisfying the energy dominant condition these surfaces are absent [41].

In the case of two vacuum scales the curvature scalars $\mathcal{R}$ and $\mathcal{P}$ can have two zero points since $\mathcal{R} \to 4(\Lambda + \lambda)$, $\mathcal{P} \to 3(\Lambda + \lambda)$ as $r \to 0$, while $\mathcal{R} \to 4\lambda$, $\mathcal{P} \to 3\lambda$ as $r \to \infty$.

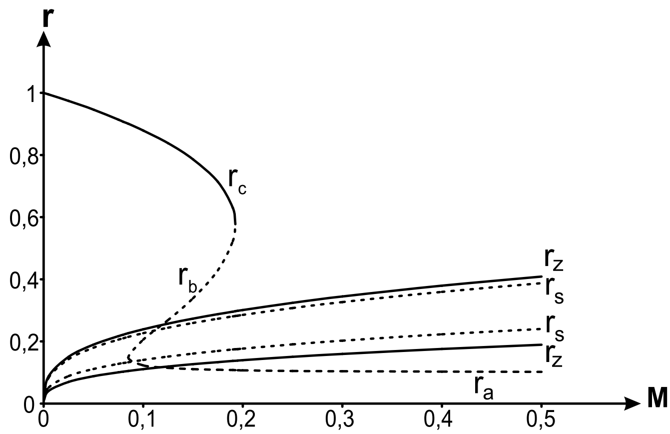

Fig. 4. Horizons, zero gravity surfaces $r = r_z$ and zero 4-curvature surfaces $r = r_s$, for $q = 10$.

For geometry with the density profile (10) the energy dominant condition is not satisfied. Zero gravity surfaces $r = r_z$ and zero 4-curvature surfaces $r = r_s$ are plotted in Fig. 4 for the case $q = 10$.

3. Thermodynamics of horizons

The Hawking temperature of a horizon r_h is given by [11]

$$kT_h = \frac{\hbar}{2\pi c}\kappa_h = \frac{\hbar}{4\pi c}\left|g'(r_h)\right| \tag{13}$$

where κ_h is the surface gravity and k is the Boltzmann constant.

We apply here the Padmanabhan approach based on the thermodynamical identity $T\,dS - dE = p\,dV$ where the pressure p is provided by a source term in the Einstein equations, and dV is the change in the volume when the horizon is displaced infinitesimally [13].

On a black hole horizon where $g'(r_h) > 0$, temperature and thermodynamical energy are given by (in the units $\hbar = G = c = k = 1$) [13]

$$T_b = \frac{g'(r_b)}{4\pi}; \qquad E_b = \frac{1}{2}r_b \tag{14}$$

On a cosmological horizon $g'(r_h) < 0$, and [13]

$$T_h = -\frac{g'(r_h)}{4\pi}; \qquad E_h = -\frac{1}{2}r_h \tag{15}$$

In our case it is valid for $r_h = r_a$ and $r_h = r_c$.

Entropy and free energy are given by

$$S_h = 4\pi r_h^2; \qquad F_h = E_h - T_h S_h \tag{16}$$

An observer in the region $r_b < r < r_c$ can detect radiation from a black hole horizon r_b and cosmological horizon r_c. An observer in the region $0 \leqslant r < r_a$ can detect radiation from his cosmological horizon r_a.

For geometry specified by (4) the temperature of a horizon is given by

$$T_h = \frac{1}{4\pi}\left|\frac{1}{r_h} - \frac{3r_h}{l^2} - 8\pi\rho(r_h)r_h\right|; \quad r_h = r_a, r_b, r_c \tag{17}$$

A specific heat, $C_h = dE_h/dT_h$ is calculated from

$$C_h^{-1} = \frac{dT_h}{dr_h}\frac{dr_h}{dE_h} \tag{18}$$

which gives

$$C_h^{-1} = -\frac{1}{2\pi}\left[8\pi\rho'(r_h)r_h + 8\pi\rho(r_h) + \lambda + \frac{1}{r_h^2}\right] \tag{19}$$

It is easy to check that C_h^{-1} can be written as

$$C_h^{-1} = \frac{1}{2\pi}\left(\frac{g'(r_h)}{r_h} + g''(r_h)\right) \tag{20}$$

On a double horizon $r_h = r_d$ it gives

$$C_d^{-1} = \frac{1}{2\pi}g''(r_d) \tag{21}$$

As we shall see in the next section, the basic formulae (13) and (20)–(21) determine the dynamics of evaporation as dictated by generic behavior of the metric function $g(r)$ for any density profile $\rho(r)$ whose behavior is governed by the weak energy condition (non-negative density for any observer which is satisfied if and only if $\rho \geqslant 0$ and $\rho + p_k \geqslant 0$) which requires it (by Eq. (2)) to decrease monotonically from $\rho_0 = (8\pi G)^{-1}\Lambda$ for $r = 0$ to $\rho = 0$ as $r \to \infty$ [24].

In particular, the simple general formula (21) tells us unambiguously that an extreme state with a double horizon is thermodynamically stable when it appears in a minimum of the metric function $g(r)$, and thermodynamically unstable when it appears in its maximum.[2]

Derivative of the metric function $g(r)$ is negative on the cosmological horizons and positive on a black hole horizon. As a results temperatures on cosmological horizons $r_h = r_a$, $r_h = r_c$ are

$$T_{c,a} = \frac{1}{4\pi}\left(8\pi\rho(r_h)r_h + \frac{3r_h}{l^2} - \frac{1}{r_h}\right) \tag{22}$$

and on a black hole horizon r_b it is

$$T_b = \frac{1}{4\pi}\left(\frac{1}{r_b} - \frac{3r_b}{l^2} - 8\pi\rho(r_b)r_b\right) \tag{23}$$

Let us note that in the case of three horizons there exists a certain range of parameters for which temperatures on a black hole and cosmological horizons are equal, i.e. the case when one can speak about a global temperature for an observer in the region between the black hole horizon r_b and the cosmological horizon r_c. The equation $g'(r_b(M,q)) = |g'(r_c(M,q))|$ gives the dependence $M(q)$ so that for each value of the parameter q there exists such a value of the mass parameter M at which surface gravities and thus temperatures are equal. Dependence of global temperature T_G on q and M is shown in Fig. 5.

4. Evolution during evaporation

The first question is – where to move horizons?

For an observer in the region $0 \leqslant r \leqslant r_a$, the horizon r_a is the boundary of his manifold, and the second law of thermodynamics reads $dS_a \geqslant 0$. It requires $dr_a \geqslant 0$.

Looking at the horizon-mass diagram plotted for the density profile (10) we see that on the black hole horizon $dr_h/dM > 0$ while on both cosmological horizons $dr_h/dM < 0$, so that

$$\frac{dr_a}{dM} \leqslant 0; \qquad \frac{dr_b}{dM} \geqslant 0; \qquad \frac{dr_c}{dM} \leqslant 0 \tag{24}$$

It follows then that when r_a increases, mass M decreases by (24), hence black hole horizon shrinks, $dr_b < 0$, and cosmological horizon moves outward, $dr_c \geqslant 0$.

[2] In [44] a specific heat was calculated as vanishing at the double horizon by identifying the thermodynamical energy with a black hole mass M, which is true for the Schwarzschild black hole but cannot be applied directly in the case of two horizons and non-zero pressure.

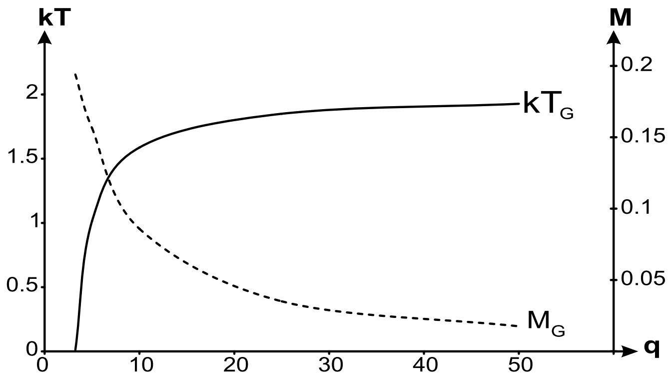

Fig. 5. Dependence of the global temperature kT_G and mass M_G on the parameter q.

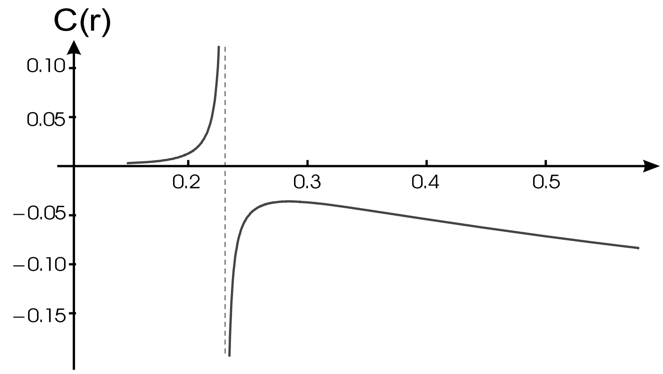

Fig. 6. Dependence of C_b on r_b for the case $q = 10$.

The next question is – how general is the relation (24)?

Following Teitelboim we can take derivative of the relation $g(r_h, M) = 0$ keeping λ fixed [15]. It gives

$$\frac{dr_h}{dM} = -\frac{\partial g(r_h)}{\partial M}\frac{1}{g'(r_h)} \tag{25}$$

With taking into account behavior of g' on the horizons, we should have to have

$$\frac{\partial g}{\partial M} < 0 \tag{26}$$

on each horizon in order to get (24).

It is straightforward to check that (26) holds everywhere for the regular metrics with $\rho \to \rho_0$ as $r \to 0$, asymptotically Schwarzschild–de Sitter, Eq. (5) as $r \to \infty$ with the mass parameter M given by (7). To ensure the needed behavior (7) for the mass function in (4), an r-dependence of a density profile should involve scaling r/r^*, with the characteristic scale r_* given by (9).

Then $\rho(r) = \rho_0 \tilde{\rho}(r/r_*)$, and the mass function reads

$$\mathcal{M}(r) = 3M\phi(y); \quad y = \frac{r}{r_*};\ \phi(y) = \int_0^y \tilde{\rho}(z) z^2\, dz \tag{27}$$

The metric function takes the form

$$g = 1 - M^{2/3}\frac{1}{r_0^{2/3}}\left[\left(\frac{3}{2^{1/3}}\right)\frac{\phi(y)}{y} + \frac{(2^{2/3})}{q^2}y^2\right] \tag{28}$$

and we see that (26) holds everywhere. On the horizons

$$\frac{\partial g}{\partial M} = -\frac{2}{3M} \tag{29}$$

As a result, for any density profile evolution of a black hole goes towards a double-horizon state with $r_a = r_b$.

Near the double horizon $r_b = r_c$ the specific heat C_b is negative by (21). The same relation requires C_b be positive near the double horizon $r_a = r_b$. Therefore it should occur a second-order phase transition during evaporation where C_b is broken and changes sign. Dependence of a specific heat on the black hole horizon r_b is shown in Fig. 6.

The temperature of a black hole horizon is shown in Fig. 7 for several values of the parameter q. At the transition it acquires the maximum value

$$T_{b\,\max} = -\frac{1}{4\pi}g''(r_b)r_b \tag{30}$$

Before the transition, $C_b < 0$, hence $dT_b/dr_b < 0$, when r_b decreases, E_b decreases too, temperature increases to a maximum

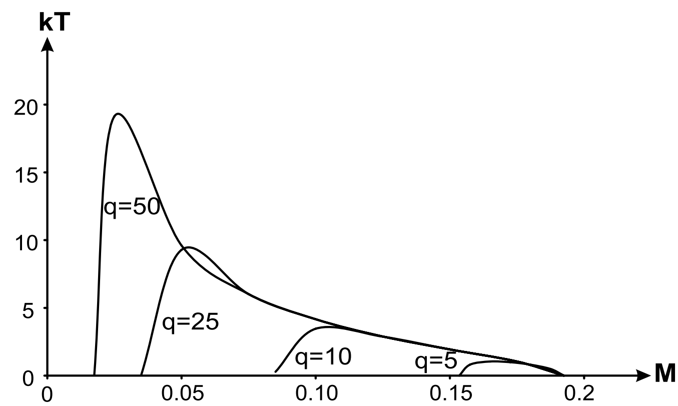

Fig. 7. Temperature on a black hole horizon.

(30) where C_b changes sign, so that after transition we have $dT_b/dr_b > 0$, and thus decreasing r_b leads to decreasing T_b until it vanishes at the double horizon. At this point of evolution specific heat C_b is positive by (21), the free energy is equal E_b which achieves its minimum, so that the double-horizon state $r_b = r_a$ is the thermodynamically stable endpoint of evolution during evaporation.

On the internal horizon thermodynamical energy is

$$E_a = -\frac{r_a}{2} \tag{31}$$

By (24), $dr_a/dM < 0$ and M decreases when r_a grows. Since $g' < 0$, we have

$$\frac{dT_a}{dr_a} = -\frac{T_a}{r_a} + \frac{1}{4\pi}g''(r_a) \tag{32}$$

Specific heat C_a is positive near the double horizon, so that $dT_a/dE_a > 0$ and $dT_a/dr_a < 0$. Hence T_a decreases with increasing r_a, the mass M decreases too, $dT_a/dM > 0$ and $dT_a/dr_a < 0$, so the growth in r_a leads to monotonic decreasing the temperature T_a until it vanishes at the double horizon where the energy E_a, and hence free energy F_a achieve the minimum.

The cosmological horizon r_c moves outwards during evaporation. The specific heat is negative near the double horizon $r_b = r_c$. Hence $dT_c/dE_c < 0$, $dT_c/dr_c > 0$, and T_c increases with increasing r_c, and since the mass decreases by virtue of (24), $dT_c/dM < 0$. Starting from the double horizon $r_b = r_c$ evolution must thus occur as follows: M decreases and r_c increases by (24), hence E_c decreases by (15), and the specific heat remains negative. The tem-

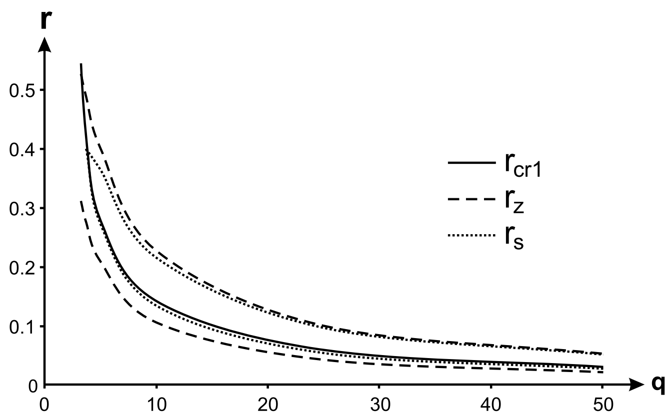

Fig. 8. Double horizon r_{cr1} and characteristic surfaces r_z and r_s in the remnant.

perature on the cosmological horizon is almost insensitive to the parameter q.

At approaching the double horizon state, the nearest to the center surfaces of zero gravity and zero curvature get consequently inside an internal horizon r_a (see Fig. 4).

The double horizon $r_a = r_b = r_{cr1}$ and characteristic surfaces are shown in Fig. 8 as functions of q. We see that the horizon fits inside zero gravity surfaces. It is intrinsic behavior for any density profile. At this double horizon $g'' > 0$. Two other extrema of a metric function (4) are maxima at $r = 0$ and $r > r_{cr1}$, so that g'' changes sign at certain values $r < r_{cr1}$ and $r > r_{cr1}$. As a result a double horizon is confined between surfaces of zero gravity.

5. Summary and discussion

We have studied thermodynamics of horizons for a spherically symmetric regular space–time with two vacuum scales, Λ as $r \to 0$ as a certain fundamental symmetry scale and the background $\lambda < \Lambda$, applying the Padmanabhan approach relevant for the multi-horizon space–time with non-zero pressure. We deduced basic thermodynamical formulae valid for any density profile satisfying the weak energy condition (needed for replacing a Schwarzschild singularity with a de Sitter vacuum interior).

We have shown that in the case of three horizons there exists a certain range of parameters for which there can exist a global temperature for an observer between the black hole horizon r_b and cosmological horizon r_c.

We found that a regular spherically symmetric black hole in de Sitter space evolves to a double-horizon thermodynamically stable remnant with the positive specific heat. Its stability to small perturbations is currently under investigation, preliminary results suggest stability in a wide range of density profiles.

During evaporation the second-order phase transition occurs where the specific heat C_b is broken and changes its sign, and the black hole temperature achieves its maximum.

In the considered case of two vacuum scales such that $\Lambda > \lambda$ and hence $q > 1$, there exists a certain critical value q_{cr} (for the density profile (10) $q_{cr} \simeq 3.24$) at which, for a certain value M_{cr}, the metric function, its first and second derivatives (and hence also tangential pressure) vanish. As a result, the temperature and the specific heat vanish, so this configuration is thermodynamically stable. It is the case of the triple horizon. It is interesting that near $q = q_{cr}$, any metric function has only one zero and a "plateau", distinguished by two conditions $g'(r) = 0$; $g''(r) = 0$. These two equations give two dependences $r_i(M)$ and $q_i(M)$, the second appears to be close to q_{cr} up to 10^{-5}. Horizons and zero-gravity surfaces are shown in Fig. 9.

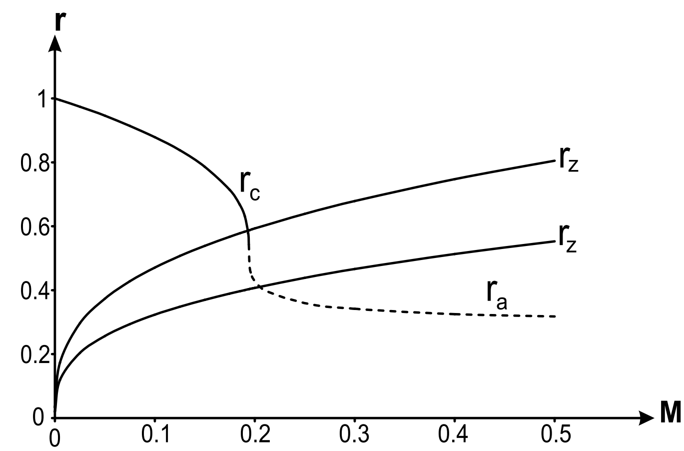

Fig. 9. Horizons and zero-gravity surfaces for $q = q_{cr}$.

The detailed study of this case and its applications will be published in a separate paper.

References

[1] S.W. Hawking, Nature 248 (1974) 30;
S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[2] R.M. Wald, Quantum Field Theory in Curved Space and Black Hole Thermodynamics, Univ. Chicago Press, 1994.
[3] J.H. MacGibbon, Nature 329 (1987) 308.
[4] K. Rajagapol, M.S. Turner, F. Wilczek, Nucl. Phys. B 358 (1991) 447.
[5] B.J. Carr, J.H. Gilbert, J.E. Lidsey, Phys. Rev. D 50 (1994) 4853.
[6] R.J. Adler, P. Chen, D. Santiago, Gen. Relativ. Gravit. 33 (2001) 2101.
[7] P. Chen, R.J. Adler, Nucl. Phys. B 124 (2003) 103.
[8] B.J. Carr, in: Proc. of 22nd Texas Symp., 2004, ECONFCO41213 (2004) 0204;
B.J. Carr, astro-ph/0504034, 2005.
[9] K. Nozari, S.H. Mehdipour, Mod. Phys. Lett. A 20 (2005) 2937;
K. Nozari, S.H. Mehdipour, gr-qc/0809.3144, 2008.
[10] B. Koch, M. Bleicher, S. Hossenfelder, JHEP 0510 (2005) 053;
G.C. Nayak, arXiv:0901.3358 [hep-ph], 2009.
[11] G.W. Gibbons, S.W. Hawking, Phys. Rev. D 15 (1977) 2738.
[12] R. Bousso, JHEP 0011 (2000) 038;
R. Bousso, JHEP 0104 (2001) 035.
[13] T. Padmanabhan, Class. Quantum Grav. 19 (2002) 5387;
T. Padmanabhan, Gen. Relativ. Gravit. 34 (2002) 2029;
T.R. Choudhury, T. Padmanabhan, Gen. Relativ. Gravit. 39 (2007) 1789.
[14] R.G. Cai, Phys. Lett. B 525 (2002) 331.
[15] C. Teitelboim, hep-th/0203258, 2002.
[16] A. Gomberoff, C. Teitelboim, Phys. Rev. D 67 (2003) 104024.
[17] R. Aros, Phys. Rev. D 77 (2008) 104013.
[18] A.G. Riess, et al., Astron. J. 116 (1998) 1009;
A.G. Riess, et al., Astron. J. 117 (1999) 707;
S. Perlmutter, et al., Astrophys. J. 517 (1999) 565;
N.A. Bahcall, et al., Science 284 (1999) 1481;
L. Wang, et al., Astrophys. J. 530 (2000) 17;
D.N. Spergel, et al., Astrophys. J. Suppl. 148 (2003) 175;
M. Schubnell, SNAB Collaboration, in: Proc. of the CIPANP, 2003;
M. Schubnell, SNAB Collaboration, astro-ph/0308404.
[19] R. Scranton, et al., astro-ph/0307335;
P.S. Corasaniti, E.J. Copeland, Phys. Rev. D 65 (2002) 043004;
P.S. Corasaniti, et al., Phys. Rev. D 70 (2004) 083006;
S. Hannestad, E. Mortsell, Phys. Rev. D 66 (2002) 063508;
K. Takahashi, et al., astro-ph/0305260;
J.L. Tonry, et al., Astrophys. J. 594 (2003) 1;
J. Ellis, Philos. Trans. R. Soc. London A 361 (2003) 2607;
E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) 1753.
[20] A. Strominger, JHEP 0110 (2001) 034;
A. Strominger, JHEP 0111 (2001) 049.
[21] A. Perez, Class. Quantum Grav. 20 (2003) R43;
C. Rovelli, Quantum Gravity, Cambridge Univ. Press, 2004.
[22] L. Modesto, in: Proc. of the XVII SIGRAV Conf., Turin, 2006;
L. Modesto, hep-th/0701239.
[23] I.G. Dymnikova, Phys. Lett. B 472 (2000) 33.
[24] I. Dymnikova, Class. Quantum Grav. 19 (2002) 725.
[25] A.D. Sakharov, Sov. Phys. JETP 22 (1966) 241.
[26] E.B. Gliner, Sov. Phys. JETP 22 (1966) 378.

[27] E. Poisson, W. Israel, Class. Quantum Grav. 5 (1988) L201.
[28] I. Dymnikova, Gen. Relativ. Gravit. 24 (1992) 235;
I. Dymnikova, CAMK preprint 216 (1990).
[29] I.G. Dymnikova, Int. J. Mod. Phys. D 5 (1996) 529.
[30] R.P. Kerr, A. Schild, Am. Math. Soc. Symp., New York, 1964.
[31] M. Gürses, F. Gürsey, J. Math. Phys. 16 (1975) 2385.
[32] I. Dymnikova, Phys. Lett. B 639 (2006) 368.
[33] I. Dymnikova, Int. J. Mod. Phys. D (2003) 1015.
[34] I. Dymnikova, E. Galaktionov, Phys. Lett. B 645 (2007) 358.
[35] L.D. Landau, E.M. Lifshitz, Classical Theory of Fields, Pergamon Press, 1975.
[36] I. Dymnikova, in: E. Tropp (Ed.), Woprosy Matematicheskoj Fiziki i Prikladnoj Matematiki, St. Petersburg, 2000, p. 29;
I. Dymnikova, gr-qc/0010016.
[37] I. Dymnikova, Gravit. Cosmol. Suppl. 8 (2002) 131, gr-qc/0201058.
[38] I. Dymnikova, in: G. Marmo, C. Rubano, P. Scudellaro (Eds.), General Relativity, Cosmology and Gravitational Lensing, Bibliopolis, Napoli, 2002, p. 95.
[39] I. Dymnikova, in: H.V. Klapdor-Kleinhaus (Ed.), Beyond the Desert 2003, Springer-Verlag, 2004, p. 485.
[40] I. Dymnikova, in: H.V. Klapdor-Kleinhaus (Ed.), Beyond the Desert 2003, Springer-Verlag, 2004, p. 521.
[41] I. Dymnikova, E. Galaktionov, Class. Quantum Grav. 22 (2005) 2331.
[42] I. Dymnikova, in: L.M. Burko, A. Ori (Eds.), Internal Structure of Black Holes and Spacetime Singularities, in: Annals of The Israel Physical Society, vol. 13, 1997, p. 422.
[43] I. Dymnikova, B. Soltysek, Gen. Relativ. Gravit. 30 (1998) 1775;
I. Dymnikova, B. Soltysek, in: J. Rembielinsky (Ed.), Particles, Fields and Gravitation, 1998, p. 460.
[44] Y.S. Myung, Y.-W. Kim, Y.-J. Park, Phys. Lett. B 656 (2007) 221.
[45] S.A. Hayward, Phys. Rev. Lett. 96 (2006) 031103.
[46] Feng-Li Lin, hep-th/9807084;
Feng-Ki Kin, Chopin Soo, Class. Quantum Grav. 16 (1999) 551.
[47] Y. Sekiwa, Phys. Rev. D 73 (2006) 084009, and references therein.
[48] K.A. Bronnikov, A. Dobosz, I.G. Dymnikova, Class. Quantum Grav. 20 (2003) 3797.
[49] A. Gomberoff, M. Henneaux, C. Teitelboim, F. Wilczek, Phys. Rev. D 69 (2004) 083520.