
Figures and figure supplements

Flagellar synchronization through direct hydrodynamic interactions

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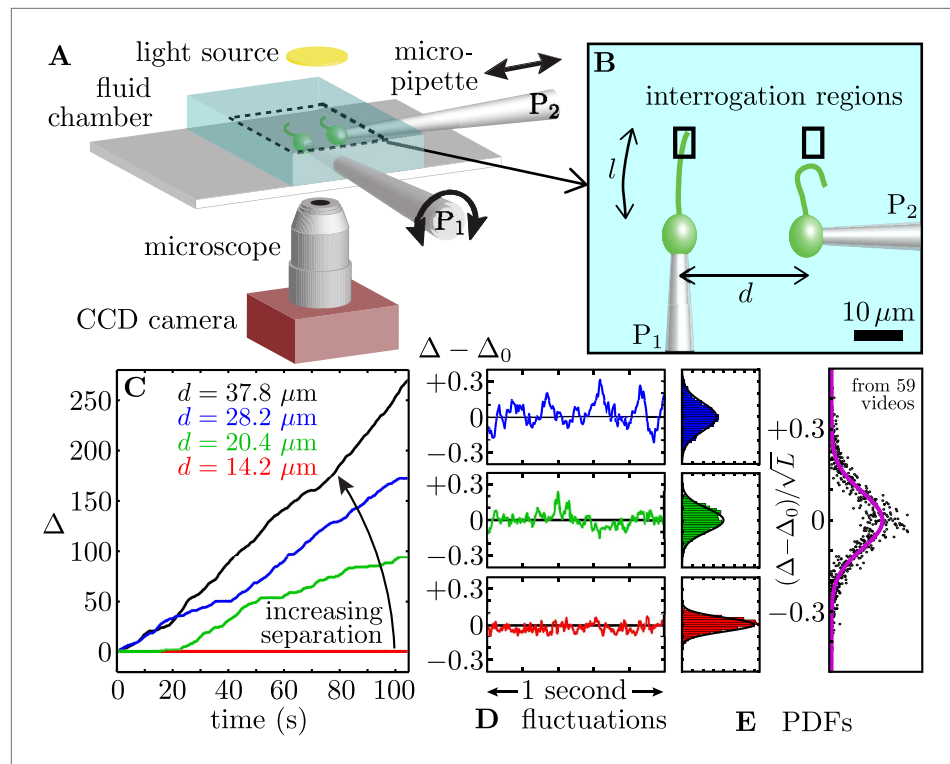


Figure 1. Synchronized pairs of beating flagella. **(A)** Experimental apparatus and **(B)** cell configuration. **(C)** Extracted phase difference $\Delta = (\varphi_1 - \varphi_2)/2\pi$ at four different interflagellar spacings, as indicated. These separations correspond to scaled spacing $L = d/l$ of 0.85, 1.22, 1.69, and 2.27. **(D)** fluctuations during phase-locked periods around the average phase lag, Δ_0 , and **(E)** the fluctuations' probability distribution functions (PDFs), each cast in terms of the rescaled separation-specific variable $(\Delta - \Delta_0)/\sqrt{L}$. Solid lines represent Gaussian fits. Further details of the phase extraction procedure can be found in **Figure 1—figure supplement 1**. Samples of the four processed videos corresponding to the cells in **Figure 1C** are shown in **Video 1**.

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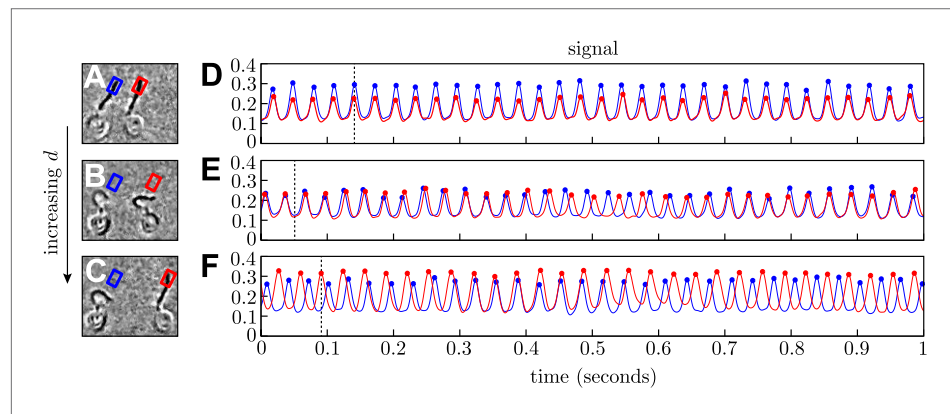


Figure 1—figure supplement 1. Phase extraction.

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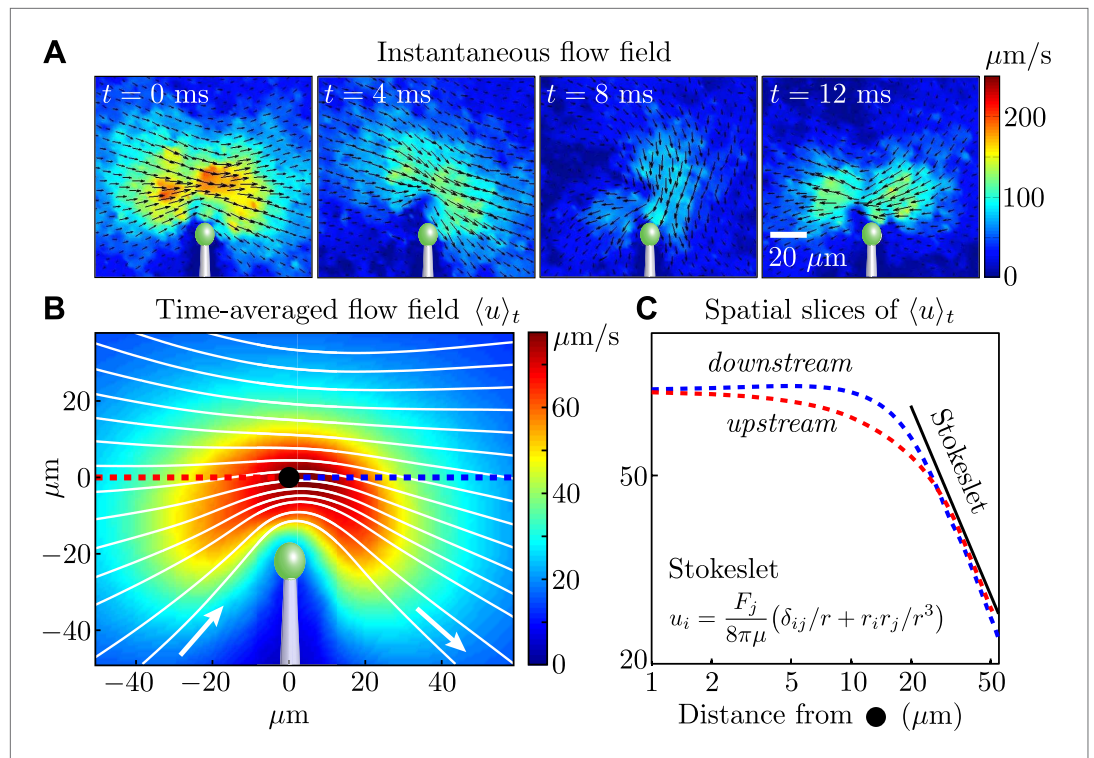


Figure 2. Measured flagellar flow field. **(A)** Time-dependent flow field for an individual cell measured using particle image velocimetry. Results are shown for the first half of the beating cycle. **(B)** Time-averaged flow field $\langle u \rangle_t = (1/r) \int_0^r |\mathbf{u}(\mathbf{x}, t)| dt$ (averaged across 4 cells with $r \sim 1000$ beats for each). The velocity magnitude (colour) and streamlines (white) are shown. **(C)** Velocity magnitude upstream (red) and downstream (blue) of the origin (black dot in **B**).

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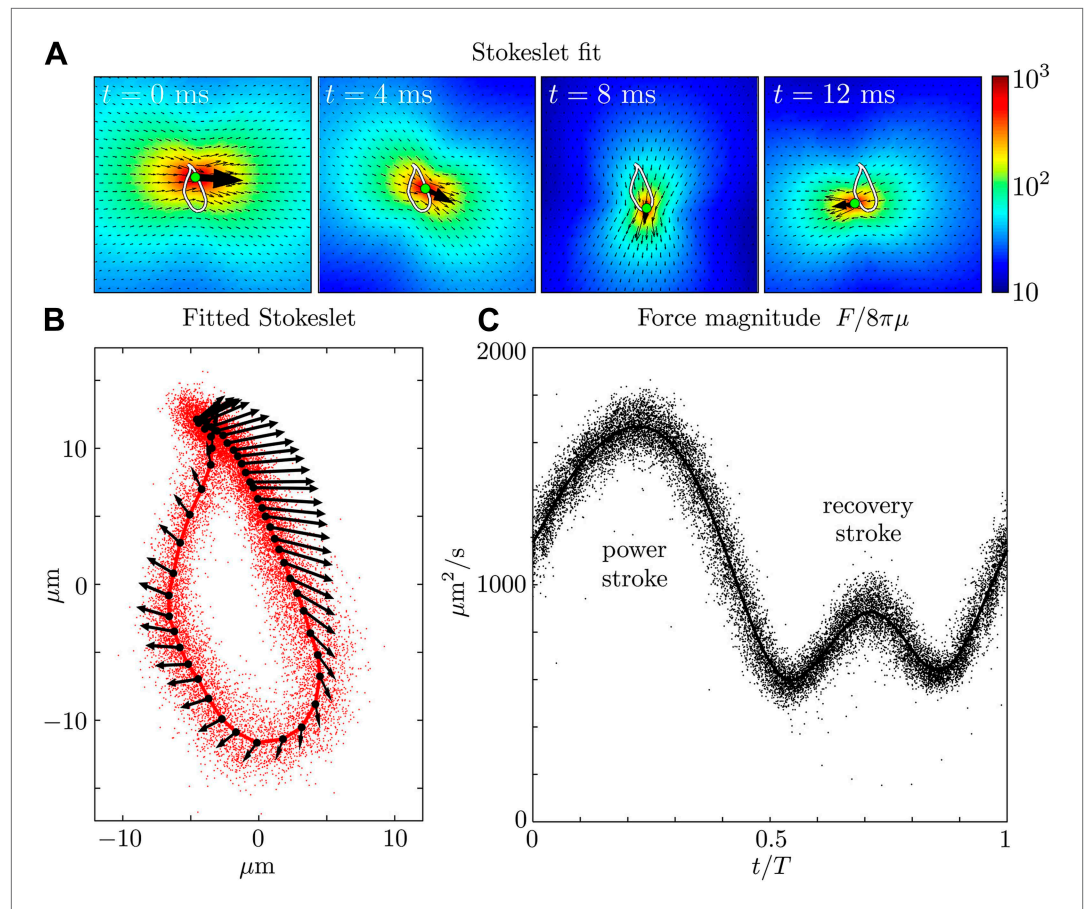


Figure 3. Force amplitude of flagellum. **(A)** Fitted instantaneous velocity field at various stages during the first half of one representative flagellar beat. **(B)** The fitted Stokeslet is shown at evenly-spaced times throughout the average flagellar beat cycle. The red dots indicate the Stokeslet position extracted from every frame. **(C)** Amplitude of the fitted point force as a function of time throughout the flagellar beat period T .

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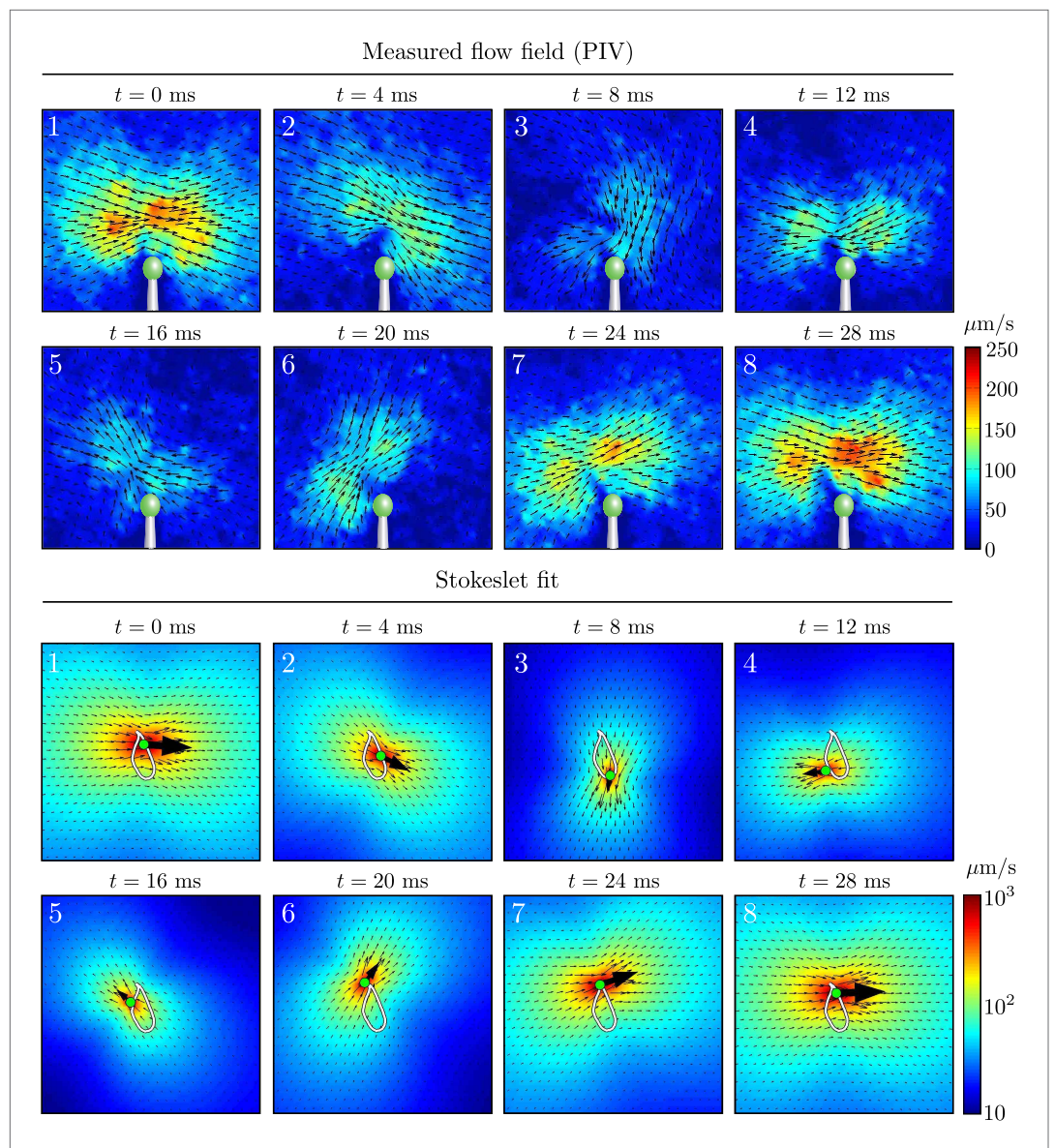


Figure 3—figure supplement 1. Time-dependent flow fields.

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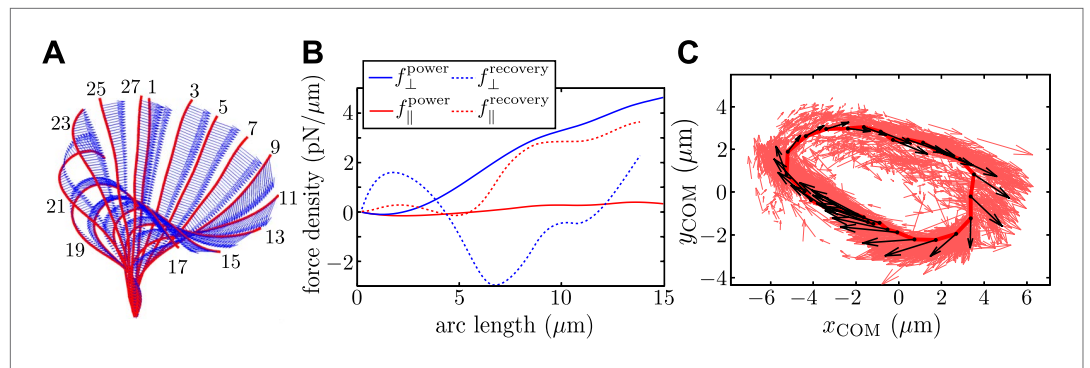


Figure 4. Resistive force theory analysis. **(A)** Instantaneous velocity distribution along the flagellum during one complete beat cycle (indexed by frame number, imaged at 1000 fps). **(B)** Components of integrated force density produced by a flagellum executing characteristic power and recovery strokes, as a function of arclength along the flagellum measured from the basal to the distal end. **(C)** Integrated vector forces $\mathbf{F}(t)$ shown localised at centre-of-mass coordinates $\mathbf{x}(t)$ (red: per frame, black: averaged over $O(10^3)$ frames), evolve cyclically around an average trajectory. The average value is $|\mathbf{F}|/8\pi\mu \sim 1910 \mu\text{m}^2/\text{s}$.

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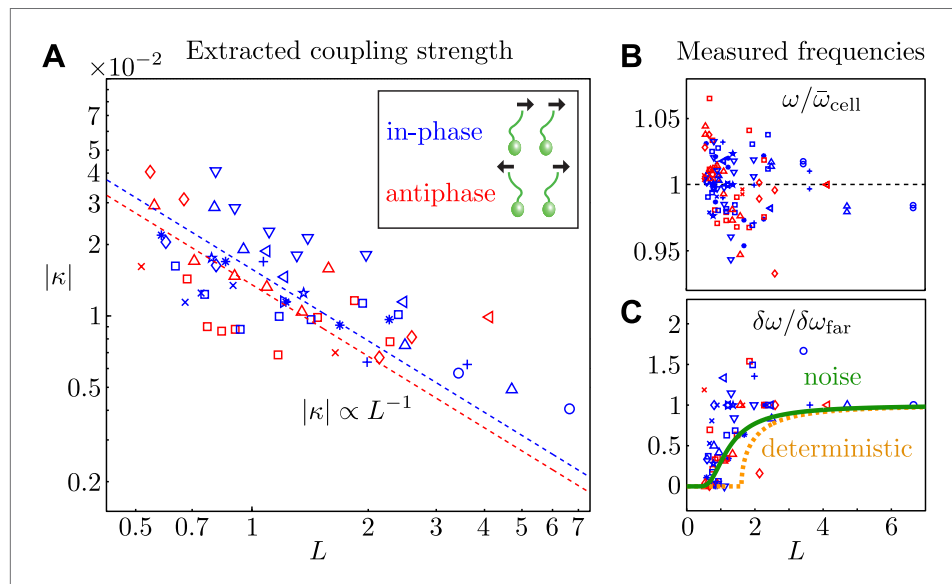


Figure 5. Coupling strength. **(A)** Dimensionless interflagellar coupling strength $\kappa = \varepsilon/\bar{\omega}$ as a function of the scaled spacing $L = d/l$ (log-log scale). The dotted lines represent fits of the form $|\kappa| = k \times L^{-1}$ with $k = 0.016$ (in-phase) and $k = 0.014$ (antiphase). **(B)** Measured beat frequency $\omega/\bar{\omega}_{\text{cell}}$ of each flagellum, nondimensionalised by the average value for that cell across several videos. **(C)** Measured frequency difference $\delta\omega/\delta\omega_{\text{far}}$ as a function of spacing L . The curves represent the predictions based on the average extracted model parameters in the absence (orange) and presence of noise (green). Symbols represent different pairs of cells, with the in-phase (blue) and antiphase (red) configurations shown.

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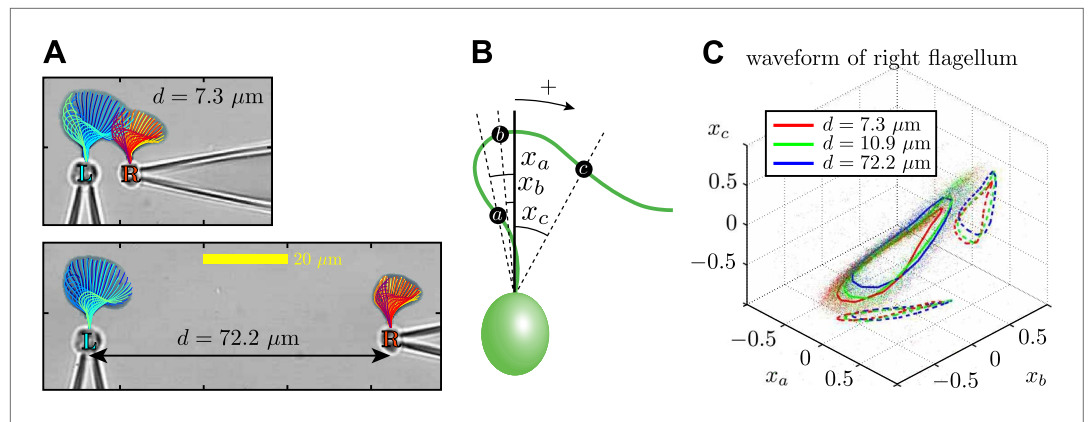


Figure 6. Waveform characteristics. **(A)** Logarithmically-scaled residence time plots of the entire flagella. The displayed waveforms correspond to 1 ms time intervals over several successive flagellar beats. **(B)** Angles x_a , x_b , x_c (in radians) measured and **(C)** their characteristic 3D trajectories. Results are shown for the right flagellum, corresponding to three different interflagellar spacings. As the spacing d is increased, the flagellar waveform exhibits a systematic change.

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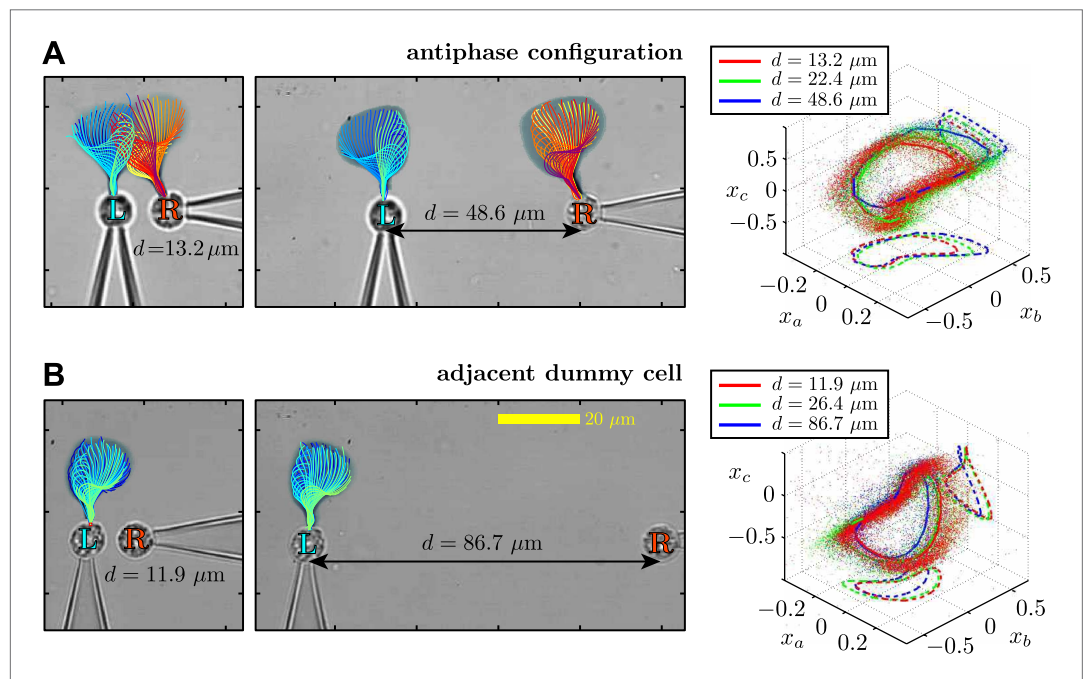


Figure 6—figure supplement 1. Flagellar filaments are tracked for cells in the (A) antiphase state, as well as (B) the situation in which one of the cells does not possess a flagellum (dummy cell).

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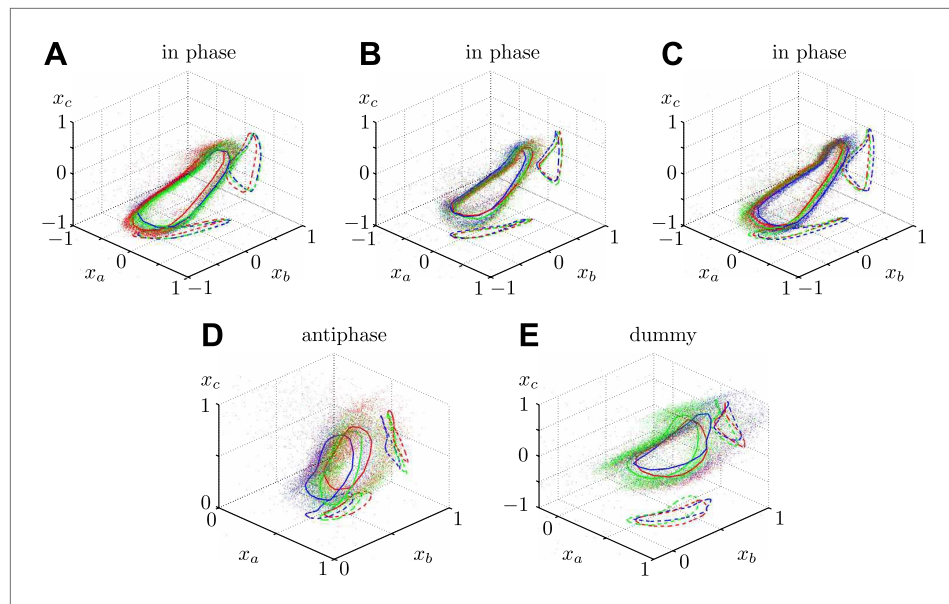


Figure 6—figure supplement 2. Additional waveform data collected for 5 different cells in various geometric configurations.

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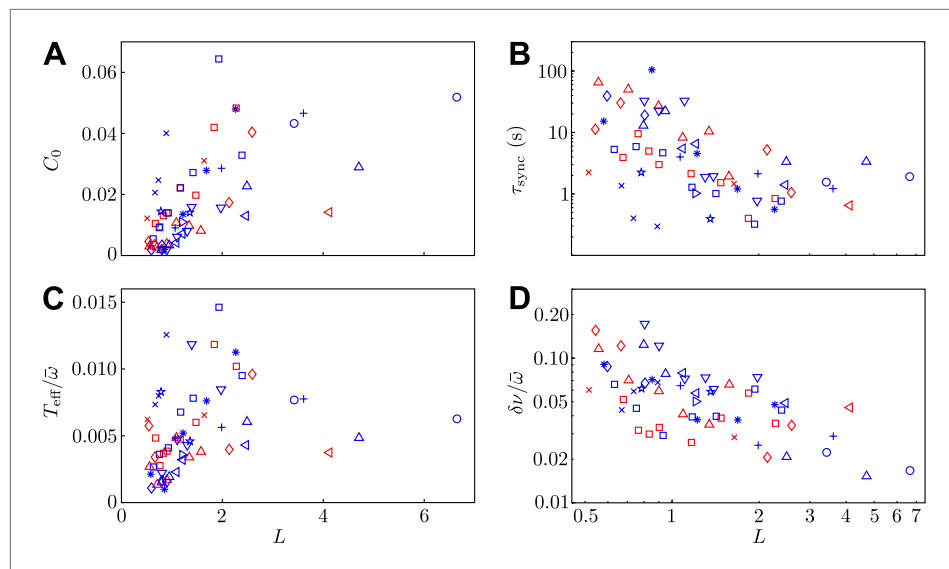


Figure 7. Model parameters. Two of the experimental observables **(A)** C_0 and **(B)** τ_{sync} , and the two additional model parameters **(C)** $T_{\text{eff}}/\bar{\omega}$ and **(D)** $\delta\nu/\bar{\omega}$ are shown as functions of interflagellar spacing for all experiments conducted.

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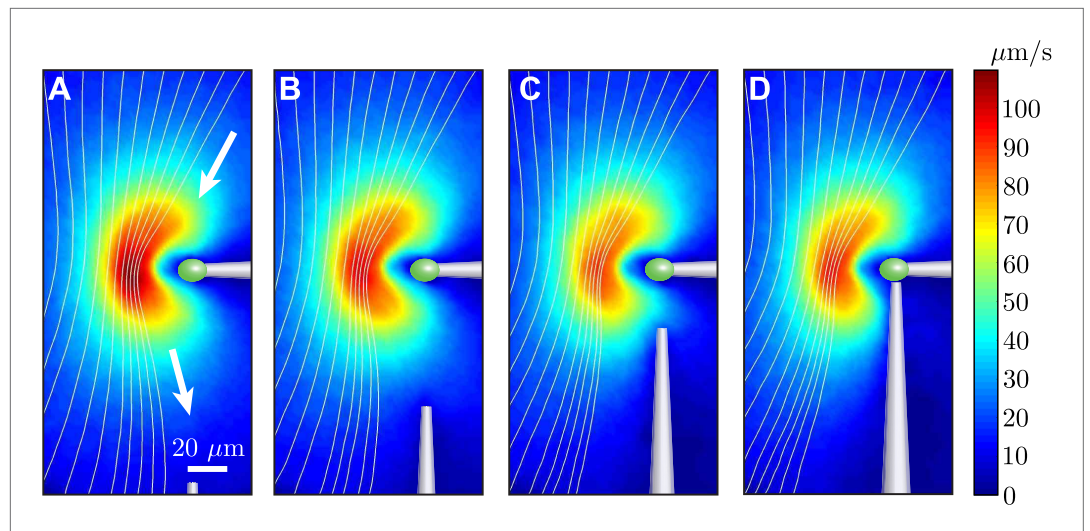


Figure 8. Effect of nearby pipette. The time-averaged flow field associated with one captured cell is measured as a second pipette slowly approaches. This demonstrates that the precise angle from which the cell is held by the micropipette has very little effect on the resultant flow field.

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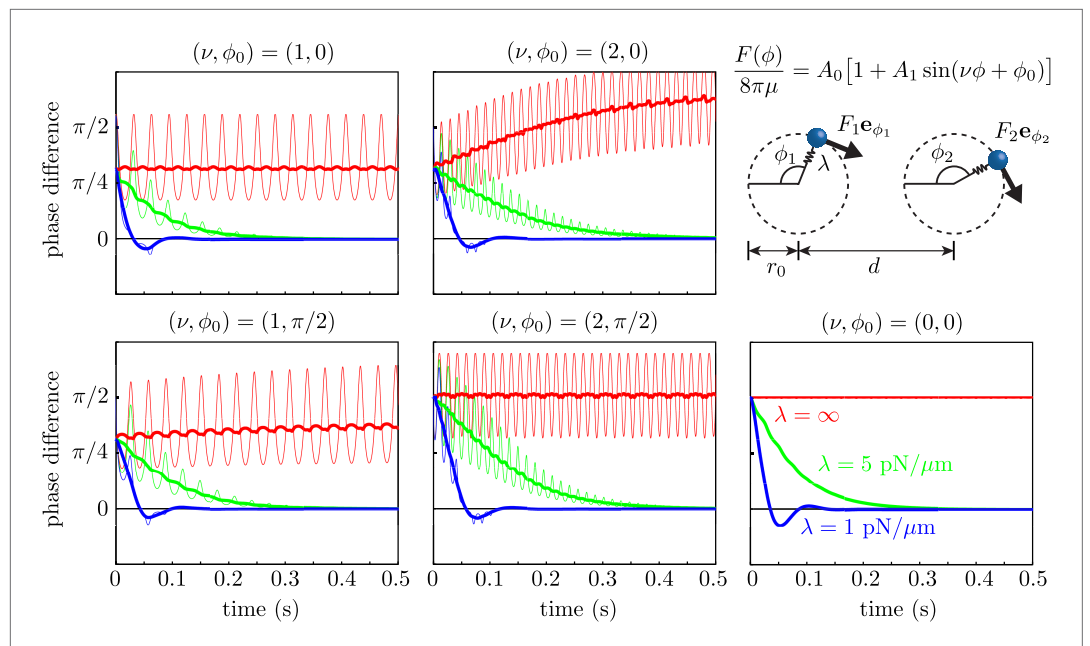


Figure 9. Effect of force modulation. Evolution of the phase difference $\delta = \phi_1 - \phi_2$ among two identical model oscillators, each composed of a sphere driven around a circular trajectory by a tangential driving force. The trajectories each possess a radial stiffness λ . Smaller values of λ yield rapid convergence towards synchrony ($\delta = 0$), in a manner essentially independent of the functional form of the driving force. Parameters used are given by $a = 0.75 \mu\text{m}$, $r_0 = 8 \mu\text{m}$, $d = 20 \mu\text{m}$, $A_0 = 1076 \mu\text{m}^2/\text{s}$ and $A_1 = 0.56$.

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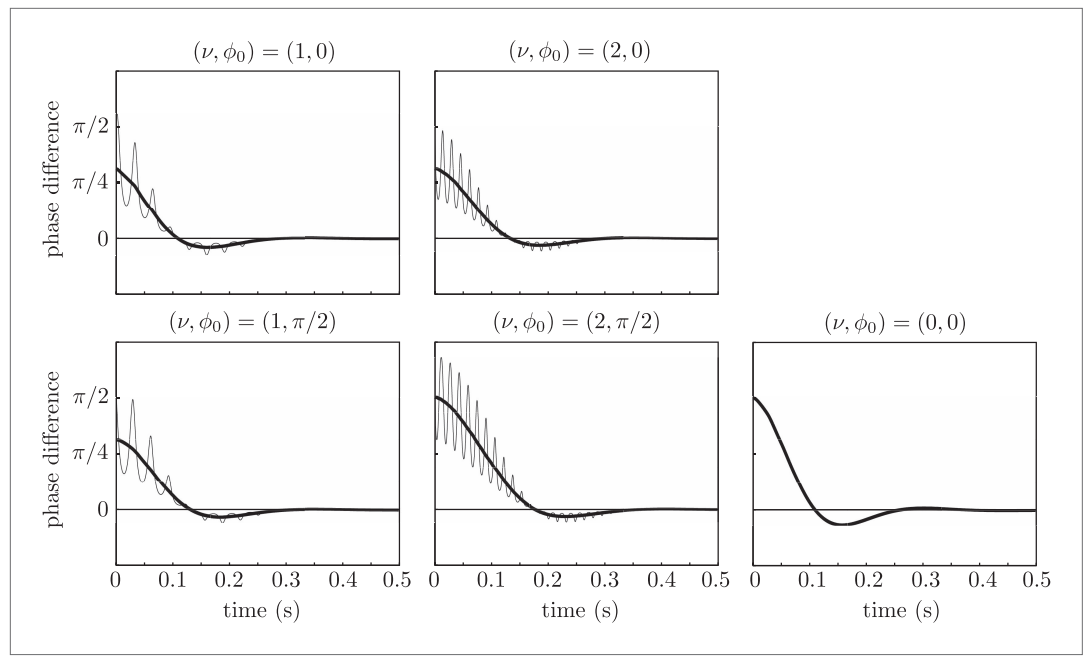


Figure 10. Effect of force modulation. Re-run of the simulations in **Figure 9** with properties inspired by real flagella.
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