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Technical Note

Thermal Conductivity of Granular Soils with Contrasting Particle Shapes

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Abstract: Particle shape is known to affect the mechanical behavior of sands as it influences packing density and particle contacts. Even though the thermal conductivity of sands also depends on packing density and particle contacts, the effects of particle shape on thermal conductivity are not well understood. A series of thermal needle tests were conducted on five granular soil mixtures with different proportions of rounded and angular glass particles having the same mineral composition and gradation. The maximum and minimum void ratios and the packing density of these mixtures were found to depend on the overall regularity, defined as the average value of the particle's aspect ratio, convexity and sphericity. For a given overall regularity, the thermal conductivity increases with decreasing void ratio or increasing relative density. Interestingly, the overall regularity has a small effect on the thermal conductivity at a given void ratio but has a significant effect on the thermal conductivity at a given relative density. A particle shape-dependent empirical equation is proposed to quantify the effects of relative density and overall regularity on the thermal conductivity of the tested sand.

Keywords: thermal conductivity; particle shape; overall regularity; relative density; void ratio

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Introduction

Particle shape has well-known effects on the characteristics of granular soils including packing density (Cho et al. 2006; Shin and Santamarina 2013; Zheng and Hryciw 2016) and fabric (Maeda et al. 2010; Turner et al. 2016; Zhao and Zhou 2017), and also can have significant effects on mechanical properties like compressibility (Rousé et al. 2008; Shin and Santamarina 2013; Gong and Liu 2017), shear modulus (Cho et al. 2006; Shin and Santamarina 2013), shear strength (Varadarajan et al. 2003; Guo and Su 2007; Rousé et al. 2008; Yang and Wei 2012; Shin and Santamarina 2013; Yang and Luo 2015; Xiao et al. 2019), dilation (Guo and Su 2007; Yang and Luo 2015; Xiao et al. 2019), susceptibility to liquefaction (Tsomokos and Georgiannou 2010; Yang and Wei 2012; Yang and Luo 2015), particle breakage during shearing (Afshar et al. 2017; Cavarretta et al. 2017), and failure mode during shearing (Alshibli et al. 2017). Particle shape may also affect the hydraulic conductivity of granular soils (Côté et al. 2011).

Although it would be logical that particle shape has an effect on the thermal conductivity, conflicting trends have been noted in the limited studies performed on this topic (Côté and Konrad 2005; Yun and Santamarina 2008; Gan et al. 2017; Lee et al. 2017; Fei et al. 2019). Accordingly, the main purpose of this study is to perform thermal needle tests on mixtures of angular and rounded glass particles having different particle shapes under a range of relative densities. The advantage of using glass particles is that they possess the same mineral composition, particle size, particle surface, and gradation, permitting the effect of contrasting particle shape to be isolated. The particle shape is quantified in this study using the overall regularity, defined as the average value of the particle's aspect ratio, convexity and sphericity. Empirical relationships are developed between the thermal conductivity, overall regularity, and relative density for the glass particles tested in this study that may give insight into the importance of particle shape on the thermal properties of granular materials in general.

Materials and Methods

Materials

Two kinds of glass particles having rounded and angular shapes are used in this study, both of which were obtained from the Hong Kong Siu Tung Glass & Plastic Group Co., Ltd. The average specific gravity for both kinds of particles is 2.35. They have similar uniform gradations with grain sizes ranging from 0.6 to 0.8 mm and can be classified as poorly graded sands according to the unified soil classification scheme (ASTM 2006). The two kinds of glass particles were combined to form five mixtures with different proportions by weight (**Figs. 1(a)-1(e)**). The designators for the mixtures are A0R100, A25R75, A50R50, A75R25 and A100R0, for example, A25R75 designates 25% angular particles plus 75% rounded particles by weight.

Thermal Conductivity Measurement

Before preparation of the specimens, the maximum and minimum void ratios of each of the mixtures were determined (ASTM 2016a, b) and the results are summarized in **Table 1**. The specimens were prepared using the undercompaction method proposed by Ladd (1978) to achieve relative densities of 0.3, 0.5, 0.6, 0.7 and 0.8 in a specially-designed mold having a diameter of 50 mm and a height of 137 mm. Thermal conductivity was measured using a vertically-oriented thermal needle probe (single needle model TR-1 from Decagon Devices, Pullman, WA) along with a KD2 Pro thermal properties analyzer. The TR-1 probe was inserted vertically from the center of the surface of the specimen after preparation. The thermal conductivity of all these specimens was measured five times at room temperature (25 °C) with their average values listed in **Table 2**. More details on thermal needle testing of granular materials can be found in Xiao et al. (2018).

Definition of Particle Shape

Particle shape can be characterized by three parameters, the aspect ratio (A_R), convexity (C_X), and sphericity (S_C) (Yang and Luo 2015; Xiao et al. 2019). These parameters were determined from binary images of two kinds of glass particles using ImageJ (Schneider et al. 2012), as shown in **Figs. 1(f)-1(g)**. These particle shape parameters are defined as

$$A_R = D_{\min}^F / D_{\max}^F \quad \textbf{(1a)}$$

$$C_X = A/(A+B) \quad \textbf{(1b)}$$

$$S_C = P_{eq}/P_r \quad \textbf{(1c)}$$

where $D_{\min}^F$ and $D_{\max}^F$ are minimum and maximum Feret diameters, respectively, as shown in **Fig. 2(a)**; A is the particle area, and $(A+B)$ is the area of particle's convex hull, as defined in **Fig. 2(b)**; P_{eq} is the perimeter of the equivalent circle which has the same area as the projected area of the particle, and P_r is the perimeter of the projected area of the particle, as shown in **Fig. 2(c)**.

Yang and Luo (2015) proposed a particle shape parameter referred to as the overall regularity O_R which can consider the impacts of three particle shape parameters, defined as follows:

$$O_R = (A_R + C_X + S_C)/3 \quad \textbf{(2)}$$

The images in **Figs. 2(d)-2(f)** show typical examples of angular particles with specific values of A_R, C_X, S_C and O_R. The particle in **Fig. 2(d)** possesses the largest O_R although its value of C_X is slightly smaller than that of the particle in **Fig. 2(e)**. The particle in **Fig. 2(f)** possesses the smallest A_R, C_X, S_C and O_R.

It should be noted that the aspect ratio, convexity, sphericity and overall regularity can only reflect the particle shape of a single particle but not an assembly of particles. Following the studies of (Yang and Luo 2015; Xiao et al. 2019), 500 rounded particles were randomly selected for testing their particle shape parameters and the cumulative distribution curve for each particle shape parameter was obtained for the rounded particle assembly. The particle-shape cumulative distribution curve for the angular particle assembly was obtained in the same way. **Fig. 3** presents the cumulative distribution curves for A_R, C_X, S_C and O_R of two assemblies. The values at 50% of the cumulative distribution are regarded as the representative values for two assemblies according to Yang and Luo (2015) and Xiao et al. (2019). The representative values of particle shape parameters for the mixtures are determined by the combination of the corresponding particle-shape values of the component materials and their percentages by weight (Yang and Luo 2015). For example, the O_R value for A25R75 is determined as follows:

$$\begin{aligned} \mathrm{O}_{R\_A25R75} &= O_{R\_A100R0} \times 25\% + O_{R\_A0R100} \times 75\% \\ &= 0.822 \times 25\% + 0.960 \times 75\% = 0.926 \end{aligned} \quad \textbf{(3)}$$

The representative values of particle shape parameters for the five mixtures are listed in **Table 1**. Mixtures with higher proportions of rounded particles have larger values of A_R, C_X, S_C and O_R.

Experimental Results and Discussion

Effect of Particle Shape on Void Ratio-Relative Density Relationship

The maximum void ratio $e_{\max}$ and minimum void ratio $e_{\min}$ were found to decrease with increasing overall regularity, as shown in **Fig. 4(a)**. For example, $e_{\max}$ decreases from 0.871 to 0.619, as O_R increases from 0.822 to 0.960. In addition, the difference between $e_{\max}$ and $e_{\min}$ decreases from 0.204 to 0.058 with an increase in O_R from 0.822 to 0.960. The relationship between $e_{\max}$ and O_R can be described by a linear regression equation:

$$e_{\max} = e_{\max}^{0} + \chi_{\max}^{e} O_R \quad \textbf{(4)}$$

where $e_{\max}^{0}$ (= 2.308) and $\chi_{\max}^{e}$ (= -1.729) are fitting parameters. Similarly, $e_{\min}$ can be estimated by

$$e_{\min} = e_{\min}^{0} + \chi_{\min}^{e} O_R \quad \textbf{(5)}$$

where $e_{\min}^{0}$ (= 1.301) and $\chi_{\min}^{e}$ (= -0.792) are fitting parameters. The same trend for $e_{\max}$ and $e_{\min}$ with respect to particle shape was reported by Cho et al. (2006), Rousé et al. (2008), Shin and Santamarina (2013), Altuhafi et al. (2016), Zheng and Hryciw (2016), and Suh et al. (2017).

Figure 4(b) shows the variations of the measured void ratios with the relative density at different O_R values. For a given relative density, the void ratio is dependent on the particle shape. For example, for I_D=0.5, the void ratios decrease from 0.769 to 0.590, as O_R increases from 0.822 to 0.960. Based on **Eqs. (4)** and **(5)**, the relationship between the void ratio and relative density incorporating the effect of particle shape can be given as

$$\begin{aligned} e &= e_{\max} - I_D\left(e_{\max} - e_{\min}\right) \\ &= e_{\max}^{0} + \chi_{\max}^{e} O_R - I_D\left(e_{\max}^{0} - e_{\min}^{0}\right) - I_D O_R\left(\chi_{\max}^{e} - \chi_{\min}^{e}\right) \end{aligned} \quad \textbf{(6)}$$

As shown in **Fig. 4(c)**, **Eq. (6)** can reasonably capture the variations of the void ratio with the relative density and overall regularity, according to the maximum error of 0.012 and R^2 =0.989.

Effect of Particle Shape on Thermal Conductivity-Void Ratio Relationship

Figure 5(a) shows variations of thermal conductivity (λ) with void ratio at different values of O_R. The test data projected on the λ - e plane for different O_R values can be fitted by the following empirical relationship:

$$\lambda = \lambda_{e0} - \chi_e^{\lambda} e \qquad \textbf{(7)}$$

where λ_{e0} (= 0.324) and χ_e^{λ} (= 0.24) are fitting parameters.

The fitting curve obtained from **Eq. (7)** reasonably describes the variations in thermal conductivity as the data all are encapsulated within a narrow band defined by two lines ($\lambda = (\lambda_{e0} \pm 0.005) - \chi_e^{\lambda} e$), as shown in **Fig. 5 (a)**. **Fig. 5 (b)** shows the fitting surface obtained from **Eq. (7)**, which adequately describes the variations in thermal conductivity with void ratio and overall regularity with a maximum absolute error of 0.005 W/m/K.

The results in **Fig. 5** indicate that the variations in thermal conductivity are mainly affected by the changes in void ratio. At the same void ratio, the influence of the particle shape on thermal conductivity is limited, with an effect of less than 5%. This finding is consistent with the observation from the work by Yun and Santamarina (2008) but inconsistent with the observation by Côté and Konrad (2005) who reported that the thermal conductivity of angular particles is higher than that of rounded particles at a given void ratio. Lee et al. (2017) reported that the dependence of thermal conductivity on the void ratio for rounded particles is higher than that for angular particles, whereas in the current study the dependence of thermal conductivity on the void ratio is hardly affected by the particle shape. The main reason behind the above differences may be the origin of the data. For example, the data for Yun and Santamarina (2008) include results for six sands tested using the same approach, the data in Côté and Konrad (2005) are from seven published papers, and the data for Lee et al. (2017) include results for nine sands tested using the same approach. However, these three studies ignored some important factors between the sands included in the comparison, such as the particle mineralogy, mean particle size, and particle size distribution which can all affect the thermal conductivity of sands to some extent (Aduda 1996; Abuel-Naga and Bouazza 2013; Dong et al. 2015; Zhang and Wang 2017; Xiao et al. 2018). In the current study, the mixtures possess the same mineral composition, mean particle size and particle size distribution, so the finding that the effect of the particle shape on thermal conductivity at a given void ratio is more reliable than the conclusions from the other studies. In addition, Fei et al. (2019) found that thermal conductivity increases with increasing particle regularity through an analysis with the finite element method, primarily due to that observation that higher regularity may lead to a higher average coordination number. Gan et al. (2017) use the discrete element method to find that the effective thermal conductivity of granular soils increases with the aspect ratio of ellipsoids. However, it should be noted that the conclusions from Gan et al. (2017) and Fei et al. (2019) are not on the basis of the same void ratio for all mixtures.

Effect of Particle Shape on Thermal Conductivity-Relative Density Relationship

It should be noted that it is not possible to obtain the same void ratio for the five mixtures, as shown in **Fig. 4(b)**, as the maximum void ratio (0.619) of the specimen A0R100 is smaller than the minimum void ratio (0.667) of the specimen A100R0, as listed in **Table 1**. The relative density, I_D, is a common controlling parameter for granular soils, and the same value of I_D is available for each mixture, as listed in **Table 1**. **Fig. 6(a)** shows variations of the thermal conductivity with the relative density and overall regularity. The test data projected on the λ-I_D plane for different O_R values in **Fig. 6(a)** are not in a narrow band, different from the data projected to the λ-e plane in **Fig. 5(a)**. It is found from the results in **Fig. 6(a)** that an increase in I_D at a given O_R, or an increase in O_R at a given I_D leads to an increase in λ which is in line with the findings of Fei et al. (2019). For example, λ increases from 0.154 to 0.172 W/m/K as I_D increases from 0.3 to 0.8 for mixtures with O_R=0.891; or λ increases from 0.149 to 0.189 W/m/K as O_R increases from 0.822 to 0.960 for mixtures with I_D=0.8. It is useful to evaluate the thermal conductivity directly from the relative density at different values of overall regularity. Substituting **Eq. (6)** into **Eq. (7)** gives a relationship between the thermal conductivity and relative density by incorporating the effect of particle shape, as follows:

$$\lambda = \lambda_{e0} - \chi_e\left(e^0_{\max} + \chi^e_{\max} O_R\right) + \chi_e I_D\left(e^0_{\max} - e^0_{\min}\right) + \chi_e I_D O_R\left(\chi^e_{\max} - \chi^e_{\min}\right) \qquad \textbf{(8)}$$

Fig. 6(b) shows that **Eq. (8)** can describe variations of thermal conductivity with the relative density and overall regularity with R^2=0.97 and a maximum absolute error of 0.006 W/m/K.

Conclusions

The effect of particle shape on thermal conductivity is investigated through a series of thermal needle tests on five mixtures that contain different proportions of rounded and angular particles. The main conclusions are summarized as follows:

(1) The overall regularity was used to quantify the particle shape and determine the effect of this variable on thermal conductivity. Both the maximum and minimum void ratios of these mixtures decreased with increasing overall regularity. An equation incorporating the overall regularity in the relationship between the void ratio and relative density was proposed for these mixtures.

(2) The thermal conductivity of these mixtures with the same mineral composition and grading increased with decreasing void ratio. However, the thermal conductivity for a given void ratio was less affected by the particle shape. The relationship between the thermal conductivity and void ratio was the same for these mixtures.

(3) The thermal conductivity of these mixtures increased with increasing relative density at a given overall regularity or with increasing overall regularity at a given relative density, which implies that the thermal conductivity at a given relative density increased as the proportion of rounded particle increased. An empirical equation incorporating the overall regularity was proposed to estimate the variation of the thermal conductivity with the relative density for these mixtures.

Data Availability

All data and models generated and used during the study appear in the submitted article.

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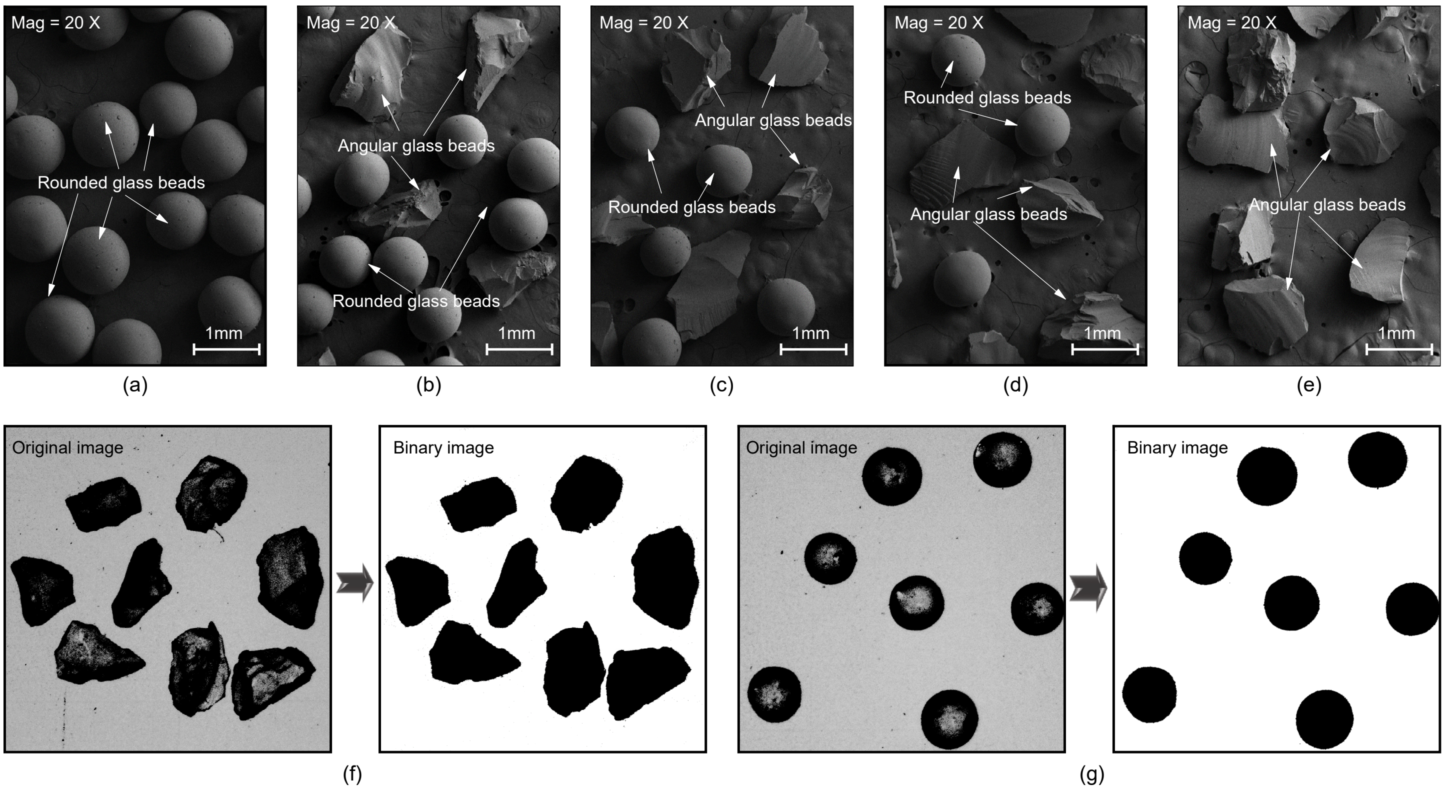

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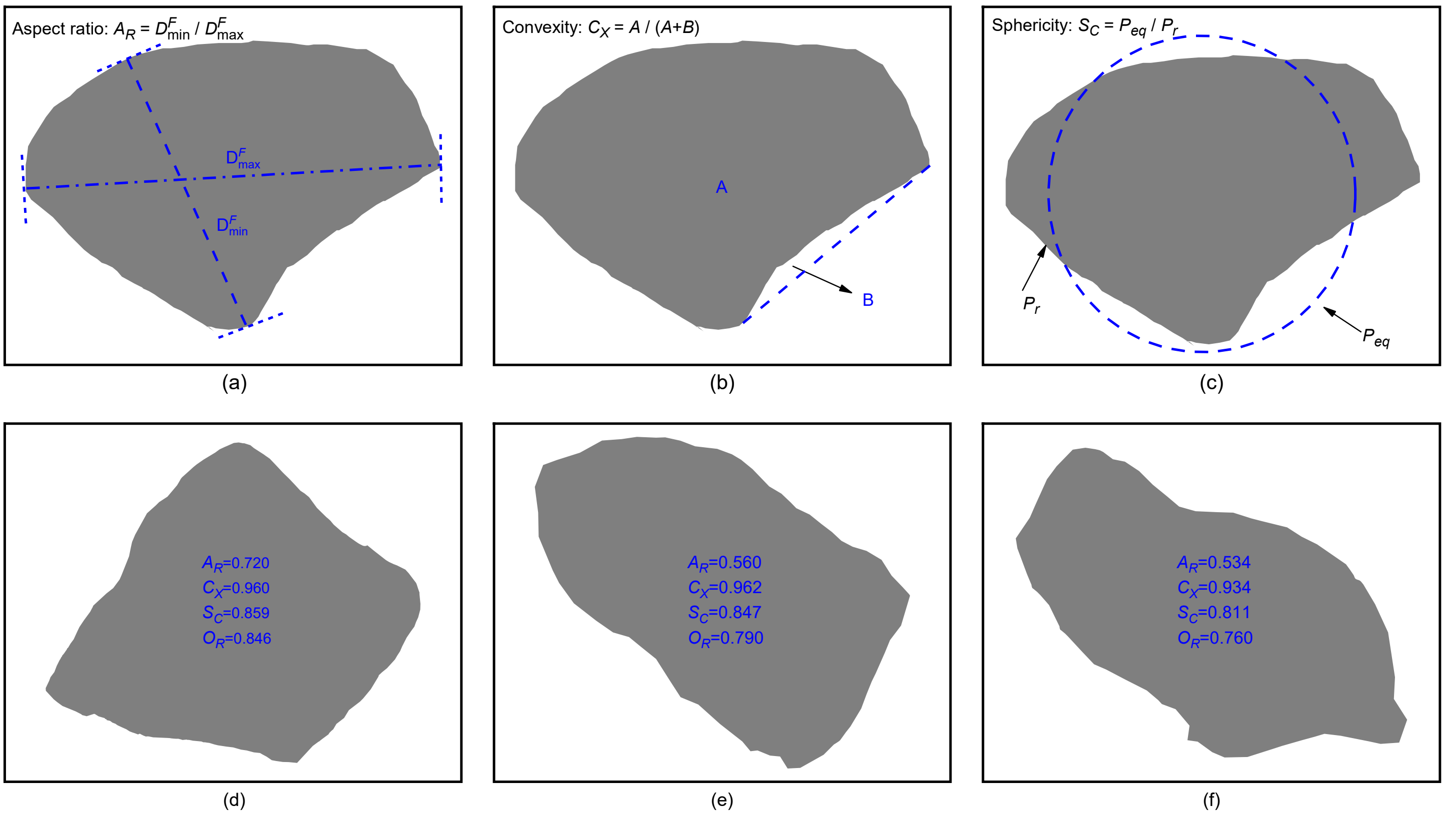

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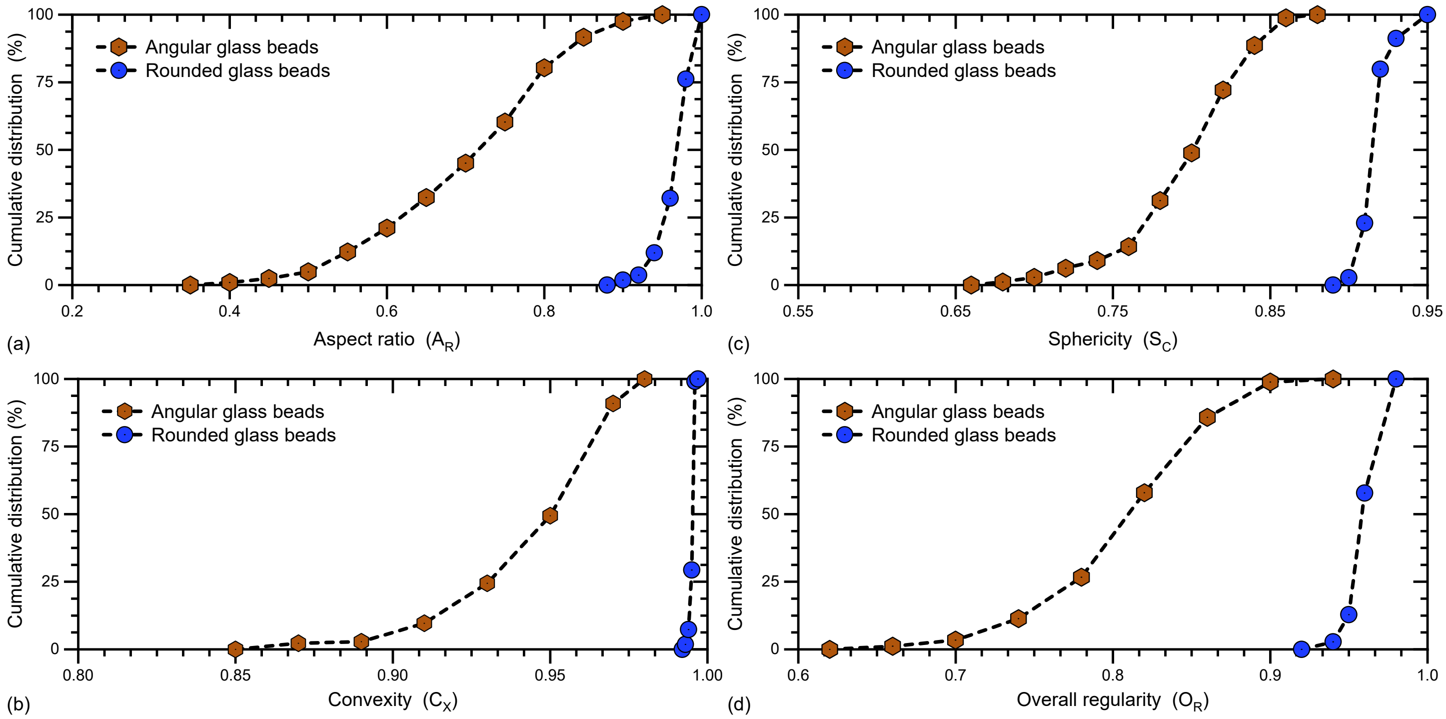

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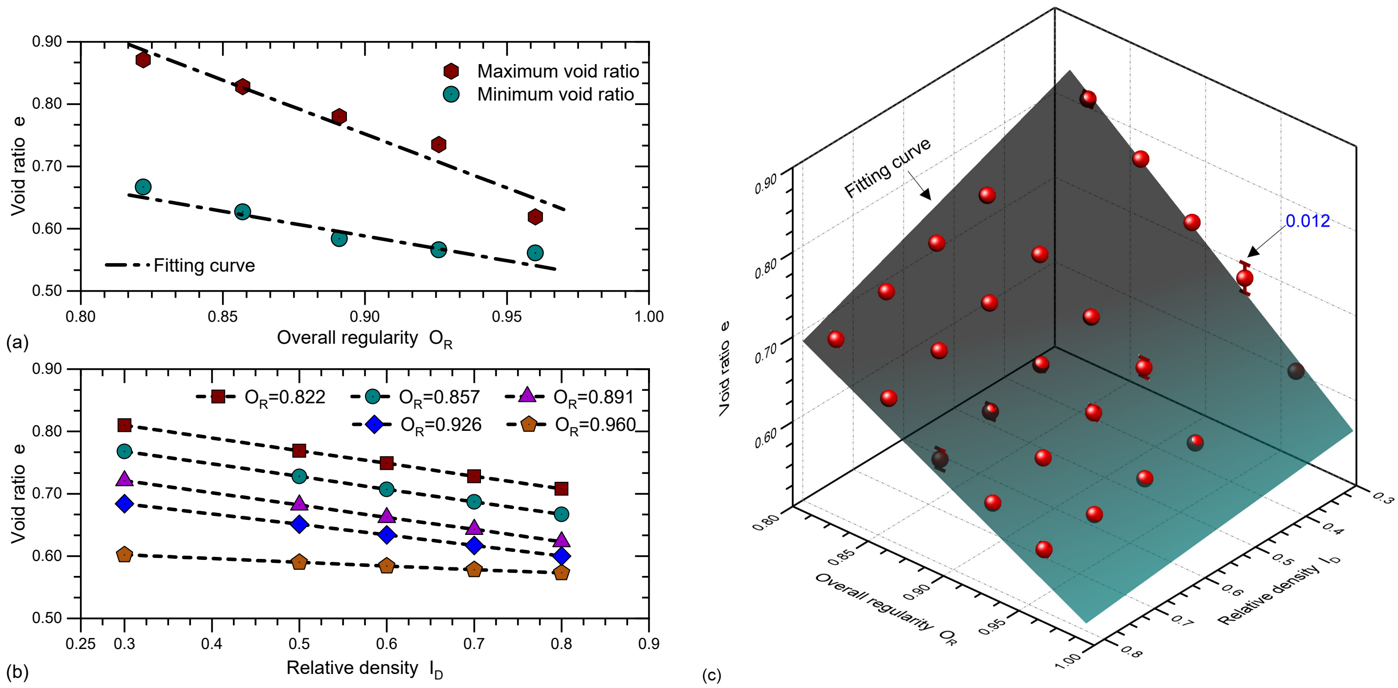

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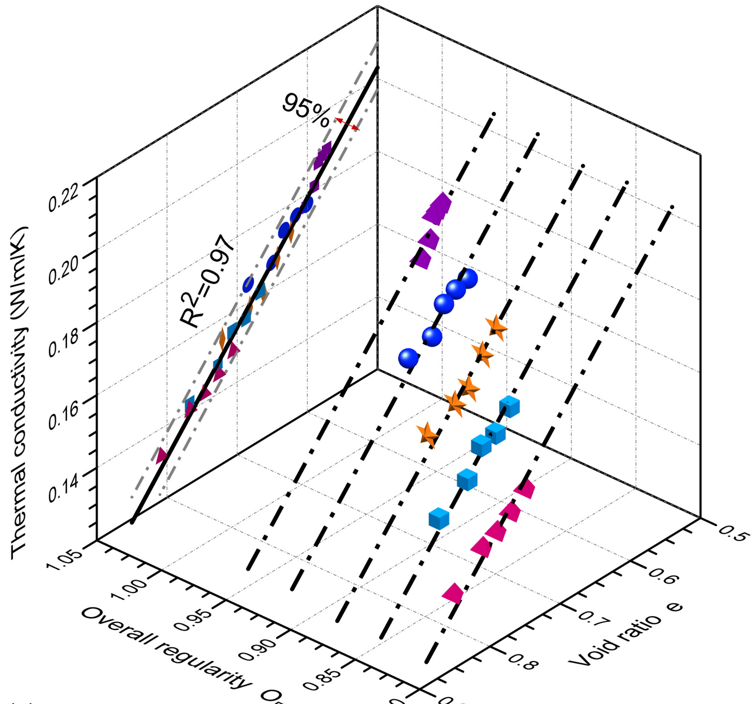

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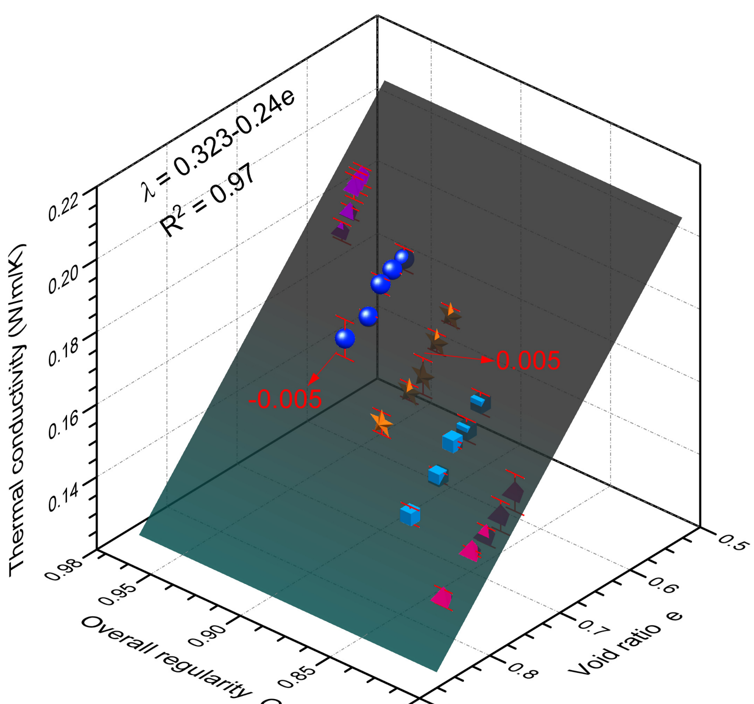

(b)

Fig 6 size: Width = One column (i.e., 8.8 cm); DPI = 600