Quantitative Models of Sovereign Debt Crises[*]

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Abstract

This chapter is on quantitative models of sovereign debt crises in emerging economies. We interpret debt crises broadly to cover all of the major problems a country can experience while trying to issue new debt, including default, sharp increases in the spread and failed auctions. We examine the spreads on sovereign debt of 20 emerging market economies since 1993 and document the extent to which fluctuations in spreads are driven by country-specific fundamentals, common latent factors and observed global factors. Our findings motivate quantitative models of debt and default with the following features: (i) trend stationary or stochastic growth, (ii) risk averse competitive lenders, (iii) a strategic repayment/borrowing decision, (iv) multi-period debt, (v) a default penalty that includes both a reputation loss and a physical output loss and (vi) rollover defaults. For the quantitative evaluation of the model, we focus on Mexico and carefully discuss the successes and weaknesses of various versions of the model. We close with some thoughts on useful directions for future research.

Keywords: Quantitative models, emerging markets, stochastic trend, capital flows, rollover crises, debt sustainability, risk premia, default risk

JEL Codes: D52, F34, E13, G15, H63

1 Introduction

This chapter is about sovereign debt crises, instances in which a government has trouble selling new debt. An important example is when a government is counting on being able to roll over its existing debt in order to service it over time. When we refer to trouble selling its debt, we include being able to sell new debt but only with a large jump in the spread on that debt over comparable risk-free debt, failed auctions, suspension of payments, creditor haircuts and outright default. So our notion of a debt crisis covers all of the major negative events that one associates with sovereign debt issuance.

We focus on debt crises in developing countries because the literature has focused on them and because these countries provide the bulk of our examples of debt crises and defaults. However, the recent debt crises in the European Union remind us that this is certainly not always the case. While the recent crises in the EU are of obvious interest, they come with a much more complicated strategic dimension, given the role played by the European Central Bank and Germany in determining the outcomes for a country like, say, Greece. For this reason we will hold to a somewhat more narrow focus. Despite this, we see our analysis as providing substantial insight into sovereign debt crises in developed countries as well.

This chapter will highlight quantitative models of the sovereign debt market. We will focus on determining where the current literature stands and where we need to go next. Hence, it will not feature an extensive literature survey, though we will of course survey the literature to some extent, including a brief overview at the end of the chapter. Instead, we will lay out a fairly cutting-edge model of sovereign debt issuance and use that model and its various permutations to gauge the successes and failures of the current literature as we see them.

The chapter will begin by considering the empirical evidence on spreads. We will examine the magnitude and volatility of spreads on sovereign debt among developing countries. We will seek to gauge the extent to which this debt features a risk premium in addition to default risk. We will also seek to characterize the extent to which the observed spread is driven by country-specific fundamentals, global financial risk and uncertainty factors, or other common drivers. To do this, we will estimate a statistical model of the spread process in our data, and this statistical model will feature several common factors that we estimate along with the statistical model. The facts that emerge from this analysis will then form the basis on which we will judge the various models that we consider in the quantitative analysis.

The chapter will then develop a quantitative model of sovereign debt that has the following key features: risk-averse competitive lenders, since it will turn out that risk premia are substantial, and a strategic sovereign who chooses how much to borrow and whether or not to repay, much as in the original Eaton and Gersovitz (1981) model. The sovereign will issue debt that has multi-period maturity. While we will take the maturity of the debt to be parametric, being able to examine the implications of short and long maturity is an important aspect of the analysis. Default by the sovereign will feature two punishments: a period of exclusion from credit markets and a loss in output during the period of exclusion. Pure reputation effects are known to fail (Bulow and Rogoff (1989)) and even coupling them with a loss of saving as well as borrowing does not generate a sufficient incentive to repay the sorts of large debts that we see in the data. Hence, we include the direct output cost as well.

Our model will feature both fundamental defaults, in which default is taking place under the best possible terms (fixing future behavior). The model will also allow for rollover or liquidity defaults, in which default occurs when lending takes place under the worst possible terms (again, fixing future behavior) as in Cole and Kehoe (2000). We include both types of defaults since they seem to be an important component of the data. Doing so, especially with multi-period debt maturity, will require some careful modeling of the timing of actions within the period and a careful consideration of both debt issuance and debt buybacks. In addition, the possibility of future rollover crises will affect the pricing of debt today and the incentives to default, much as in the original Calvo (1988) model.

We will consider two different growth processes for our borrowing countries. The first will feature stochastic fluctuations around a deterministic trend with constant growth. The second will feature stochastic growth shocks. We include the deterministic trend process because the literature has focused on it. However, the notion that we have roughly the same uncertainty about where the level of output of a developing country will be in 5 years and in 50 years seems sharply counterfactual, as documented by Aguiar and Gopinath (2007). Hence our preferred specification is the stochastic growth case and, so, we discuss this case as well.

There will be three shocks in the model. The first is a standard output shock that will vary depending on which growth process we assume. The second is a shock to lender wealth. The third is a belief-coordination shock that will determine whether a country gets the best or the worst possible equilibrium price schedule in a period. An important question for us

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will be the extent to which these shocks can generate movements in the spread that are consistent with the patterns we document in our empirical analysis of the data.

The chapter will examine two different forms of the output default cost. The first is a proportional default cost as has been assumed in the early quantitative analyses and in the theoretical literature on sovereign default. The second form is a nonlinear output cost such as was initially pioneered by Arellano (2008). In this second specification, the share of output lost in default depends positively on (pre-default) output. Thus, default becomes a more effective mechanism for risk sharing compared to the proportional cost case. As noted in Chatterjee and Eyigungor (2012), adding this feature also helps to increase the volatility of sovereign spreads.

2 Motivating Facts

2.1 Data for Emerging Markets

We start with a set of facts that will guide us in developing our model of sovereign debt crises. Our sample spans the period 1993Q4 through 2014Q4 and includes data from 20 emerging markets: Argentina, Brazil, Bulgaria, Chile, Colombia, Hungary, India, Indonesia, Latvia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Turkey, and Ukraine. For each of these economies, we have data on GDP in US dollars measured in 2005 domestic prices and exchange rates (real GDP), GDP in US dollars measured in current prices and exchange rates (nominal GDP), gross external debt in US dollars (debt), and market spreads on sovereign debt.[1]

Tables 1 and 2 report summary statistics for the sample.[2] Table 1 documents the high and volatile spreads that characterized emerging market sovereign bonds during this period. The standard deviation of the level and quarterly change in spreads 676 and 229 basis points, respectively. Table 2 reports an average external debt-to-(annualized)GDP ratio of 0.46. This level is low relative to the public debt levels observed in developed economies. The fact that emerging markets generate high spreads at relatively low levels of debt-to-GDP reflects one aspect of the "debt intolerance" of these economies documented by Reinhart, Rogoff,

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[1]Data source for GDP and debt is Haver Analytics' Emerge database. The source of the spread data is JP Morgan's Emerging Market Bond Index (EMBI).

[2]Note that Russia defaulted in 1998 and Argentina in 2001, and while secondary market spreads continued to be recorded post default, these do not shed light on the cost of new borrowing as the governments were shut out of international bond markets until they reached a settlement with creditors. Similarly, the face value of debt is carried throughout the default period for these economies.

and Savastano (2003).

The final column concerns "crises," which we define as a change in spreads that lie in the top 5 percent of the distribution of quarterly changes. This threshold is a 158 basis-point jump in the spread. By construction, 5 percent of the changes are coded as crises; however, the frequency of crises is not uniform across countries. Nearly 20 percent of Argentina's quarter-to-quarter changes in spreads lie above the threshold, while many countries have no such changes.

While many of the countries in our sample have very high spreads, only two - Russia in 1998 and Argentina in 2001 - ended up defaulting on their external debt, while a third, Ukraine, defaulted on its internal debt (in 1998). This highlights the fact that periods of high spreads are more frequent events than defaults. Nevertheless, it is noteworthy that the countries with the highest mean spreads are the ones that ended up defaulting during this period. This suggests that default risk and the spread are connected.

Table 1: Sovereign Spreads: Summary Statistics

Country	Mean $r - r^\star$	Std Dev $r - r^\star$	Std Dev $\Delta(r - r^\star)$	95th pct $\Delta(r - r^\star)$	Frequency Crisis
Argentina	1,525	1,759	610	717	0.18
Brazil	560	393	174	204	0.09
Bulgaria	524	486	129	155	0.03
Chile	146	57	34	34	0.00
Colombia	348	206	88	245	0.05
Hungary	182	154	57	88	0.02
India	225	54	47	85	0.00
Indonesia	285	137	98	73	0.02
Latvia	157	34	16	17	0.00
Lithuania	246	92	48	98	0.00
Malaysia	175	122	75	81	0.03
Mexico	345	253	134	127	0.05
Peru	343	196	84	182	0.06
Philippines	343	153	75	136	0.04
Poland	191	138	54	67	0.01
Romania	271	102	49	68	0.00
Russia	710	1,096	478	175	0.06
South Africa	226	116	68	99	0.03
Turkey	395	217	95	205	0.05
Ukraine	760	607	350	577	0.11
Pooled	431	676	229	158	

Table 2: Sovereign Spreads: Summary Statistics

Country	Mean $\frac{B}{4*Y}$	Corr $(\Delta(r - r^\star), \Delta y)$	Corr $(r - r^\star, \%\Delta B)$	Corr $(\Delta(r - r^\star), \%\Delta B)$
Argentina	0.38	-0.35	-0.22	0.08
Brazil	0.25	-0.11	-0.18	-0.01
Bulgaria	0.77	0.09	-0.20	0.06
Chile	0.41	-0.16	-0.18	-0.11
Columbia	0.27	-0.29	-0.40	-0.07
Hungary	0.77	-0.24	-0.56	-0.05
India	0.82	-0.32	0.04	-0.65
Indonesia	0.18	-0.43	-0.03	0.07
Latvia	0.49	-0.18	-0.12	-0.16
Lithuania	1.06	-0.25	-0.17	-0.31
Malaysia	0.54	-0.56	-0.33	0.24
Mexico	0.16	-0.4	0.23	-0.13
Peru	0.48	-0.01	-0.39	-0.05
Philippines	0.47	-0.16	0.06	0.09
Poland	0.57	-0.09	-0.35	-0.38
Romania	0.61	0.5	0.42	-0.33
Russia	NA	-0.45	-0.30	0.02
South	0.26	-0.14	-0.38	-0.24
Turkey	0.38	-0.34	-0.20	0.08
Ukraine	0.64	-0.49	-0.60	-0.07
Pooled	0.46	-0.27	-0.19	0.01

2.2 Statistical Spread Model

To further evaluate the empirical behavior of emerging market government bond spreads, we fit a statistical model to our data. In this model a country's spread is allowed to depend on country-specific fundamentals as well as several mutually orthogonal common factors (common across emerging markets) that we will implicitly determine as part of the estimation. To do this, we use EMBI data at a quarterly frequency. We have data for $I = 20$ countries from 1993:Q4-2015:Q2 (so $T = 87$), with sporadic missing values. If we index a country by i and a quarter by t, then we observe spreads, debt-to-GDP ratios, and real GDP growth: $\{s_{it}, b_{it}, g_{it}\}_{i=1,t=1}^{I,T}$. We also suppose that there are a set of J common factors that impact all the countries (though perhaps not symmetrically): $\{\alpha_t^j\}_{j=1}^J$.

We specify our statistical model as follows:

$$s_{it} = \beta_i b_{it} + \gamma_i g_{it} + \sum_{j=1}^{J} \delta_i^j \alpha_t^j + \kappa_i + \epsilon_{it}, \tag{1}$$

where ϵ_{it} is a mean-zero, normally distributed shock with variance σ_i^2. Notice that we allow for the average spread and innovation volatility to vary across countries. In the estimation we impose the constraint that $\delta_i^j \geq 0$ for all i, so we are seeking common factors that cause all spreads to rise and fall together.

These common factors are permitted to evolve as follows. Let α_t be the J-dimensional vector of common factors at time t. Then

$$\alpha_t = \Gamma \alpha_{t-1} + \eta_t \tag{2}$$

where η_t is a J-dimensional vector of normally distributed i.i.d. innovations orthogonal to each other. Because we estimate separate impact coefficients for each common factor, we normalized the innovation volatilities to 0.01. We restrict Γ to be a diagonal matrix, i.e., our common factors are assumed to be orthogonal and to follow AR(1) processes.

To estimate this model, we transform it into state-space form and apply MLE. We apply the (unsmoothed) Kalman Filter to compute the likelihood for a given parameterization. When the model encounters missing values, we will exclude those values from the computation of the likelihood and the updating of the Kalman Filter. Thus, missing values will count neither for nor against a given parameterization.

Table 3: Country-Specific Variance Decomposition
Average Marginal R^2

Country (i)	b_{it}	g_{it}	α_t^1	α_t^2	R^2	Obs.
Argentina	0.16	0.01	0.20	0.02	0.39	39
Brazil	0.28	0.01	0.52	0.05	0.87	81
Bulgaria	0.18	0.01	0.44	0.27	0.90	59
Chile	0.05	0.13	0.38	0.21	0.77	63
Colombia	0.20	0.05	0.55	0.16	0.95	55
Hungary	0.28	0.19	0.05	0.12	0.64	63
India	0.10	0.26	0.32	0.32	1.00	8
Indonesia	0.09	0.07	0.38	0.45	0.99	43
Latvia	0.03	0.03	0.86	0.08	1.00	9
Lithuania	0.06	0.01	0.67	0.25	0.99	20
Malaysia	0.23	0.11	0.46	0.16	0.96	24
Mexico	0.01	0.23	0.59	0.17	0.99	51
Peru	0.34	0.04	0.52	0.07	0.97	71
Philippines	0.26	0.05	0.50	0.01	0.83	84
Poland	0.06	0.10	0.23	0.32	0.71	42
Romania	0.15	0.03	0.47	0.23	0.87	12
Russia	0.12	0.05	0.21	0.51	0.90	62
South Africa	0.03	0.32	0.25	0.36	0.96	48
Turkey	0.05	0.09	0.77	0.04	0.94	74
Ukraine	0.02	0.26	0.20	0.41	0.89	44

Table 3 reports the explanatory power of the country-specific fundamentals as well as the two global factors. Specifically, we construct a variance decomposition following the algorithm of Lindeman, Merenda, and Gold (1980) as outlined by Gromping (2007). This procedure constructs the average marginal R^2 in the case of correlated regressors by assuming a uniform distribution over all possible permutations of the regression coefficients. We can see from this table first that our regressors explain much of the variation for many of the countries (as high as 99.88 percent for India). We can also see that country-specific fundamentals, here in the form of the debt-to-GDP ratio and the growth rate of output, explain only a modest amount of the variation in the spreads; typically less than 20 percent. This means that much of the movement in the spreads is explained by our two orthogonal factors.

Figure 1 plots our two common factors.[3] Given the importance our estimation ascribes

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[3]See Longstaff, Pan, Pedersen, and Singleton (2011) for a related construction of a global risk factor.

Figure 1: Estimated Common Factors
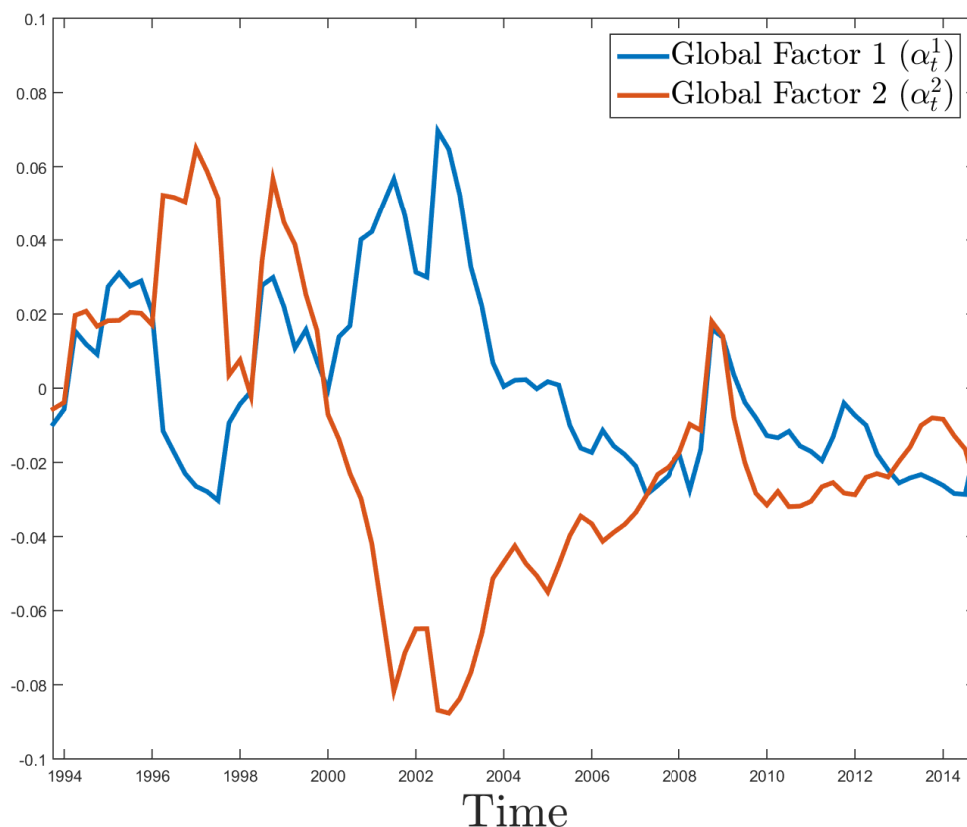
to them, we sought to uncover what is really driving their movements. To do this, we use a regression to try to construct our estimated common factors from the CBOE VIX, S&P 500 Diluted Earnings P/E ratio, and the LIBOR.[4] These regressors are standard measures of foreign financial-market uncertainty, price of risk and borrowing costs, respectively. These results are reported in table 4. The top panel reports the results from regressing the level of the factors on the level of foreign financial variables and the bottom reports the comparable regressions in first differences. We find that the foreign financial variables explain a modest amount of the variation in the level of the common factors: Each has an R^2 less than 0.3. To the extent that these objects do explain the common factors, however, it seems as if common factor 1 is driven primarily by measures of investor uncertainty and the price of risk, while common factor 2 is driven primarily by world interest rates. In first differences, the foreign

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[4]The LIBOR is almost perfectly correlated with the fed funds rate, so for precision of estimates we exclude the latter.

factors explain a third of the variation in the first factor but very little of the second factor.

There is an additional surprising finding about how risk pricing impact our spreads. The coefficient on the P/E ratio for the level specification is *positive* in common factor 1, where it has a substantial impact. Since an increase in the price of risk will drive down the P/E ratio, this means that our spreads are rising when the market price of risk is falling. This is the opposite of what our intuition might suggest. This coefficient reverses sign in the first-difference specification, reflecting that the medium run and longer correlation between the P/E ratio and our first factor has the opposite sign of the quarter-to-quarter correlation. The first-difference specification is what has been studied in the literature (Longstaff, Pan, Pedersen, and Singleton (2011); Borri and Verdelhan (2011)). These results show that the foreign risk premium may influence spreads differentially on impact versus in the longer run.

Table 4: Common Factor Regressions: Levels

Index		VIX	PE Ratio	LIBOR	R^2
			Levels		
α_t^1	Coefficient	$8.32e{-}4$ $(3.36e{-}4)$	$2.00e{-}3$ $(6.31e{-}4)$	$9.75e{-}4$ $(1.1e{-}3)$	
	Var Decomp	0.10	0.17	0.02	0.29
α_t^2	Coefficient	$6.1383e{-}4$ $(5.0460e{-}4)$	-0.0017 $(9.4742e{-}4)$	0.0088 (0.0017)	
	Var Decomp	$-4.0795e{-}5$	-0.0058	0.2722	0.27
			First Differences		
α_t^1	Coefficient	0.001 (0.002)	-0.001 (0.001)	-0.001 (0.002)	
	Var Decomp	0.30	0.06	0.00	0.35
α_t^2	Coefficient	0.001 (<0.001)	0.001 (0.001)	0.002 (0.003)	
	Var Decomp	0.05	< 0.01	0.01	0.06

2.3 Excess Returns

We turn next to the relationship between spreads and defaults. One of the striking facts here is that spreads "over-predict" future defaults in that ex post returns exceed the return

Table 5: Realized Bond Returns

Period	EMBI+	2-Year Treasury	5-Year Treasury
1993Q1–2014Q4	9.7	3.7	4.7
1993Q1–2003Q4	11.1	5.4	6.3
2004Q1–2014Q4	8.2	2.0	3.1

on risk-free assets. Hence, risk premia play an important role.

The fact that spreads are compensating lenders for more than the risk-neutral probability of default is suggested by the statistics reported in Table 1. The average spread is relatively high, and there are significant periods in which spreads are several hundred basis points. However, the sample contains only two defaults: Russia in 1998 and Argentina in 2001.

To explore this more systematically, we compute the realized returns on the EMBI+ index, which represents a value-weighted portfolio of emerging country debt constructed by JP Morgan. In Table 5, we report the return on this portfolio for the full sample period the index is available, as well as two sub-periods. The table also reports the returns to the portfolio U.S. Treasury securities of 2 years and 5 years maturity. We offer two risk-free references, as the EMBI+ does not have a fixed maturity structure and probably ranges between 2 and 5 years.

The EMBI+ index paid a return in excess of the risk-free portfolio of 5 to 6 percent. This excess return is roughly stable across the two sub-periods as well. Whether the realized return reflects the ex ante expected return depends on whether our sample accurately reflects the population distribution of default and repayment. The assumption is that by pooling a portfolio of bonds, the EMBI+ followed over a 20 year period provides a fair indication of the expected return on a typical emerging market bond. Of course, we cannot rule out the possibility that this sample is not representative. Nevertheless, the observed returns are consistent with a fairly substantial risk premium charged to sovereign borrowers.

2.4 Deleveraging

The data from emerging markets can also shed light on debt dynamics during a crisis. Table 2 documents that periods of above-average spreads are associated with reductions in the face

value of gross external debt. The pooled correlation of spreads at time t and the percentage change in debt between $t-1$ and t is -0.19. The correlation of the *change* in spread and debt is roughly zero. However, a large change in the spread (that is, a crisis period) is associated with a subsequent decline in debt. In particular, regressing the percent change in debt between t and $t+1$ on the indicator for a crisis in period t generates a coefficient of -1.6 and a t-stat of nearly 3. This relationship is robust to the inclusion of country fixed effects. This implies that a sharp spike in spreads is associated with a subsequent decline in the face value of debt.

2.5   Taking Stock

Our empirical analysis has led us to a set of criteria that we would like our model to satisfy. Specifically:

1. Crises, and particularly defaults, are low probability events;

2. Crises are not tightly connected to poor fundamentals;

3. Spreads are highly volatile;

4. Rising spreads are associated with de-leveraging by the sovereign; and

5. Risk premia are an important component of sovereign spreads.

In considering which features of real-world economies are important in generating these patterns, the first thing to recognize is that sovereign debt lacks a direct enforcement mechanism: most countries default despite having the physical capacity to repay. Yet, countries seem perfectly willing to service significant amounts of debt most of the time (rescheduling of debts and outright default are relatively rare events). Without any deadweight costs of default, the level of debt that a sovereign would be willing to repay is constrained by the worst punishment lenders can inflict on the sovereign, namely, permanent exclusion from all forms of future credit. It is well known that this punishment is generally too weak, quantitatively speaking, to sustain much debt (this is spelled out in a numerical example in Aguiar and Gopinath, 2006). Thus, we need to posit substantial deadweight costs of default.

Second, defaults actually occurring in equilibrium reflect the fact that debt contracts are not fully state-contingent, and default provides an implicit form of insurance. However, with rational risk-neutral lenders who break even, on average, for every loan they make to

sovereigns, the deadweight cost of default (which does not accrue to lenders) makes default an actuarially unfair form of insurance against bad states of the world for the sovereign. And, with risk-averse lenders, this insurance-through-default becomes even more actuarially unfair. Given fairly substantial deadweight costs of default and substantial risk aversion on the part of lenders, the insurance offered by the possibility of default appears to be quite costly in practice. The fact that countries carry large external debt positions despite the costs suggests that sovereigns are fairly impatient.

However, while myopia can explain in part why sovereigns borrow, it does not necessarily explain why they default. As noted already, default is a very costly form of insurance against bad states of the world. This fact – via equilibrium prices – can be expected to encourage the sovereign to stay away from debt levels for which the probability of default is significant. This has two implications. First, when crises/defaults do materialize, they come as a surprise, which is consistent with these events being low probability. Unfortunately, the other side of this coin is that getting the mean and volatility of spreads right is a challenge for quantitative models. Getting high and variable spreads means getting periods of high default risk as well as substantial variation in expected future default risk. This will be difficult to achieve when the borrower has a strong incentive to adjust his debt-to-output level to the point where the probability of future default is (uniformly) low.

3 Environment

The analysis focuses on a sovereign government that makes consumption and savings/borrowing decisions on behalf of the denizens of a small open economy facing a fluctuating endowment stream. The economy is small relative to the rest of the world in the sense that the sovereign's decisions do not affect any world prices, including the world risk-free interest rate. However, the sovereign faces a segmented credit market in that it can only borrow from a set of potential lenders with limited wealth. In this section, we proceed by describing the economy of which the sovereign is in charge, the sovereign's decision problem and the lenders' decision problem. We then give the definition of an equilibrium and discuss issues related to equilibrium selection. We conclude the section by briefly describing how we compute model.

3.1 The Economy

3.1.1 Endowments

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The economy receives a stochastic endowment $Y_t > 0$ each period. We assume that

$$\ln Y_t = \sum_{s=1}^{t} g_s + z_t, \tag{3}$$

where g_t and z_t follow first-order Markov processes. This specification follows Aguiar and Gopinath (2006, 2007) and nests the endowment processes that have figured in quantitative studies. In particularly, setting $g_t = g$ generates a deterministic linear trend. More generally, g_t can be random, which corresponds to the case of stochastic trend. In either case, z_t is transitory (but potentially persistent) fluctuations around trend growth. In this chapter we will study both specifications in some detail.

3.1.2 Preferences

The economy is run by an infinitely-lived sovereign government. The utility obtained by the sovereign from a sequence of aggregate consumption $\{C_t\}_{t=0}^{\infty}$ is given by:

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \, 0 < \beta < 1 \tag{4}$$

and

$$u(C) = \begin{cases} C^{1-\sigma}/1 - \sigma & \text{for } \sigma \geq 0 \text{ and } \sigma \neq 1 \\ \ln(C) & \text{for } \sigma = 1 \end{cases} \tag{5}$$

It is customary to assume that the sovereign has enough instruments to implement any feasible consumption sequence as a competitive equilibrium and, thus, abstract from the problem of individual residents of the economy. This does not mean that the government necessarily shares the preferences of its constituents, but rather that it is the relevant decision maker vis-a-vis international financial markets.[5]

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[5]In particular, one interpretation of the environment is that C_t represents public spending and Y_t the available revenue that is allocated by the government.

3.1.3 Financial Markets and the Option to Default

The sovereign issues noncontingent bonds to a competitive pool of lenders. Bonds pay a coupon every period up to and including the period of maturity, which, without loss of generality, we normalize to r^* per unit of face value, where r^* is the (constant) international risk-free rate. With this normalization, a risk-free bond will have an equilibrium price of one. For tractability, bonds are assumed to mature randomly as in Leland (1994).[6] Specifically, the probability that a bond matures next period is a constant $\lambda \in [0, 1]$. The constant hazard of maturity implies that all bonds are symmetric before the realization of maturity at the start of the period, regardless of when they were issued. The expected maturity of a bond is $1/\lambda$ periods and so $\lambda = 0$ is a consol and $\lambda = 1$ is a one-period bond. When each unit of a bond is infinitesimally small and any given unit matures independently of all other units, a fraction λ of any nondegenerate portfolio of bonds will mature with probability 1 in any period. With this setup, a portfolio of sovereign bonds of measure B gives out a payment (absent default) of $(r^* + \lambda)B$ and has a continuation face value of $(1 - \lambda)B$.

We will explore the quantitative implications of different maturities, but in any given economy, bonds with only one specific λ are traded. The stock of bonds at the start of any period – inclusive of bonds that will mature in that period – is denoted B. We do not restrict the sign of B, so the sovereign could be a creditor ($B < 0$) or a debtor ($B > 0$). If $B < 0$, the sovereign's (foreign) assets are assumed to be in risk-free bonds that mature with probability λ and pay interest (coupon) of r^* until maturity. The net issuance of bonds in any period is $B' - (1 - \lambda)B$, where B' is the stock of bonds at the end of the period. If the net issuance is negative, the government is either purchasing its outstanding debt or accumulating foreign assets; if it is positive, it is either issuing new debt or de-accumulating foreign assets.

If the sovereign is a debtor at the start of a period, it is contractually obligated to pay λB in principal and r^*B in interest (coupon) payments. The sovereign has the option to default on this obligation. The act of default immediately triggers exclusion from international financial markets (i.e., no saving or borrowing is permitted) starting in the next period. Following the period of mandatory exclusion, exclusion continues with constant probability $(1 - \xi) \in (0, 1)$ per period. Starting with the period of mandatory exclusion and continuing for as long as exclusion lasts, the sovereign loses a proportion $\phi(g, z)$ of (nondefault state)

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[6]See also Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012) and Arellano and Rama-narayanan (2012).

output Y. When exclusion ends, the sovereign's debts are forgiven and it is allowed to access financial markets again.
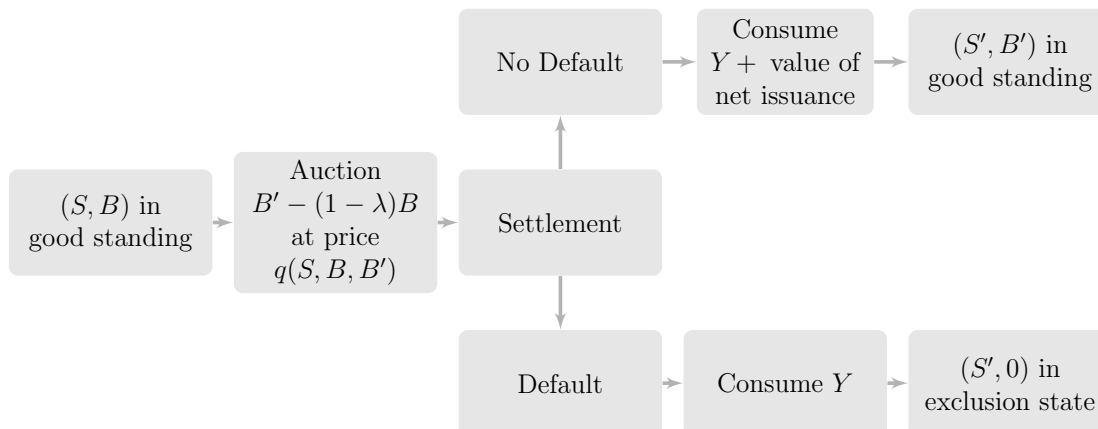
3.1.4 Timing of Events

Figure 2: Timing within a Period

The timing of events within a period is depicted in Figure 2. A sovereign in good standing observes S, the vector of current-period realizations of all exogenous shocks, and decides to auction $B' - (1 - \lambda)B$ units of debt, where B' denotes the face value of debt at the start of the next period. If the sovereign does not default at settlement, it consumes the value of its endowment plus the value of its net issuance (which could be positive or negative) and proceeds to the next period in good standing.

If the sovereign defaults at settlement, it does not receive the auction proceeds and it is excluded from international credit markets. Thus it consumes its endowment and proceeds to the next period in which it is also excluded from borrowing and lending. We assume that the amount raised via auction, if any, is disbursed to all existing bondholders in proportion to the face value of their bond positions, i.e., each unit of outstanding bonds is treated equally and receives $q(S, B, B')(B' - (1 - \lambda)B)/B'$. The implication is that as long as $B > 0$ purchasers of newly issued bonds suffer an immediate loss following default. If the sovereign defaults at settlement after purchasing bonds (i.e., after a buyback of existing debt), we assume that it defaults on its new payment obligations along with any remaining outstanding debt. Thus the sovereign consumes its endowments in this case as well (and moves on to the next period in a state of financial exclusion).

Our timing regarding default deviates from that of Eaton and Gersovitz (1981), which

has become the standard in the quantitative literature. In the Eaton-Gersovitz timing, the bond auction occurs after that period's default decision is made. That is, the government is the Stackelberg leader in its default decision in a period. Thus newly auctioned bonds do not face any within-period default risk and, so, the price of bonds depend only on the exogenous states S and the amount of bonds the sovereign exits a period with, B'. Our timing expands the set of equilibria relative to the Eaton-Gersovitz timing, and in particular allows a tractable way of introducing self-fulfilling debt crises, as explained in (sub)section 3.5 below.[7] It is also worth pointing out that implicit in the timing in Figure 2 is the assumption that there is only one auction per period. While this assumption is standard, it does allow the sovereign to commit to the amount auctioned within a period.[8]

3.2   The Sovereign's Decision Problem

We will state the sovereign's decision problem in recursive form. To begin, the vector $S \in \mathscr{S}$ of exogenous state variables consists of the current endowment Y and current period realizations of the endowment shocks g and z; it also contains W, the current period wealth of the representative lender, as this will affect the supply of foreign credit; and it contains $x \in [0, 1]$, a variable that indexes investor beliefs regarding the likelihood of a rollover crisis (explained more in section 3.5). Both W and x are stochastic and assumed to follow first-order Markov processes. We assume that all conditional expectations of the form $\mathbb{E}_S f(S', \cdot)$ encountered below are well-defined.

Let $V(S, B)$ denote the sovereign's optimal value conditional on S and B. Working backwards through a period, at the time of settlement the government has issued $B' - (1-\lambda)B$ units of new debt at price $q(S, B, B')$ and owes $(r^* + \lambda)B$. If the government honors its obligations at settlement, its payoff is:

$$V^R(S, B, B') = \begin{cases} u(C) + \beta\mathbb{E}_S V(S', B') & \text{if } C \geq 0 \\ -\infty & \text{otherwise} \end{cases}. \tag{6}$$

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[7]The timing in Figure 2 is adapted from Aguiar and Amador (2014b), which in turn is a modification of Cole and Kehoe (2000). The same timing is implicit in Chatterjee and Eyigungor's (2012) modeling of a Cole-Kehoe type rollover crisis. In both setups, the difference relative to Cole and Kehoe is that the sovereign is not allowed to consume the proceeds of an auction if it defaults. This simplifies the off-equilibrium analysis without materially changing the results. See Auclert and Rognlie (2014) for a discussion of how the Eaton-Gersovitz timing in some standard environments has a unique Markov equilibrium, thus ruling out self-fulfilling crises.

[8]For an exploration of an environment in which the government cannot commit to a single auction, see Lorenzoni and Werning (2014) and Hatchondo and Martinez (undated).

where

$$C = Y + q(S, B, B')[B' - (1 - \lambda)B] - (r^* + \lambda)B. \tag{7}$$

If the sovereign defaults at settlement, its payoff is:

$$V^D(S) = u(Y) + \beta \mathbb{E}_S V^E(S') \tag{8}$$

where

$$V^E(S) = u(Y(1 - \phi(g, z))) + \beta \mathbb{E}_S \left[\xi V(S', 0) + (1 - \xi) V^E(S') \right] \tag{9}$$

is the sovereign's value when it is excluded from financial markets and incurs the output costs of default. Recall that ξ is the probability of exiting the exclusion state and, when this exit occurs, the sovereign re-enters financial markets with no debt. Note also that the amount of new debt implied by B' is not relevant for the default payoff as the government does not receive the auction proceeds if it defaults at settlement.

Finally, the current period value function solves:

$$V(S, B) = \max \left\langle \max_{B' \leq \theta Y} V^R(S, B, B'), V^D(S) \right\rangle, \ \forall \ S \text{ and } B. \tag{10}$$

The upper bound θY on the choice of B' rules out Ponzi schemes.

Let $\delta(S, B, B')$ denote the policy function for default at settlement conditional on B'. For technical reasons, we allow the sovereign to randomize over default and repayment when it is indifferent, that is, when $V^R(S, B, B') = V^D(S)$. Therefore, $\delta(S, B, B') : \mathscr{S} \times \mathbb{R} \times (-\infty, \theta Y] \to [0, 1]$ is the probability the sovereign defaults at settlement, conditional on (S, B, B'). Let $A(S, B) : \mathscr{S} \times \mathbb{R} \to (-\infty, \theta Y]$ denote the policy function that solves the inner maximization problem in (10) when there is at least one B' for which C is strictly positive. The policy function of consumption is implied by those for debt and default.

3.3 Lenders

We assume financial markets are segmented and only a subset of foreign investors participates in the sovereign debt market. This assumption allows us to introduce a risk premium on sovereign bonds as well as to explore how shocks to foreign lenders' wealth influence equilibrium outcomes in the economy, all the while treating the world risk-free rate as given.

For simplicity, all period t lenders participate in the sovereign bond market for one period and are replaced by a new set of lenders.

We assume there is a unit measure of identical lenders each period. Let W_i be the wealth of an individual lender in the current period (W is the *aggregate* wealth of investors and is included in the state vector S in this capacity). Each lender allocates his wealth across two assets: the risky sovereign bond and an asset that yields the world risk-free rate r^*. Lenders must hold nonnegative amounts of the sovereign bond but can have any position, positive or negative, in the risk-free asset. The lender's utility of next period (terminal) wealth, $\widetilde{W_i}$, is given by

$$k(\widetilde{W_i}) = \begin{cases} \widetilde{W_i}^{1-\gamma}/1 - \gamma & \text{for } \gamma \geq 0 \text{ and } \gamma \neq 1 \\ \ln(\widetilde{W_i}) & \text{for } \gamma = 1 \end{cases}.$$

Note that $\widetilde{W_i}$ is distinct from the W' that appears in S' (next period's exogenous state vector) as the latter refers to the aggregate wealth of next period's new cohort of lenders.

The one-period return on sovereign bonds depends on the sovereign's default decision within the current period as well as on next period's default decision. Let $\widetilde{D}$ and $\widetilde{D}'$ denote the sovereign's realized default decisions, either 0 (no default) or 1 (default), at settlement during the current and next period, respectively. A lender who invests a fraction (or multiple) μ of his current wealth W_i has random terminal wealth $\widetilde{W_i}$ given by

$$(1 - \mu)W_i(1 + r^*) + \mu W_i/q(S, B, B') \left[(1 - \widetilde{D})(1 - \widetilde{D}')\right] \left[r^* + \lambda + (1 - \lambda)q(S', B', B'')\right], \quad (11)$$

where,

$$\begin{aligned} \tilde{D} &= 1 \text{ with probability } \delta(S, B, B') \\ \tilde{D}' &= 1 \text{ with probability } \delta(S', B', A(S', B')) \\ B'' &= A(S', B'). \end{aligned} \quad (12)$$

The wealth evolution equation omits terms that are only relevant off equilibrium; namely, it omits any payments from the settlement fund after a default. These will always be zero in equilibrium.

The representative lender's decision problem is how much sovereign debt to purchase at

auction. Specifically:

$$L(W_i, S, B, B') = \max_{\mu \geq 0} \mathbb{E}_S \left[k\left(\widetilde{W}_i \right) \Big| B, B' \right],$$

subject to (11) and the expressions in (12). The solution to the lender's problem implies an optimal $\mu(W_i, S, B, B')$.

The market-clearing condition for sovereign bonds is then

$$\mu(W, S, B, B') \cdot W = q(S, B, B') \cdot B' \text{ for all feasible } B' > 0, \qquad (13)$$

where W is the aggregate wealth of the (symmetric) lenders. The condition requires that the bond price schedule be consistent with market clearing for any potential $B' > 0$ that raises positive revenue. This is a "perfection" requirement that ensures that when the sovereign chooses its policy function $A(S, B)$, its beliefs about the prices it will face for different choices of B' are consistent with the "best response" of lenders. There are no market-clearing conditions for $B' \leq 0$; the sovereign is a small player in the world capital markets and, thus, can save any amount at the world risk-free rate.

Differentiation of the objective function of the lender with respect to μ gives an FOC that implies

$$q(S, B, B') = \frac{\mathbb{E}_S[\widetilde{W}^{-\gamma}(1 - \widetilde{D})(1 - \widetilde{D}')(r^* + \lambda + (1 - \lambda)q(S', A(S', B')))]}{(1 + r^*)\mathbb{E}_S[\widetilde{W}^{-\gamma}]} \qquad (14)$$

where $\widetilde{W}$ is evaluated at $\mu(W, S, B, B')$.

Equation (14) encompasses cases that are encountered in existing quantitative studies. As noted already, in the Eaton-Gersovitz timing of events there is no possibility of default at settlement. This means $\delta(S, B, B') = 0$ and the pricing of bonds at the end of the current period reflects the possibility of default in future periods only. This means $\delta(S', B', B''(S', B'))$ does not depend on B'', only on (S', B'). Thus, q depends on (S, B') only. If lenders are risk neutral and debt is short term ($\gamma = 0$ and $\lambda = 1$), $q(S, B, B')$ is simply the probability of repayment on the debt next period; if lenders are risk neutral but debt is long term ($\gamma = 0$ and $\lambda > 0$)

$$q(S, B, B') = \frac{\mathbb{E}_S(1 - D(S', B'))(r^* + \lambda + (1 - \lambda)q(S', A(S', B')))]}{(1 + r^*)}. \qquad (15)$$

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3.4 Equilibrium

Definition 1 (Equilibrium). *Given a first-order Markov process for S, an equilibrium consists of a price schedule $q : \mathscr{S} \times \mathbb{R} \times (-\infty, \theta Y] \to [0,1]$; sovereign policy functions $A : \mathscr{S} \times \mathbb{R} \to (-\infty, \theta Y]$ and $\delta : \mathscr{S} \times \mathbb{R} \times (-\infty, \theta Y] \to [0,1]$; and lender policy function $\mu : \mathbb{R}^+ \times \mathscr{S} \times \mathbb{R} \times (-\infty, \theta Y] \to \mathbb{R}$; such that: (i) $A(S,B)$ and $\delta(S,B,B')$ solve the sovereign's problem from Section 3.2, conditional on $q(S,B,B')$ and the representative lender's policy function; (ii) $\mu(W,S,B,B')$ solves the representative lender's problem from Section 3.3 conditional on $q(S,B,B')$ and the sovereign's policy functions; and (iii) market clearing: equation* (13) *holds.*

3.5 Equilibrium Selection

Because the default decision is made at the time of settlement, the equilibrium of the model features defaults that occur due to lenders' refusal to roll over maturing debt. To see how this can occur, consider the decision problem of a lender who anticipates that the sovereign will default at settlement on new debt issued in the current period, i.e., the lender believes $\delta(S,B,B') = 1$ for all (feasible) $B' > (1-\lambda)B$. Then, the lender's optimal μ is 0 and the market-clearing condition (13) implies that $q(S,B,B') = 0$ for $B' > (1-\lambda)B$. In this situation, the most debt the sovereign could exit the auction with is $(1-\lambda)B$ and consistency with lender beliefs requires that $V^D(S) \geq V^R(S,B,(1-\lambda)B)$.[9] On the other hand, for a given stock of debt and endowment, there may be a positive price schedule that can also be supported in equilibrium. That is, if $q(s,B,\tilde{B}) > 0$ for some $\tilde{B} > (1-\lambda)B$ (which necessarily implies that lenders do not anticipate default at settlement for $B' = \tilde{B}$) and $V^D(S) < V^R(S,B,\tilde{B})$, the sovereign would prefer issuing new bonds to help pay off maturing debt and thus find it optimal to repay at settlement. Defaults caused by lenders offering the adverse equilibrium price schedule when a more generous price schedule that induces repayment is also an equilibrium price schedule are called a *rollover crisis*. A default that occurs because there is no price schedule that can induce repayment (because endowments are too low and/or debt is too high) is called a *fundamental default*.

We incorporate rollover crises via the belief shock variable x. We assume that x is uniformly distributed on the unit interval, and we denote values of $x \in [0, \pi)$ as being in the crisis zone and values of $x \in [\pi, 1]$ as being in the noncrisis zone. In the crisis zone, a

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[9]If this condition is violated, the sovereign would strictly prefer to honor its obligation even after having acquired some small amount of new debt, contrary to lender beliefs

rollover crisis occurs *if* one can be supported in equilibrium. That is, a crisis occurs with $q(S, B, B') = 0$ for all $B' > (1-\lambda)B)$ if $V^R(S, B, (1-\lambda)B) < V^D(S)$ *and* $x(S) \in [0, \pi)$. On the other hand, if a positive price of the debt can be supported in equilibrium, conditional on the sovereign being able to roll over its debt, then this outcome is selected if $x(S) \in [\pi, 1]$. If S is such that $V^R(S, B, (1-\lambda)B) \geq V^D(S)$, then no rollover crisis occurs even if $x(S) \in [0, \pi)$. We let π index the likelihood a rollover crisis, if one can be supported in equilibrium.

We end this section with a comment on the incentive to buy back debt in the event of a failed auction, defined as a situation where lenders believe that $\delta(S, B', B) = 1$ for all $B' > (1-\lambda)B$ (either because of a rollover crisis or because of a solvency default). With a failed auction and long-term debt, the government has an incentive to buy back its debt on the secondary market if the price is low enough and then avoid default at settlement. For instance, this incentive will be strong if $q(S, B, B') = 0$ for $B' < (1-\lambda)B$. In this case, the sovereign could purchase its outstanding debt at zero cost and if

$$u(Y + (r^* + \lambda)B) + \beta \mathbb{E}_S V^R(S', B, 0) > u(Y) + \beta \mathbb{E}_S V^E(S'),$$

the sovereign's incentive to default at settlement will be gone. But, then, a lender would be willing to pay the risk-free price for the last piece of debt and outbid the sovereign for it.

To square the sovereign's buyback incentives with equilibrium, we follow Aguiar and Amador (2014b) and assume that in the case of a failed auction, the price of the debt $q(S, B, B')$ for $B' \leq (1-\lambda)B$, is high enough to make the sovereign just indifferent between defaulting on the one hand and, on the other, paying off its maturing debt and buying back $(1-\lambda)B - B'$ of its outstanding debt. Given this indifference, we further assume that the sovereign randomizes between repayment and default following a buyback, with a mixing probability that is set so that current period lenders are willing to hold on to the last unit of debt in the secondary market in the event of a buyback (more details on the construction of the equilibrium price schedule are provided in the computation section).

3.6 Normalization

Since the endowment Y has a trend, the state vector S is unbounded. To make the model stationary for computation we normalize the nonstationary elements of the state vector S by the trend component of Y_t,

$$G_t = \exp(\sum_1^t g_s). \tag{16}$$

The elements of the normalized state vector s are (g, z, w, x), where w is W/G. Since Y/G is a function of z only and z already appears in S, s contains one less element than S. It will be convenient to use the same notation defined above for functions of S for functions of the normalized state vector s. Normalizing both sides of the budget constraint (7) by G and denoting C/G by c, B/G by b and B'/G by b' yields the normalized budget constraint

$$c = \exp(z) + q(s, b, b')[b' - (1 - \lambda)b] - (r^* + \lambda)b. \tag{17}$$

Here we are imposing the restriction that the pricing function is homogeneous of degree 0 in the trend endowment G and, so, denote it by $q(s, b, b')$.[10]

Next, since $u(C) = G^{1-\sigma}u(c)$, we guess $V^R(S, B, B') = G^{1-\sigma}V^R(s, b, b')$ and $V(s, b) = G^{1-\sigma}V(S, B)$. This gives

$$V^R(s, b, b') = u(c) + \beta\mathbb{E}_s g'^{1-\sigma}V(s', b'/g'). \tag{18}$$

Analogous guesses for the value functions under default and exclusion yield

$$V^D(s) = u(\exp(z)) + \beta\mathbb{E}_s g'^{1-\sigma}V^E(s') \tag{19}$$

and

$$V^E(s) = u(\exp(z)(1 - \phi(g, z))) + \beta\mathbb{E}_s g'^{1-\sigma}\left[\xi V(s', 0) + (1 - \xi)V^E(s')\right]. \tag{20}$$

So,

$$V(s, b) = \max\left\langle \max_{b' \leq \theta \exp z} V^R(s, b, b'), V^D(s)\right\rangle, \ \forall\ s \text{ and } b. \tag{21}$$

We denote the sovereign's default decision rule from the stationarized model by $\delta(s, b, b')$ and we denote by $a(s, b)$ the solution to $\max_{b' \leq \theta \exp z} V^R(s, b, b'))$, provided repayment is feasible at (s, b).

Turning to the lender's problem, observe that given constant relative risk aversion, the optimal μ (the fraction devoted to the risky bond) is independent of the investor's wealth.

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[10]In particular, we are assuming that prices are functions of the ratios of debt and lenders' wealth to trend endowment but not of the level of trend endowment G itself. One could conceivably construct equilibria where this is not the case by allowing lender beliefs to vary with the level of trend endowment, conditional on these ratios. We are ruling out these sorts of equilibria.

Let $\mu(1, s, b, b')$ be the optimal μ of a lender with unit wealth. The FOC associated with the optimal choice of μ implies a normalized version of (14), namely,

$$q(s, b, b') = \frac{\mathbb{E}_s[\tilde{w}^{-\gamma}(1 - D)(1 - D')(r^* + \lambda + (1 - \lambda)q(s', b', a(s', b')))]}{(1 + r^*)\mathbb{E}_s[\tilde{w}^{-\gamma}]}, \qquad (22)$$

where $\tilde{w}$ is the terminal wealth of the lender with unit wealth evaluated at $\mu(1, s, b, b')$ and the expectation is evaluated using the sovereign's (normalized) decision rules.

The normalized version of the key market-clearing condition is then

$$\mu(1, s, b, b') \cdot w = q(s, b, b') \cdot b' \text{ for all feasible } b' > 0. \qquad (23)$$

For a given pricing function $0 \le q(s, b, b') \le 1$, standard Contraction Mapping arguments can be invoked to establish the existence of all value functions. For this, it is sufficient to bound b' from below by some $\underline{b} < 0$, i.e., impose an upper limit on the sovereign's holdings of foreign assets (in addition to the upper limit on its issuance of debt to rule out Ponzi schemes), and assume that $\beta \mathbb{E} g'^{1-\sigma}|g < 1$ for all $g \in \mathcal{G}$.

3.7 Computation

Computing an equilibrium of this model means finding a price function $q(s, b, b')$ and associated optimal stationary decision rules $\delta(s, b, b')$, $a(s, b)$ and $\mu(1, s, b, b')$ that satisfy the stationary market-clearing condition (23). That is, it means finding a collection of functions that satisfy

$$\mu(1, s, b, b') \cdot w = \qquad (24)$$
$$\left[\frac{\mathbb{E}_s[\tilde{w}^{-\gamma}(1 - \widetilde{D})(1 - \widetilde{D}')(r + \lambda + (1 - \lambda)q(s', b', a(s', b')))]}{(1 + r^*)\mathbb{E}_s[\tilde{w}^{-\gamma}]} \right] b' \; \forall \, s, \, b \text{ and } b'.$$

If such a collection can be found, an equilibrium in the sense of Definition 1 will exist in which all the nonstationary decision rules are scaled versions of the stationary decision rules, i.e., $A(S, B) = a(s, b)G$, $\delta(S, B, B') = \delta(s, b, b')$ and $\mu(W, S, B, B') = \mu(1, s, b, b')wG$.

On the face of it, this computational task seems daunting given the large state and control space. It turns out, however, that (24) can be solved by constructing the solution out of the solution of a computationally simpler model. This simpler model adheres to the Eaton-Gersovitz timing, so $\delta(s, b, b') = 0$, and thus q is a function of s and b' only. But, unlike the

standard Eaton-Gersovitz model, it is modified to have rollover crises.[11] The modification is as follows: If s is such that the belief shock variable $x(s)$ is in $(\pi, 1]$ (i.e., it is not in the crisis zone), the sovereign is offered $q(s, b')$ where b' can be any feasible choice of debt (think of this as the price schedule in "normal times"). But if $x(s)$ is in $[0, \pi]$, the sovereign is offered a truncated *crisis* price schedule in which $q(s, b') = 0$ for all $b' > (1 - \lambda)b$ *provided* default strictly dominates repayment under the crisis price schedule; if the proviso is not satisfied, the sovereign is offered the normal (nontruncated) price schedule.

To see how this construction works, let $q(s, b')$ be the equilibrium price function of this rollover-modified EG model. That is, $q(s, b')$ satisfies

$$\mu(1, s, b') \cdot w = \left[\frac{\mathbb{E}_s[\tilde{w}^{-\gamma}(1 - D(s', b'))(r + \lambda + (1 - \lambda)q(s', a(s', b')))]}{(1 + r)\mathbb{E}_s[\tilde{w}^{-\gamma}]} \right] b' \qquad (25)$$

where $D(s, b)$ and $a(s, b)$ are the associated equilibrium policy functions. And let $V(s, b)$ and $V^D(s)$ be the associated value functions. Next, let $G(Q; s, b, b')$ be defined as the utility gap between repayment and default at settlement when the auction price is Q:

$$u\left[\exp(z(s)) - (r^* + \lambda)b + Q(b' - (1 - \lambda)b)\right] + \beta \mathbb{E}_s g'^{1-\sigma} V(s', b'/g') - V^D(s).$$

G encapsulates the incentive to default or repay at settlement in a model in which default at settlement is not permitted. The logic underlying the construction of the price schedule for the model in which default at settlement *is* permitted is this: If $G(s, b, b')$ evaluated at $Q = q(s, b')$ is nonnegative, $q(s, b, b')$ is set equal to $q(s, b')$, as there is no incentive to default at settlement; if $G(s, b, b')$ evaluated at $Q = q(s, b')$ is negative, $q(s, b, b')$ is set to 0 if the incentive to default is maintained at an auction price of zero, or it is set to some positive value between 0 and $q(s, b')$ for which the sovereign is indifferent between default and repayment.

(i) For $b' \geq (1 - \lambda)b$

$$q(s, b, b') = \begin{cases} 0 \text{ if } G(q(s, b'); s, b, b') < 0 \\ q(s, b') \text{ if } G(q(s, b'); s, b, b') \geq 0. \end{cases}$$

The top branch deals with the case where the sovereign's incentive to default at settlement is strictly positive after having issued debt at price $q(s, b')$. Since G is (weakly)

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[11]This model is described in section E of Chatterjee and Eyigungor (2012)).

increasing in Q in this case, the incentive to default at settlement is maintained at $Q = 0$ and, so, we set $q(s, b, b') = 0$. The bottom branch deals with the case where the sovereign (weakly) prefers repayment over default. In this case, the price is unchanged at $q(s, b')$.

(ii) For $b' < (1 - \lambda)b$:

$$q(s, b, b') = \begin{cases} 0 \text{ if } G(0; s, b, b') < 0 \\ Q^*(s, b, b') \text{ if } Q \in [0, q(s, b')) \\ q(s, b') \text{ if } G(q(s, b'); s, b, b') \geq 0. \end{cases}$$

The bottom branch offers $q(s, b')$ if $G(q(y, b'); s, b, b') \geq 0$. If $G(q(y, b'); s, b, b') < 0$, then two cases arise. Since G is weakly decreasing in Q, it is possible that there is a $Q \in [0, q(s, b'))$ for which the $G(Q; s, b, b') = 0$. In this case, we set $q(s, b, b') = Q$. If there is no such Q, then $G(0; s, b, b') < 0$ and we set $q(s, b, b') = 0$.

Next, we verify that given $V(s, b)$ and $V^D(s)$ (the value functions under $q(s, b)$), the optimal action under $q(s, b)$ is also an optimal action under $q(s, b, b')$. First, consider (s, b) for which the optimal action is to choose $a(s, b)$. This implies that $G(q(s, b); s, b, a(b, s)) \geq 0$. Then, by construction, $q(s, b, b') = q(s, b)$ and the payoff from choosing $a(s, b)$ is the same as under $q(s, b)$ and this payoff will (weakly) dominate the payoff from choosing any other b' for which $q(s, b, b') = q(s, b')$ (by optimality). Furthermore, the payoff from any b' for which $q(s, b, b') \neq q(s, b)$ is never better than default. It follows that $a(s, b)$ (coupled with $\delta(s, b, a(s, b)) = 0$) is an optimal choice under $q(s, b, b')$. Next, consider (s, b) for which it is optimal to default under $q(s, b)$. This implies $G(q(s, b); s, b, b') < 0$ for all feasible b'. Then, by construction, default at settlement is the best option, or one of the best for all b' under $q(s, b, b')$.

Finally, we have to verify that $q(s, b, b')$ is consistent with market clearing. For (s, b, b') such that $q(s, b, b') = q(s, b)$, market clearing is ensured because the market clears (by assumption) under $q(s, b)$. For (s, b, b') such that $q(s, b, b') = 0$, market clearing is ensured trivially. For (s, b, b') such that $q(s, b, b') \in (0, q(s, b))$, market clearing can be ensured by selecting $\delta(s, b, b')$ appropriately. For instance, if lenders are risk-neutral, $\delta(s, b, b')$ is set to satisfy $q(s, b, b') = [1 - \delta(s, b, b')]q(s, b')$. Then, with probability $\delta(s, b, b')$ the sovereign defaults and the bonds are worthless, and with probability $1 - \delta(s, b, b')$, the sovereign repays

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and the bonds are worth $q(s, b')$. With risk-averse lenders, $\delta(s, b, b')$ can be similarly set to make lenders willing to lend b' at $q(s, b, b')$.[12]

We conclude the description of the construction of $q(s, b, b')$ by noting how it modifies the rollover price schedule under $q(s, b')$. Under $q(s, b')$, a rollover crisis is a price schedule with (a) $x(s) \in [0, \pi]$, (b) for $b \geq ((1-\lambda)b$, $q(s, b') = 0$, and (c) $D(s, b) = 1$. Under $q(s, b, b')$, a rollover has (a) $x(s) \in [0, \pi]$, (b) for $b' \geq (1-\lambda)b$, $q(s, b, b') = 0$ (which, in this case, is also $q(s, b')$) and (c) for $b' < (1-\lambda)b$, $q(s, b, b')$ is given by the construction under (ii). Thus, the only modification to the crisis price schedule is to lower the prices associated with buybacks (as discussed earlier in section 3.5).

In the rest of this section, we describe the iterative process by which the (stationary) equilibrium of the rollover-modified EG model is computed. First, the space of feasible b' is discretized. Second, the space of x (the belief shock variable) is also discretized with "crisis" equal to a value of 1, taken with probability π, and "normal" equal to a value of 0, taken with probability $(1-\pi)$. Suppose that $\{q^k(s, b')\}$ is the price schedule at the start of iteration k. Let $a(s, b; q^k), D(s, b; q^k)\}$ be the sovereign's decision rules conditional on $q^k(s, b')$. Then, for every feasible $b' > 0$ for which $q^k(s, b')b' > 0$, the price implied by the lender's optimal choice of μ and market clearing is

$$J^k(s, b')) = \frac{\mathbb{E}_s[\tilde{w}^{-\gamma}(1 - D(s', b'; q^k))(r + \lambda + (1-\lambda)q^k(s', a(s', b'; q^k)))]}{(1+r^*)\mathbb{E}_s[\tilde{w}^{-\gamma}]}, \tag{26}$$

where, using (23), the $\mu(1, s, b'; q^k)$ that appears in $\tilde{w}$ is replaced by $[q^k(s, b) \cdot b']/w(s)$. If $|\max J^k(s, b') - q^k(s, b')|$ is less than some chosen tolerance $\epsilon > 0$, the iteration is stopped and the collection $\{q^k(s, b'), a(s, b; q^k), D(s, b; q^k), \mu(1, s, b'; q^k)\}$ is accepted as an approximation of the equilibrium. If not, the price schedule is updated to

$$q^{k+1}(s, b') = \xi q^k(s, b') + (1-\xi)J^k(s, b'), \tag{27}$$

where $\xi \in (0, 1)$ is a damping parameter (generally close to 1).

In a purely discrete model in which all shocks and all choices belong to discrete sets, the iterative procedure described above typically fails to converge for a wide choice of parameter

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[12]If $\delta(s, b, b') = 0$ lenders would be just willing to lend b' at the price $q(s, b')$ (because they are willing to do so under $q(s, b')$). If the probability of default at settlement is kept at zero and the price of the bond is lowered to $q(s, b, b')$, there will be an excess demand for bonds. This excess demand can be choked off by lowering $\delta(s, b, b')$ sufficiently.

values. The reason is that the equilibrium we are seeking is, in effect, a Nash equilibrium of a game between the sovereign and its lenders and we should not expect the existence of an equilibrium in *pure* strategies, necessarily. To remedy the lack of convergence, it is necessary to let the sovereign randomize appropriately between two actions that give virtually the same payoff. The purpose of the continuous i.i.d. shocks (z in the SG model and m in the DG model) is to provide this mixing. We refer the reader to Chatterjee and Eyigungor (2012) for a discussion of how continuous i.i.d. shocks allow robust computation of default models.

4 Benchmark Models

We calibrate two versions of the basic model (under the assumption that rollover crises never happen). In one version, labeled DG, the endowment process of the sovereign and the wealth process of investors are modeled as independent stationary fluctuations around a common deterministic growth path. In the second version, labeled SG, the growth rates of endowments and investor wealth follow independent stationary processes with a common mean growth.

To calibrate the endowment process we use quarterly real GDP data for Mexico for the period 1980Q1 to 2015Q2. For the DG model, $G_t = (1 + g)^t$ and income is a stationary process plus a linear trend. The stationary component, z_t, is assumed to be composed of two parts: a persistent part e_t that follows an AR1 process and a purely transitory part m_t:

$$z_t = e_t + m_t, \quad m_t \sim N(0, \sigma_m^2) \text{ and } e_t = \rho_e e_{t-1} + v_t \quad v_t \sim N(0, \sigma_v^2) \tag{28}$$

As explained at the end of the previous section, the transitory shock m_t is required for robust computation of the equilibrium bond price function. We set $\sigma_m^2 = 0.000025$ and estimate (28) using standard state-space methods. The estimation gives $\rho_e = 0.85$ (0.045) and $\sigma_v^2 = 0.000139$ ($1.08E - 05$) (standard errors in parenthesis). The slope of the trend line implies a long-run quarterly growth rate of 0.56 percent (or annual growth rate of 2.42 percent).

For the SG model, the growth rate g_t is stochastic. Now, $\ln(Y_t) = \sum_0^t g_t + z_t$ and the growth rate of the period t endowment, $\ln(Y_t) - \ln(Y_{t-1}) \equiv \Delta y = g_t + z_t - z_{t-1}$. We assume

$$g_t = \alpha + \rho_g g_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \text{ and } z_t \sim N(0, \sigma_z^2) \tag{29}$$

and use the observed growth rate of real GDP to estimate (29) using state-space methods.

Table 6: Parameters of Endowment Processes

Parameter	Description	DG	SG
$-$	Average annual growth rate of endowments	2.42	2.45
ρ_e	Autocorrelation of y	0.85	$-$
σ_v	Standard deviation of innovations to e or g	0.012	0.011
σ_m	Standard deviation of m	0.005	$-$
ρ_g	Autocorrelation of g	$-$	0.45
σ_z	Standard deviation of z	$-$	0.003

The estimation yields $\alpha = 0.0034$ (0.0012), $\rho_g = 0.45$ (0.12), $\sigma_v^2 = 0.000119$ (0.0000281) and $\sigma_z^2 = 0.000011$ $(8.12e - 06)$. The estimates of α and ρ_g imply an average growth rate of 2.45 percent at an annual rate. These estimates are summarized in Table 6

Regarding $\phi(g, z)$, which determines the level of output under exclusion from credit markets, we assume

$$\text{for DG: } \phi(g, z) = d_0 \exp(z)^{d_1} \text{ and for SG: } \phi(g, z) = d_0 \exp(g)^{d_1}. \tag{30}$$

In either model, setting $d_1 = 0$ leads to default costs that are proportional to output in both models. If $d_1 > 0$, then default costs rise more than proportionately with z in the DG model, and more than proportionately with g in the SG model.

We assume that g takes values in a finite set $\mathscr{G}$. In the deterministic growth case $\mathscr{G}$ is a singleton. The specification of z depends on what is being assumed for g. When g is stochastic, z is drawn from a distribution H with compact support $[-\bar{h}, \bar{h}]$ and continuous CDF. When g is deterministic, $z = e + m$, where e follows a first-order Markov process with values in a finite set $\mathscr{E}$ and m is drawn from H. In either case, z is first-order Markov in its own right (in the stochastic g case, trivially so) but it is *not* finite-state.

Aside from the parameters of the endowment process, there are 12 parameters that need to be selected. The model has 3 preference parameters, namely, β (the sovereign's discount factor), σ (the curvature parameter of the sovereign's utility function) and γ (the curvature parameter of the investors utility function). It has 2 parameters with respect to the bond market, namely, λ (the probability with which a bond matures), and r_f (the risk-free rate of return available to investors). It has 3 parameters with respect to the default state, namely, d_0 and d_1, the parameters of the $\phi(g, z)$, and ξ, the probability of re-entry into credit markets from the exclusion state. Finally, there are 3 parameters governing the stochastic evolution

of investor wealth w_t. For the DG version, w_t is defined as $\ln\left(W_t/\omega(1+g_Y)^t\right)$ and for the SG version as $\ln\left(W_t/\omega Y_t\right)$, where ω controls the average wealth of investors relative to the sovereign. In either case w_t follows an AR1 process with persistence parameter ρ_w and unconditional variance σ_w^2.

Turning first to preference parameters, σ is set to 2, which is a standard value in the literature. The curvature parameter of the investor's utility function, γ, affects the compensation required by investors for default risk (risk premium). However, for any γ, the risk premium also depends on ω, as this determines the fraction of investor wealth that must reside in sovereign bonds in equilibrium. Thus, we can fix γ and vary ω to control the risk premium. With this in mind, γ was also set equal to 2.

With regard to the bond market parameters, we set the (quarterly) risk-free rate to 0.01. This value is roughly the average yield on a 3-month U.S. Treasury bill over the period 1983-2015.[13]. The probability of a bond maturing, λ, is set to $1/8 = 0.125$ which implies that bonds mature in 2 years, on average. This is roughly consistent with the data reported in (Broner, Lorenzoni, and Schmukler, 2013) which show that the average maturity of bonds issued by Mexico during the Brady bonds era prior to the Tequila crisis (1993-1995) was 2.5 years (post-crisis, the average maturity lengthened substantially).

The exclusion state parameters, d_0, d_1 and ξ, affect the value of the default option. The value of ξ was set to 0.125, which implies an average exclusion period of 2 years, on average. Settlements following default have generally been quick in the Brady era, so a relatively short period of exclusion seems appropriate.

Finally, we use the U.S. P/E ratio as a proxy for investor wealth. We set the autocorrelation of the investor wealth process to 0.91, which is the autocorrelation of the P/E ratio at a quarterly frequency for the period 1993Q1-2015Q2. We assume that w takes values in a finite set $\mathscr{W}$ and its (first-order) Markov process has an unconditional mean $\omega > 0$, where ω determines the relative wealth of investors via-a-vis the sovereign.

These parameter choices are summarized in Table 7.

The remaining five parameters $(\beta, d_0, d_1, \omega, \sigma_w^2)$ are jointly determined to match moments in the data. The moments chosen are the average debt-to-GDP ratio for Mexico, the average EMBI spreads on Mexican sovereign debt, the standard deviation of the spread, the fraction of variation in Mexican spreads accounted for by the variation in investor wealth proxied by

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[13]We use constant maturity yield computed by the Treasury and this data series begins in 1983Q3.

Table 7: Other Parameters Selected Independently

Parameter	Description	Value
σ	Risk aversion of sovereign	2.000
γ	Risk aversion of investors	2.000
r_f	Risk-free rate	0.010
λ	Reciprocal of average maturity	0.125
ξ	Probability of exiting exclusion	0.125
ρ_w	Autocorrelation of wealth process	0.910

the variation in the U.S. P/E ratio, and an annualized default frequency of 2 percent.[14]

We do the moment matching exercise in two steps. First, we set the curvature parameter for default costs, d_1, to 0 so that default costs are simply proportional to output and we drop the standard deviation of spreads as a target. The results are shown in Table 8. The finding is that the SG model can be calibrated to the data quite well but the DG model could not. The DG model could get the debt-to-GDP ratio and the R^2 of the spreads on P/E regression, but the average spread and the average default frequency are an order of magnitude below their targets. These results echo those in Aguiar and Gopinath (2006).

Table 8: Targets and Model Moments with Proportional Default Costs

Description	Target	DG	SG
Debt-to-annual GDP	0.66	0.66	0.66
Average default freq	0.02	0.003	0.02
Average EMBI spread	0.03	0.001	0.03
R^2 of spreads on P/E	0.22	0.20	0.27

Given the poor quantitative performance of the DG model with proportional costs, the rest of this chapter focuses on models with asymmetric default costs. We return to the proportional default cost and discuss its shortcomings in the next section after presenting our benchmark results.

[14]If we date the beginning of private capital flows into emerging markets in the postwar era as the mid-1960s, Mexico has defaulted once in 50 years.

5 Benchmark Results with Nonlinear Default Costs

Table 9 reports the results of the moment matching exercise when all five parameters are chosen to match the four targets above and the standard deviation of spreads. As is evident, the performance of the DG model improves substantially and it can now deliver the target level of average spreads and default frequency.

A surprising finding is that neither model can match the observed spread volatility, which is an order of magnitude larger in the data than in the models. The finding is surprising because asymmetric default cost models have been been successful in matching the volatility of spreads on Argentine sovereign bonds (the case that is most studied in the quantitative default literature). As explained later in the paper, the reason for the models' inability to match spread volatility is that neither z nor g is sufficiently volatile for Mexico (compared to Argentina) for the asymmetry in default costs to matter. Given this, the curvature parameter for default costs cannot be pinned down and we simply set it to a relatively large value and chose the remaining four parameters to match the other four targets.

Table 9: Targets and Model Moments with Asymmetric Default Costs

Description	Target	DG	SG
Debt-to-Annual GDP	0.66	0.66	0.66
Average default freq	0.02	0.02	0.02
Average EMBI spread	0.03	0.03	0.03
R^2 of spreads on P/E	0.22	0.23	0.26
S.D. of EMBI spread	0.03	0.005	0.002

The parameter values implied by this moment matching is reported in Table 10.

Table 10: Parameters Selected Jointly

Parameter	Description	DG	SG
β	Sovereign's discount factor	0.892	0.842
d_0	Level parameter for default costs	0.075	0.068
d_1	Curvature parameter for default costs	10.0	10.0
ω	Wealth of investors relative to mean endowment	2.528	2.728
σ_w	S.D. of innovations to wealth	2.75	0.275

5.1 Equilibrium Price and Policy Functions

In this subsection we characterize the equilibrium bond price schedules and policy functions for debt issuance. We discuss the benchmark stochastic-growth (SG) and deterministic-growth (DG) versions of the model.

The price schedules and policy functions for our two growth cases are depicted in Figure 3. As one can see from the first panel of the figure, the price schedules for the two different growth processes are quite similar. In both cases the price schedules are highly nonlinear, reflecting the positive feedback between the value of market access and q: the option to default lowers q for any B'/Y, which, in turn, lowers the value of market access and further increases the set of states in which default is optimal. Careful inspection will show that the DG schedule responds slightly less to an increase in debt right at the bend point.

The government's policy functions for debt issuance are depicted in the second panel of Figure 3. These two functions exhibit an important difference. The striking fact about the SG debt policy functions is that it is quite flat around the 45-degree line: This implies that the optimal policy features sharp leveraging and deleveraging that offsets the impact of good and bad growth shocks, respectively, and returns B'/Y to the neighborhood of the crossing point quite rapidly. Notice also that the crossing point is not very far from the levels of debt for which default is triggered. This "distance to default," and therefore the equilibrium spreads, are essentially determined by the output costs of default.

In contrast, the policy function for debt issuance for the DG economy depicts a significantly more modest leveraging and deleveraging response to deviations in the debt-to-output ratio around the 45-degree line. As we will see below, this will lead to sharp differences in the predicted outcomes of the two versions of our model.

We turn next to trying to understand how our model will respond to shocks. To do that we examine how our bond demand schedule responds to output and wealth shocks. These are plotted in Figures 4 and 5 respectively. With respect to output shocks, we see a fairly stark difference between our two models. Growth shocks have very little impact on the bond demand schedule in the SG model. But shocks that move output away from its deterministic trend have a fairly large effect in the DG version. This suggests that the stochastic growth version of our model will be much less responsive to output shocks than the deterministic growth version.

The reason for the difference in the response to output shocks between our two models

Figure 3: Pricing Schedules and Policy Functions

(a) Pricing Schedules   (b) Policy Functions
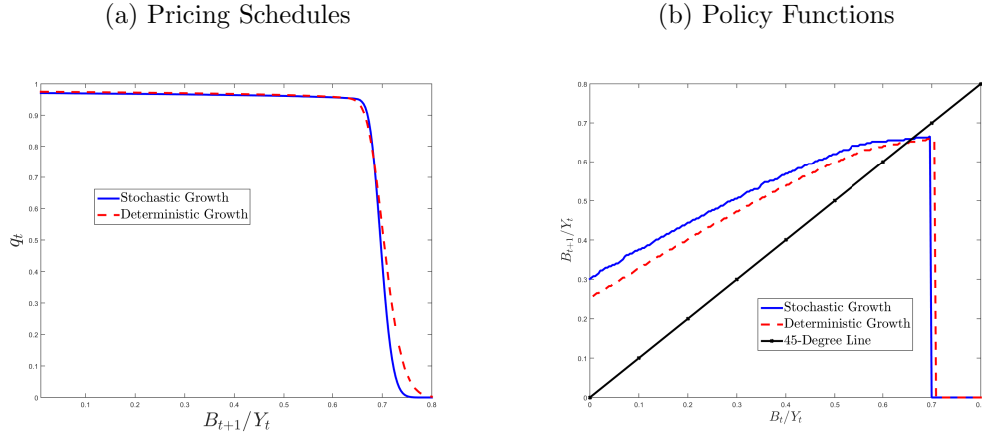
stems from the interaction of two factors. First, when output is substantially below trend in the DG model, the agents in the economy anticipate that a recovery to trend is highly likely, making the future level of output look positive relative to the present. At the same time, our assumption of asymmetric default costs means that defaulting when output is below trend is less costly than defaulting when output has recovered to trend. Overall this creates a stronger incentive to default in the near term for given levels of B/Y and B', and this shifts in (out) the pricing schedule in response to a negative (positive) output shock. The shift in the price schedule offsets the country's desire for smoothing, but, at the same time, generates movement in the spread. Below we compare this to proportional default cost case and show that the shifts result mostly from the asymmetric default cost.

In contrast, negative growth shocks in the stochastic growth model make the expectation of future growth lower because these growth shocks are positively autocorrelated. Thus nonlinear output costs makes delaying default more attractive. In addition, the negative trajectory of output encourages the country to save, not borrow. The first effect dampens the shift in the price schedule, while the second effect dampens the incentive to borrow. Together this means that there is little or no increase in the spread today. As we will see, these differences will lead to differences in equilibrium outcomes such as the dispersion in debt-to-output levels and spreads.

Both models are quite unresponsive to wealth shocks. Interestingly, a wealth shock tends to twist the price schedule. For example, a positive wealth shock pushes out the price for

high borrowing levels and but pulls it down for low borrowing levels. This last part arises from the increased incentive to dilute the current bonds in the future since the "price" of such dilution is not as high. We graphed the SG schedule on a magnified scale in order to make this twisting more apparent. This mechanism is explored in detail in Aguiar, Chatterjee, Cole, and Stangebye (2016).

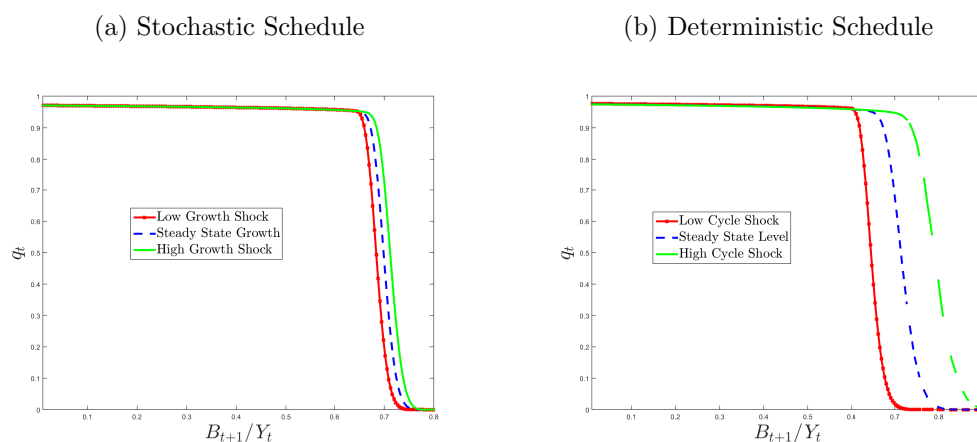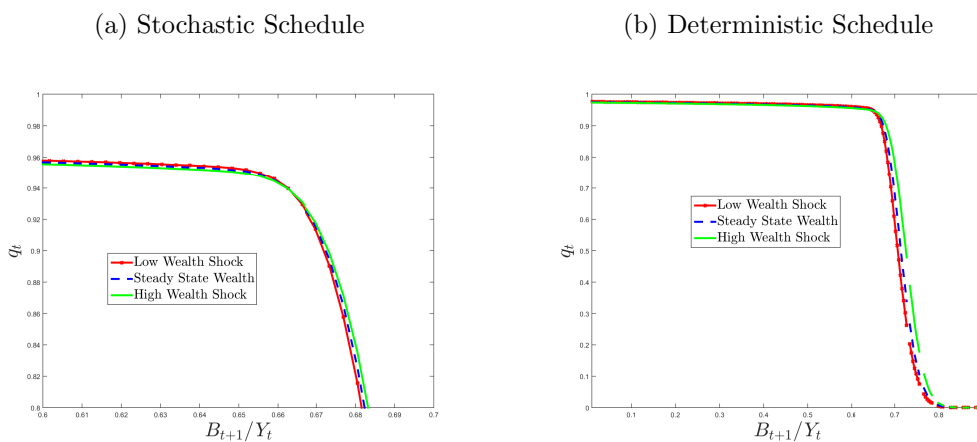Figure 4: Pricing Schedules and Output Shocks

(a) Stochastic Schedule   (b) Deterministic Schedule

Figure 5: Pricing Schedules and Wealth Shocks

(a) Stochastic Schedule   (b) Deterministic Schedule

In the deterministic case, we see relatively large movements in the pricing schedule with

shocks. In figure 6 we plot the pricing schedules for the proportional default cost case. In the DG model the price schedule does not respond to the output shock. This is because the expected positive trajectory of output makes the current debt-to-output ratio less onerous, while the proportionate default costs do not generate as strong an incentive to default today relative to the nonlinear case. Hence, the incentive to default is fairly stable and the price schedule does not shift in. At the same time, the feedback effect in the DG model with proportionate costs is so strong that the price schedule completely collapses past a certain B/Y ratio. This leads the country to stay sufficiently far inside of the collapse point that the probability of default tomorrow is virtually zero. In particular, it is very hard to generate a modest default probability and spread premium given this extreme pricing schedule. This is why this model is so hard to calibrate and why we get no volatility in the spread.

Figure 6: DG Model Pricing Schedule and Policy Function with Proportionate Costs

(a) Price Schedule   (b) Policy Function
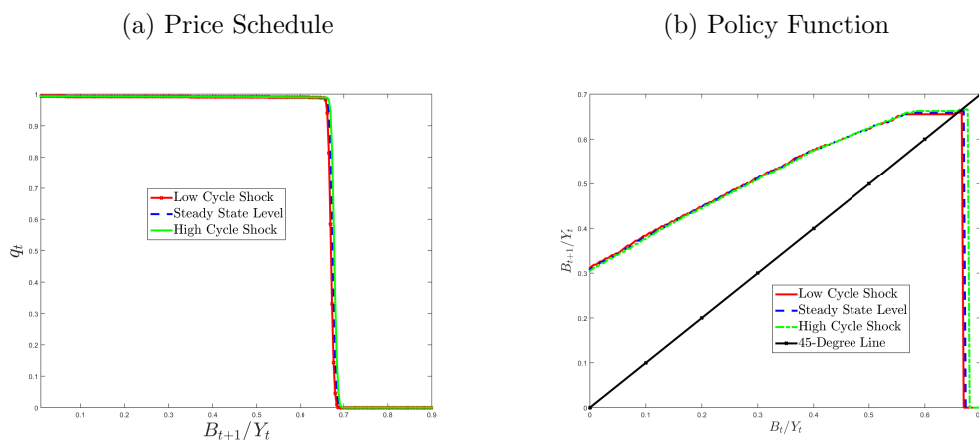
## 5.2 Boom-and-Bust Response

The sharp difference between our models comes from their responses to output shocks. To further understand the response of our models to growth rate shocks, we consider what happens after a sequence of positive shocks terminates in an negative shock. We refer to this as a boom-and-bust cycle.

In Figure 7 we show the policy response to a series of positive output shocks of varying length, followed by a bad output shock. We also show the impact on the equilibrium spread. In both cases, the fairly high degree of persistence in our output shocks leads the government

to borrow into a boom, raising the debt-to-output ratio. In the SG model, the government chooses to immediately delever in response to the negative output shock if it comes early enough in the boom; if it comes late, it defaults. The government in the DG model behaves similarly, except that it chooses to delever slightly more slowly in the case of a boom of intermediate length.

The spread behaves somewhat differently across the two versions of our model. In the SG version, the spread initially falls in respond to a positive output shock, but then it bounces back to essentially the same level as before in response to continued positive growth rate shocks because of the government's decision to lever up. More important, even in the period in which a negative growth rate shock first occurs, the government's decision to sharply delever means that the spread does not change in response to the negative shock. While the policy response of the government in the DG model is very similar to that of the SG model, the slightly slower deleveraging in response to a negative output shock leads to a sharp temporary rise in the spread.

Figure 7: Boom-and-Bust Cycle

(a) Stochastic   (b) Deterministic
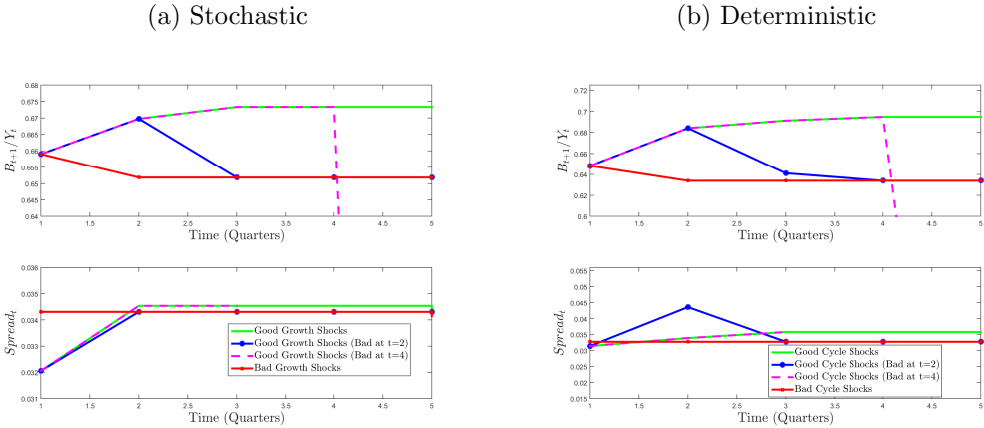
5.3 Equilibrium Outcomes

In this section we lay out the results for both versions of our model with nonlinear output loses. Our first set of results are presented in Table 11. The first three statistics, which were targeted, match the long-run data for Mexico and are in the ball park for other emerging economies. The sixth statistic we report is the R^2 of a regression of the spread on the investor

wealth shock w. This too is targeted to match the results of the regression of the spread on the U.S. price-earnings ratio and is roughly in line with the data.

There are two nontargeted moments in Table 11. The first is the correlation of the average excess return and the growth rate of output. For the stochastic growth economy, the sign of this correlation is positive, which is surprising, since one would expect positive growth rate shocks to lower the spread. However, the magnitude of this correlation is in the ball park in that the correlation is quite weak as it is in the data. In the DG model this correlation is both of the wrong sign and also substantially higher. This reflects that economy's the greater responsiveness to output shocks, which we discussed earlier in reference to Figure 4. Below we more closely examine the evidence on spreads and shocks using regression analysis to compare model and data results.

The other nontargeted moment is the standard deviation of the spread. This moment is too low, since it should be roughly equal to the average level of the spread. The fact that the spread's relative variation was still so low even with nonlinear default costs is surprising given that the literature has found that such costs can generate relatively realistic variation levels. However, the papers that have found this result have been calibrated to Argentina, which has a much more volatile output series.

To examine whether this might be at the root of our failure, we examined the implications of the DG model when we calibrate output to Argentina. When we calibrate our output process to Argentina, the autocorrelation coefficient for our output deviation from trend, z_t, rises from 0.853 to 0.930, thereby becoming more persistent. In addition, the standard deviation of z rises from 0.023 to 0.074, so the output deviations from trend are more volatile overall. All of the other model parameters are left unchanged. We report the results from this experiment in the last column of Table 11.

When we switch to the Argentine growth process for the deterministic model, the average debt-to-output level falls sharply, to 0.28, which is somewhat inconsistent with the fact that Argentina has a much higher value of this ratio than Mexico. In addition, the average spread rises sharply, to 0.06, and the volatility of the spread increases to 0.07. Both of these changes are consistent with the data in that Argentina has a much higher average spread and a much more volatile spread. This last finding indicates that the key to the literature's positive finding on spread volatility is the combination of nonlinear default costs and quite high output volatility. However, this story cannot explain the spread volatility in a country like Mexico with lower output volatility.

Table 11: Basic Statistics: Stochastic and Deterministic Growth Models

	Stochastic Benchmark	Deterministic Benchmark	Deterministic Argentina*
Debt-to-GDP	0.66	0.66	0.28
Average default freq.	0.02	0.02	0.04
Average spread	0.03	0.03	0.06
S.D. of spreads	0.002	0.004	0.07
Corr of spreads with Δy or z	0.15	0.46	-0.76
R^2 of spreads on w	0.26	0.17	0.01

One other stark difference between the results with the Mexico and the Argentina output calibrations concerns the correlation of the spread and the percent deviation of output from trend. This has now become very negative. In Table 2 the average correlation in our sample was -0.27, and the highest value was only -0.56 for Malaysia. The correlation in Argentina was -0.35 and in Mexico it was -0.4. So a value of -0.76 with the Argentine calibration for output looks too high. Below in the regression analysis, we examine more closely the extent to which this success comes at the price of making spreads too dependent on output fluctuations.

The ergodic distributions of the debt-to-income ratio and the spread is depicted in Figure 8. For the stochastic growth case, both the debt-to-income and the spread distributions are very tight and symmetric around their mean. The distribution of the debt-to-income ratio for the DG case is also symmetrical, but it is substantially more dispersed. For the spread distribution, the deterministic growth distribution is not completely symmetric and is again substantially more dispersed than the stochastic case. The greater dispersion in the debt-to-GDP ratio and the spread in the deterministic growth model is consistent with our earlier observation that the deterministic economy was more responsive to output shocks.

This spread can be decomposed into a default premium and a risk premium. Specifically, the risk premium is the standard difference between the expected implied yield on sovereign bonds and the risk-free interest rate. The default premium is the promised yield that would equate the expected return on sovereign bonds (inclusive of default) to a risk-free bond; that is, the yield that would leave a risk-neutral lender indifferent. The top panel of Figure 9 depicts the risk premium and the bottom panel depicts the default premium. In both cases the risk and default spreads quite similar to each other, suggesting that the two are moving

Figure 8: Ergodic Distributions

(a) Debt-to-Income
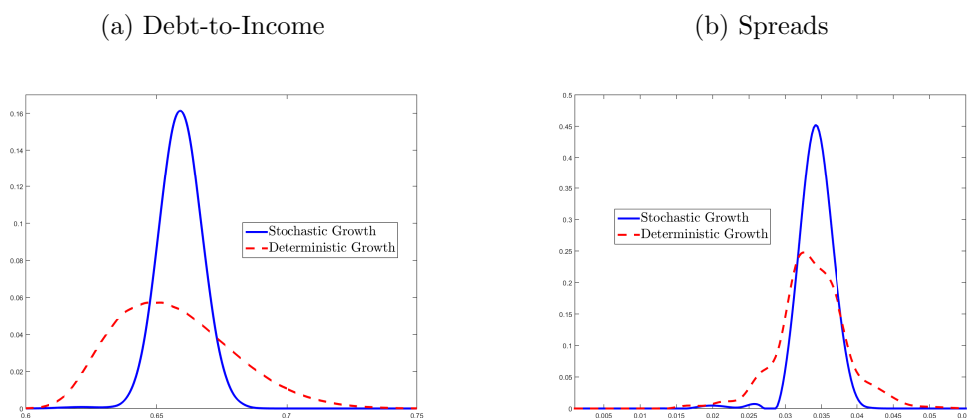
(b) Spreads

closely in parallel. On average, roughly 60 percent is the default premium and the rest is risk premium. This reflects our calibration target of 3 percent average spread and 2 percent default probability.

Figure 9: Decomposition of the Spread

(a) Stochastic Growth

(b) Deterministic Growth
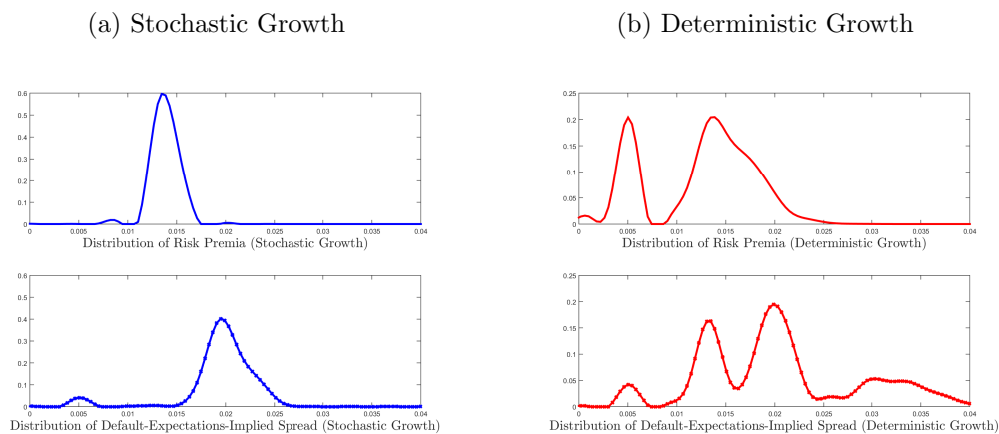
To understand the circumstances in which we are getting defaults and crises in our models, we examine the share of defaults and crises with large negative output changes and large negative investor wealth shocks. These negative changes are 1.5 standard deviations relative

Table 12: Default and Crisis Statistics for the Nonlinear Default Cost Economies

	Def. Share with Output Collapse	Def. Share with w Collapse	Crisis Sh. with Output Collapse	Crisis Share with w Collapse
Stochastic	0.80	0.02	0.31	0.01
Deterministic	0.60	0.06	0.66	0.03

to the unconditional distribution. We use negative growth rate realizations for output so we are using the same metric for both models. The results are reported in Table 12. The results imply that, in the SG model, defaults are almost always associated with negative growth rate shocks and almost never with negative wealth shocks. In the deterministic growth model, the dependence of defaults on negative output shocks is a bit weaker and investor wealth shocks play essentially no role. When we turn to spread crises, we see much less dependence on growth shocks in the SG model and again essentially no dependance on wealth shocks. This is because a very negative growth shock leads to either an immediate default or rapid deleveraging. In contrast, in the DG model, the dependence of spread crises on growth shocks is even higher than it is for defaults.

5.4 Simulation Regressions

To compare the model to the data more closely, we take our model-simulated data and regress the spread on a constant and our three shocks. Besides the benchmark versions of SG and DG, we also included the results when we calibrate the output process to Argentina in the DG case. The results are in Table 13. We have already reported the results of estimating our statistical model in Table 3. However, those regressions included our two common factors. To make a closer comparison with the model regressions, we examine regressions for several of our countries with just the financial controls we considered in decomposing the common factors. We believe that including these financial controls as important in making this comparison. In our model data the output and wealth shocks are orthogonal by construction. In the actual data, an important concern is the feedback from interest rate or risk premium shocks to growth (as emphasized by Neumeyer and Perri, 2005).

While our two benchmark models were calibrated to Mexico, which we view as representative of countries subject to sovereign debt crises, the data series are fairly short to evaluate these somewhat rare events. Hence, it is useful to compare our model regression results to a

range of countries in the data. To aid in this comparison, we also consider the DG version of our model with a growth process calibrated to Argentina.

In the SG model the output shock is the growth rate, or g_t, while in the DG model it is the deviation from trend, or z_t. To make a consistent comparison to the data-based regressions, we did them both with the growth rate of output as the shock and with the deviation of log output from a linear trend. These results are reported in Tables 14 and 15.

Table 13: Spread Regressions with Wealth (Simulated Data)

	B_t/Y_t	g_t or z_t	w_t	R^2
	SG Benchmark Calibration			
Coefficient	0.0286	0.0191	0.0070	
Var Decomp	0.3850	0.0154	0.1660	0.5663
	DG Benchmark Calibration			
Coefficient	0.0412	-0.0707	$9.2928e{-4}$	
Var Decomp	0.3016	0.1145	0.1323	0.5484
	DG Argentina Calibration			
Coefficient	0.307443	-0.77599	-0.00067	
Var Decomp	0.030814	0.532024	0.000354	0.563191

When we examine the results for the SG benchmark model with nonlinear default costs, one sees that the debt-to-output ratio has a positive coefficient and is explaining 38 percent of the movements in the spread as measured by the marginal R^2. This finding is consistent with the data regressions where this variable always has a positive coefficient and explains almost half of the spread in three of our countries and virtually nothing in two of them. The marginal R^2 for the growth rate shock is 0.01, which is very consistent with our growth rate regressions in Table 14 and the sign of that coefficient is positive. The wealth shock explains 17 percent of the variation according to the marginal R^2. This too is consistent with the data, since in some countries the financial variables explain very little, and in several others, particularly Mexico, they explain a great deal.

There are two major surprises in the SG model regression. First, the sign of the output shock is positive in the SG model, indicating that positive growth rate shocks raise the spread. This is contrary to the sign of this term in the data regressions. However, this result

seems consistent with the results we showed for a boom-bust cycle in Figure 7. There, only the initial response to a good output shock was negative while a sequence of good output shocks led the government to raise its debt-to-output ratio and thereby induce an increase in the spread. Note that this response is not present in the DG model. Instead, because the government was slower to delever, a sequence of positive shocks followed by a negative one led to a temporary jump upwards in the spread.

Second, the sign of the wealth factor is positive, indicating that an increase in investor wealth, which should lower risk pricing holding everything else fixed, actually raises the spread. This result is consistent, however, with our earlier surprise finding that the sign of the P/E ratio in the data regressions is positive, indicating that a fall in the risk premium in the data also raises the spread. We will seek to better understand this finding in our quantitative exercises below.

In the simulated data regressions from the DG benchmark and DG Argentine models we also see that the debt-to-output ratio explains 30 percent of the variation in output and that the sign of this term is positive. However, if we compare this explanatory power to the regressions in Table 15, this is high relative to what we find when we take the output shock to be a deviation from trend. The sign on the deviation is negative, as one would expect and as we see in the data. In the DG benchmark the expiatory power of the output shock is only 11 percent which is consistent with the regression results. However, the explanatory power of this variable under the Agrentine growth process is over 50 percent, which is much higher than anything we see in the data regressions. Thus it does seem like the ability of the nonlinear output cost element to increase the spread volatility when the variability of output is sufficiently high comes at the expense of tying the spread much too closely to output fluctuations. In addition, the sign of the wealth term changes when we move from the benchmark to the Argentinian output calibration. However, the positive sign in the benchmark case is consistent with the positive sign of the P/E ratio in the data regressions.

Table 14: Spread Regressions (Data): output shock = growth rate

Country	B_t/Y_t	g_t	VIX	PE Ratio	LIBOR	R^2
Argentina:						
Coefficients	0.0067	-1.0480	$7.8592e{-}4$	0.0034	-0.0372	
	$(9.9307e{-}4)$	(0.6770)	(0.0013)	(0.0046)	(0.0072)	
Var Decomp	0.4962	0.0120	0.0059	0.0085	0.0880	0.6105
Brazil:						
Coefficients	0.0026	-0.3134	0.0013	$1.8695e{-}4$	0.0023	
	$(3.1092e{-}4)$	(0.2297)	$(4.4568e{-}4)$	$(8.1841e{-}4)$	(0.0014)	
Var Decomp	0.4943	0.0150	0.0537	0.0093	0.0482	0.6204
Colombia:						
Coefficients	0.0018	-0.1535	0.0011	$7.5692e{-}4$	$5.2909e{-}4$	
	$(1.5892e{-}4)$	(0.1102)	$(1.2462e{-}4)$	$(3.0964e{-}4)$	$(5.3118{-}4)$	
Var Decomp	0.4900	0.0236	0.2594	0.1017	0.0062	0.8809
Mexico:						
Coefficients	$7.2889{-}4$	-0.1595	$6.2858e{-}4$	$6.4423e{-}4$	$1.1697e{-}4$	
	$(2.2988{-}4)$	(0.0467)	$(6.0880e{-}5)$	$(1.2801e{-}4)$	$(3.5179e{-}4)$	
Var Decomp	-0.0226	0.1350	0.6598	0.1212	-0.0087	0.8847
Russia:						
Coefficients	$2.5708e{-}4$	-0.7400	0.0025	0.0117	0.0058	
	$(8.0133{-}4)$	(0.7191)	(0.0015)	(0.0031)	(0.0075)	
Var Decomp	0.0210	0.0109	0.0540	0.3217	0.0696	0.4771
Turkey:						
Coefficients	0.0012	-0.2489	$7.3488e{-}4$	0.0028	$4.8343e{-}4$	
	$(1.7406{-}4)$	(0.0660)	$(1.9433e{-}4)$	$(3.4963e{-}4)$	$(7.1599e{-}4)$	
Var Decomp	0.1520	0.0911	0.1413	0.3847	0.0068	0.7759

Table 15: Spread Regressions (Data): output shock = deviation from trend

Country	B_t/Y_t	z_t	VIX	PE Ratio	LIBOR	R^2
Argentina						
Coefficients	0.0058 (0.0016)	−22.2293 (38.4213)	0.0014 (0.0013)	0.0011 (0.0062)	−0.0384 (0.0080)	
Var Decomp	0.2463	0.1060	0.0138	0.0098	0.2080	0.5839
Brazil						
Coefficients	0.0027 (0.0004)	11.6322 (12.8857)	0.0015 (0.0004)	0.0005 (0.0009)	0.0021 (0.0014)	
Var Decomp	0.3778	0.0638	0.0512	0.0657	0.0564	0.6150
Colombia						
Coefficients	0.0015 (0.0002)	−19.6663 (7.9572)	0.0011 (0.0001)	0.0009 (0.0003)	0.0013 (0.0006)	
Var Decomp	0.3178	0.1130	0.2353	0.2000	0.0245	0.8903
Mexico						
Coefficients	0.0007 (0.0002)	−4.8005 (3.3338)	0.0007 ($6.0951e{-}5$)	0.0006 (0.0001)	0.0006 (0.0005)	
Var Decomp	0.0371	0.1085	0.5613	0.1058	0.0473	0.8599
Russia						
Coefficients	−7.0e−4 (0.0006)	−96.9253 (17.1416)	0.0027 (0.0013)	0.0024 (0.0030)	0.0185 (0.0051)	
Var Decomp	0.0705	0.2624	0.0494	0.1642	0.1072	0.6536
Turkey						
Coefficients	0.0009 (0.0002)	−18.3784 (4.1594)	0.0008 (0.0002)	0.0013 (0.0005)	0.0027 (0.0008)	
Var Decomp	0.0956	0.2719	0.1433	0.2271	0.0519	0.7898

5.5 Comparative Experiments

We want to examine how the equilibrium predictions of our two benchmark models respond to changes in several key parameters. This will help us understand exactly what is driving our outcomes. In these experiments we change only the parameter in question, and we explicitly do not recalibrate the other parameters. The results are given in Table 16.

The first set of results in column 2 examines the impact of shortening the average maturity from 2 years to 1 quarter. In both the SG and the DG versions, this shortening of the maturity sharply reduces the default rate and the average spread almost to zero. This occurs because with debt that matures in a single period, future debt issuance has no effect on the value of bonds currently being issued. With longer maturity bond this is not the case and future issuances diultes the value of current debt. Since capital loss on outstanding bonds from new issuance of debt is not borne by the sovereign, long maturity bonds induce over-borrowing and higher default risk. Put differently, twith short maturity debt, the government is forced to internalize the full cost of a rise in default risk and therefore chooses to constrains its borrowing.

The second set of results concerns the impact of risk aversion on our equilibrium outcomes. In both the SG and DG cases, the frequency of default falls sharply. However, the increase in the price of risk just offsets this drop, so the average spread stays roughly unchanged. This indicates the greater discipline imposed on sovereign's borrowing behavior from a higher risk aversion on part of lenders. The greater discipline comes from the fact that the spread required per unit of default risk is higher with greater risk aversion, making default risk much more expensive for the sovereign. As a result, the sovereign optimally chooses to lower its expected future default risk. This result can also sheds light on why an increase in w raised the spread rather than lower it. Future risk pricing can discipline future behavior. How strong that is will determine the extent to which it shows up as an increase or a decrease in the spread today. But it will increase the frequency of defaults.

The third set of results concerns the impact of making wealth shocks i.i.d. In this case, the disciplining affect of having a high future price of risk because of a low value of w today is removed. In the benchmark cases, this future discipline led to a twisting of the price schedules. When the debt-to-output is low, the future disciplinary effect dominates the static risk pricing effect and, as a result, a high w shocks lowers the price of debt. When the debt-to-output ratio is high, the static pricing effect dominates and rise in w increases the price of debt (see Figure 5). With i.i.d. w, this twisting effect is gone and an increase

in w strictly increases q where it is below the risk-free rate. In both the SG and DG models this leads to a sharp fall in the impact of wealth shocks on the spread as measured by the R^2. Consistent with this, the correlation of the wealth shock and the spread goes from 0.15 in the benchmark to 0.002 with i.i.d. w in the SG model and from 0.40 to 0.03 in the DG model.

The final set of results concerns the impact of autocorrelation of output shocks. For the SG model we reduce the correlation in the output growth rate g from 0.45 to 0, and in the DG model we reduce the correlation in the deviations from trend from 0.85 to 0.45. In both models the debt-to-output ratio goes up as the hedging motive goes up. In both models the default frequency goes down as the likelihood of a sequence of bad shocks driving a country into default goes down. In addition, in both models the incentive to borrow into a boom goes down as the likelihood of the good times continuing is reduced. As a result, the correlation of the spread and the growth rate of output is now negative in both models. At the same time spreads and default frequencies fall in both models.

Table 16: Comparative Statistics: Stochastic and Deterministic Growth Models

| | Stochastic Growth | | | | |
	benchmark	short maturity	high risk aversion	i.i.d. w	i.i.d. g
Debt-to-GDP	0.66	0.68	0.66	0.66	0.78
Average default freq.	0.02	0.007	0.001	0.02	0.006
Average spreads	0.03	0.002	0.03	0.03	0.01
S.D. of spreasds	0.002	0.001	0.002	0.002	0.002
Corr of spreads with Δy	0.15	0.15	0.14	0.17	-0.23
R^2 of spreads on w	0.26	0.008	0.43	0.003	0.29

| | Deterministic Growth | | | | |
	benchmark	short maturity	high risk aversion	i.i.d. w	low auto. z
Debt-to-GDP	0.66	0.67	0.65	0.66	0.87
Average default freq.	0.02	0.002	0.01	0.02	0.003
Average spreads	0.03	0.003	0.03	0.03	0.007
S.D. spreads	0.004	0.001	0.005	0.004	0.001
Corr of spreads with z	0.46	0.09	0.39	0.51	-0.21
R^2 of spreads on w	0.17	0.01	0.36	0.001	0.23

5.6   Taking Stock

Our models of sovereign borrowing, default and the spread can match a number of key facts in the data. They can match the overall borrowing level, but this comes at the expense of assuming that default costs are large so that we can get the sovereign to repay, and that the sovereign is fairly myopic since borrowing and occasionally defaulting is, as we noted earlier a poor way of getting insurance.

Risk aversion on part of lenders leads to the average spread being greater than the average frequency of default, hence lenders earn a positive risk premium of about 1 percent.

The sovereign tends to borrow into booms, which is consistent with the boom-bust cycle we observe in many emerging economies. Also, the end of the boom is associated with a sudden shift in the price schedule for debt, which resembles the lending cutoff (suddent stops) observed in the data. This borrowing into booms depends on future optimism, which here comes through the autocorrelation in output shocks. If we make growth rates i.i.d. in the SG model or reduce the persistence of deviations from trend in the DG model, borrowing-into-booms effect largely goes away. This in turn leads to a sharp fall in the frequency of default and therefore the spread.

When we compare the spread regressions in the model simulated data with those in the data, the overall behavior is broadly consistent with that observed in the data. For both the SG and DG benchmark models, the importance of the debt-to-output ratio and the output shock is consistent with the regression results. However, the positive impact of a growth shock on the spread in the SG model is not consistent with the negative sign of the coefficient on this variable in the regression. This indicates that the reliance on a boom-bust cycle as opposed to the smoothing of consumption is excessive in this version of the model.

Global risk pricing shocks, which we model as shocks to the wealth of investors, have a surprisingly limited impact in our model. Interestingly, an increase in lenders' risk aversion that stems from a decrease in their wealth leads to a *fall* in the spread. A similar impact occurs when we increase investors' risk aversion in our comparative statics exercises. This result comes through the higher price of debt issuance, which lowers the extent to which current lenders need to worry about the dilution of the value of their claims in the future. The threat of future dilution goes away with short maturity debt. This is why we see a sharp fall in default rates and spreads when we switch to one-period debt.

The impact of persistent wealth shocks stemming from changes in borrowing discipline

in the future leads to one of the surprising empirical successes of our models. In our spread regressions, a decrease in the price of risk increases the P/E ratio, but increases in the P/E ratio are associated with increases, not decreases, in the spread on emerging market sovereign bonds. This inverse relationship between the price of risk and spreads is predicted by both models. In our comparative statics exercise we saw that this correlation essentially goes away when wealth shocks become i.i.d., confirming that the inverse relationship is driven by anticipation of changes in future borrowing behavior.

The major failure of our benchmark models is with respect to the volatility of the spread. It is much too low in the model relative to the data. This indicates that the levering/delevering response to output shocks is too strong, resulting in a spread that is too smooth. This was particularly true in the initial version of our model with proportionate output costs, but is still true when we switch to nonlinear output costs of default (which improves the insurance offered by defaulting).

Increasing the variance of the output process in the DG model can substantially increase the variance of the spread, bringing it in line with the data for most countries. However, this positive result comes at a cost. First, it implies that the model cannot account for counties in our sample, such as Mexico, which have less volatile output processes. Additionally, relative to the data, higher volatility leads to too strong a dependence of spreads on output shocks.

These results suggest that what is needed is:

1) An additional shock to the pricing of debt that is not tied to country fundamentals or global risk pricing factors. This is indicated by the importance of the two common factors in the spread regressions and their lack of dependance on global asset pricing factors.

2) A reduction in the levering/delevering incentive or at least a drawing out of debt crises, which leads to high levels of the spread in response to these crises.

6   Rollover Crises

Our model was constructed to allow for rollover crises along the lines of Cole and Kehoe (2000). Here we conduct a preliminary investigation of the potential for rollover crises to add to the volatility of the spread in our models without tying this volatility too tightly to country fundamentals.

Rollover crises emerge from investors' failure to coordinate their beliefs on the good equilibrium outcome in which the government is offered a generous price schedule and therefore

chooses to not default. Instead, investors adopt pessimistic beliefs about government's be-
havior, which leads them to offer an adverse price schedule – specifically, a zero price for new
issuance of bonds – and this, in turn, induces the government to default. The government's
default then validates the investors' pessimistic beliefs. What is empirically attractive about
this mechanism is that while requiring that the country's fundamentals be bad enough to
generate a default under the adverse price schedule, it allows relatively wide latitude in the
timing of a sovereign debt crises.

In constructing a quantitative model of rollover crises, the first question is: what is a
plausible process for beliefs? Beliefs, unlike, say, output, cannot be directly observed, and
hence its impact and its stochastic evolution must be inferred. Aguiar, Chatterjee, Cole, and
Stangebye (2016) estimate shifts in beliefs from spreads. Another alternative is to adopt a
state-space approach in which the belief process and it's realizations are estimated jointly
along with other parameters of the model as in Bocola and Dovis (2015). A related al-
ternative would be to construct belief processes that replicated the impact and time series
properties of the common factors estimated in the spread regressions reported earlier. How-
ever, undertakings such as these are beyond the scope of a handbook chapter. So, instead,
we follow Cole and Kehoe (2000) and its quantitative implementation in Chatterjee and
Eyigungor (2012) and assume that there is a constant probability of a crisis. This lim-
its the empirical scope of self-fulfilling rollover crises, but does allow us to partially gauge
their potential impact. Also, we do not recalibrate the models so this too is a quantitative
comparative statics exercise.

The results are presented in Table 17 along with our baseline results (for the nonlinear
output loss from default). Here, we assume that if a country is in the crisis zone (i.e., a
rollover crisis can be supported in equilibrium) then a rollover crisis transpires with a 20
percent probability. Several results stand out. First, the possibility of rollover crises reduces
the average debt-to-output ratio. This makes sense because rollover defaults are generally
more costly than fundamental defaults (they can occur even when output is relatively high)
and this makes the sovereign wary about borrowing too much. In constrast, the average
default frequency does not change much with the adddition of rollover crisis and, as a result,
the impact on the average spread is fairly small (in the SG model it stays the same, while in
the DG model it rises slightly). However, there is significant change in the nature of defaults
since many of them are now being induced by rollover crises. This is particularly pronounced
in the case of the SG model, where 70 percent are now rollover-induced defaults. Along with

Table 17: Stochastic and Deterministic Growth Models: Benchmark vs. Rollover Crises

	Stochastic Benchmark	Stochastic w. RC	Deterministic Benchmark	Deterministic w. RC
Debt-to-GDP	0.66	0.63	0.66	0.65
Average default freq.	0.02	0.02	0.02	0.02
Average spread	0.03	0.04	0.03	0.04
S.D. of spreads	0.002	0.002	0.004	0.004
Corr of spreads with Δy or z	0.15	0.06	0.46	0.11
R^2 of spreads on w	0.26	0.18	0.17	0.09
Share of rollover defaults	0	0.70	0	0.30

this change in the nature of the defaults comes a change in the relationship between the spread and our fundamental shocks. In both models the correlation of the spread and the output shocks falls. This is particularly pronounced in the DG model, where it falls from 0.46 to 0.11.[15] In a similar fashion, the R^2 of the regressions of our spread on our wealth shock w falls in both models. In the SG model it falls by one-third, while in the DG model it falls by one-half. At the same time, the standard deviation of the spread hardly changes with belief shocks.

The lack of increase in the spread's volatility is surprising. To understand a bit better what is going on, we plot default indifference curves for both the benchmark SG model and the SG model with rollover crises in response to belief shocks in Figure 10. We start first with the benchmark model. The indifference condition between defaulting and not defaulting traces out combinations of the debt-to-output ratio and the current growth rate. Since growth is positively autocorrelated in this model, high growth today is good news about future output and hence reduces the incentive to default. Of course, a high debt burden encourages default. This gives us the trade-off we see in the first panel of the figure. We have also plotted the stationary debt levels (i.e., the debt level where where $b = a(s, b)$) as a function of the current growth rate of output. These debt levels are important because the government finds it optimal to lever/delever back to this point in response to a shock. The fact that the stationary points are positively sloped reflects the tendency to borrow into

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[15]Another feature of rollover defaults is that they can occur for fundamentals that are, on average, better than in the case of fundamental defaults. Thus, the correlation between defaults and fundamentals is also weakened, consistent with evidence reported in Tomz and Wright (2007). See also Yeyati and Panizza (2011) for an empirical evaluation of the timing of output losses surrounding default episodes.

a boom discussed earlier. Defaults occur in equilibrium largely because a sufficiently low growth rate shock from a debt position close to the stationary points last period generated a current debt-to-output level that is on the wrong side of the indifference curve. In which case, the government optimally chooses to default. The fact that the gap between the indifference curve and the stationary point is increasing in g illustrates why default is closely associated with low output shocks.

In the second panel of Figure 10, we see a similar graph for the SG model with rollover crises. Only now there are two indifference curves: one for fundamental defaults as in the benchmark model and one for rollover crises defaults. Since the lending terms are worse, the rollover indifference curve lies below the fundamental curve, indicating that a rollover crisis is possible for a given growth rate g_t at a strictly lower level of b_t. Note that the fundamental indifference curve is lower than in the benchmark model. This is because the future prospect of rollover crises lowers the payoff even when these crises do not occur today and this has shifted down the solvency indifference curve. As a result, defaults will occur at lower debt levels fixing g than in the benchmark model. Next, note that the stationary debt level curve has also been shifted down. This is because the increased likelihood of a default and its adverse consequences means that the optimal level of borrowing has decreased. The fact that 70 percent of the defaults occur under the crisis pricing schedule means that the likelihood of drawing a sufficiently bad output shock to force the government over the fundamental indifference curve has gone down substantially. In this sense the gap between the solvency indifference curve and the stationary debt levels has widened.

There is a sense in which virtually all of the defaults in the model with rollover crises are driven by beliefs. This is because, if we asked whether the states in which realized defaults are in equilibrium, very few of them are on the wrong side of the benchmark indifference schedule. It is also worth noting that if we suddenly switch from a situation in which the benchmark pricing schedule, policy function and beliefs applied, to one in which the rollover ones did, then the government would have to sharply delever in the face of a worse price schedule, even if a crisis did not formally occur in the current period. This sort of transition might be a way to generate more volatility in the spread, especially if the government could be induced to slow down the rate at which it delivered.

Figure 10: Default Indifference Curves and Stationary Policy Choices

(a) Benchmark

(b) w. Rollover Crises
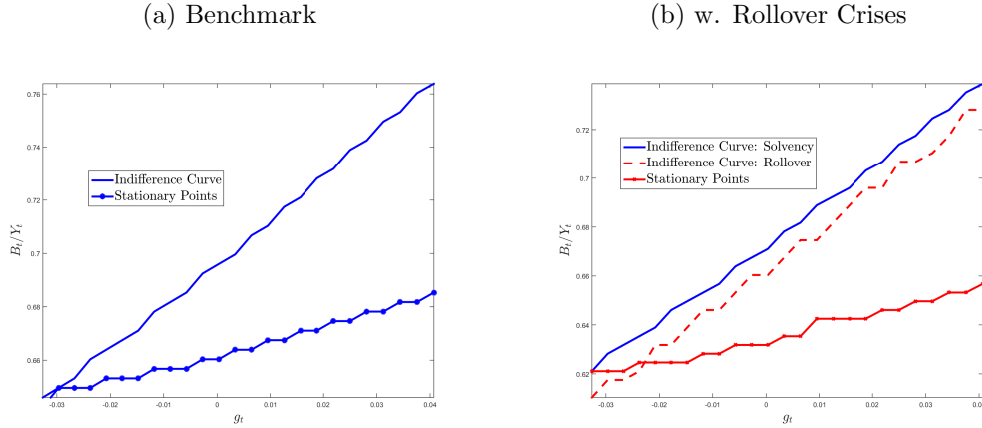
7 Extensions and Literature Review

Beginning with Aguiar and Gopinath (2006) and Arellano (2008), there is now a substantial body of work drawing on the Eaton-Gersovitz framework. Aguiar and Amador (2014a) discuss the theoretical and conceptual issues in this area. This section provides a brief guide to the evolving quantitative literature (the reader is encouraged to consult the studies mentioned here for additional related work).

Existence and Uniqueness of Equilibrium: The existence of an equilibrium when both endowments and assets are continuous is an open question.[16] Aguiar and Amador (2014a) discuss that the operator whose fixed point characterizes the equilibrium (with permanent autarky as punishment) is monotone and note how this can be useful to compute an equilibrium. When both b and the non i.i.d. component of endowments are discrete, Chatterjee and Eyigungor (2012) establish the existence of an equilibrium for debt with arbitrary maturity and temporary or permanent autarky following default.

The issue of uniqueness of equilibrium is more subtle. For the case where default is punished with permanent autarky, Auclert and Rognlie (2014) prove uniqueness for the Eaton-Gersovitz model with one-period debt. Passadore and Xandri (2015) study the mul-

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[16]Eaton and Gersovitz (1981) pointed out that *if* the probability of default $\mathbb{E}_s(D(s', b'))$ is differentiable in b', the solution to the bond pricing equation amounts to the solution of a first-order nonlinear differential equation. However, differentiability of $\mathbb{E}_s D(s', b')$ requires everywhere differentiability of the value function, which is not true in a model with default.

tiplicity that arises when the state space for debt is restricted to be nonnegative (that is, no saving). Stangebye (2015a) and Aguiar and Amador (2016) discuss how multiplicity in the Eaton-Gersovitz model arises in the absence of one-period debt due to the vulnerability to dilution. More generally, one can often construct multiple equilibria with variations on the standard set up. Cole and Kehoe (2000) alter the Stackelberg nature of the government's default decision in order to generate self-fulfilling rollover crises. Chatterjee and Eyigungor (2012) exploits a similar variation to generate (investor) belief-driven rollover crises in a model that otherwise resembles the Eaton-Gersovitz setup.

The Strategic Structure of the Debt Market: In the Eaton-Gersovitz setup, the sovereign accesses the debt market at most once within a period. If the sovereign may access the market as many times within a period as it wishes, lenders at any given round of borrowing must anticipate the sovereign's future within-period borrowing decisions (Bizer and DeMarzo, 1992). As shown in Hatchondo and Martinez (undated) equilibrium implications of this is that investors will offer the sovereign a state-dependent pair of bond price and debt limit, $\{\bar{q}(y,b), \bar{x}(y,b)\}$, with the sovereign free to borrow any $b' \leq \bar{x}(y,b)$ at the price $\bar{q}(y,b)$. Interestingly, the bond *price* depends on inherited debt b (while in the standard setup the bond price *schedule* $q(y,b')$ is independent of b) and, so, borrowing history matters for the terms of credit. Lorenzoni and Werning (2014) and Ayres, Navarro, Nicolini, and Teles (2015) discuss this issue in detail.

Contract Choice: In the standard setup, the structure of a unit bond is fixed and described by the pair (z, λ). At the cost of enlarging the state space, more flexible contractual structures are possible. Bai, Kim, and Mihalache (2014) define a unit bond by (T, δ), where the bond pays $(1 + \delta)^{-\tau}$, $0 \leq \tau \leq T$ periods from maturity. Sanchez, Sapriza, and Yurdagul (2015) consider the case where $\delta = 0$. Both relax the fixity of the contractual structure by letting the sovereign replace old debt each period with new debt with a different contractual structure.

Maturity Choice: Cole and Kehoe (1996) discuss the role of maturity in the presence of self-fulfilling debt crises. In the standard setup, market incompleteness is extreme in that only one type of debt contract can be issued at any time. Arellano and Ramanarayanan (2012) consider the case where the sovereign can simultaneously buy and sell bonds of different maturities and show that the average maturity shortens as fundamentals weaken. Aguiar and Amador (2014b) show that when default probabilities are high, the sovereign has an incentive to reduce its stock of one-period debt. Shorter maturity provides the sovereign the correct incentives to minimize the inefficiencies represented by default. Bocola and Dovis

(2015) discuss the role of maturity choice in the presence of both fundamental and rollover crises and analyze their separate roles in the recent Eurozone debt crisis.

Exchange Rates, Default Risk, and Currency Denomination: Sovereign defaults are generally preceded by a depreciation of the country's currency, with a further sharp depreciation occurring soon after default. Asonuma (2014) documents these facts and develops a two-country model with traded and nontraded goods in which one country is the borrower and the other the lender. Negative shocks to productivity in the borrowing country can trigger a real exchange rate depreciation which, in turn, can raise the likelihood of a default on sovereign debt. Gumus (2013) examines the currency denomination of debt in a similar model with two types of debt: In one, the payoff is linked to the domestic price index (a proxy for local currency debt) and in the other to the price of the tradeable good (a proxy for foreign currency debt). Although the default risk on "local currency debt" is not uniformly lower than the default risk on "foreign currency debt," the former is found to be the better (higher welfare) arrangement.

Explicit Treatment of the Government: For some purposes, it is important to model the sovereign separately from private-sector agents. Cuadra and Sapriza (2008) analyze borrowing and default behavior when redistributive conflict and the risk of political turnover impart myopia (present-bias) *a lá* Alesina and Tabellini (1990). In their model, the sovereign discounts the future more than citizens do, which helps to partially rationalize the low discount factors often used in quantitative models. Hatchondo, Martinez, and Sapriza (2009) consider two types of governments that differ in their discount factors with the goal of analyzing how political risk affects default probabilities and the volatility of spreads. Cuadra, Sanchez, and Sapriza (2010) model the government sector in order to give an account of the strongly pro-cyclical nature of fiscal policy in emerging economies.[17]

Settlement Following Default: Sovereign defaults end with a settlement on the defaulted debt, wherein creditors accept a haircut and the sovereign regains (unencumbered) access to credit markets. Generally, settlement occurs after a significant amount of time has elapsed since default. In the context of one-period debt and equal treatment of all creditors in default (the so-called *pari passu* clause), Yue (2010) models settlement as the outcome of a one-shot Nash bargain between the sovereign and the representative creditor in the period of default. Following agreement, the sovereign is assumed to repay the renegotiated debt over time, with

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[17]Amador (2012) shows that once the equilibrium of the political game between different groups comprising the government is taken into account, it becomes possible to sustain positive levels of debt even when punishment for default is limited to exclusion from future credit (*contra* Bulow and Rogoff, 1989)

no possibility of default or access to new borrowing. This produces a theory of haircuts but not of delays. Bi (2008) assumes that defaulted debts must be settled in cash but employs the stochastic alternating-offers game developed in Merlo and Wilson (1995) to produce a theory of both haircuts and delays. (Benjamin and Wright, 2009) observe that settlement is typically done with new debt (rather than just cash) and allow for this possibility within the context of the stochastic alternating-offers game. In both models, delays arise because it is optimal for both parties to defer settlement until the sovereign's endowment is sufficiently high.[18]

Restructuring Without Default: Default and debt restructuring is a form of ex-post state contingency. Logically, and in practice, ex-post state contingency need not involve default. Hatchondo, Martinez, and Sosa-Padilla (2014) point to voluntary debt exchanges as debt write-offs that occur when a sequence of bad endowment shocks places the sovereign on the wrong side of the revenue Laffer curve. Relatedly, Asonuma and Trebesch (2015) document that about a third of all restructurings in the last several decades occurred in the absence of default, termed *preemptive* restructuring. They extend the Eaton-Gersovitz model to allow for such restructurings and show that they occur when the likelihood of a future default is high. Salomao (2014) has analyzed how the presence of a credit default swap (CDS) market impacts debt renegotiation, when the outcome of the negotiation determines whether a "credit event" is triggered.

Partial Default: Default is typically modeled as a binary event on a single type of debt. In reality, sovereigns have a range of external obligations outstanding at any point in time, including trade credit, bank loans, bonds, bilateral (government-to-government) loans, loans from multilateral agencies (IMF, World Bank and other agencies) and they may choose to default on some types of loans but not on others. Thus, in the aggregate, default tends to be partial. Based on this observation, Arellano, Mateos-Planos, and Rios-Rull (2013) develop a one-period debt model in which the sovereign can partially default on existing debt. Unpaid debts accumulate arrears and there is an output loss that is increasing in the ratio of unpaid to total debts. In their model, moderately bad output shocks trigger partial default that gets "cured" as output recovers.

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[18]Bai and Zhang (2012) explore the role of asymmetric information in creating delays in reaching settlement in a stylized environment. The uninformed party (the sovereign) screens creditors (who privately know their reservation value) by making successively attractive offers over time. They show that delay is shorter when the defaulted debt is traded in the secondary market because the price partially reveals the creditors' reservation value.

Reputation: Quantitative sovereign debt models generally do not give any role to reputation in sustaining debt, although the idea that reputation matters is invoked in Eaton and Gersovitz, and, more comprehensively, in Tomz (2007). D'Erasmo (2012) extends the Eaton-Gerovitz model to the case where investors are uncertain about the sovereign's discount factor (degree of patience), which is taken to be stochastic. Investors' perception of the likelihood that the sovereign is the patient type now appears as an additional state variable in the sovereign's dynamic program. The patient type's desire to separate itself from the impatient type encourages more disciplined borrowing behavior on its part. In equilibrium, the patient type can sustain a higher level of debt on average. Generally speaking, the impatient type defaults and the patient type reaches settlement on the defaulted debt.

Sudden Stops: There is a large literature on "sudden stops" that focuses on the macroeconomic implications of a halt of capital inflows into emerging markets. This literature does not base the "sudden stop" on a rollover problem and abstracts from the possibility of sovereign default induced by the sudden stop (see, for instance, Mendoza (2010) and the references cited therein). Bianchi, Hatchondo, and Martinez (2014) make the connection to sovereign default by extending the Eaton-Gersovitz model to allow for an exogenous stop in capital inflows and study the role of international reserves (which cannot be grabbed by foreign investors in the event of default) as a hedge against such stops.[19]

Fiscal Rules and Default: There is a literature aimed at understanding the equilibrium implications of fiscal policy rules. Ghosh, Kim, Mendoza, Ostry, and Qureshi (2011) analyze a model where the government adheres to some given fiscal rule as long as the deficit implied by the rule can be financed at a finite interest rate. In terms of our notation, this is a setup in which there is some function $c(y, b)$ (the fiscal policy rule) and b' is chosen each period to satisfy $q(y, b') \cdot [b' - (1 - \lambda)b] = y - (r^* + \lambda)b - c(y, b)$. Because the revenue curve $q(y, b')b'$ is an inverted U, there may be no b' that satisfies this equation in which case the sovereign defaults. Furthermore, if there is one b' that satisfies the budget constraint, there will always exist another b' on the "wrong side" of the revenue Laffer curve that will also satisfy this equation. Ghosh *et al.* assume that the sovereign and investors avoid the wrong side of the Laffer curve and compute the highest debt level $\bar{b}$ beyond which default is certain. Lorenzoni and Werning (2014) and Stangebye (2015b) study a similar setup but the focus is on the

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[19]The accumulation of foreign reserves to mitigate rollover risk has been examined from an optimal contracting perspective in Hur and Kondo (2014). They point to the drop-off in the frequency of sudden stops following reserve accumulation by emerging markets as evidence that reserves affect the likelihood of a rollover crisis.

rise in interest rates if investors temporarily coordinate on the low price (and therefore high debt) equilibrium path. These authors focus on the recent Eurozone experience.

Debt Dilution and Alternative Trading Arrangements: In quantitative models with long-term debt, "debt-dilution" is an important force leading to excesive borrowing and default. This leads to consideration of alternative trading arrangements that mitigate the adverse effects of debt dilution. Chatterjee and Eyigungor (2015) analyze how respecting seniority during (post-default) debt renegotiations can improve incentives and the welfare of the sovereign. Hatchondo, Martinez, and Roch (2015) analyze how adherence to a fiscal policy rule that binds future sovereigns' borrowing decisions can improve the welfare of the current sovereign.

Decentralized Borrowing and Centralized Default: A growing portion of a country's external debt is debts incurred by private borrowers. Kim and Zhang (2012) analyze an Eaton-Gersovitz model in which private agents choose how much to borrow but the sovereign chooses whether to default. Because private borrowers act as price-takers, the equilibrium resembles one in which the sovereign can access the credit market unboundedly many times within a period.

Contagion and Correlated Defaults: Lizarazo (2009) studies how the terms of credit offered to sovereigns are affected if sovereigns share a common risk-averse lender. Correlated defaults may occur because a default by one sovereign lowers the wealth of the lender and reduces the supply of credit to all sovereigns. The reduction in supply could push another sovereign into default. Arellano and Bai (2014) study a similar environment but include renegotiation on the defaulted debt and show that bargaining protocols (independent versus coordinated bargaining with sovereigns following default) differentially affect the likelihood of correlated defaults.

Inflation and Default: The bulk of the quantitative-theoretic literature on debt and default models real economies. Two exceptions are Nuno and Thomas (2015) and Du and Schreger (2015). The former compares (in a continuous-time setting) outcomes where sovereign debt is denominated in real terms (with the possibility of outright default) to one where it is nominal and the sovereign chooses monetary and fiscal policy under discretion. The latter studies default risk on sovereign debt denominated in local currency, when private borrowers issue debt denominated in foreign currency. The existence of foreign currency private debt makes inflating away local currency sovereign debt expensive and, thus, keeps default risk on local currency sovereign debt positive (as observed in the data).

News Shocks: Sovereign defaults do not occur only when fundamentals are weak. One pos-

sible explanation of this fact could be that they occur when the sovereign and investors receive bad news about the future. Durdu, Nunes, and Sapriza (2013) extend the standard Eaton-Gersovitz set up to include news shocks about future TFP. In addition to default triggered by bad news, the precision of news about future TFP is shown to have quantitatively significant effects the bond pricing schedule.

Default Costs: Quantitative-theoretic models of debt and default typically take the structure of the output costs of default as given. Two exceptions to this practice are Mendoza and Yue (2012) and Perez (2015). In the former, the default costs are grounded in producers' inability to import foreign intermediate inputs when the country is in default. The key implication of this setup is asymmetric default costs: the output costs of default are proportionally higher when TFP is high because that is when the loss of foreign intermediate inputs is proportionately more costly. In the latter, the output costs of default are grounded on the loss of net worth of financial intermediaries (who hold sovereign debt) that occurs with default and the consequent fall in the level and efficiency of financial intermediation, which then depresses output.

Investment and Default: The quantitative debt and default literature has uniformly examined endowment economies. An exception is Gordon and Guerron-Quintana (2016) who extend – both substantively and computationally – the long-term debt model of Chatterjee and Eyigungor (2012) to include capital accumulation (with costly adjustment) and labor-leisure choice. Their goals are a more complete understanding of emerging market business cycles and of the impact of phyiscal capital on debt sustainability.

8   Conclusion: Where We've Been and Where We Need to Go?

This chapter has documented a number of important facts about sovereign default crises, including:

1. Average spreads, spread volatility and the frequency of spread crises vary quite a bit across developing countries.

2. Fundamentals explain only a limited share of spread movements.

3. Spreads have some common factors driving them. However, these factors do not seem tightly connected to standard measures of risk pricing, uncertainty or the risk-free rate.

We have also examined alternative versions of the standard model of sovereign borrowing and defaults. Some of these versions explain many of the main facts, such as the average

spread, the default frequency and average debt-to-GDP ratios. However, all of these models struggle to simultaneously explain the volatility of spreads and its apparent lack of connection to country fundamentals. Specifically:

1. In our model countries engage in very limited borrowing and saving to smooth consumption. While this leveraging and deleveraging behavior is found in the data, it seems much less pronounced. As a result, the variation in the debt-to-output ratio is smaller in the model than in the data. This leads to much less variation in the models' implied spread.

2. Nonlinear default costs can increase the volatility of the spread in the DG model when the volatility of output is high. But this increase in the spread comes at the expense of tightly tying movements in the spread to country fundamentals.

3. The SG model is much less sensitive to including nonlinear default costs in part because the low current output realizations do not stimulate much borrowing as growth rates are modestly positively persistent and because the volatility in growth rates is small relative to volality in the deviation from trend.

Both increases in the risk aversion of our lenders and negative shocks to their wealth did not lead to sharp increases in the spread as simple intuition might suggest. Instead the disciplinary effect of the increase in the price of default risk reduces the future incentive of the government to issue debt into the range that will generate a positive probability of default. This increase in future discipline lower creditors' anticipation of future dilution of their claims by the government and can actually reduces spreads. This negative relationship between the pricing of default risk and the equilibrium spread also appears to be an important factor in the data, thus, validating this surprising implication of our models.

The failure of our models to explain the volatility of spreads stems from the fact that the debt-to-output ratio is largely pinned down by a couple of key features. First, because the government is quite myopic, smoothing plays a limited role in it's optimal policy choice; instead, borrowing is driven by impatience that is ultimately held in check by lack of commitment. Second, because of the strong feedback effect of default risk and risk premia on the government's incentive to default, the debt price schedule is highly nonlinear in the relevant region. As a result, the location of the kink in the price schedule interacts with the sovereign's myopia to almost completely determine its borrowing behavior. In the end, this leads to sharp leveraging/deleveraging in response to positive and negative output shocks and very little variation in the spread. These forces are somewhat ameliorated in cases where the output shock is sufficiently volatile (so the nonlinearity in the default cost can play a role),

but even in those cases the sovereign's behavior responds sharply to the contemporaneous shock realization and does not display the history dependance that expenditure-smoothing would have implied. As a result, only the current output shock matters for spreads and this ends up overloading its importance relative to the data.

Rollover crises are a promising way of generating debt crises, particularly since they don't imply an overly tight connection to country fundamentals. However, the sort of stationary rollover risk that we have considered here is not sufficient to produce the kind of variability in the spread that we see in the data. Instead, they seem to simply crowd out standard fundamental crises. What is needed is a more dynamic version of time-varying risks. At the same time, we need to rationalize a reduction in the speed with which the government chooses to undo the impact of negative shocks on the spread by borrowing less and yet not default on the debt.

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