Fiscal policy in debt constrained economies ☆

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Abstract

We study optimal fiscal policy in a small open economy (SOE) with sovereign and private default risk and limited commitment to tax plans. The SOE's government uses linear taxation to fund exogenous expenditures and uses public debt to inter-temporally allocate tax distortions. We characterize a class of environments in which the tax on *labor* goes to zero in the long run, while the tax on capital income may be non-zero, reversing the standard prediction of the Ramsey tax literature. The zero labor tax is an optimal long run outcome if the economy is subject to sovereign debt constraints and the domestic households are impatient relative to the international interest rate. The front loading of tax distortions allows the economy to build a large (aggregate) debt position in the presence of limited commitment. We show that a similar result holds in a closed economy with imperfect inter-generational altruism, providing a link with the closed-economy literature that has explored disagreement between the government and its citizens regarding inter-temporal tradeoffs.
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1. Introduction

Economies frequently pursue policies that lead to fiscal crises, usually typified by sustained deficits that eventually lead to an inability to increase or roll over debt (without paying an historically abnormal premium), and an associated sharp increase in tax rates and decline in government expenditure. The recent experience of Greece, Ireland, and Portugal, are only the latest examples of such crises.[1] Policies that run up debt, and eventually encounter borrowing constraints, may be the rational response of citizens (and their politicians) who face a world interest rate that is below their subjective rate of time preference. However, the normative question of optimal tax policy in an economy that faces long run binding debt constraints has not been thoroughly studied. To this end, this paper studies optimal fiscal policy in economies that are debt constrained, with a specific interest in relatively "impatient" economies for which the debt constraints are particularly relevant.

We consider optimal fiscal policy in a linear-tax framework. The canonical Ramsey formulation of optimal fiscal policy is quite simple: a government funds fiscal expenditures using linear taxes, and chooses (under full commitment) the sequence of taxes that maximizes the welfare of the representative domestic household. A well known result in this framework is that capital taxes should be zero in the long run if the economy converges to a steady state (Judd, 1985; Chamley, 1986; Atkeson et al., 1999; Straub and Werning, 2014), and that taxes on labor income should be "smoothed" using government debt (Lucas and Stokey, 1983; Ljungqvist and Sargent, 2004). That is, if a steady state exists, the government will rely solely on labor taxes.[2] This prediction is robust to dropping the Ramsey assumption of full commitment, as shown by Dominguez (2007) and Reis (2013), if debt constraints do not bind in the steady state. There are a number of alternative environments in which capital is taxed in the steady state, but our focus is not solely the role of capital taxes in a steady state, but also the role of labor taxes.

This paper explores several variations of the canonical framework under limited commitment. At their core, each variation shares the fact that the inter-temporal marginal rate of substitution (MRS) of private agents may differ from the marginal rate of transformation (MRT) in the long run. In particular, private households are impatient relative to the inter-temporal price of resources. The greater impatience may reflect higher mortality in developing economies, imperfect altruism, or simple preference heterogeneity (with large/rich countries being rich because they have patient agents). Alternatively, countries with weaker domestic financial markets may export their savings, putting downward pressure on the world interest rate faced by citizens and governments in the rest of the world (Caballero et al., 2008; Mendoza et al., 2009). In this spirit, our primary scenario is a small open economy in which domestic households discount at a rate that differs from the world interest rate. If the economy faces a borrowing constraint, agents would like to pursue a declining path of consumption given the world interest rate, but are eventually constrained from doing so. Our main result is that in such an environment the tax on labor income converges to zero. That is, the optimal response to impatience and binding borrowing constraints is to *front load* taxes, driving labor taxes to zero in the limit. This result is quite general. In particular, we show that as long as (i) the private inter-temporal MRS is impatient relative to the inter-temporal MRT, (ii) the allocation remains interior (what we refer to as "no immiseration"), and (iii) there is no disagreement between the govern-

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[1] See Reinhart and Rogoff (2009) for many more examples from the historical record.

[2] If labor is a type of capital, as in environments in which human capital can be accumulated, then labor taxes may also go to zero for the same reason that capital taxes go to zero. See Jones et al. (1997).

ment and the private agents regarding the static trade-off between consumption and leisure, then the labor tax converges to zero in the long run. This holds regardless of whether the economy converges to a steady state or not, and regardless of the discount factor of the government.

The zero-labor-tax result arises because the private agents wish to front load consumption and leisure. As a result, the optimal fiscal policy front-loads private agent utility, subject to the debt constraints. However, this does not necessarily imply large public debt positions. Indeed, the fact that labor taxes are zero in the long run requires the fiscal authority to fund government expenditures from net claims on domestic households (and foreigners) plus any net tax receipts from capital income. In Section 4 we explore an example economy in which the government runs fiscal surpluses on the transition to the steady state. It is the household sector that is indebted in the long run, not the fiscal authority.

Our analysis has antecedents in the closed economy literature. Reis (2012) explores a closed economy model in which the government lacks commitment and may be more or less patient than the private agents. Reis shows that a relatively patient government will implement a labor tax of zero in the steady state. Reis' main focus, however, is on the case of a relatively impatient government, which does not result in a zero long-run labor tax. We shed light on her results by showing that the key element for the long-run behavior of the labor tax is not the discount factor of the government relative to the private households, but instead, the discount factor of private households relative to the inter-temporal MRT. In her closed economy analysis, the government's discount factor determines the economy's long-run capital stock, and hence, determines the inter-temporal MRT given the neoclassical production function. But, as our open economy illustrates, the government's discount factor and the inter-temporal MRT are not necessarily linked. We expand on the relation to the closed-economy literature in Section 6.3.[3]

On a methodological note, there are well known technical issues in problems with an infinite sequence of debt constraints (for example, Dechert, 1982; Benhabib and Rustichini, 1997; Rustichini, 1998). The fact that our economy may not converge to a steady state requires that we need to allow for the fact that the sequence of Lagrange multipliers on the debt constraints do not necessarily converge. Our approach to this issue differs from the literature cited above, and may be of use in other applications.

We stress that we hew fairly closely to the standard framework to highlight the role of binding debt constraints on tax smoothing. We do allow for limited commitment and consider taxes supported by reputational equilibria, as well as incorporate alternative political economy frictions. However, we do not address issues of private information, heterogeneity, non-linear taxes, and incompleteness of asset markets.

The remainder of the paper is organized as follows: Section 2 presents the model environment; Section 3 characterizes the optimal fiscal policy in an open economy setting; Section 4 presents a numerical analysis of the optimal policy; Section 5 extends the benchmark model to include private debt constraints; Section 6 considers several extensions including alternative objective and government preferences as well as a closed economy environment; and Section 7 concludes. The appendix contains all proofs.

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[3] The front loading of labor taxes may require the fiscal authority to pay down debt and/or accumulate claims on the private sector. This is reminiscent of Aiyagari et al. (2002) and related papers in the optimal taxation literature. However, the crucial friction in those models that motivates fiscal assets is the incompleteness of markets. This is not a relevant force in our deterministic setting.

2. Environment

In this section we describe the environment faced by households, firms, and the government. The key departure from the standard framework concerns limited commitment to tax and debt promises. We restrict attention to a deterministic environment. In this section, we focus on a small open economy, which faces an exogenous world interest rate $r_t^\star$. We characterize the closed economy model in Section 6.3.

2.1. Representative household

The representative domestic household has time-separable preferences with utility over consumption c (our numeraire) and labor n represented by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \tag{1}$$

with $\beta \in (0, 1)$.

We impose the following restrictions:

Assumption 1. The utility function u satisfies the following conditions: (i) $u : X \to \mathbb{R}$ where $X \equiv (0, \infty) \times (0, \bar{n})$ with $0 < \bar{n} \leq \infty$; (ii) u is twice differentiable with $u_c > 0$, $u_n < 0$, $u_{cc} < 0$, $u_{nn} < 0$ and $u_{cc}u_{nn} - (u_{cn})^2 > 0$ for all $c, n \in X$; (iii) consumption and leisure are normal goods, $u_{cc}u_n - u_{cn}u_c \geq 0$ and $u_{nn}u_c - u_{cn}u_n \leq 0$ for all $c, n \in X$; and (iv) u satisfies the following boundedness assumption on preferences: both $u_c c(u_{cc}/u_c - u_{cn}/u_n)$ and $u_c n(u_{cn}/u_c - u_{nn}/u_n)$ are bounded functions in $(c, n) \in (\epsilon_c, \infty) \times (0, \epsilon_n)$ for some $\epsilon_c > 0$, $\epsilon_n \in (0, \bar{n})$.

The first three assumptions are standard. The last assumption ensures that certain key expressions remain well behaved as consumption becomes large or labor approaches zero. This latter assumption holds for several of the preferences commonly used in the macroeconomics literature. For example, if utility takes the usual Cobb–Douglas form $(c^\gamma (1-n)^{1-\gamma})^{1-\sigma}/(1-\sigma)$ with $\gamma \in (0, 1)$ and $\sigma > 0$, or the power-separable form $c^{1-\sigma}/(1-\sigma) - \psi n^{1+\gamma}/(1+\gamma)$ with $\sigma, \gamma, \psi > 0$, then the conditions in Assumption 1 are satisfied.

For what follows, we will assume that the consumers are more impatient that the world financial markets in the limit:

Assumption 2 (*Impatient consumers*). There exists $M > 0$ and T such that $1 > M > \beta(1 + r_t^\star)$ for all $t > T$.

The focus of the paper is to understand the role that binding sovereign debt constraints play in tax smoothing. Relative impatience implies that such constraints remain relevant in the long run, and we will contrast the results with those from the standard assumption $\beta(1 + r_t^\star) = 1$ as we proceed.

The household provides labor in a competitive domestic labor market at a wage w_t, and labor is immobile across borders. Without loss of generality, we assume labor taxes are levied on the firms, so w_t represents wages after taxes.

Let r_t denote the net interest rate (before-taxes) received by consumers on their financial assets from time $t - 1$ to t. No arbitrage implies $r_t = r_t^\star$ in an open economy. Let r_t^k denote the

rental rate of the domestic capital stock owned by consumers, and δ its depreciation rate. Let ϕ_t^k represents the residence-based tax on capital income received in time t, where "residence-based" refers to the fact that domestic agents pay this tax regardless of the source of the capital income or its location. We introduce "source-based" taxation in the firm's problem below. No arbitrage between bonds and physical capital implies that the after tax return is equalized:

$$1 + (1 - \phi_t^k)r_t = 1 + (1 - \phi_t^k)(r_t^k - \delta).$$

It follows that:

$$r_t^k = r_t + \delta. \tag{2}$$

We define the (after-tax) period-0 price of consumption at time t as:

$$p_t = \prod_{s=1}^{t} \frac{1}{1 + (1 - \phi_s^k)r_s} \tag{3}$$

We normalize $p_0 = 1$.

We denote a_t as the wealth of the household in period t after interest and tax payments are made. This wealth in principle includes both financial wealth (domestic and foreign bonds) and capital holdings. To be precise about the timing, if x is invested at the end of period $t - 1$, then the pre-tax wealth in period t is $(1 + r_t)x$ and the after-tax wealth is $a_t = (1 + (1 - \phi_t^k)r_t)x$. This after-tax wealth, plus labor income and government transfers, represents resources available to the household when consumption occurs. The flow budget constraint governing the evolution of a_t is:

$$a_{t+1} = (1 + (1 - \phi_{t+1}^k)r_{t+1})(a_t + w_t n_t + T_t - c_t), \tag{4}$$

where T_t represents non-negative lump-sum transfers from the government.

From the household's perspective, all assets will be perfect substitutes in equilibrium and we do not need to track the individual portfolio allocation within a period. That is, a_t is a sufficient state variable for the household's problem conditional on tax rates and prices.

Beginning from an initial asset holdings a_0 and imposing a No Ponzi condition, we solve (4) forward to obtain the present-value budget constraint of the consumer:

$$\sum_{t=0}^{\infty} p_t \left(c_t - w_t n_t - T_t\right) \leq a_0. \tag{BC}$$

Note that our notation implies that a_0 is net of period-0 capital taxes. As is well known, distortionary taxation may be avoided with a large enough initial capital levy. By starting from an a_0 such that distortionary taxes are still required, we are implicitly following the standard practice of assuming a bound on the initial capital levy. To be explicit, assume $\phi_0^k = 0$, which is without loss of generality given that a_0 is treated as an arbitrary initial condition.[4]

Our benchmark environment assumes that households do not face constraints on borrowing. We revisit this assumption in Section 5. To anticipate, we show that it is sufficient to consider a representative domestic household who ignores the presence of debt constraints. In particular, we show that any competitive equilibrium allocation that satisfies a private debt constraint can be

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[4] This is without loss of generality for the consumer's problem. For the government's problem introduced below, we can adjust period-0 fiscal requirements to reflect any initial capital tax revenue, making the zero tax without loss of generality for that problem as well.

implemented as an equilibrium in which households do not directly face a borrowing constraint. This is true because the effect of a private debt constraint on household behavior can be replicated by a tax on borrowing plus a lump sum rebate of this tax revenue. However, the presence of private debt constraints alters the government's problem, as discussed below.

The household's problem is to choose sequences $\{c_t\}$ and $\{n_t\}$ to maximize (1) subject to (BC). This is a standard convex problem that can be solved using Lagrange multiplier techniques. Let θ be the multiplier on the budget constraint. The first order conditions for each t can be written as:

$$\beta^t u_c(c_t, n_t) = \theta p_t \tag{5}$$

$$\beta^t u_n(c_t, n_t) = -\theta p_t w_t. \tag{6}$$

These conditions plus the constraints are necessary and sufficient for the unique solution to the household's problem.

Note that $\theta > 0$, given the strict monotonicity of u, and so the budget constraint holds with equality. We can then write the budget constraint of the consumer as:

$$\sum_{t=0}^{\infty} \beta^t \left(u_c(c_t, n_t) \left(c_t - T_t \right) + u_n(c_t, n_t) n_t \right) = u_c(c_0, n_0) a_0. \tag{7}$$

As usual, this equation will later form the basis of the "implementability condition" used in Proposition 1.

2.2. Firms

The representative firm operates a constant returns to scale production function $F(k, n)$ and hires labor and rents capital in competitive factor markets to maximize profits. It pays a linear source-based tax τ_t^n on its wage bill and a source-based tax τ_t^k on its rental payments. The firms' problem in period t is therefore

$$\max_{k_t, n_t} \left\{ F(k_t, n_t) - (1 + \tau_t^n) w_t n_t - (1 + \tau_t^k) r_t^k k_t \right\},$$

where w_t is the wage and r_t^k the rental rate.

The first order conditions, necessary and sufficient, are:

$$F_k(k_t, n_t) = (1 + \tau_t^k) r_t^k \tag{8}$$

$$F_n(k_t, n_t) = (1 + \tau_t^n) w_t. \tag{9}$$

2.3. Government budget constraints

The government has to fund a sequence of expenditures g_t, which we take to follow a deterministic and exogenous process.[5]

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[5] It is possible to make g_t an endogenous choice with some (potentially time varying) utility value. As our main result holds for arbitrary sequences of government expenditure (subject to boundedness conditions on the equilibrium allocation), we simply treat public expenditures as a primitive.

The relevant inter-temporal price for the government is the before tax price, q_t:

$$q_t \equiv \prod_{s=1}^{t} \frac{1}{1+r_s} \tag{10}$$

with $q_0 = 1$. Therefore, the government's present value budget constraint is:

$$\sum_{t=0}^{\infty} q_t \left(g_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1+(1-\phi_t^k)r_t} a_t \right) \leq -b_0, \tag{11}$$

where b_0 is the period-0 liabilities of the government inclusive of interest payments. Recall that a_t is after-tax period-t wealth, so the initial amount invested at the end of period $t-1$ is $a_t/(1+(1-\phi_t^k)r_t)$, which is the tax base for residence-based capital taxation in the expression above.

The inequality in (11) reflects that we are allowing free disposal of government income. If it holds with equality, we can recover the evolution of government liabilities. Specifically, letting b_t denote the government's liabilities in period t inclusive of interest rate payments, we have

$$b_{t+1} = (1+r_{t+1}) \left(g_t + b_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1+(1-\phi_t^k)r_t} a_t \right). \tag{12}$$

2.4. Aggregate resource constraint

We can sum the budget constraints of the representative household and the government to obtain an aggregate resource constraint for the economy:

$$\sum_{t=0}^{\infty} q_t \left(c_t + g_t + (r_t^\star + \delta)k_t - F(k_t, n_t) \right) \leq A_0, \tag{RC*}$$

where $A_0 = a_0 - b_0$ is the household's private wealth minus government liabilities. The state variable A_0 captures the "net wealth" of the economy, while a_0 represents the wealth of private households.[6] These two state variables are sufficient to define the feasible allocations of the economy: any allocation that satisfies the household's budget constraint (BC) and the aggregate resource constraint (RC*) will also satisfy the government's budget constraint (11).

It may be helpful to note that k_0 is not a separate state variable of the problem. As capital is freely mobile and reversible at the start of the period, all that matters for the aggregate resource constraint is the wealth of the households minus the liabilities of the domestic government. To the extent that capital exceeds the net wealth of the economy, capital can be financed by either the household borrowing from foreigners to purchase physical capital or by direct purchase of capital by the foreign resident. By the same logic, the ownership of the capital stock is also indeterminate, as the domestic household and foreign investors are both indifferent between investing in either type of bonds and physical capital. Conditional on A_0 and a_0, one does not need to know who owns physical capital or domestic government bonds to determine whether an allocation

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[6] More generally, we can define the "net wealth" of the economy at any time t as $A_t \equiv \frac{(1+r_t)a_t}{1+(1-\phi_t^k)r_t} - b_t$. This adjusts end-of-period household assets for capital taxes, which are within country transfers. The expression for period-0 is simplified by our normalization $\phi_0^k = 0$.

satisfies the budget sets. The composition of the gross claims across different economic agents is irrelevant.[7]

While we refer to A_0 as the aggregate net wealth of the economy, it is not equal to the net foreign asset position of the economy. A_0 includes domestic capital owned by the domestic household, which is not included in the definition of net foreign assets. To link the state variable A_0 to the net foreign asset position of the economy, we can express net foreign assets at the end of period 0 by $NFA_0 = a_0 - b_0 - (1 + r_0)k_0$. To see why this expression equals net foreign assets, recall that a_0 includes the domestic household's net position in foreign bonds plus its holding of domestic government bonds and physical capital. The economy's foreign liabilities consist of foreign residents' ownership of domestic bonds and domestic capital. The above expression is thus equal to the household's net holding of foreign bonds minus foreign residents' net positions in domestic bonds and capital, which is the definition of net foreign assets. Our state variable A_0 is therefore net foreign assets plus the stock of domestic capital.[8]

2.5. Government's lack of commitment

The Ramsey approach to optimal fiscal policy assumes the government can commit to a sequence of tax, transfers and debt repayment promises. We are interested, however, in environments in which the government lacks commitment to debt and tax promises. Towards that goal, we consider the following *sovereign constraints*, that take the form:

$$\hat{W}_t(\{c_s, n_s\}_{s=t}^\infty, k_t) \geq 0, \text{ for } t \in \{0, 1, \dots\}.$$

We restrict attention to forward-looking constraints that place a lower bound on utility, rather than an upper bound. We also restrict attention to sovereign constraints where the government evaluates the static trade-off between consumption and leisure in the same way as the households.[9] Specifically, we assume that

Assumption 3. The exists a function W_t such that $\hat{W}_t(\{c_s, n_s\}_{s=t}^\infty, k_t) = W_t(\{u(c_s, n_s)\}_{s=t}^\infty, k_t)$. The function W_t is differentiable in all its arguments and $\partial W_t / \partial u_s \geq 0$ for all $t \geq 0$ and $s \geq t$; with $\partial W_t / \partial u_t > 0$ for all $t \geq 0$.

We can now rewrite the sovereign constraints as:

$$W_t(\{c_s, n_s\}_{s=t}^\infty, k_t) \geq 0, \text{ for } t \in \{0, 1, \dots\} \tag{13}$$

where $u_t = u(c_t, n_t)$.

A straightforward interpretation of these sovereign constraints is a limited commitment environment in which the government cannot commit to its promises on taxes and debt. Consider

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[7] For example, the following two initial asset positions are observational equivalent in terms of equilibrium allocations: (i) the government owes a unit of consumption to domestic households, and domestic households have no other asset or liabilities; and (ii) the government owes a unit of consumption to foreign lenders, but domestic households have foreign assets valued at a unit of consumption. In the first case, the government borrows from the domestic households directly, while in the second case, it borrows indirectly through the foreign lenders. Both cases have the same initial wealth for the government and the domestic households, and as a result, have no effect on the equilibrium outcome.

[8] An alternative approach to the aggregate resource constraint is to start with the economy's net exports: $NX_t = F(k_t, n_t) - c_t - g_t - (k_{t+1} - k_t)$. The present discounted value of net exports must equal the initial net foreign liability position. Rearranging the sum and using $A_0 = NFA_0 + (1 + r)k_0$ gives (RC⋆).

[9] In Section 6.2 we relax this assumption and discuss its implications.

thus the game played by the government, the representative household and the international financial markets. Suppose that if the government in power at time t deviates from a prescribed allocation (for example, by defaulting on the debt or expropriating capital), it can guarantee itself a deviation utility $\underline{U}_t(k_t)$. Then, a constraint of the type above ensures that the allocation is indeed a subgame perfect equilibrium. In Section 5 we show that such a constraint also arises when the government has the ability (but not necessarily the desire) to enforce private debt contracts with international financial markets. With this interpretation in mind, we shall refer to these constraints as "sovereign debt constraints" (SDC).

Moreover, the constraints can accommodate political economy distortions that interact with limited commitment. For example, Aguiar and Amador (2011) introduce constraints on allocations of the form:

$$W_t = \sum_{j=0}^{\infty} \beta^j \left(\theta \delta^j + 1 - \delta^j \right) u(c_{t+j}, n_{t+j}) - \underline{U}_t(k_t) \geq 0, \forall t \geq 0, \tag{14}$$

where θ and δ reflect the fact that political incumbents may face political turnover risk and prefer consumption to occur during incumbency. Setting $\theta = 1$ and $\delta = 0$ generates the limited commitment-benevolent government framework.[10] Similarly, Aguiar et al. (2009) model a limited-commitment government with a geometric discount factor $\tilde{\beta} \neq \beta$, with associated constraints:

$$W_t = \sum_{j=0}^{\infty} \tilde{\beta}^j u(c_{t+j}, n_{t+j}) - \underline{U}_t(k_t) \geq 0, \forall t \geq 0. \tag{15}$$

The fact that constraints of the type (13) arise in models of endogenous default motivate the title of the paper. Nevertheless, limited commitment is not the only motivation for such constraints. In a full commitment environment, there may be reason to incorporate constraints of that form. For example, consider a dynastic model in which agents live for one period and bequeath assets to the next generation of the dynasty. Inter-generational altruism is governed by β. Now consider a Pareto problem in which the government maximizes the initial generation's utility as given by equation (1), subject to giving generation t a utility level of at least $\underline{U}_t$.[11] In this environment, the constraint set takes the form:

$$W_t = \sum_{s=0}^{\infty} \beta^s u(c_{t+s}, n_{t+s}) - \underline{U}_t \geq 0, \forall t \geq 1.$$

More generally, the notation allows for arbitrary discounting between any two periods by the incumbent government. This includes discounting at the world interest rate, at the private-household's discount rate, or non-geometric discounting. This highlights that subsequent analysis does not depend on the relative impatience of the government.

These sovereign constraints motivate environments in which the optimal fiscal policy distorts inter-temporal tradeoffs from the perspective of domestic households, whose decisions underlie the implementability condition. While fairly general, constraints of the type (13) do not encompass environments in which the government has a direct incentive to distort the household's static labor-leisure choice. That is, W_t depends on consumption and labor through the utility function $u(c, n)$, but not directly. This rules out situations in which labor supply and labor taxes are not chosen simultaneously (so n_t affects off-equilibrium payoffs, as in Acemoglu et al., 2008, 2011),

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[10] See Battaglini and Coate (2008) for a similar representation of the government's utility.

[11] See Phelan (2006) and Farhi and Werning (2007).

or environments in which the government wants to manipulate relative factor prices (see the discussion of this in regard to private debt constraints in Section 5 below).

With this, we are now ready to define a competitive equilibrium in the next subsection.

2.6. Competitive equilibrium

We define a competitive equilibrium in the standard fashion, augmented by the presence of the sovereign debt constraints:

Definition 1. An **open economy competitive equilibrium** with an initial asset position (A_0, a_0), consists of sequences of prices $\{r_t, w_t, r_t^k\}$; taxes $\{\phi_t^k, \tau_t^n, \tau_t^k\}$ with $\phi_0^k = 0$; non-negative lump-sum transfers $\{T_t\}$; and quantities $\{c_t, n_t, k_t\}$ such that **(i)** inter-temporal prices correspond to the small open economy assumption: $r_t = r_t^\star$ for all t, and r_t^k, p_t, q_t satisfy equations (2), (3), and (10); **(ii)** $\{c_t, n_t\}$ solve the constrained household problem given initial wealth a_0, prices and taxes; **(iii)** $\{n_t, k_t\}$ solve the firm's problem given prices and taxes, that is, equations (8) and (9) hold; **(iv)** the government budget constraint (11) holds; **(v)** the open-economy aggregate resource constraint (RC*) holds; and **(vi)** the sovereign debt constraints (13) hold for all t.

Given that the sovereign debt constraints are restrictions on an allocation's quantities (c_t, n_t, k_t), it follows that competitive equilibria can be characterized using a primal approach, as usual:

Proposition 1. *An allocation $\{c_t, n_t, k_t\}_{t=0}^{\infty}$ can be implemented as a competitive equilibrium if and only if: (a) the resource constraint (RC*) holds; (b) the sovereign debt constraints (13) hold for all t; and (c) the following implementability condition is satisfied:*

$$\sum_{t=0}^{\infty} \beta^t (u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t) \geq u_c(c_0, n_0)a_0. \tag{IC}$$

There are in principle many competitive equilibria. We discuss equilibrium selection in the next section.

3. Efficient allocations

Let us start by defining our notion of efficiency:

Definition 2. An **efficient allocation** is a competitive equilibrium allocation that maximizes household's utility as of time 0.

Note that the economy is populated by a representative household as well as a sequence of political incumbents characterized by W_t. This objective function, plus a corresponding sequence of constraints $W_t \geq 0$, implies that our notion of efficiency can be viewed as choosing a competitive equilibrium that lies along the Pareto frontier of the initial representative household and the sequence of political incumbents.[12] It should be clear that efficiency does not imply the lack of political economy distortions in the choice of allocations.

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[12] In Section 6.3 we reinterpret the Pareto problem as one spanning different generations of private households, rather than political incumbents, with the objective function representing the first generation's welfare.

Efficient allocations are solutions to the following problem:

$$\max_{\{c_t, n_t, k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \tag{P}$$

subject to:

$$\sum_{t=0}^{\infty} q_t \big(F(k_t, n_t) - c_t - g_t - (r_t^{\star} + \delta)k_t \big) + A_0 \geq 0, \tag{P.1}$$

$$\sum_{t=0}^{\infty} \beta^t \big(u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t \big) - u_c(c_0, n_0)a_0 \geq 0, \tag{P.2}$$

$$W_t(\{u_s\}_{s \geq t}, k_t) \geq 0, \ \text{for all } t \geq 0. \tag{P.3}$$

As can be seen from the problem above, only the initial wealth positions of the domestic households and the government matter for the determination of an efficient allocation. In addition, as long as constraint (P.2) is binding, an efficient allocation will determine the associated wealth positions of the households and the government in all subsequent periods.[13] However, and similar to our discussion in Section 2.6, the gross asset positions across all the economic agents (foreign markets, domestic households and the government) are irrelevant in the initial period, and remain indeterminate afterwards.

Given our notion of efficiency, the objective in (P) uses the discount factor of the representative household. Nevertheless, it will be important to keep in mind that the choice of discount factor in the objective does not affect the result of a zero labor tax in the long-run. We discuss in Subsection 6.1 that the result can be extended to more general discount factors in the objective.

3.1. Necessary conditions for an efficient allocation

Note that implementability and sovereign debt constraints, (P.2) and (P.3), are not necessarily convex. We proceed by characterizing necessary conditions for an interior optimum. We use Lagrangian techniques to characterize the optimal policy. There are well known technical issues regarding the validity of these techniques when there is an infinite sequence of constraints (Dechert, 1982; Benhabib and Rustichini, 1997; Rustichini, 1998). Our approach, which is detailed in the appendix, avoids these complications without requiring that the economy or the multipliers converge to a steady state. The technique involves proving that while the economy may not converge, the labor tax implied by the optimal allocation does converge. We can then characterize the optimum labor tax by considering an arbitrarily large but bounded sequence of perturbations. For expositional ease, in the text we include sequences of multipliers of arbitrary length and leave the technical justification to the appendix.

Let η denote the multiplier on (P.2), μ denote the multiplier on (P.1), and $\beta^t \lambda_t$, $t = 0, 1, \ldots$ denote the sequence of multipliers on constraints (P.3). The first order necessary condition with respect to consumption at time $t \geq 1$ is:

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[13] This follows because a binding (P.2) implies that transfers are zero, and a version of equation (7) can be used to determine the wealth position of the domestic households at all times. The resource constraint (P.1), which will be binding in equilibrium, can be used to determine the aggregate wealth position of the country.

$$\beta^t \left[u_c(c_t, n_t) + \eta \left(u_c(c_t, n_t) + u_{cc}(c_t, n_t)c_t + u_{cn}(c_t, n_t)n_t \right) \right]$$
$$+ \left[\sum_{s=0}^{t} \beta^s \lambda_s \frac{\partial W_s}{\partial u_t} \right] u_c(c_t, n_t) = q_t \mu. \tag{17}$$

The first term $\beta^t u_c(c_t, n_t)$ represents the value of consumption in period t to the consumer (the objective function). The term following the multiplier η reflects the need to satisfy the implementability condition (P.2). The terms involving W_s represent the fact that increasing consumption in period t relaxes the sovereign constraints in that period, and all preceding periods to the extent that W is forward looking. The final term to the right of the equal sign is the price of consumption at time t, translated into utility terms via the multiplier μ. Note that for period $t = 0$, we have the same condition but replace the term multiplying η with

$$u_c(c_0, n_0) + u_{cc}(c_0, n_0)(c_0 - a_0) + u_{cn}(c_0, n_0)n_0,$$

given the presence of $u_c(c_0, n_0)$ on the right hand side of the implementability condition (P.2).

The first order condition for labor is similar:

$$\beta^t \left[u_n(c_t, n_t) + \eta \left(u_{cn}(c_t, n_t)c_t + u_n(c_t, n_t) + u_{nn}(c_t, n_t)n_t \right) \right]$$
$$+ \left[\sum_{s=0}^{t} \beta^s \lambda_s \frac{\partial W_s}{\partial u_t} \right] u_n(c_t, n_t) = -F_n(k_t, n_t) q_t \mu, \tag{18}$$

for $t \geq 1$. For $t = 0$, replace the term multiplying η with $u_{cn}(c_0, n_0)(c_0 - a_0) + u_n(c_0, n_0) + u_{nn}(c_0, n_0)n_0$.

The first order condition for capital is:

$$F_k(k_t, n_t) = r^\star + \delta + \frac{\beta^t}{q_t \mu} \lambda_t \frac{\partial W_t}{\partial k_t}. \tag{19}$$

Note that capital may be distorted away from the efficient level ($F_k = r^\star + \delta$) as the sovereign's debt constraint may depend on the level of installed capital. In particular, a large capital stock may tempt the government to renege on implicit capital tax promises ($\partial W_t / \partial k_t > 0$), leading to downward distortion of investment when (P.3) binds.[14]

3.2. Optimal taxation

Using the first order conditions, we can characterize properties of optimal fiscal policy; that is, the nature of taxation in an efficient allocation. Recall that the tax on labor τ^n can be expressed in terms of quantities using the household's and firm's first order conditions:

$$\tau_t^n = \frac{F_n(k_t, n_t) u_c(c_t, n_t)}{-u_n(c_t, n_t)} - 1.$$

Dividing (18) by (17) and rearranging, we have:

$$1 + \tau_t^n = \frac{1 + \eta \left(1 + \frac{u_{cn}c_t + u_{nn}n_t}{u_n} \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t}}{1 + \eta \left(1 + \frac{u_{cc}c_t + u_{cn}n_t}{u_c} \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t}}. \tag{20}$$

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[14] See Benhabib and Rustichini (1997), Dominguez (2007), Reis (2013), Aguiar et al. (2009), and Aguiar and Amador (2011) for studies of capital taxation in limited commitment environments.

From this expression, we see that to the extent the right hand side differs from one, labor will be distorted by taxation.

Distortionary taxes in (20) stems from $\eta > 0$, as the expression in parentheses multiplying η is different in the denominator and numerator of (20). Recall that η is the multiplier on the implementability condition, which ensures that fiscal policy is implemented in a manner consistent with household and firm optimization. If the government had access to lump sum taxation, or had sufficient initial assets to cover expenditures without distortionary taxes, then η is zero.

However, the fact that $\eta > 0$ is not sufficient to conclude that labor is distorted in the limit. We must first analyze the remaining term in (20). In particular, in both the numerator and denominator there is the cumulative sum of non-negative numbers reflecting the sovereign debt constraints: $\sum_{s=0}^{t} \beta^{s-t} \lambda_s \partial W_s / \partial u_t$. If this sum grows without bound, and the terms involving η are bounded, the expression on the right of (20) converges to one; that is, the labor tax converges to zero. However, for this sum to diverge, it must be that the sovereign debt constraints are binding in the long run, and in principle must be ruling out the Ramsey (full commitment) allocation from the equilibrium set. Note that the Ramsey allocation prescribes a marginal utility of consumption that increases without bound; that is, the country immiserates itself. This obviously requires an extreme form of commitment to the country's foreign liabilities. As an alternative, we consider allocations where this does not happen:

Definition 3 *(No immiseration).* An allocation satisfies **no-immiseration** if $\liminf_{t \to \infty} c_t > 0$ and $\limsup_{t \to \infty} n_t < \bar{n}$.

The lack of commitment provides the natural intuition that a country would rather repudiate its debt obligations than see consumption and leisure go to zero. A sufficient condition on the sovereign debt constraints to ensure no-immiseration is:

Lemma 1. *Suppose that there exist $\hat{c} > 0$, $\hat{n} \in (0, \bar{n})$ and finite T such that for all $t > T$:*

$$\sup_{\{c_s, n_s\}_t^\infty, k_t} \left\{ W_t(\{u(c_s, n_s)\}_{s=t}^\infty, k_t) \middle| c_t = \hat{c} \right\} < 0, \ and$$

$$\sup_{\{c_s, n_s\}_t^\infty, k_t} \left\{ W_t(\{u(c_s, n_s)\}_{s=t}^\infty, k_t) \middle| n_t = \hat{n} \right\} < 0,$$

then an efficient allocation must satisfy no-immiseration.

Many standard frameworks satisfy this condition. For example, suppose $W_t \equiv \sum_{s=t}^\infty \theta^s u_s - \underline{U}(k_t)$ for some discount factor $\theta \in (0, 1)$ and outside option $\underline{U}(k_t)$ such that (i) $\underline{U}(k_t)$ is bounded below, and (ii) u is bounded above with $\lim_{c \to 0} u(c, n) = \lim_{n \to \bar{n}} u(c, n) = -\infty$; then an efficient allocation satisfies no-immiseration.[15]

We now show that a non-immiserizing efficient allocation must feature a labor tax that goes to zero:

Proposition 2 *(Zero labor tax in the long run). Suppose that Assumptions 1, 2 and 3 hold. If an interior efficient allocation exists and satisfies no-immiseration, then $\lim_{t \to \infty} \tau_t^n = 0$.*

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[15] For example, if utility takes the usual Cobb–Douglas form $(c^\gamma (1-n)^{1-\gamma})^{1-\sigma}/(1-\sigma)$ with $\gamma \in (0, 1)$ and $\sigma > 0$, or the power-separable form $c^{1-\sigma}/(1-\sigma) - \psi n^{1+\gamma}/(1+\gamma)$ with $\sigma, \gamma, \psi > 0$, then conditions (ii) and (iii) hold for $\sigma > 1$, the empirically relevant value.

The proof relies on the fact that Assumption 2 implies that β^t/q_t converges to zero, as domestic consumers are impatient relative to the world interest rate. To see why this is important, consider the first order condition for consumption, (17), where we have rearranged terms:

$$\frac{\beta^t}{q_t}\left[1 + \eta\left(1 + \frac{u_{cc}c_t + u_{cn}n_t}{u_c}\right) + \sum_{s=0}^{t}\beta^{s-t}\lambda_s\frac{\partial W_s}{\partial u_t}\right] = \frac{\mu}{u_c(c_t, n_t)}. \qquad (21)$$

The fact that consumption is bounded away from zero and labor is bounded away from $\bar{n}$ (no-immiseration), together with $\mu > 0$,[16] implies the right hand side of the above equation is strictly positive. The fact that β^t/q_t is converging to zero implies the limit of the sum must be unbounded (given that Assumption 1 implies that the term multiplying η is bounded). The unbounded sum reflects that the debt constraints are binding in the limit, with the strictly positive multipliers accumulating over time. Looking back at equation (20), if the infinite sum diverges, and the terms in brackets remain bounded, then the labor tax must be going to zero.

The standard Ramsey solution can be recovered by setting $\lambda_s = 0$ for all s. In this case, both sides of (21) converge to zero (while the ratio of q_t to $\beta^t u_c$ may be bounded), and if $\eta \neq 0$ the labor tax wedge may be non-zero. This highlights the fact that the Ramsey full-commitment allocation without debt constraints does not call for zero labor taxes in the long run. If $\beta(1 + r_t^\star) < 1$, the Ramsey allocation calls for increasing marginal utility of consumption, which violates the no-immiseration condition in the proposition. In the presence of limited commitment, such a path of consumption is not self-enforcing and the growing sum of non-zero multipliers reflects the presence of the debt constraints.

Note as well that if $\beta(1 + r_t^\star) = 1$, we have $\beta^t/q^t = 1$ and the terms involving η remain relevant in the long run. Therefore, labor taxes do not in general converge to zero when households discount at the world interest rate (while capital taxes typically do converge to zero). See Dominguez (2007) and Reis (2013) for the equivalent closed economy analysis in which the marginal rate of inter-temporal substitution equals the return on capital in the steady state. When agents are patient, there is no counter-weight to the incentive to relax the debt constraints by saving, and the constraints become irrelevant in the long run (this is an application of the well known "back loading" result of Ray, 2002).

3.2.1. A perturbation

A simple perturbation sheds light on why the economy converges to a zero labor tax. To simplify the perturbation analysis, we restrict attention to an economy with no capital (that is, production is given by $F(n) = n$); separable utility ($u_{cn} = 0$); and a constant interest rate ($r_t^\star \equiv R^\star - 1$).

We perturb a candidate equilibrium allocation $\{c_t, n_t\}_{t \geq 0}$. No-immiseration (Definition 3) implies that there exists a T and a pair $(\hat{c}, \hat{n})$ such that $n_t < \hat{n} < \bar{n}$ and $c_t > \hat{c} > 0$, for all $t > T$. Consider now a perturbation in the allocation at some $t > T$ and $s > t$ such that n_s changes by $\Delta n_s \neq 0$, c_s changes by $\Delta c_s \neq 0$, and n_t changes by Δn_t. We make the sign of Δn_s equal to the sign of τ_s^n. We perturb c_s such that the instantaneous utility at time s remains unchanged:

$$u_c(s)\Delta c_s + u_n(s)\Delta n_s = 0.$$

To ensure the resource constraint continues to hold, Δn_t must satisfy:

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[16] The fact that the resource constraint binds ($\mu > 0$) is an intuitive result that we prove in the appendix.

$$\Delta n_t = -(R^\star)^{-(s-t)}(\Delta n_s - \Delta c_s) = -(R^\star)^{-(s-t)}\frac{\tau_s^n}{1+\tau_s^n}\Delta n_s$$

As long as $\tau_s^n \neq 0$, the fact $\Delta n_s \tau_s > 0$ implies that $\Delta n_t < 0$.

This perturbation leaves utility unchanged in period s and strictly increases leisure in period t. The perturbation therefore increases the objective in (P). That we can find a welfare-improving perturbation that satisfies the resource constraint when labor is taxed is not surprising: A non-zero labor-tax is a static distortion, and relaxing it generates the possibility of a welfare gain. The perturbation satisfies the sovereign constraints as these are weakly increasing in utility flows. This leaves the key question of whether the perturbation can be financed by the government; that is, whether the IC constraint continues to hold.

To answer this, let us look at the effect of this perturbation on the IC constraint. The left hand side of the IC constraint, equation (P.2), changes with the perturbation by the following amount

$$\Delta IC \equiv \beta^t \left\{ D_n(t)\Delta n_t + \beta^{(s-t)}D_c(s)\Delta c_s + \beta^{s-t}D_n(s)\Delta n_s \right\}$$

where $D_n(t) \equiv u_{nn}(t)n_t + u_n(t) < 0$ and $D_c(t) \equiv u_c(t) + u_{cc}(t)c_t$.

Substituting the previous expressions, we can rewrite ΔIC as

$$\Delta IC = \left\{ -D_n(t) - \frac{(\beta R^\star)^{s-t}}{\tau_s^n}\left[-u_{cc}(s)c_s + \frac{u_{nn}(s)u_c(s)}{u_n(s)}n_s \right] \right\}(-\beta^t \Delta n_t).$$

Efficiency of the original allocation requires that $\Delta IC \leq 0$, otherwise, the perturbation delivers a feasible and implementable improvement. Efficiency therefore requires (using $\Delta n_t < 0$ and $D_n(t) < 0$) that

$$0 \leq \tau_s^n \leq (\beta R^\star)^{s-t}\left[\frac{-1}{D_n(t)}\left(-u_{cc}(s)c_s + \frac{u_{nn}(s)u_c(s)}{u_n(s)}n_s \right) \right]. \tag{22}$$

The term in square brackets is positive and bounded given the non-immiseration requirement. Assumption 2 implies that $\beta R^\star < 1$, and as a result, the right-hand side of equation (22) converges to zero as s tends to infinity. Efficiency then requires that $\lim_{s\to\infty} \tau_s^n = 0$.

The perturbation highlights some key features behind the long-run result. In particular, it exploits the fact that resources are discounted at a rate $1/R^\star$, while private agents discount at $\beta < 1/R^\star$. The perturbation generates a reduction in revenue to the government in period s (as the future labor tax is reduced), while generating an increase in revenue in period t, that more than compensates the later loss. The perturbation explicitly uses the non-immiseration assumption: The necessity of zero long-run labor taxation no longer holds if we can implement paths in which leisure and consumption approach zero in the limit. The sovereign debt constraints (or alternatively, as we discuss below, a patient objective function) ensures that non-immiseration must hold in an efficient allocation. A final key feature of the perturbation is that the discount factors in the objective function (P) and the sovereign debt constraints play no role; namely, all that is required is that raising private utility in period t and leaving it unchanged in period in $t + s$ is strictly preferable from the perspective of the objective and the sequence of political incumbents. This relies on the fact that conditional on $u(c, n)$, the objective and debt constraints are invariant to the particular mix of consumption and leisure.

4. An example economy with sovereign debt constraints

We now study a specific open-economy environment to render the constraints (13) concrete and fully characterize the efficient allocation.

4.1. Preferences and technology

Consider a small open economy without capital and with a linear production technology: $F(n) = n$. Assume also that g_t is constant and that the economy can borrow from abroad at a constant risk-free interest rate of $R^\star \equiv 1 + r^\star$. We impose that $\beta R^\star \leq 1$. The per-period utility function of the domestic representative agent is:

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \omega \frac{n^{1+\nu}}{1+\nu}. \tag{23}$$

At any point, the government of the economy can decide to cut all access from the international financial markets by defaulting on their external obligations. Such a decision guarantees a utility level of $\underline{U}$ to the representative agent, where this payoff $\underline{U}$ can include the costs of default imposed on the country by the foreigners in a default event. For our analysis, we do not need to specify anything more about how $\underline{U}$ gets determined, except that it places a lower bound on equilibrium consumption and leisure:

Assumption 4. There exists $\tilde{c} > 0$ and $\tilde{n} < \bar{n}$ such that $\underline{U} > u(\tilde{c}, \tilde{n})/(1 - \beta)$.

The sovereign constraints (P.3) ensure that domestic citizens are never made better off by default:

$$W_t \equiv \sum_{j=t}^{\infty} \beta^{j-t} \left(\frac{c_j^{1-\sigma}}{1-\sigma} - \omega \frac{n_j^{1+\nu}}{1+\nu} \right) \geq \underline{U}; \quad \text{for all } t. \tag{24}$$

The implementability condition takes the following form:

$$\sum_{t=0}^{\infty} \beta^t \left\{ c_t^{1-\sigma} - \omega n_t^{1+\nu} \right\} \geq c_0^{-\sigma} a_0 \tag{25}$$

4.2. The efficient allocation

The efficient allocation for this economy can then be characterized as follows. Suppose that constraint (24) is not binding. Then the first order condition for consumption, equation (17), can be written in this environment as

$$\left(\beta R^\star \right)^t (1 + \eta (1 - \sigma)) = \mu c_t^\sigma, \quad \text{while } W_t > \underline{U}, t > 0.$$

If $\beta R^\star = 1$, then c_t is constant, while if $\beta R^\star < 1$, c_t is strictly declining due to impatience. Similarly, the first order condition for labor, equation (18), takes the form:

$$\left(\beta R^\star \right)^t (1 + \eta (1 + \nu)) = \omega^{-1} \mu n_t^{-\nu}, \quad \text{while } W_t > \underline{U}, t > 0.$$

The right hand side is strictly declining in n_t. If $\beta R^\star = 1$, then n_t is constant, while if $\beta R^\star < 1$, n_t is strictly increasing due to impatience.

The labor tax wedge along this unconstrained path is:

$$\tau_t^n = \frac{\eta (\nu + \sigma)}{1 + \eta(1 - \sigma)}, \quad \text{while } W_t > \underline{U}, t > 0.$$

Regardless of relative impatience, while unconstrained by the borrowing constraint, labor tax is a constant. If $\beta R^\star = 1$, we see that consumption, labor, and taxes are all constant, which accords with tax smoothing.
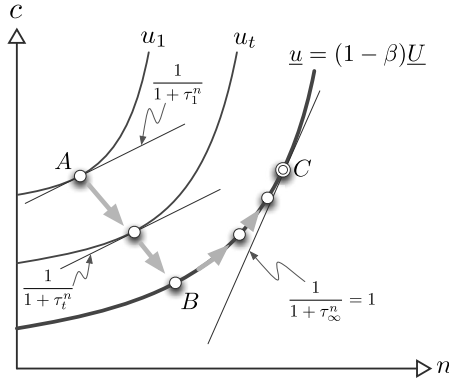
Fig. 1. Path of the efficient allocation when $\beta R < 1$. The curves represent intra-period indifference curves in the (c, n) space, and movements to the southeast represent reductions in utils. The arrows represent time, and the white circles the static allocation at a given point in time. Point C represents the steady state.

If $\beta R^\star < 1$ taxes are constant while unconstrained, but the fact that c_t is falling and n_t is increasing over time implies that $u_t < u_{t-1}$ and this path is not sustainable in the long run. It must be then that at some point $W_t = \underline{U}$ by Assumption 4. The following lemma states that $W_t = \underline{U}$ is an absorbing state for the utility level:

Lemma 2. *Suppose that $W_t = \underline{U}$ for $t > 0$, then $W_{t+s} = \underline{U}$ for all $s > 0$.*

We can also obtain an explicit solution for the behavior of the labor tax. In our current environment, $\partial W_s / \partial u_t = \beta^{t-s}$ for $s \leq t$, and zero otherwise. Substituting this into (17) and (18), we have

$$(\beta R^\star)^t \left(1 + \eta(1-\sigma) + \sum_{j=0}^{t} \lambda_j \right) = \mu c_t^\sigma$$

$$(\beta R^\star)^t \left(1 + \eta(1+\nu) + \sum_{j=0}^{t} \lambda_j \right) = \omega^{-1} \mu n_t^{-\nu},$$

where we have used that $f'(n) = 1$. Taking the ratio of these two expressions to solve for the labor tax, we have:

$$\tau_t^n = \frac{\eta(\nu+\sigma)}{1 + \eta(1-\sigma) + \sum_{j=0}^{t} \lambda_j}.$$

The fact that $\lambda_t > 0$ whenever the borrowing constraint binds implies that the summation $\sum_{j=0}^{t} \lambda_j$ is increasing over time along a constrained path, and the labor tax is falling. In the limit, $\tau_t^n = 0$, as implied by Proposition 2.

The efficient allocation is graphically depicted in Fig. 1. The figure represents indifference curves in consumption-labor space. The curve $\underline{u} \equiv (1-\beta)\underline{U}$ denotes the flow utility that delivers $\underline{U}$. The tax wedge can be read off the slope of the indifference curve: $\frac{1}{1+\tau^n} = -\frac{u_n}{u_c} = \frac{dc}{dn}\big|_u$. We consider the initial allocation (after time 0) at point A, which is associated with some initial state (a_0, A_0). Recall that in this case, because of the absence of physical capital, A_0 represents the economy's aggregate (public and private) net foreign asset position at point A.

We overlay the dynamics of the efficient allocation on the indifference curves, using the figure as a phase diagram. For $u_t > \underline{u}$, the allocation is unconstrained by the borrowing limit. As noted above, the unconstrained allocation features falling consumption, increasing labor, and a constant tax wedge. The constant tax wedge implies parallel shifts across indifference curves as utility falls. Point B is the first point at which utility reaches $\underline{u}$. Once $u_t = \underline{u}$, utility is constant and we move along the indifference curve. As we move along, consumption and labor both increase and the tax wedge falls, eventually reaching zero in the limit. Point C represents the steady state of the economy.

The intuition for the result while the economy is unconstrained by the borrowing limit is straightforward. Impatience relative to $r^\star$ makes a declining path of consumption and leisure optimal. Standard tax smoothing insights make a constant tax rate efficient as well. The fact that the economy ultimately hits the constraint is also straightforward. However, the question remains why point B is not a steady state. That is, once the economy hits the borrowing constraint, why are there further dynamics, and why do these involve a declining labor tax and increasing consumption?

The preceding discussion emphasized that private agents wish to borrow, which is reflected in the relatively low discount factor in the implementability condition. The dynamics from point B to C shed some light on how this desire to front load influences the path of taxation. Note that private utility is constant as the economy transits from B to C. However, output net of consumption is not constant. As the economy moves along the debt constraint, the tax on labor is reduced and output increases more than consumption.

The fact that output increases more than consumption holding constant government expenditures implies that payments to foreign lenders (net exports) increase. That is, by moving along the indifference curve to the zero-tax steady state C, the government increases the aggregate debt servicing capacity of the economy. In fact, point C is the solution to the problem of maximizing steady state net exports subject to the lower bound on utility, and therefore the economy converges to the steady state with the maximal sustainable net foreign liability position.

The benefit of this policy for an impatient household is that consumption and leisure can be front loaded when the debt is incurred, which relaxes the implementability condition. A declining path of labor taxes allows the economy to credibly commit to service a large stock of aggregate debt in the future. With endogenous debt constraints, the efficient allocation manipulates the timing of tax distortions to relax the debt constraints. In contrast, if we were to model limited commitment as an ad hoc maximum value of sovereign debt, the government has no scope to relax this constraint by front loading distortions.

We should emphasize that aggregate net foreign liabilities are maximized in the long run, not the government liabilities. Indeed, the fact that the government does not tax labor income in the long run requires the government to hold enough claims against domestic households or foreigners to fund government expenditures plus any net subsidy to capital income. This generates the somewhat surprising implication that the government of an impatient economy may efficiently choose to run fiscal surpluses on the transition path.

4.3. Managing the portfolio of sovereign and domestic debt

We now explore the time path of aggregates of the example economy described above. Starting from a positive labor tax economy, we have that labor taxes are falling over time and both the domestic households and the economy as a whole are running down assets. It is not clear, however, what is happening to government debt itself, which is the difference between household

and aggregate wealth.[17] In particular, the government may be borrowing from abroad to retire domestic debt.

Fig. 2 depicts the path of a particular parameterization of the example economy.[18] Given that the model is quite simple, we present Fig. 2 as a guide to intuition rather than a quantitative exercise. Time is the horizontal axis in each panel, and period T denotes the first period in which the borrowing constraint binds. The first panel depicts consumption and labor, and the second panel depicts utility relative to $\underline{u}$. Consumption and leisure decline prior to period T, as the borrowing constraint is slack and agents are relatively impatient. At period T, utility has fallen to $\underline{u}$, as seen in panel (b), and remains there. Nevertheless, as discussed above, dynamics continue, with leisure continuing to fall but consumption now rising. This is supported by a declining tax on labor. In particular, as depicted in panel (c), labor taxes are constant prior to T, but then begin to fall. This is the front loading of taxes that is optimal when the economy is constrained. Eventually, the labor taxes converge to zero, as stated in Proposition 2.

The tax on capital income is zero when the economy is unconstrained, and then becomes negative (a tax on borrowing) after period T. The tax on borrowing is necessary to keep the domestic households from violating the aggregate borrowing limit. In the steady state, we have $\phi_\infty^k = -\frac{1-\beta R^\star}{r^\star \beta}$, so that agents choose a constant path of consumption at the after-tax interest rate. The fact that consumption is increasing after period T implies that ϕ^k undershoots its steady state level.

Panel (d) depicts the corresponding path of assets and liabilities. The country's aggregate net foreign asset position A falls rapidly while the borrowing constraint is slack. Once constrained, the economy continues to draw down assets and starts to accumulate foreign liabilities, although the process is slower after period T. Household wealth a also falls, both before and after T. The "deceleration" at period T is less pronounced for household wealth than it is for aggregate wealth. This reflects that household wealth needs to be reduced in order to make labor efficiency consistent with the implementability constraint in the steady state. That is, the flip side of front-loading labor taxes is that the government pays down domestically held public debt. The government's total debt, b, is also depicted in panel (d), which represents the difference between household wealth and aggregate wealth. As might be expected, government debt is initially increasing while the economy is unconstrained. However, at some point *before* the economy becomes constrained, the government starts paying down its debt, and continues to do so after period T. In the limit, debt is reduced to the point that labor taxes are no longer necessary to fund government expenditures.

For this numerical example, we pick parameters so that $A_\infty < 0$; that is, net foreign assets eventually become negative, as the economy as a whole becomes indebted. Accounting implies that the private household's assets are therefore less than government liabilities in the long run ($a_\infty < b_\infty$). Specifically in this example, in the long run the government holds net claims against foreign and domestic agents ($b_\infty < 0$). The government must eventually accumulate assets in order to finance government expenditures as the tax on private debt that decentralizes the debt constraints is not sufficient to completely finance g in the absence of labor taxation. The private

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[17] More precisely, recall that for $t > 0$, $A_t = R^\star/(1 + (1-\phi_t^k)r^\star)a_t - b_t$.

[18] Specifically, let $u(c, n) = \log c - \omega \frac{n^{1+\nu}}{1+\nu}$. We set $\nu = 0.5$ and $g = 0.20$, and select ω and $\underline{u}$ so that steady state labor/income is 1 and steady state net foreign liabilities are 65 percent of aggregate income. The international interest rate is 0.05, and $\beta = 0.94$, so households discount at a higher rate than the world interest rate.
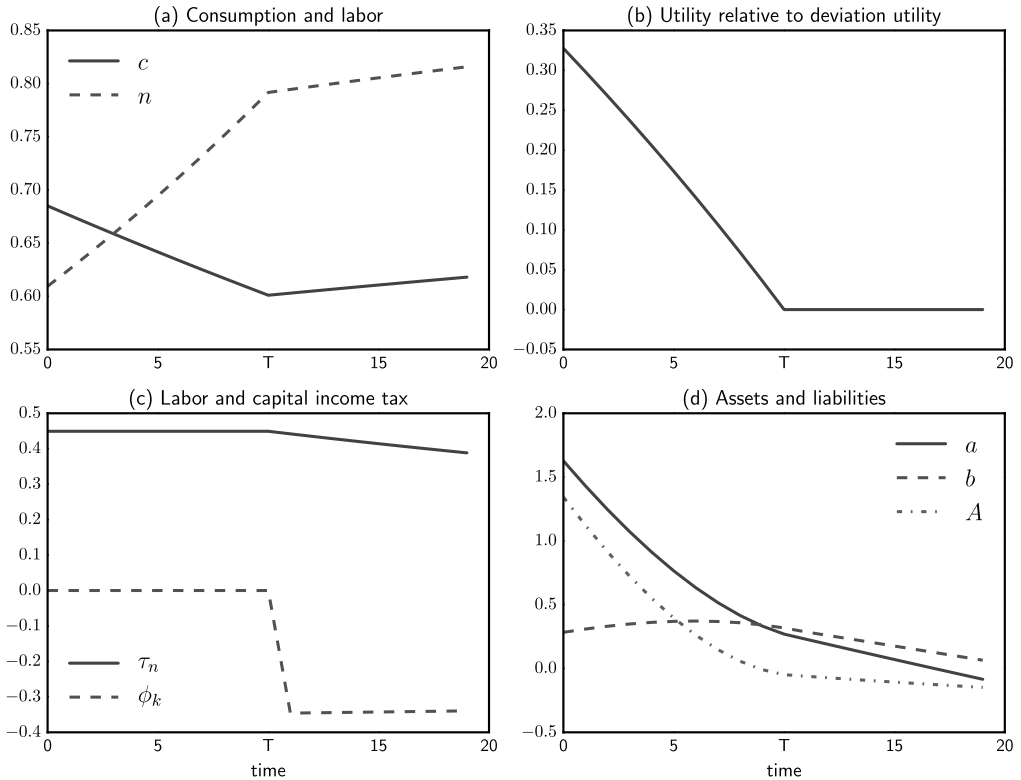
Fig. 2. Time path of economic aggregates. Panel (a) shows the paths of consumption (c) and labor (n). Panel (b) shows the utility relative to $\underline{u}$. Panel (c) shows the paths of labor and capital income taxes (τ_n and ϕ_k). Panel (d) shows the paths of the household's wealth (a), the government's total debt (b), and the country's aggregate net foreign asset position (A).

agents are net debtors in the long run ($a_\infty < 0$), reflecting that the efficient allocation responds to the domestic household's desire to borrow, not the fiscal authority's.[19]

The dynamics depicted in Fig. 2 highlight the important role that government debt and net foreign assets play in supporting the convergence to first best labor. As discussed previously in footnote 7, the model does not make a clean prediction for the relative quantities of public debt held domestically and abroad. In fact, for many developing economies, private residents do not hold large net foreign asset positions, and the vast majority of international borrowing and lending is implemented by the government. Motivated by this, we can select, among the many possible implementations of an efficient allocation, the one where the amount of foreign assets held by the domestic household is equal to zero. Under this selection, we can interpret domestic assets, a in the model, as domestically held government debt; and the country's net foreign assets, A in the model, as government foreign reserves minus sovereign debt.[20] With this, panel (d) of

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[19] It is possible to obtain the steady state values directly. Consumption and labor in steady state are given by the solution to $u_c(c_\infty, n_\infty) = -u_n(c_\infty, n_\infty)$ and $u(c_\infty, n_\infty) = (1 - \beta)\underline{U}$. Given this, $A_\infty = R^\star(n_\infty - c_\infty - g)/r^\star$ and $a_\infty = (c_\infty - n_\infty)/(1 - \beta)$.

[20] This follows from the absence of physical capital, but even in an economy with capital, we continue to have a sharp prediction for domestically held debt as long as private foreign assets are zero.

Fig. 2 indicates that domestically held government debt is falling (*a*), while net foreign liabilities are increasing (*A* is falling). Therefore, the path to efficient labor is one in which the public debt held abroad is increasing while the public debt held domestically is falling. As a normative prediction, the model states that the government should pay off domestically held debt first, while continuing to borrow from abroad. This is the optimal path to zero labor taxes for an impatient economy where the private sector does not hold any foreign assets or liabilities.

5. Household debt constraints

In this section we extend our benchmark model to incorporate limited commitment on the part of the domestic household. In addition to extending the optimality of front-loading tax distortions to this richer environment, a methodological contribution of this subsection is to demonstrate the validity of the primal approach despite private-agent borrowing constraints that distort individual Euler equations.

Specifically, we assume that agents face "household debt constraints" (HHDC) of the form:

$$V_t(\{u_s\}_{s=t}^\infty | \{w_s, T_s, \phi_s\}_{s=t}^\infty) \equiv \sum_{s=t}^\infty \beta^{s-t} u_s - \underline{V}_t\left(\{w_s, T_s, \chi_s\}_{s \geq t}\right) \geq 0 \text{ for all } t \geq 1,$$

(HHDC)

where $\underline{V}_t$ represents the value of repudiating debt at time t. Following Kehoe and Levine (1993), the constraint captures a limited commitment environment in which agents that default are permanently excluded from assets markets, but continue to participate in spot labor markets. In our environment, this implies working in the labor market. We also allow for additional enforcement mechanisms that the government may have available, such as wage garnishment, which we parameterize by $\chi_t \in [0, 1]$. In particular, let

$$\underline{V}_t\left(\{w_s, T_s, \chi_s\}_{s \geq t}\right) \equiv \sum_{s=t}^\infty \beta^{s-t} \max_{n_s} u(\chi_s w_s n_s + T_s, n_s). \tag{26}$$

Note that after a default, consumption is labor income minus wage garnishment ($\chi_s w_s n_s$) plus transfers if any (T_s).

The presence of equilibrium prices in the constraint set opens up room for externalities stemming from households' borrowing constraints, a feature of limited commitment environments emphasized by Kehoe and Levine (1993) and a large subsequent literature. For a given wage and transfer sequence, note that V_t is differentiable and concave in household consumption and labor.

We do not include an initial period debt constraint given that maximizing (1) will satisfy the initial constraint, if feasible, and we assume that it is always feasible. In particular, we assume that the constraint set has a non-empty interior. That is, a_0 is such that for any bounded sequence of positive prices there exists an interior point in the constraint set. A sufficient condition for this is that it is feasible for the agent to supply $n < \bar{n}$ in the initial period and save part of that period's labor income.

The household's problem is now the same as before, with the addition of the sequence of constraints (HHDC). Let $\beta^t \gamma_t \geq 0$ be the multiplier on period t's constraint, and, as before, θ be the multiplier on the household budget constraint (BC).[21]

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[21] We solve the household's and government's problem using Lagrange multiplier techniques. As noted before, there are technical issues regarding the nature of the multipliers in infinite dimensional problems. We proceed by assuming the

The revised first order conditions are:

$$\beta^t u_c(c_t, n_t) \left(1 + \sum_{s=0}^{t} \gamma_s \right) = \theta p_t \tag{27}$$

$$\beta^t u_n(c_t, n_t) \left(1 + \sum_{s=0}^{t} \gamma_s \right) = -\theta p_t w_t. \tag{28}$$

These conditions, the constraints, and the complementary slackness conditions characterize the unique solution to the household's problem.

Note that as before $\theta > 0$, and so the debt constraints affect inter-temporal decisions, but not the static consumption-leisure choice:

$$\frac{-u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t.$$

The debt constraints (and their associated multipliers) can be viewed as an inter-temporal wedge. In particular, it will be convenient to define

$$\Gamma_t \equiv 1 + \sum_{s=1}^{t} \gamma_s,$$

with $\Gamma_0 = 1$ as the first period's constraint is slack. The first order conditions can then be written as

$$\beta^t \Gamma_t u_c(c_t, n_t) = \theta p_t$$
$$\beta^t \Gamma_t u_n(c_t, n_t) = -\theta p_t w_t.$$

Using that $T_t \geq 0$, it follows that we can write the budget constraint of the consumer as:

$$\sum_{t=0}^{\infty} \beta^t \Gamma_t (u_c(c_t, n_t) c_t + u_n(c_t, n_t) n_t) \geq u_c(c_0, n_0) a_0, \tag{29}$$

which is the revised implementability condition. This expression differs from (P.2) due to the presence of Γ_t, reflecting the shadow costs of the household's borrowing constraint.

The definition of a competitive equilibrium remains the same as the benchmark, with the stipulation that prices and allocations are consistent with the household's optimization in the presence of debt constraints, given a wage-garnishment policy χ_t.

Recall that the primal approach to solving for efficient allocations treated the implementability condition as a constraint. The difficulty here is that (29) depends on the multipliers Γ_t as well as the allocation. The presence of debt constraints implies that an equilibrium allocation may be supported by multiple price sequences as the Euler equation holds only with weak inequality. Fiscal policy determines not only the allocation, but also the prices conditional on an allocation. For each of these candidate price sequences, we have associated multipliers γ_t to satisfy the household's first order conditions. Nevertheless, the following proposition states we can purse the primal approach despite the household's borrowing constraints:

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existence and validity of these multipliers. A sufficient condition in this context is that u is bounded. See the appendix for more details.

Proposition 3. *An allocation $\{c_t, n_t, k_t\}_{t=0}^{\infty}$ can be implemented as a competitive equilibrium if and only if: (a) the resource constraint* (RC*) *holds; (b) the household debt constraint holds at the equilibrium wage sequence and zero transfers:*

$$V_t\left(\{u_s\}_{s=t}^{\infty} | \{w_s, 0, \chi_s\}\right) \geq 0, \text{for all } t \geq 1, \tag{30}$$

where $u_s = u(c_s, n_s)$, $w_s = -u_n(c_s, n_s)/u_c(c_s, n_s)$; (c) the sovereign debt constraints hold; and (d) the implementability condition of Proposition 1 is satisfied:

$$\sum_{t=0}^{\infty} \beta^t (u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t) \geq u_c(c_0, n_0)a_0. \tag{IC}$$

This proposition states that we can search for efficient allocations without concerning ourselves with Γ_t, as long as we restrict attention to allocations that satisfy the household debt constraints. The reason we can ignore the wedge in the household's Euler equation stems from their equivalence to a tax on borrowing. In particular, consider an agent that wishes to borrow more but is constrained from doing so. The multiplier γ_t is the shadow cost of this constraint. The government can replace γ_t with a tax on borrowing, so the agent is indifferent to the constraint at the original allocation (that is, the Euler equation holds with equality given the tax). This replaces the shadow cost with an actual tax. This is always feasible for the government as long as the tax raises revenue, that is, as long as asset positions are non-positive when the debt constraint binds. As the deviation utility balances the budget period-by-period by definition (as agents lose access to financial markets), and delivers the same utility when the constant binds, this will always be the case. In this manner, the government can raise revenue without distorting the allocation by taxing borrowing at the debt constraint. The intuition is the same as the well known equivalence of an import tariff and an import quota – both restrict the quantity of imports, but differ only in how the tariff revenue is shared.

The only difference between the problem with and without the household's debt constraints is therefore the constraint (30) and the potential availability of additional enforcement tools χ_t. Absent these additional tools, the government will have an incentive to distort spot wages to relax the household's borrowing constraint. In this way, labor taxes will, in general, not be zero, even in the absence of fiscal expenditures. This is a crude mechanism to enforce private debt contracts, as the distortion affects all workers along the equilibrium path.

A more realistic scenario is where additional tools (ranging from wage garnishment to debtor's prisons) may be available. The question then becomes one of whether the government is *willing* to enforce contracts directly. If the household accumulates enough foreign debt, the government may not choose to enforce these contracts, even if it can. However, this is the same question as whether the government repays its own debt, and thus equivalent to the type of sovereign debt constraints explored in our benchmark formulation. We conclude by formally stating this equivalence in regard to long run labor taxes:

Corollary 1 (*Corollary to Proposition 2 with private debt constraints*). *Suppose the government can enforce private debt contracts directly (i.e., $\chi_t = 0$ is feasible for all t), then under the assumptions of Proposition 2 $\lim_{t \to \infty} \tau_t^n = 0$.*

6. Extensions

In this section we consider how our results translate to alternative assumptions and environments. In Definition 2, we imposed that an efficient allocation is one that maximizes the

household's utility as of time 0. As a result, the discount factor that appears in the objective of Problem (P) is the same as the one that discounts the implementability condition (P.2). In addition, we restricted attention to a government's felicity function that shares the private household's static trade-off between consumption and leisure. In subsections 6.1 and 6.2 we relax both of these. In subsection 6.3 we study a closed economy. Given the focus on the relatively low world interest rate, it may not be clear at this point that the result regarding front loading labor taxes carries over to a closed economy. It is therefore useful to show that the result rests on the wedge between the inter-temporal marginal rate of substitution and the marginal rate of transformation, whether the MRT is determined by the domestic capital stock or the world interest rate. This closed economy extension allows us to relate to the related literature on optimal taxation under limited commitment.

6.1. An alternative objective

Consider problem (P) with the alternative objective:

$$\sum_{t=0}^{\infty} \tilde{\beta}^t u(c_t, n_t).$$

The counterpart to (21) is:

$$\frac{\tilde{\beta}^t}{q_t} + \frac{\beta^t}{q_t} \left[\eta \left(1 + \frac{u_{cc} c_t + u_{cn} n_t}{u_c} \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t} \right] = \frac{\mu}{u_c(c_t, n_t)}. \tag{31}$$

By the same logic presented in relation to (21), no-immiseration implies that either $\tilde{\beta}^t/q_t$ is bounded away from zero in the limit and/or $(\beta^t/q_t) \lim_{t \to \infty} \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t} = \infty$. Note that if the objective is relatively patient ($\tilde{\beta}^t = q_t$), the debt constraints may not bind. This reflects that a patient objective and debt constraints both provide an incentive to delay consumption, and in this sense serve a similar role in preventing long-run immiseration.

The first-order condition with respect to labor for $t > 0$ is:

$$\frac{\tilde{\beta}^t}{q_t} + \frac{\beta^t}{q_t} \left[\eta \left(1 + \frac{u_{nn} n_t + u_{cn} c_t}{u_n} \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t} \right] = -\frac{\mu F_n}{u_n(c_t, n_t)}. \tag{32}$$

It is interesting in regard to (32) that even if the objective function discounts at the world interest rate ($\tilde{\beta}^t = q_t, \forall t$) and the sovereign debt constraints never bind ($\lambda_t = 0, \forall t$), it is not the case that consumption and leisure are necessarily smoothed over time. For example, with separable power utility, the term multiplying η will be a constant. Setting $\tilde{\beta}^t = q_t$ and $\lambda_t = 0$, the left hand side will be strictly decreasing over time as $\beta^t/q_t \to 0$. The right hand side correspondingly implies that n_t must be increasing over time. This highlights the fact that the private household's impatience provides an incentive to front load leisure even if the objective function is patient. The front loading relaxes the implementability condition and so reduces the overall impact of tax distortions.

Dividing (32) by (31) and rearranging, the corresponding expression for the labor tax is:

$$1 + \tau_t^n = \frac{\left(\frac{\tilde{\beta}}{\beta} \right)^t + \eta \left(1 + \frac{u_{cn} c_t + u_{nn} n_t}{u_n} \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t}}{\left(\frac{\tilde{\beta}}{\beta} \right)^t + \eta \left(1 + \frac{u_{cc} c_t + u_{cn} n_t}{u_c} \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial u_t}}. \tag{33}$$

This differs from (20) to the extent that $\tilde{\beta} \gtrless \beta$. If $\tilde{\beta} \le \beta$, then $\tilde{\beta}^t / q_t \to 0$, and the above discussion indicates that the summation terms in the numerator and denominator diverge to infinity. Alternative, if $\tilde{\beta} > \beta$, the first term in the numerator and denominator grows without bound. In this case, regardless of the long-run properties of λ_t, the labor tax converges to zero.

This discussion highlights that it is the private household's impatience that leads to zero long-run labor taxation. The debt constraints and/or a patient objective are necessary to ensure that the no immiseration condition is satisfied. Conditional on no immiseration, the discount factor of the objective and the sequence of incumbent politicians is not relevant to the long-run result concerning the labor tax.

6.2. Government preferences

While the discount factor implicit in the sequence of incumbent government value functions, W_t, can be fairly general, Proposition 2 does rely on the fact that the government shares the private household's static utility $u(c, n)$. Suppose instead that the government's felicity function differs from the private household's. That is, incumbent t's value function is $W_t(\{\tilde{u}(c_s, n_s)\}_{s=t}^\infty, k_t)$ with

$$\frac{\tilde{u}_c(c, n)}{\tilde{u}_n(c, n)} \neq \frac{u_c(c, n)}{u_n(c, n)}.$$

That is, all else equal, the government differs from the household in the static tradeoff between consumption and leisure.[22]

In this case, with our benchmark objective function, (20) becomes:

$$1 + \tau_t^n = \frac{1 + \eta\left(1 + \frac{u_{cn}c_t + u_{nn}n_t}{u_n}\right) + \frac{\tilde{u}_n(c_t, n_t)}{u_n(c_t, n_t)} \sum_{s=0}^t \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial \tilde{u}_t}}{1 + \eta\left(1 + \frac{u_{cc}c_t + u_{cn}n_t}{u_c}\right) + \frac{\tilde{u}_c(c_t, n_t)}{u_c(c_t, n_t)} \sum_{s=0}^t \beta^{s-t} \lambda_s \frac{\partial W_s}{\partial \tilde{u}_t}}.$$ (34)

As in the benchmark, no immiseration implies that the summation terms go to infinity, and we have:

$$\lim_{t\to\infty} \left(1 + \tau_t^n\right) = \lim_{t\to\infty} \left(\frac{\tilde{u}_n}{\tilde{u}_c} \frac{u_c}{u_n}\right) \neq 1.$$

This states that the long-run labor tax wedge is equal to the gap between the private household's and political incumbents' static MRS. The fact that the incumbent disagrees about the static consumption-leisure tradeoff implies that distorting this margin (from the perspective of the household/objective) is one way to relax the sovereign's participation constraint. At zero labor tax, this has second-order consequences for the household's welfare, but generates a first-order gain due to the relaxation of the binding debt constraint.

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[22] We could make the point more generally by considering government utility directly over sequences of consumption and labor, $\tilde{W}_t(\{c_s, n_s\}_{s \ge t}, k_t)$, without intermediating through an alternative felicity function. The expressions become more cumbersome, but the same conceptual point made below holds. A similar point can be made using the context of Acemoglu et al. 2008, 2011, in which incumbent politicians have an incentive to seize final output after production. This creates an incentive to distort private labor supply down to relax the incumbent's participation constraint (even when lump-sum taxation is available). Mapping that environment to our framework would involve having labor enter the debt constraints via the production function rather than private utility.

We can link this directly to our benchmark result by recalling that the private households' and firms' problems imply:

$$-\frac{u_n}{u_c} = \frac{F_n}{1+\tau^n}$$

at all t. Substituting into the expressions above and taking limits, we have that

$$\lim_{t\to\infty} -\frac{\tilde{u}_n}{F_n \tilde{u}_c} = 1.$$

That is, from the perspective of the future political incumbents there is no labor distortion.

A similar long-run distortion holds if the objective function has an alternative felicity function. In this case, efficiency directly implies the distortion of the household's static tradeoff, as would be the case in a standard Pigouvian tax problem. In these alternative scenarios, there is an incentive to distort private labor supply even if lump sum taxation were available.

6.3. A closed economy

In this subsection, we show how our results carry over to a closed economy. In a closed economy, all capital is owned by domestic agents, and the source tax on capital, τ_t^k, is equivalent to the residence based tax on capital, ϕ_t^k, so we can set the last one to zero without loss of generality. It follows then that $p_t = q_t$ and the aggregate resource constraint is the national income accounting identity:

$$c_t + g_t + k_{t+1} - (1-\delta)k_t \leq F(k_t, n_t). \tag{RC}$$

Market clearing requires that the households' initial wealth corresponds to their holdings of government debt plus the domestic capital stock: $a_t = b_t + p_{t-1}k_t/p_t$, where the fact that k_t is adjusted by the inter-temporal price p_{t-1}/p_t reflects the fact that a_t and b_t are in period-t (after tax) units, while k_t is the amount invested at the end of period $t-1$. We restrict $\tau_0^k = 0$ to eliminate the initial capital levy solution.

For simplicity, we assume that the households can commit to debt contracts, and the only constraints are on the sovereign. This will avoid the need to discuss the additional notation of Section 5.

The requirements of a closed economy competitive equilibrium are the same as their open economy counterparts, with the appropriate adjustment to the resource constraint, and recognition that initial capital is a state variable in a closed economy:

Definition 4. A **closed economy competitive equilibrium** from an initial position (k_0, b_0) consists of prices $\{r_t, w_t, r_t^k\}$, taxes $\{\phi_t^k = 0, \tau_t^n, \tau_t^k\}$ with $\tau_0^k = 0$, non-negative lump-sum transfers $\{T_t\}$, and quantities $\{c_t, n_t, k_t\}$ such that **(i)** r_t, r_t^k, p_t, q_t satisfy equations and (2), (3), (10); **(ii)** households optimize given prices and taxes subject to their budget constraint (BC); **(iii)** firms maximize profits given prices and taxes; that is, equations (8) and (9) hold; **(iv)** the government budget constraint (11) holds; **(v)** the sequence of closed-economy aggregate resource constraints (RC) hold; and **(vi)** the sovereign debt constraints (13) hold for all t.

As before, we use the primal approach. The closed economy version of Proposition 1 is:

Proposition 4. *An allocation is consistent with a closed economy competitive equilibrium if and only if it satisfies the implementability condition*

$$\sum_{t=0}^{\infty} \beta^t \left(u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t \right) \geq u_c(c_0, n_0)\Big(b_0 + \big(F_k(k_0, n_0) + 1 - \delta \big)k_0 \Big) \qquad \text{(IC}')$$

and the sequence of closed-economy resource constraints (RC) *hold.*

Equation (13) continues to define the additional sovereign constraints on the problem. The natural interpretation of (13) in a closed economy is one of a lower bound on aggregate savings, rather than an upper bound on aggregate debt. In this regard, the dynastic model with insufficient private altruism (relative to the government's Pareto weights) is perhaps the most relevant interpretation. For the closed economy, we add an additional assumption on the functions W_t:

Assumption 5. $\partial W_s / \partial k_t \leq 0 \, \forall s, t$.

This assumption ensures that additional capital (weakly) tightens the constraint. This is consistent, for example, with more capital raising the incentive of the government to renege on its tax promises.

The definition for an efficient allocation given in Definition 2 continues to hold with the relevant notion of equilibrium being a closed economy competitive equilibrium. To characterize (interior) efficient allocations, let $\beta^t \psi_t$ denote the multiplier on the time t aggregate resource constraint.[23] The first order conditions for $t \geq 1$ are:

$$\frac{1}{\psi_t}\left[1 + \eta + \eta\left(\frac{u_{cc}(c_t, n_t)}{u_c(c_t, n_t)}c_t + \frac{u_{cn}(c_t, n_t)}{u_c(c_t, n_t)}n_t \right) + \sum_{s=0}^{\infty} \beta^{s-t}\lambda_s \frac{\partial W_s}{\partial u_t} \right] = \frac{1}{u_c(c_t, n_t)} \qquad (35)$$

$$\frac{1}{\psi_t}\left[1 + \eta + \eta\left(\frac{u_{cn}(c_t, n_t)}{u_n(c_t, n_t)}c_t + \frac{u_{nn}(c_t, n_t)}{u_n(c_t, n_t)}n_t \right) + \sum_{s=0}^{\infty} \beta^{s-t}\lambda_s \frac{\partial W_s}{\partial u_t} \right] = -\frac{F_n(k_t, n_t)}{u_n(c_t, n_t)} \qquad (36)$$

$$F_k(k_t, n_t) + 1 - \delta + \frac{1}{\psi_t}\sum_{s=0}^{\infty} \beta^{s-t}\lambda_s \frac{\partial W_s}{\partial k_t} = \frac{\psi_{t-1}}{\beta \psi_t}. \qquad (37)$$

The initial period first order conditions ($t = 0$) are adjusted in the same way as they were in the open economy formulation.

We can now state the closed economy version of Proposition 2:

Proposition 5. *[Zero labor tax in the long-run – closed economy] Suppose that Assumptions 1, 3 and 5 hold. If an (interior) efficient allocation satisfies no-immiseration and if:*

(a) *there exist $M > 0$ and T such that $1 > M > \beta(F_k(k_t, n_t) + 1 - \delta)$ for all $t > T$;*

then $\lim_{t \to \infty} \tau_t^n = 0$.

The impatience condition (a) replaces Assumption 2 in Proposition 2. Intuitively, an impatience condition for a closed economy involves $\beta(F_k(k_t, n_t) + 1 - \delta)$, as the relevant marginal rate of transformation in a closed economy is the marginal product of capital.

An important antecedent of our analysis is Reis (2012), which considers optimal taxation under limited commitment when the government and private agents discount at different rates. In

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[23] As in the open economy case, we leave to the appendix discussion of the existence of Lagrange multipliers.

the case of a relatively patient government, Reis shows that if the economy converges to a steady state the long-run labor tax is zero. As was the case in our open-economy analysis, Proposition 5 does not impose explicit conditions on the discount factor of the government. Nevertheless, there is a natural connection between the proposition and the Reis result.

Specifically, Proposition 5 involves a restriction on the long-run marginal product of capital. This condition could be met due to a feature of the technology, for example, if the marginal product of capital is bounded above as $k \to 0$. This is a direct parallel to the exogenous world interest rate of the open-economy model.

Alternatively, the condition may involve a restriction on the long-run allocation itself. In particular, condition (a) implies that, in the long run, capital should remain above the level consistent with the private households' "modified golden rule." In a standard neoclassical framework, this implies a lower bound on capital above the laissez-faire steady state. Such a lower bound on capital can arise in a closed economy due to the limited commitment of the government. The sequence of sovereign debt constraints places a floor on the utility of future governments. In the open-economy model, this restricted the sustainable level of net foreign liabilities for the economy. The closed-economy counterpart is that sufficient capital must be accumulated and retained in the long run. Note that limited commitment requires a certain level of consumption and leisure, but this does not place an explicit lower bound on capital. Recall that in the current environment, we assume $\partial W_s / \partial k_t \leq 0$. All else equal, reducing capital (weakly) relaxes the constraint, so the over-accumulation of capital is not hard-wired.[24] Rather, the lower bound on capital arises to ensure the feasibility of an allocation that does not violate an incumbent's debt constraint.

The above analysis helps draw the connection between our analysis and that in Reis (2012), which considers optimal taxation under limited commitment when the government and private agents discount at different rates. Reis shows that a relatively patient government will drive the long-run labor tax to zero if the economy converges to a steady state. She considers an objective function that maximizes the government's welfare rather than the private household's. As discussed in Section 6.1, when the objective is patient, the labor tax goes to zero in the long run regardless of whether the debt constraints bind in the long run or not. While this echoes the Reis result, our long-run tax result holds for general discount factors for the government or the objective function of the tax problem. Proposition 5 highlights that what drives the labor tax to zero in the long run is the combination of no immiseration and a long-run inter-temporal MRT that is below the private household's discount rate. Whether the low MRT is willingly implemented by a patient government (as in Reis), or forced upon it by debt constraints, is not relevant for the long-run labor tax, although it may matter for the transition.

7. Conclusion

This paper characterized the optimal fiscal policy when agents are relatively impatient and thus debt constraints bind even in the long run. A defining feature of the optimal policy is that labor taxes are front loaded. A consequence of such a policy is that the transition to the steady involves a conservative fiscal policy in which the government accumulates enough assets to finance expenditures in the absence of labor tax revenue. While the fiscal authority may run surpluses,

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[24] This rules out such constraints as $k \geq \underline{k} > k^\star$, where $k^\star$ is the modified golden rule capital. It allows the standard limited commitment constraints in which more capital makes deviation more profitable. Such a restriction on W was unnecessary in the open economy case as any over-accumulation of capital did not affect inter-temporal prices, which were pinned down by international financial markets.

the economy as a whole is accumulating foreign liabilities. The mechanism rationalizes why taxes are front loaded rather than smoothed in response to an unanticipated fiscal shock. In particular, front loading allows the economy to aggressively access international debt markets in the presence of limited commitment, which is the relevant concern in our benchmark scenario.

One potential drawback of our deterministic environment (or an extension with state-contingent debt) is the inability to speak to spikes in sovereign risk premia as a country's borrowing capacity becomes saturated. In our framework, limited commitment is manifested through quantity rationing. The increase in the cost of borrowing is reflected in the accumulating Lagrange multipliers on the debt constraints; that is, via the shadow cost of debt rather than a change in the market price. Given this symmetry, the desire to lower the cost of borrowing in a model with explicit default may also call for front loading of labor taxes. Similarly, the model does not encompass other potential frictions in the asset markets, such as illiquidity of public or private debt, that may play a role in fiscal crises. Such additional complications do not a priori challenge the argument for front loading tax revenues, but we leave for future research a full analysis. Nevertheless, the intuition behind the result is likely to play a role in other environments. Namely, an economy that wishes to front load consumption and leisure should operate as efficiently as possible in the long run to maximize its debt servicing capacity. As private indebtedness does not undermine efficiency to the same extent as public indebtedness, this requires the fiscal authority to "make room" by front loading its labor tax revenue.

Appendix A. Proofs

A.1. Proof of Proposition 1

Proof. The only if part: Take the budget constraint of the consumer (BC) and substitute p_t by $\beta^t u_c(c_t, n_t)/u_c(c_0, n_0)$ and substitute w_t by $-u_n(c_t, n_t)/u_c(c_t, n_t)$. Then you get that:

$$\sum_{t=0}^{\infty} \beta^t \Big(u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t \Big) - u_c(c_0, n_0)a_0 = u_c(c_0, n_0)T \geq 0$$

where the last inequality follows from $T \geq 0$ and that $u_c \geq 0$. The necessity that the resource constraint and the sovereign debt constraints hold follows from the definition of equilibria.
The if part: Given an allocation, let us define the following objects:

$$w_t = -u_n(c_t, n_t)/u_c(c_t, n_t),$$
$$r_t^k = r_t^* - \delta,$$
$$\tau_t^n = -\frac{F_n(k_t, n_t)u_c(c_t, n_t)}{u_n(c_t, n_t)} - 1,$$
$$\tau_t^k = F_k(k_t, n_t)/r_t^k - 1,$$

for all $t \geq 0$. Define as well:

$$\phi_t^k = 1 - \frac{1}{r_t^*}\left(\frac{u_c(c_{t-1}, n_{t-1})}{\beta u_c(c_t, n_t)} - 1\right),$$
$$p_t = \beta^t u_c(c_t, n_t)/u_c(c_0, n_0),$$
$$T_t = 0,$$

for all $t \geq 1$; and

$$T_0 = \sum_{t=0}^{\infty} \beta^t \frac{u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t}{u_c(c_0, n_0)} - a_0 \geq 0,$$

$$\theta = u_c(c_0, n_0).$$

Now note that p_t satisfies (3) by construction. So part (i) of the equilibrium definition is satisfied. Given the definition of $\theta \geq 0$, w_t and p_t, we get that conditions (5) and (6) are satisfied as well. Note that the budget constraint of the consumer is satisfied with T_t as defined above. So part (ii) is satisfied.

For part (iii), note that from the definition of r_t^k and τ_t^n and τ_t^k it follows that equations (8) and (9) hold. So part (iii) is satisfied. Part (v) holds as well, by hypothesis of the proposition. We now show that part (iv) holds. We don't appeal directly to Walras' law because the budget constraint of the government is not necessarily holding with equality. However, rewriting equation (RC*), we get:

$$\sum_{t=0}^{\infty} q_t \left(g_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k)r_t} a_t \right)$$

$$+ \sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t + \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k)r_t} a_t \right) \leq A_0 \qquad (38)$$

where we used the first order conditions of the firm together with $F = F_n n + F_k k$. Now, note that:

$$\sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t + \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k)r_t} a_t \right)$$

$$= \sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t - a_t + \frac{1 + r_t}{1 + (1 - \phi_t^k)r_t} a_t \right)$$

$$= \sum_{t=0}^{\infty} q_t \left(c_t - w_t n_t - T_t - a_t + \frac{1}{1 + (1 - \phi_t^k)r_t} a_{t+1} \right) + a_0 = a_0$$

where we have used the definition of a_t. Then, plugging back into (38):

$$\sum_{t=0}^{\infty} q_t \left(g_t + T_t - \tau_t^n w_t n_t - \tau_t^k r_t^k k_t - \frac{\phi_t^k r_t}{1 + (1 - \phi_t^k)r_t} a_t \right) \leq A_0 - a_0 = -b_0 \qquad (39)$$

And thus part (iv) holds.

Taken together, the above implies that the sequences of prices, quantities and taxes we have constructed is a competitive equilibrium. So the allocation is consistent with an open economy competitive equilibrium. □

A.2. Proof of Lemma 1

Proof. Consider a possible efficient allocation $\{c_t^\star, n_t^\star, k_t^\star\}_{t=0}^{\infty}$. Let us first prove that consumption is bounded above zero in the limit: $\liminf_{t \to \infty} c_t^\star > 0$. Towards a contradiction, suppose that this is not true. Hence, we can find a $t_0 > T$ such that $c_{T_0}^\star < \epsilon_c$. But then we know that:

$$W_{t_0}(\{u(c_s^\star, n_s^\star)\}_{s=t_0}^\infty, k_{t_0}) < W_{t_0}(\{u(c_s, n_s^\star)\}_{s=t_0}^\infty, k_{t_0}) < 0$$

with $c_{t_0} = \epsilon_c$ and $c_t = c_t^\star$ for $t \geq t_0$. The first inequality follows from strict monotonicity of u_{t_0} with respect to c_{t_0} and of W_{t_0} with respect to u_{t_0}, while the second follows from the hypothesis of Lemma 1. But then this implies that the sovereign debt constraint is violated at time t_0, generating a contradiction.

The proof of $\limsup_{t\to\infty} n_t^\star < \bar{n}$ is similar, and we omit it. $\quad\square$

A.3. Proof of Proposition 2

Let $\{c^\star, n^\star, k^\star\}$ be an interior efficient allocation. To eliminate the need to work in an infinite dimensional space,[25] consider the subproblem (P^T) which equals problem (P) with the additional restriction that $\{c_t, n_t\} = \{c_t^\star, n_t^\star\}$ for all $t > T$ and $k_t = k_t^\star$ for all $t \geq 0$. Clearly, $\{c^\star, n^\star, k^\star\}$ is a solution to subproblem (P^T). Note that subproblem (P^T) has a finite number of constraints, and to avoid dealing with the regularity conditions,[26] we use a version of the Lagrangian theorem stated in Luenberger (1969, chapter 9.4, problem 3). Applied to our setting, it implies that there exists a non-negative vector $\{r^T, \mu^T, \eta^T, \lambda_0^T, \ldots, \lambda_T^T\}$, *not identically zero*, where $\{\mu^T, \eta^T, \lambda_0^T, \ldots, \lambda_T^T\}$ represent the multipliers associated to constraints (P.1), (P.2), and (P.3) respectively; and are such that the standard complementary conditions hold, together with the following first order conditions:

$$\beta^t \left[r^T u_c(c_t, n_t) + \eta^T \left(u_c(c_t, n_t) + u_{cc}(c_t, n_t)(c_t - \mathbb{1}_{\{0\}}(t)a_0) + u_{cn}(c_t, n_t)n_t \right) \right] +$$
$$\left[\sum_{s=0}^t \beta^s \lambda_s^T \frac{\partial W_s}{\partial u_t} \right] u_c(c_t, n_t) = q_t \mu^T, \tag{40}$$

$$\beta^t \left[r^T u_n(c_t, n_t) + \eta^T \left(u_{cn}(c_t, n_t)(c_t - \mathbb{1}_{\{0\}}(t)a_0) + u_n(c_t, n_t) + u_{nn}(c_t, n_t)n_t \right) \right] +$$
$$\left[\sum_{s=0}^t \beta^s \lambda_s^T \frac{\partial W_s}{\partial u_t} \right] u_n(c_t, n_t) = -F_n(k_t, n_t) q_t \mu^T, \tag{41}$$

for $t \in \{0, 1, \ldots, T\}$ and where $\mathbb{1}_{\{0\}}$ is an indicator function taking value of 1 when evaluated at 0. The first equation is the first order condition with respect to consumption and the second with respect to labor. Note that the difference with the usual necessity theorem is the presence of $r^T \geq 0$ that multiplies the objective function's contribution to the first order conditions (r^T is a multiplier of the objective function).

Using (40) and (41), together with interiority of the efficient allocation, we get that:

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[25] Working directly in infinite dimensional space is not an issue if we restrict attention to bounded utility functions (that is, $u(0, n) > -\infty$ and $u(c, \bar{n}) > -\infty$). In this case, the constraint set maps our commodity space into the space of bounded sequences (ℓ_∞), and it can be shown that the associated multipliers are summable sequences of non-negative numbers. In order to extend our results to unbounded utility functions, we pursue an alternative approach.

[26] The difficulty with the standard regularity conditions stems from the nature of the sovereign debt constraint. In particular, the optimal allocation may be such that the gradient of the constraint set does not have full rank. For example, an extreme case that cannot be ruled out in our setting is if there is only one allocation that satisfies all the constraints. Adding a multiplier r^T in front of the objective function addresses this issue.

$$\eta^T \left(\frac{\beta^t}{q_t} \right) \left[\left(u_{cc}(t) - \frac{u_{cn}(t)u_c(t)}{u_n(t)} \right) (c_t^\star - \mathbb{1}_{\{0\}}(t)a_0) + \left(u_{cn}(t) - \frac{u_{nn}(t)u_c(t)}{u_n(t)} \right) n_t^\star \right]$$

$$= \mu^T \left(1 + \frac{F_n(t)u_c(t)}{u_n(t)} \right) \tag{42}$$

for all $t \in \{0, \dots, T\}$.

We first proof the following simple lemma:

Lemma A3. *Suppose that Assumption 1 holds. Then*

$$c \left(u_{cc} - \frac{u_{cn}u_c}{u_n} \right) + n \left(u_{cn} - \frac{u_{nn}u_c}{u_n} \right) < 0$$

as long as $c > 0$ and $n > 0$.

Proof. Normality guarantees that the terms inside the brackets are non-positive. Our strict concavity assumptions guarantee that at least one of them is strictly negative. To see this, suppose not. Then we have that $u_c/u_n = u_{cn}/u_{nn}$ from the second term, and from the first we get $u_{cc} - \frac{u_{cn}^2}{u_{nn}} = 0$ which is a contradiction of our strict concavity assumption. □

We can now state that the multiplier in the resource constraint is always strictly positive:

Lemma A4. *Under Assumptions 1 and 3, in any subproblem (P^T), with $T \geq 1$, the resource constraint binds (i.e., $\mu^T > 0$).*

Proof. Suppose that $\mu^T = 0$. Then, using (42) for some $t \geq 1$, we get:

$$\frac{\eta^T}{u_c(t)} \left[c_t \left(u_{cc}(t) - \frac{u_{cn}(t)u_c(t)}{u_n(t)} \right) + n_t \left(u_{cn}(t) - \frac{u_{nn}(t)u_c(t)}{u_n(t)} \right) \right] = 0 \tag{43}$$

Using Lemma A3, it follows that, for the above equation to hold, η^T must be zero. Using (40) and interiority, we get

$$r^T = -\sum_{s=0}^{t} \beta^{s-t} \lambda_s^T \frac{\partial W_s}{\partial u_t}$$

for all $t \geq 0$. Given that $\lambda_t^T \geq 0$ and that $\partial W_t/\partial u_t > 0$ by Assumption 3, it follows then that $r^T = \lambda_t^T = 0$ for all $t \leq T$. But this is a contradiction, as $\{r^T, \mu^T, \eta^T, \lambda_0^T, \dots, \lambda_T^T\}$ cannot be identically zero. □

Then, we have the following lemma:

Lemma A5. *There exists a non-negative constant C_1 such that $\eta^T/\mu^T = C_1$ for any subproblem (P^T) with $T \geq 1$.*

Proof. Note that for any subproblem with $T \geq 1$ we have that:

$$\frac{\eta^T}{\mu^T} = \left(\frac{\beta}{q_1}\right)\left[\left(u_{cc}(1) - \frac{u_{cn}(1)u_c(1)}{u_n(1)}\right)c_1^\star + \left(u_{cn}(1) - \frac{u_{nn}(1)u_c(1)}{u_n(1)}\right)n_1^\star\right]^{-1}$$
$$\times \left(1 + \frac{F_n(1)u_c(1)}{u_n(1)}\right)$$

which follows from equation (42), Lemma A3 and Lemma A4. The left hand side of the above equation is a constant for all $T \geq 1$, as it is only a function of the allocation at time $t = 1$: $\{c_1^\star, n_1^\star, k_1^\star\}$. Hence, η^T/μ^T is constant. Non-negativity follows from the non-negativity of the Lagrange multipliers. $\quad\square$

We can now prove Proposition 2:

Note that $\frac{\beta^t}{p_t^\star} = \Pi_{s=0}^t \beta\left(1 + r_s^\star\right)$. Remember that $\beta(1 + r_t^\star) > 0$ for all t. And then, $\beta(1 + r_t^\star) < M_1 < 1$ for sufficiently large t implies that $\lim_{t\to\infty} \beta^t/q_t = 0$. Using Lemma A5 and equation (42), we have that:

$$C_1\left(\frac{\beta^t}{q_t}\right)\left[\left(u_{cc}(t) - \frac{u_{cn}(t)u_c(t)}{u_n(t)}\right)c_t + \left(u_{cn}(t) - \frac{u_{nn}(t)u_c(t)}{u_n(t)}\right)n_t\right]$$
$$= 1 + \frac{F_n(t)u_c(t)}{u_n(t)} = -\tau_t^n$$

Now note that for sufficiently large t, $c_t > \epsilon_c$ for some $\epsilon_c > 0$ and $n_t < \epsilon_n$ for some $\epsilon_n < \bar{n}$, by the no-immiseration condition. Assumption 1 (iv) then guarantees that the terms inside the square brackets are bounded. Taking the limits as $t \to \infty$, it follows then that $\lim_{t\to\infty} \tau_t^n = 0$. $\quad\square$

A.4. Proof of Lemma 2

Proof. Let's proceed by contradiction. Suppose that for some $t > 0$, $W_t = \underline{U}$ and $W_{t+1} > \underline{U}$. Then, from the first order conditions we know that $c_t \geq c_{t+1}$ and $n_t \leq n_{t+1}$. This implies that $u_t \equiv u(c_t, n_t) \geq u(c_{t+1}, n_{t+1}) \equiv u_{t+1}$. However note that $u_t + \beta(u_{t+1} + \beta u_{t+2} + \dots) = \underline{U}$ and that $u_{t+1} + \beta u_{t+2} + \beta^2 u_{t+3} + \dots > \underline{U}$ by the hypothesis of the lemma. Then, it must be that $u_t + \beta\underline{U} < \underline{U}$, and thus $u_t < (1 - \beta)\underline{U}$. From above, we know then that $u_{t+1} \leq u_t < (1 - \beta)\underline{U}$. Using that $u_{t+1} + \beta u_{t+2} + \dots > \underline{U}$, we have that $u_{t+2} + \beta u_{t+3} + \dots > (\underline{U} - u_{t+1})/\beta > (\underline{U} - (1 - \beta)\underline{U})/\beta = \underline{U}$. It follows then that at $t + 2$ the borrowing constraint is not binding and thus, $u_{t+2} \leq u_{t+1}$. Proceeding in this fashion we can show that $u_{t+s} \leq (1 - \beta)\underline{U}$ for all $s > 1$. But this violates the borrowing constraint as it implies that $\sum_{s=0}^\infty \beta^s u_{t+s+1} < \underline{U}$. $\quad\square$

A.5. Proof of Proposition 3

We solve the household's problem in the presence of a sequence of debt constraints using Lagrangian techniques. If u is bounded, then the constraint set maps the commodity space into ℓ_∞, the space of bounded sequences. Standard results (see Luenberger, 1969) imply that necessary conditions for an optimum is that the associated Lagrangian is at a stationary point. One technical detail is that the multipliers are from the dual of ℓ_∞, which is larger than ℓ_1, the space of summable sequences. However, it can be shown that for the household's problem, the multipliers are indeed sequences of summable, non-negative numbers (see Dechert, 1982 and Rustichini, 1998). For environments in which u is not bounded over the relevant commodity space (e.g.,

$u(0, n) = u(c, \bar{n}) = -\infty)$, we cannot ensure the validity of the Lagrangian with summable multipliers. We proceed assuming that if u is unbounded, such multipliers do indeed exist and the usual first order and complementary slackness conditions are necessary at an interior optimum. This assumption is only necessary for the case in which household's face debt constraints. The main propositions in the case with only sovereign debt constraints are proven without these additional assumptions.

Proof. The if part: The "if part" follows from the proof of Proposition 1 above. With $\gamma_t = 0$ for all t, the household's first order conditions are satisfied at the defined prices and allocations, as well as the complementary slackness on the household debt constraint.

The only if part: For part (a), the necessity of the resource constraint follows from the definition of equilibrium. It follows from (27) that the budget constraint of the household must hold with equality in a competitive equilibrium. The first order conditions plus the household's budget constraint imply (7) holds for a sequence of transfers $T_t \geq 0$ and a sequence of $\gamma_t \geq 0$, with $\gamma_0 = 0$ and $\gamma_t = 0$ whenever $V_t > 0$. We need to show that this allocation must satisfy the household's first order and complementary slackness conditions at $\gamma_t = 0$ for all t. We begin with the following claim:

Lemma A6. *Suppose that $\{c_t, n_t\}$ solves the household's problem for a sequence of prices (p_t, w_t), punishments χ_t, and transfers T_t. Let Γ_t denote the associated multipliers. If there exists a t_0 such that $V_{t_0} = 0$, then*

$$\sum_{t=t_0}^{\infty} \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t) n_t \right] \leq 0. \tag{44}$$

Proof. We proceed by contradiction. Suppose there exists t_0 such that $V_{t_0} = 0$ but the left hand side of (44) is strictly positive. From the household's first order conditions, we can substitute in prices and write this as:

$$\sum_{t=t_0}^{\infty} p_t \left[c_t - T_t - w_t n_t \right] > 0. \tag{45}$$

As $V_{t_0} = 0$, we have

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t, n_t) = \underline{V}_{t_0} \leq \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(w_t \tilde{n}_t + T_t, \tilde{n}_t),$$

where $\tilde{n}_t \equiv \operatorname{argmax}_{\hat{n}_t} u(w_t \hat{n}_t + T_t, \hat{n}_t)$. The last inequality follows from the upper bound assumption on deviation utility and that $\chi_t \in [0, 1]$. Moreover, defining $\tilde{c}_t \equiv w_t \tilde{n}_t + T_t$, the same inequality implies that

$$\sum_{t=t_0}^{\infty} p_t(c_t - T_t - w_t n_t) \leq \sum_{t=t_0}^{\infty} p_t(\tilde{c}_t - T_t - w_t \tilde{n}_t) = 0,$$

a contradiction of (45). □

We now show that any equilibrium allocation must be decentralizable with $\gamma_t = 0$ for all t:

Lemma A7. *If $\{c_t, n_t, k_t, T_t, p_t, w_t, \tau_t^k, \tau_t^n, \phi_t^k, \chi_t\}$ is a competitive equilibrium, then $\{c_t, n_t, k_t, \tilde{T}_t, \tilde{p}_t, w_t, \tau_t^k, \tau_t^n, \tilde{\phi}_t^k, \chi_t\}$ is also a competitive equilibrium, where*

$$\tilde{p}_t = \beta^t \frac{u_c(c_t, n_t)}{u_c(c_0, n_0)}, \forall t. \tag{46}$$

That is, prices are such that $\Gamma_t = 1$ for all t.

Proof. Define $\Delta \equiv \sum_{t=0}^{\infty} (p_t - \tilde{p}_t)(c_t - w_t n_t - T_t)$, as the impact on the budget constraint of the alternative price sequence. The household's budget constraint in the original equilibrium implies:

$$a_0 = \sum_{t=0}^{\infty} \tilde{p}_t (c_t - w_t n_t - T_t) + \Delta.$$

If $\Delta \leq 0$, then the allocation satisfies the household's budget constraint under the new prices. Note that by definition, $\tilde{p}_t = p_t / \Gamma_t$. We have

$$\Delta = \frac{1}{u_c(c_0, n_0)} \sum_{t=0}^{\infty} \left(1 - \frac{1}{\Gamma_t}\right) \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t)n_t\right].$$

Let $\lambda_t = \left(1 - \Gamma_t^{-1}\right)$. Note that $\lambda_0 = 0$, $\lambda_t = \lambda_{t-1}$ if $\gamma_t = 0$ and $\lambda_t > \lambda_{t-1}$ if $\gamma_t > 0$. Let $\{t_0, t_1, \dots\}$ denote the elements of the set $T = \{t | \gamma_t > 0\}$. We can then write:

$$\Delta = \sum_{t=t_0}^{\infty} \lambda_t \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t)n_t\right]$$

$$= \lambda_{t_0} \sum_{t=t_0}^{\infty} \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t)n_t\right]$$

$$(\lambda_{t_1} - \lambda_{t_0}) \sum_{t=t_1}^{\infty} \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t)n_t\right]$$

$$(\lambda_{t_2} - \lambda_{t_1}) \sum_{t=t_1}^{\infty} \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t)n_t\right]$$

$$+ \dots .$$

From Lemma A6, $\sum_{t=t_k}^{\infty} \beta^t \Gamma_t \left[u_c(c_t, n_t)(c_t - T_t) + u_n(c_t, n_t)n_t\right] \leq 0$ for all $t_k \in T$, and so $\Delta \leq 0$. To ensure that the household's budget constraint holds with equality at the alternative prices, define $\tilde{T}_0 = T_0 - \Delta \geq 0$. The sequence of $\tilde{\phi}_t$ can be recovered from $\tilde{p}_t$ as usual. The wages and source-based taxes can be left as in the original allocation and satisfy the conditions of equilibrium. $\quad\square$

We now show that any equilibrium with positive transfers can be supported with zero transfers:

Lemma A8. *If $\{c_t, n_t, k_t, T_t, p_t, w_t, \tau_t^k, \tau_t^n, \phi_t^k, \chi_t\}$ is a competitive equilibrium, then there exists an alternative equilibrium with $\tilde{T}_t = 0$ for all $t \geq 1$.*

Proof. From the preceding lemma, we only need consider equilibria for which $\Gamma_t = 1$ for all t, as any alternative equilibrium can be supported as a $\Gamma_t = 1$ equilibrium. As transfers only affect the household's problem, we need to check that the original allocation satisfies the household's problem at the original prices but with a new sequence of transfers. Consider $\tilde{T}_0 = \sum_{t=0}^{\infty} p_t T_t$, and $\tilde{T}_t = 0$ for all $t \geq 1$. Substituting $\tilde{T}_t$ for T_t has no impact on the household's budget constraint, by construction. Note that $\underline{V}_t\left(\{w_s, T_s\}_{s \geq t}\right) \geq \underline{V}_t\left(\{w_s, 0\}_{s \geq t}\right)$ for all t by monotonicity of deviation utility with respect to transfers. Therefore the new allocation satisfies the household debt constraints. The fact that $\Gamma_t = 1$ implies that complementary slackness continues to hold. Therefore, the original allocation with the new sequences of transfers continues to satisfy the household's problem at the original prices. □

Lemmas A6 through A8 prove that any competitive equilibrium can be implemented with zero transfers after the first period and $\Gamma_t = 1$ for all t. Using the fact that $T_0 \geq 0$, we can rewrite (7) as (IC). □

A.6. Proof of Corollary 1

Proof. Given the result of Proposition 3, the efficient allocation problem with household debt constraints is the same as the benchmark problem (P) with the addition of constraint (30) and the additional choice sequence $\{\chi_t\}_{t=0}^{\infty}$. The policy of setting $\chi_t = 0$ for all t minimizes the sequence of $\underline{V}_t$'s, and thus maximizes the set of allocations that satisfy the household's debt constraint without affecting any other aspects of the problem. Any efficient allocation must therefore satisfy this constraint, and we can thus confine attention to the problem subject to $V(c_t, n_t | 0, 0, 0) \geq 0$. Note that setting $\chi_t = 0$ is full garnishment of wages and thus removes equilibrium prices from the constraint set, making the constraint identical to a sovereign debt constraint. This renders the problem isomorphic to the benchmark model, and so the result of Proposition 2 holds. □

A.7. Proof of Proposition 4

The proof of the closed economy case follows that of the open economy (Proposition 1), and we omit the details.

A.8. Proof of Proposition 5

Proof. The proof of the closed economy result follows the open economy proof (Proposition 2). The only difference is we now face a sequence of resource constraints, rather than a single present value constraint. As before, let $\{c^\star, n^\star, k^\star\}$ be an interior efficient allocation, and we consider the subproblem (P^T). It will be useful to define a closed economy counterpart to q_t:

$$\tilde{q}_t \equiv \prod_{s=0}^{t} \left(F_k(k_s^\star, n_s^\star) + 1 - \delta\right)^{-1}.$$

Note that $F_k(k_s^\star, n_s^\star) + 1 - \delta > 0$ for $\delta \in (0, 1)$. Let η^T be the multipliers on (IC′), $\beta^t \psi_t^T$ be the multiplier on the sequence of resource constraints (RC), and $\beta^t \lambda_t^T$ be the multiplier on the sequence of debt constraints. As in the proof of Proposition 2, we let $r^T \geq 0$ be the multiplier on the objective function. The first order conditions are:

$$r^T + \eta^T + \eta^T \left(\frac{u_{cc}(c_t, n_t)}{u_c(c_t, n_t)}(c_t - \mathbb{1}_{\{0\}}(t)a_0) + \frac{u_{cn}(c_t, n_t)}{u_c(c_t, n_t)}n_t \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s^T \frac{\partial W_s}{\partial u_t}$$
$$= \psi_t^T \frac{1}{u_c(c_t, n_t)},$$

$$r^T + \eta^T + \eta^T \left(\frac{u_{cn}(c_t, n_t)}{u_n(c_t, n_t)}(c_t - \mathbb{1}_{\{0\}}(t)a_0) + \frac{u_{nn}(c_t, n_t)}{u_n(c_t, n_t)}n_t \right) + \sum_{s=0}^{t} \beta^{s-t} \lambda_s^T \frac{\partial W_s}{\partial u_t}$$
$$= -\psi_t^T \frac{F_n(k_t, n_t)}{u_n(c_t, n_t)},$$

for all $t \in \{0, \dots, T\}$, and

$$\psi_t^T (F_k(k_t, n_t) + 1 - \delta) + \sum_{s=0}^{\infty} \beta^{s-t} \lambda_s^T \frac{\partial W_s}{\partial k_t} = \frac{\psi_{t-1}^T}{\beta},$$

for all $t \geq \{1, \dots, T\}$.

As in the open economy case, in any subproblem (P^T), with $T \geq 1$, the resource constraint binds for every $1 \leq t \leq T$ (i.e., $\psi_t^T > 0$). The proof follows that of Lemma A4 and we omit it.

Define $\tilde{C}_1$ to be the closed economy counterpart of C_1 from Lemma A5. Specifically:

$$\tilde{C}_1 = \frac{\tilde{q}_1 \eta^T}{\beta \psi_1^T}$$
$$= \frac{\tilde{q}_1}{\beta} \left[\left(u_{cc}(1) - \frac{u_{cn}(1)u_c(1)}{u_n(1)} \right) c_1^\star + \left(u_{cn}(1) - \frac{u_{nn}(1)u_c(1)}{u_n(1)} \right) n_1^\star \right]^{-1}$$
$$\times \left(1 + \frac{F_n(1)u_c(1)}{u_n(1)} \right).$$

From the first order condition for capital, we have for $1 \leq t \leq T$:

$$\psi_t^T = \frac{\psi_{t-1}^T}{\beta} \left(F_k(k_t, n_t) + 1 - \delta + \frac{1}{\psi_t^T} \sum_{s=0}^{\infty} \beta^{s-t} \lambda_s^T \frac{\partial W_s}{\partial k_t} \right)^{-1}$$
$$\geq \frac{\psi_{t-1}^T}{\beta} \frac{\tilde{q}_t}{\tilde{q}_{t-1}},$$

where the last line follows from the definition of $\tilde{q}$, the equivalence of (c_t, n_t, k_t) with $(c_t^\star, n_t^\star, k_t^\star)$, and the fact that $\frac{1}{\psi_t^T} \sum_{s=0}^{\infty} \beta^{s-t} \lambda_s^T \frac{\partial W_s}{\partial k_t} \leq 0$. Iterating on this last inequality backwards in time until period 1, we have

$$\psi_t^T \geq \frac{\tilde{q}_t}{\beta^t} \frac{\psi_1^T \beta}{\tilde{q}_1}.$$

Returning to the labor tax, subtracting the first two first-order conditions, we have for $t \in \{1, \dots, T\}$:

$$-\tau_t^n = \frac{\eta^T}{\psi_t^T} \left[\left(u_{cc}(t) - \frac{u_{cn}(t)u_c(t)}{u_n(t)} \right) c_t + \left(u_{cn}(t) - \frac{u_{nn}(t)u_c(t)}{u_n(t)} \right) n_t \right],$$

which implies

$$
\left| \tau_t^n \right| =
$$

$$
\frac{\eta^T}{\psi_t^T} \left| \left(u_{cc}(t) - \frac{u_{cn}(t) u_c(t)}{u_n(t)} \right) c_t + \left(u_{cn}(t) - \frac{u_{nn}(t) u_c(t)}{u_n(t)} \right) n_t \right|
$$

$$
\leq \frac{\beta^t}{\tilde{q}_t} \frac{\eta^T \tilde{q}_1}{\psi_1^T \beta} \left| \left(u_{cc}(t) - \frac{u_{cn}(t) u_c(t)}{u_n(t)} \right) c_t + \left(u_{cn}(t) - \frac{u_{nn}(t) u_c(t)}{u_n(t)} \right) n_t \right|
$$

$$
= \frac{\beta^t}{\tilde{q}_t} \tilde{C}_1 \left| \left(u_{cc}(t) - \frac{u_{cn}(t) u_c(t)}{u_n(t)} \right) c_t + \left(u_{cn}(t) - \frac{u_{nn}(t) u_c(t)}{u_n(t)} \right) n_t \right|.
$$

The assumptions listed in Proposition 5 imply $\beta^t / \tilde{q}_t \to 0$, and that the final term is finite. Therefore $\tau_t^n \to 0$. $\square$

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