Sovereign Debt Crises and Floating-Rate Bonds*

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Abstract

Sovereign debt markets are plagued by a number of frictions; in particular, a limited commitment to repay, limited commitment to future fiscal policies (debt dilution), lack of state contingency, and vulnerability to self-fulfilling runs. We use an analytical model to explore the role of maturity in mitigating or exacerbating the respective frictions. We show that long-term debt with a variable (but capped) coupon combines many of the desirable properties of both short-term and long-term bonds. We then turn to a quantitative model to explore the welfare benefits or costs of issuing floating rate bonds.

1 Introduction

The choice of sovereign debt maturity in countries at risk of default represents a complex set of competing forces. The tradeoffs reflect the underlying frictions present in international sovereign debt markets.

A primary friction is the lack of state contingency in debt contracts. This generates two forces in terms of maturity choice. The first is that long-term bonds may be a useful tool for a government to hedge to shocks to the cost of funds, say arising from business cycle fluctuations. However, the lack of contingency opens the door to default occurring in equilibrium. Because of the government's inability to commit to future fiscal decisions, bondholders are subject to future dilution of their claims. This generates an opposite force: Short-term bonds provide protection from future dilution and, as we shall see, provide better incentives to the government to minimize the costs of default.

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This trade-off between insurance and incentives is fundamental to the maturity choice, but misses another element. The presence of a significant stock of debt in short-maturity by itself generates another potential risk: it leaves the government vulnerable to self-fulfilling roll-over crises. This is probably the main drawback of short-term debt — and perhaps the reason why so many restructurings involve lengthening the maturity structure.

In this paper, we explore the advantage of an alternative contract: *a floating rate long-term bond with a coupon cap*. Through both a simple analytical framework, as well as in a richer quantitative framework, we explore the relative benefits of such bonds. We show that having a coupon on a long-term bond indexed to one-period-ahead default probabilities provides all the incentive properties of one-period bonds, and the coupon cap reduces the vulnerability to rollover risk. This floating rate contract can be implemented by indexing the coupon to the auction price of a small amount of one-period bonds.

The framework we explore has both dilution and rollover risk. Dilution risk is well known in the literature (Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012), and Aguiar, Amador, Hopenhayn and Werning (2019) argue that when default risk is high, it is optimal for the government to issue only short-term bonds. This is the case in many real world crises, as originally documented by Broner, Lorenzoni and Schmukler (2013). Indeed, Bocola and Dovis (2019) argue that the observed shortening of maturity of new issuances of Italian bonds implies a limited role for rollover risk in the European debt crisis. This runs counter to the conventional wisdom that developed in the wake of Mario Draghi's "Whatever it takes" speech in the summer of 2012. That wisdom holds that the crisis was a self-fulfilling run by creditors that was solved by the European Central Bank stepping in as the lender of last resort.

Rollover risk was a prominent theme after Mexico's 1994-95 crisis. Cole and Kehoe (1996) and Cole and Kehoe (2000) used that crisis as a launching point for their model of rollover risk. Alesina and Tabellini (1990) provide an earlier analysis of self-fulfilling failed auctions. In fact, our discussion of dilution versus rollover risk mirrors that of Alesina and Tabellini (1990), who discuss the experience of floating rate Italian nominal bonds as the best response to weak inflation credibility and rollover risk.

Aguiar and Amador (2021b) provide additional evidence on the presence of rollover risk in the data. In particular, they analyze market swaps that involve issuing long-term bonds to repurchase short-maturity bonds. For a case involving the Dominican Republic in 2020, they show that the price of *all* bonds increases at the time of the swap, including those of the long-term bonds being issued. They use an analytical framework similar to one used below to argue that this is evidence that rollover risk is a prominent feature of the data.

The environments we study hew fairly closely to the quantitative sovereign debt literature. The main source of risk is endowment risk, to which we add the possibility of a self-fulfilling failed auction. The calibration is based on the benchmark long-term bond paper, Chatterjee and Eyigungor (2012). We find that issuing floating-rate bonds eliminates the risk of a self-fulfilling run while preserving the incentives of one-period bonds. In particular, the government's welfare in the floating-rate bond model in the presence of rollover risk is similar to that of a government with one-period bonds and zero chance of a rollover crisis. Moreover, the floating-rate model dominates the fixed-rate long-term bond model. Welfare gains of switching to floating-rate bonds at zero debt are roughly one percent of consumption.

A few caveats are in order to temper these conclusions. One is that we assume the government can auction small amounts of one-period bonds in order to index the coupon payments on the long-term floating rate bond. This abstracts from liquidity issues in bond markets. Moreover, Alesina, Prati and Tabellini (1990) argue that there is evidence that the Italian benchmark bond auctions may have been manipulated, a possibility we omit from the analysis. Finally, we incorporate the hedging benefits of long-maturity bonds by having persistent income shocks. However, this omits other sources of risk that can be hedged by long-term bonds, such as shocks to risk premia or the risk-free rate. This are potentially important drawbacks, which we leave for future work.

While we focus on floating-rate bonds, other bond covenants can be used to deal with both dilution and rollover risk. Floating rate debt is subject to its own source of multiplicity, as studied by Calvo (1988) and, more recently, Ayres, Navarro, Nicolini and Teles (2018). Calvo argues that refusing to issue at a high interest rate can help select the best equilibrium. In this spirit, a cap on the coupon can mitigate the risk of this multiplicity, something we also discuss and incorporate in our analysis. Hatchondo, Martinez and Sosa-Padilla (2016) discuss covenants that compensate legacy lenders for capital losses as a solution to dilution. Finally, beyond contract covenants, fiscal rules (for example, Hatchondo, Roch and Martinez, 2012) have been proposed as the solution to dilution, and alternative auction protocols (for example, Chamon, 2007) have been proposed to remove rollover risk. The advantage of floating-rate bonds are that they do not require commitment to enforce fiscal rules or other non-market mechanisms, relying only on competitive markets.

The paper is organized as follows. Section 2 introduces the general framework absent rollover risk; Section 3 provides some analytical results on the efficiency of one-period bonds; Section 4 introduces rollover risk; Section 5 presents results of the quantitative exercises; and Section 6 concludes.

2 Th Fundamental Risk Model

Our framework is based on the standard environment popular in the quantitative sovereign debt literature (see Aguiar and Amador, 2021a, for a textbook treatment). We extend this framework by introducing floating-rate-coupon bonds. We also alter the model to allow for rollover risk. For expositional reasons, we hold off on the rollover risk extension until after discussing key properties of the baseline model.

Consider a discrete-time, small open economy model. Time is indexed by $t = 0, 1, 2, \ldots$ and the state of nature in time t is given by $s_t \in \mathbb{S}$. The state will index output, default penalties, and, in the extension, include a sunspot that coordinates lenders' beliefs about a crisis. The state s_t follows a first-order Markov process, and we let s^t denote the history of all shocks up to an including time t. We let π denote the unconditional probability.

The economy is run by a government, which evaluates policies with payoffs given by

$$\sum_{t,s^t} \pi(s^t)\beta^t u(c(s^t)),$$

where $c(s^t)$ is consumption of a freely traded good in history s^t. We assume u is strictly increasing and strictly concave. Every period, the government controls an endowment $y(s_t) > 0$ of this tradable good, where y is assumed to lie in discrete and bounded set.

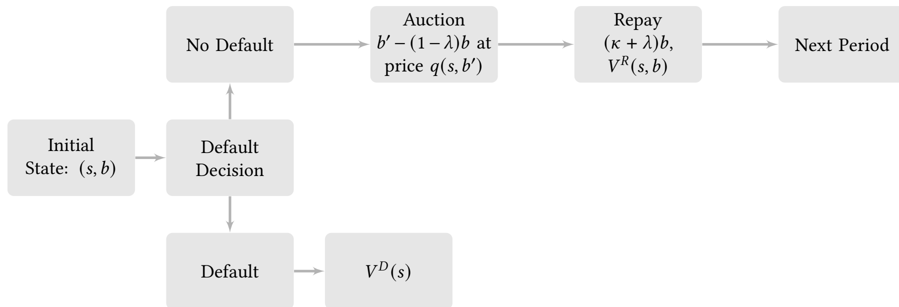
The government trades financial assets with competitive, risk-neutral lenders who discount at rate $R^{-1} = (1+r)^{-1}$. We assume $\beta R \leq 1$. Financial trade is restricted to a non-contingent bond. A bond is characterized by a maturity and a coupon. Each unit of debt matures with probability $\lambda \in [0, 1]$, which is *iid* across units. In any non-trivial portfolio, we therefore assume the fraction λ matures and the fraction $1-\lambda$ remains. The expected maturity is $1/\lambda$. When $\lambda = 1$, we have one-period bonds and when $\lambda = 0$ we have a perpetuity. Such "perpetual youth" bonds are a tractable approach to handling bonds of long maturity, and have been used by Leland (1994), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012) among others.[1]

Let b be the face value of debt at the end of period t and κ the promised coupon next period. Then, in period $t + 1$ the government owes payments of $(\kappa + \lambda)b$ in every continuation state s_{t+1}. To rule out Ponzi schemes, let $\overline{B}$ denote some arbitrary upper bound on debt issuance and restrict $b \in \mathbb{B} = (-\infty, \overline{B}]$. By making $\overline{B}$ such that promised payments are never greater than the natural debt limit, we ensure it never binds along the equilibrium path, and we will suppress the constraint from the notation going forward.

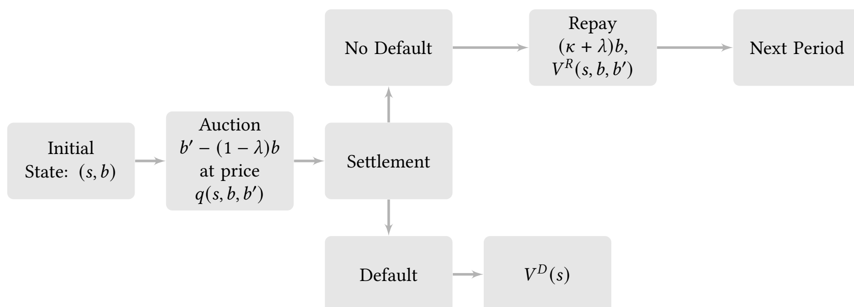
We focus on Markov equilibria in which equilibrium objects are functions of the exogenous state s_t as well as the government's indebtedness. Let $q : \mathbb{S}\times\mathbb{B} \rightarrow [0, 1]$ denote the price schedule,

[1] An alternative and equivalent formalization is that of a bond that has an exponentially declining coupon.

Figure 1: Within-Period Timing



(a) Eaton-Gersovitz Timing



(b) Cole-Kehoe Timing

and $\mathcal{K} : \mathbb{S} \times \mathbb{B} \longrightarrow [0, \overline{\kappa}]$ denote the coupon schedule. In particular $q(s, b')$ represents the price of the bonds and $\mathcal{K}(s, b')$ represents the associated coupon to be paid next period, given today's state s and the new level of debt b' chosen by the government. In the case of a fixed rate bond, the coupon will be a constant. We will discuss the determination of the floating coupon later on. Except for the potential for default, there is no ex-post contingency in the coupon payment once next period's state is realized.

We consider two timing conventions. The first is the "Eaton-Gersovitz" (EG) timing, which is the standard in the literature since Aguiar and Gopinath (2006), Arellano (2008), and Hamann (2002). Under EG timing, depicted in panel (a) of Figure 1, the government first observes nature's draw of s, then commits to either repay or default on outstanding debt, and then auctions off new bonds. In the alternative, "Cole-Kehoe" (CK) timing, the government, after observing s, first auctions new debt and then decides whether to repay or default on outstanding debt. The key distinction is whether the result of the auction plays a role in the repayment decision. In EG timing, repayment is independent of the realized auction price, while in CK repayment is contingent on the success or failure of a bond auction. We begin by discussing the equilibrium under EG timing.

2.1 The Government's Problem

If the government defaults at time t in state s, the lenders receive a payoff of zero and the government receives a payoff of $V^D(s)$. In particular,

$$V^D(s) = u(y^D(s)) + \beta \mathbb{E}_s \left[\theta V(s', 0) + (1-\theta) V^D(s') \right]. \tag{1}$$

The term $y^D(s)$ is the endowment received in default when the state is $s \in \mathbb{S}$. This captures any punishment in terms of loss of endowment due to default as well as the fact that the government must consume hand-to-mouth while excluded from international financial markets. We have assumed that with probability θ the government regains access to bond markets and starts anew with zero debt and value $V(s', 0)$ in state s'. With probability $1-\theta$, the government remains in the default state.[2]

If the government has opted to repay, the government's payoff satisfies the following Bellman equation:

$$\begin{aligned} V^R(s, b, \kappa) = \max_{b' \in \mathbb{B}} \Big\{ & u\Big(y(s) - (\kappa + \lambda) b + q(s, b')(b' - (1-\lambda) b)\Big) \\ & + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) \max \Big\langle V^R(s', b', \mathcal{K}(s, b')), V^D(s') \Big\rangle \Big\}. \end{aligned} \tag{2}$$

where the government's state includes the current realization of s, the inherited debt b, and the promised coupon κ. In solving this problem, the government takes the equilibrium schedules for the price and the coupon, q and $\mathcal{K}$, as given and optimally chooses b'. The continuation value reflects that the government has the option to default next period after observing s'. We can redefine the government's value as a function of (s, b) and the lagged state, s_{-1}, given that $\kappa = \mathcal{K}(s_{-1}, b)$ is determined in equilibrium. Henceforth, we write $V^R(s_{-1}, s, b)$, with $\kappa = \mathcal{K}(s_{-1}, b)$ being the coupon that is due in the current period.

Let $\mathcal{B} : \mathbb{S} \times \mathbb{S} \times \mathbb{B} \to \mathbb{B}$ denote the optimal policy function of the government. Implicitly in problem (2), we are assuming that there exists a b' such that it is feasible to repay; that is, $y(s) - (\kappa + \lambda) b + q(s, b')(b' - (1-\lambda) b) \geq 0$ for some $b' \in \mathbb{B}$. If this is not the case, we set $V^R = -\infty$ so that the government defaults whenever repayment is infeasible.

Define $V(s_{-1}, s, b) \equiv \max \left\langle V^R(s_{-1}, s, b), V^D(s) \right\rangle$ to be the government's value at the start of the period. The government repays if $V^R(s_{-1}, s, b) \geq V^D(s)$ and defaults otherwise. Let $\mathcal{D} : \mathbb{S} \times \mathbb{S} \times \mathbb{B} \to \{0, 1\}$ denote the optimal default policy, with the value one indicating default and zero indicating repayment.

[2] Note that we have abstracted from renegotiation by assuming a zero recovery rate.

2.2 The Lenders' Break-Even Condition

The restriction on equilibrium prices is that lenders break even in expectation. In particular:

$$q(s,b) = \frac{\sum_{s' \in \mathbb{S}} \pi(s'|s) \left[(1 - \mathcal{D}(s,s',b)) \left((\kappa + \lambda) + (1-\lambda) q(s', \mathcal{B}(s,s',b)) \right) \right]}{R}, \tag{3}$$

where $\kappa = \mathcal{K}(s,b)$. The price of the debt reflect the probability of repayment times the coupon payment, κ, the maturing payments, λ, and the value of a (non-maturing) bond next period.

2.3 Fixed and Floating Rate Bonds

We consider two alternative coupon schedules. The standard approach is a constant coupon. In particular, define the "fixed-rate coupon schedule" as $\mathcal{K}(s,b) = \kappa$ for all $(s,b) \in \mathbb{S} \times \mathbb{B}$ for some constant κ.[3]

The second is a floating-rate coupon. The coupon is determined as follows. Consider an equilibrium default probability $\mathcal{D}(s,s',b)$, and let q_S be defined as

$$q_S(s,b) \equiv R^{-1} \sum_{s'|s} \pi(s'|s)(1 - \mathcal{D}(s,s',b)). \tag{4}$$

The value q_S corresponds to the associated price of one-period bond with a promised payment of 1. Note that q_S lies between zero and R^{-1}. We define the "floating rate coupon schedule" as:

$$\mathcal{K}(s,b) \equiv \min \left\{ \frac{1}{q_S(s,b)} - 1, \overline{\kappa} \right\}. \tag{5}$$

for a given finite upper bound $\overline{\kappa}$.

The floating rate coupon corresponds to the yield of an associated one-period bond (as long as the cap is not binding). This coupon compensates the bondholder for the one-period ahead risk of default. It is important to note that this one-period bond is not actually traded in equilibrium[4], unless $\lambda = 1$. Nevertheless, given equilibrium default behavior, we can construct a q_S and $\mathcal{K}$. In particular, q_S is the price that would obtain in equilibrium if an infinitesimal amount of one-period bonds were issued along with the benchmark bonds.

The equilibrium in the floating-rate model depends on $\mathcal{K}$, which, in turn, depends on the default policy function. We are looking for a fixed point of this mapping. There may be more than one, as we discuss at the end of this section.

[3] Note that in this case, the level of the coupon is irrelevant, as just re-indexes what is meant for a unit of a bond.
[4] Or it is maintained at a zero supply.

2.4 Equilibrium

We are now ready to define an equilibrium:

Definition 1. An **Eaton-Gersovitz equilibrium** is a price schedule q, a coupon schedule $\mathcal{K}$, a value function V^R with associated policies $\mathcal{B}$ and $\mathcal{D}$, and a default value V^D such that: (i) The lenders' break-even condition (3) is satisfied given $\mathcal{B}, \mathcal{K}$, and $\mathcal{D}$; (ii) given $\mathcal{D}$, $\mathcal{K}$ is either fixed or determined by equations (4) and (5) depending on whether we have a fixed or a floating rate coupon bond; (iii) given q and $\mathcal{K}$, V^R solves the government's Bellman equation (2) with optimal policy $\mathcal{B}$, (iv) $\mathcal{D}(s, s', b) = 1$ if $V^R(s, s', b) < V^D(s')$ and zero otherwise; and (v) given V^R, V^D solves the recursion (1).

2.5 Prices and Future Fiscal Policies

The two alternative coupon structures have different implications for how future fiscal policy affects bond prices. Under the fixed-rate schedule, equation (3) indicates that for $\lambda < 1$ the debt issuance policy function $\mathcal{B}(s', b)$ affects the price of the non-maturing bonds next period, and hence affects the price of bonds today. This is the standard channel in which lack of commitment to future fiscal policy potentially "dilutes" existing bondholders and depresses the value of long-term bonds. We shall return to this below.

Now consider the floating rate coupon. Suppose that in equilibrium $\mathcal{B}$ is such that there is an upper bound on the ergodic distribution of debt, $B_{max} < \overline{B}$. Moreover, suppose that $q_{min} \equiv \min_{s \in \mathbb{S}} q_S(s, B_{max}) > 0$. That is, along the equilibrium path the government never issues debt to the point that it will default with probability one the next period. Both of these conditions are typically satisfied in standard quantitative sovereign debt models. Then, if $\overline{\kappa} > 1/q_{min} - 1$, a valid equilibrium price schedule is $q(s, b) = 1$ for all $s \in \mathbb{S}$ and $b \leq B_{max}$.[5] In this scenario, the long bond price is constant, and, more importantly, independent of future fiscal policy.

As noted above, in the floating rate case $\mathcal{K}$ is defined by q_S, which in turn depends on equilibrium behavior. And this equilibrium behavior depends on $\mathcal{K}$. There may be multiple fixed points of this mapping. For example, without a finite upper bound $\overline{\kappa}$ on the coupon, there is always and

[5] To see this, define the price operator T_q by equation (3):

$$\begin{aligned}[T_q q](s, b) &= R^{-1}\mathbb{E}\left[(1 - \mathcal{D}(s, s', b))\left(\kappa + \lambda + q(s', \mathcal{B}(s, s', b))(1 - \lambda)\right)\right] \\ &= R^{-1}\mathbb{E}\left[(1 - \mathcal{D}(s, s', b))\left(\frac{1}{q_S(s, b)} - 1 + \lambda + q(s', \mathcal{B}(s, s', b))(1 - \lambda)\right)\right] \\ &= 1 + (1 - \lambda)R^{-1}\mathbb{E}_s(1 - \mathcal{D}(s, s', b))\left(q(s', \mathcal{B}(s, s', b)) - 1\right),\end{aligned}$$

where the last line uses the definition of q_S, that $q_S > 0$, and that the coupon cap is not binding. The operator maps bounded functions on the domain $\mathbb{S} \times (-\infty, B_{max}]$ into itself, and satisfies the Blackwell conditions for a contraction. For any $\mathcal{B}$ such that $\mathcal{B}(s_{-1}, s, b) \leq B_{max}$ on this domain, $q = 1$ is the unique fixed point of the price operator.

equilibrium with zero borrowing. To see this, posit the schedule $\mathcal{K}(s,b) = \infty$ for all $s \in S$ and $b > 0$. For any $b > 0$, it is infeasible for the government to repay, and hence the government will default with probability one, validating $q_S = 0$ and $\mathcal{K} = \infty$. A finite coupon rules out this extreme equilibrium; but at this stage, we do not have sufficient conditions to ensure that there is a unique floating-rate equilibrium.

3 One-Period Bonds as a Planning Problem

With long-term fixed-rate bonds, the existing bondholders are affected by future fiscal policy decisions. One-period fixed-rate bonds do not feature this risk to bondholders: by the time the government decides on future decisions, they have already been paid. A useful way to see this advantage of one-period bonds is to consider the dual of problem (2), as done in Aguiar and Amador (2019).

Specifically, consider problem (2) for the case of $\lambda = 1$ (one period bonds) and normalize $\kappa = 0$. Then (2) can be written as:

$$V^R(s,b) = \max_{c,b' \in \mathbb{B}} \left\{ u(c) + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) \max \left\langle V^R(s',b'), V^D(s') \right\rangle \right\}$$
$$\text{subject to } c \leq y(s) - b + q(s,b')b'.$$

Because $\kappa = 0$, we can drop the lagged s_{-1} as an argument for this exercise. As shown by Aguiar and Amador (2019), on the relevant domain for bonds, $V^R(s,b)$ is strictly decreasing in b for each s.[6] Let $B(s,v)$ denote the inverse of V^R. That is, $V^R(s, B(s,v)) = v$.

Given the strict monotonicity of V^R, B solves the dual problem:

$$B(s,v) = \max_{c,b'} y(s) - c + R^{-1}b' \sum_{s' \in \mathbb{S}} \pi(s'|s) \mathbb{1}(V^R(s',b') \geq V^D(s'))$$
$$\text{subject to } v = u(c) + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) \max \left\langle V^R(s',b'), V^D(s') \right\rangle,$$

where $\mathbb{1}(x)$ is the indicator function that equals one when x is true and zero otherwise, and where we have used the equilibrium condition $q = R^{-1}\mathbb{1}(V^R \geq V^D)$. As $V^R(s,b')$ is strictly decreasing, the choice of b' is also the choice of the government's continuation value. In particular, we can think of adding $v(s')$ as a choice variable subject to the constraint that $v(s') = V^R(s',b')$ for all s' such that $V^R(s',b') \geq V^D(s')$. This constraint is equivalent to $B(s', v(s')) = b'$ for all s' such that

[6] By relevant domain, we mean the domain on which the government can feasibly repay. See Aguiar and Amador (2019) for more details.

$v(s') \geq V^D(s')$. This leads to the following problem:

$$B(s, v) = \max_{c, b', \{v(s')\}_{s' \in \mathbb{S}}} \left\{ y(s) - c + R^{-1} b' \sum_{s' \in \mathbb{S}} \pi(s'|s) \mathbb{1}(v(s') \geq V^D(s')) \right\} \tag{6}$$

subject to:

$$v = u(c) + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) \max \left\langle v(s'), V^D(s') \right\rangle$$

$$b' = B(s', v(s')) \text{ for all } s' \text{ such that } v(s') \geq V^D(s').$$

Problem (6) is similar to an optimal contracting problem. The Principal (lender) chooses a sequence of consumption and continuation values for the Agent (the government) subject to a promise keeping constraint and the "spanning" condition $b' = B(s', v(s'))$. This last condition restricts the span of continuation values and reflects that the one-period bond is non-contingent.

The spanning constraint contains an equilibrium object (the inverse value function). An alternative maturity structure would involve a different restriction on spanning. It may be the case that long-term bonds allow for better hedging of risk, and a true planning problem will not be constrained from implementing such an allocation.

Aguiar and Amador (2019) note that equation (6) defines an operator that maps a function B (which appears in the spanning constraint) into another B function (the one that maximizes the payoff to lenders). They show that this mapping is a contraction and therefore there is a unique equilibrium in the one-period bond model.[7]

Note that the Principal cannot prevent the government from walking away from the contract and taking the outside option V^D. Nevertheless, absent default, the choice of c and b' maximize the joint surplus conditional on the spanning condition. In particular, the equilibrium is the same regardless of whether the government or the lenders set fiscal policy, reflecting that incentives are aligned with one-period bonds.

This alignment of incentives does not hold for long-term bonds, and we cannot write the long-term bond equilibrium as a pseudo-planning problem like (6).[8]

[7] To do this, it is first necessary to relax the spanning condition to an inequality. See that paper for details. In addition, the result requires that there is no re-entry to financial markets after a default, so that $\theta = 0$; so that V^D is given. For an alternative contraction mapping approach, see also Bloise and Vailakis (2022).

[8] One way to see why not mechanically, is that there are three relevant variables for long-term bonds: the face value of debt b, the government's value v, and the market value of debt $q \times b$. With long-term bonds, the equilibrium q depends on future policies that are beyond the control of current actors (either lenders or the incumbent government). In the one-period bond model, absent default, the market value and face value coincide at the start of the period.

An Example

To provide more insight into why incentives are aligned regarding fiscal policy in the one-period bond model, we shut down the endowment fluctuation; that is, we set $y(s) = y$ for all $s \in S$. We set $\theta = 0$, (no re-entry after default). The only remaining risk is the value of default $V^D(s)$, which we allow to vary with the state. Let s be *iid* over time and be such that V^D is drawn from a continuous distribution with CDF $F(\cdot)$ and support $[\underline{V}, \overline{V}]$.

With this *iid* shock process, once the government decides to repay, the current value of s becomes irrelevant, and can be dropped as a state variable. That is, $V^R(s, b)$ can be written $V^R(b)$, and its inverse as $B(v)$. In the dual problem (6) for this case, there is a single continuation value v' and the spanning condition becomes $b' = B(v')$. Substituting the spanning condition into the objective and using the fact that the government repays if $v^D < v'$, we can write the dual problem as:

$$B(v) = \max_{c,v'} \left\{ y - c + R^{-1} F(v') B(v') \right\}$$
$$\text{subject to: } v = u(c) + \beta F(v') v' + \beta \int_{v'}^{\overline{V}} v^D dF.$$

A key distinction between this problem and the original (6) is that without income fluctuations or persistence in the outside option, there is no risk that can be hedged. Bonds of any maturity will either be defaulted on or will have a price that is invariant to the realized default value, v^D, conditional on repayment.

The planner's inverse Euler equation for this problem (assuming $v' \in (\underline{V}, \overline{V})$) is:[9]

$$\frac{1}{u'(c')} = \frac{\beta R}{u'(c)} + \frac{f(v')B(v')}{F(v')}, \tag{7}$$

where $f = dF/dv$ and c' is next period's consumption conditional on repayment and the optimal choice v'. To gain some intuition, set $\beta R = 1$ and let $u(c) = \log(c)$. We then have

$$c' = c + \frac{f(v')B(v')}{F(v')}.$$

The second term on the right-hand side is the marginal probability of default times the amount of debt. If this is strictly positive, then the optimal plan sets $c < c'$. That is, the optimal plan is to save. And the rate of saving is determined by the marginal decline in default probability. The greater $f(v')/F(v')$, the stronger the incentive to save at the margin. This reflects that the risk to the lender is the amount of debt outstanding times the probability of default. The optimal

[9] See Proposition 3 of Aguiar et al. (2019).

contract internalizes that saving reduces this risk.

Now recall that the optimal contracting problem is just an alternative view of the equilibrium in which the government makes all decisions. Why does the government want to reduce the risk of default? Keep in mind that the government strategically defaults, so at the moment of default it captures an increase in value. Why not just wait for a high v^D (say a bailout or forgivable default) and then default?

In equilibrium, it is the price schedule that aligns incentives. Specifically, $q(b') = R^{-1}F(V^R(b'))$. Differentiating:

$$q'(b') = R^{-1}f(v')V^{R\prime}(b'),$$

where $v' = V^R(b')$. From the envelope condition, $V^{R\prime}(b') = -u'(c')$. Substituting in, equation (7) becomes:

$$u'(c)\left(1 + \frac{q'(b')b'}{q(b')}\right) = \beta R u'(c').$$

In the equilibrium, the government saves because $q'(b) < 0$, and it understands that by saving, it will issue/roll over its bonds at a higher price. In particular, the government captures the *entire* benefit of reducing default risk via higher prices: the incentives are aligned between borrower and lender to minimize the risk of default.

Now, it is also the case that $q'(b) < 0$ with long-term bonds. However, the government is not rolling over its entire stock of debt. Thus, it does not internalize the entire cost of default to the lenders, which involves new bonds as well as legacy bonds, and hence does not capture the entire benefit of reductions in default risk. At the extreme case of a perpetuity ($\lambda = 0$), the government does not have to roll over any debt and has no incentive to reduce the risk of default in this case. This is the sense that fiscal policy is *inefficient* with long-term bonds.

3.1 Floating Rate Bonds

With the above insights in hand, we can now see one of the advantages of floating rate bonds. If the coupon on the entire stock of debt reflects the default probability, the government has the same incentive to save as in the case of one-period bonds.

Consider the case discussed previously, where the coupon cap did not bind in equilibrium, and thus $q(s, b) = 1$ for a domain that encompasses the ergodic support, $b \leq B_{max}$.

The government's value conditional on repayment is $V^R(s_{-1}, s, b)$. Recall that the original value function was written $V^R(s, b, \kappa)$. For an equilibrium $\mathcal{K}$, we replaced κ with s_{-1}. To construct a pseudo-planning problem, we do not substitute out $\mathcal{K}$ but include it explicitly as a constraint in the dual problem. Specifically, let $B(s,v,\kappa)$ be the inverse of $V^R(s,b,\kappa)$. The government's budget constraint (with $q(s,b')=1$) is:

$$c = y(s) - (\kappa+\lambda)b + b' - (1-\lambda)b.$$

Let $\tilde{b} \equiv (1+\kappa)b$ and let $\tilde{B}(s,v,\kappa) \equiv (1+\kappa)B(s,v,\kappa)$. Then, the dual problem becomes:

$$\tilde{B}(s,v,\kappa) = \max_{c,b',\kappa',\{v(s')\}} \left\{ y(s) - c + \frac{\tilde{b}'}{1+\kappa'} \right\} \tag{8}$$

subject to:

$$v = u(c) + \beta \max \sum_{s'\in\mathbb{S}} \pi(s'|s) \max \left\langle v(s'), V^D(s') \right\rangle$$

$$\tilde{b}' = \tilde{B}(s', v(s'), \kappa') \text{ for all } s' \text{ such that } v(s') \geq V^D(s')$$

$$\kappa' = \mathcal{K}(s,b'),$$

where we have suppressed the ergodic set constraint that $v(s')$ must be such that $b' = B(s', v(s'), \kappa') \leq B_{max}$, as it should not bind in this case.

Assuming that the coupon cap is not binding, a final equilibrium restriction requires that

$$\frac{1}{1+\mathcal{K}(s,b')} = R^{-1} \sum_{s'\in\mathbb{S}} \pi(s'|s) \mathbb{1}(v(s'|s,v,\kappa) \geq V^D(s')).$$

where $v(s'|s,v,\kappa)$ is an optimal policy of (8).

Note that we can drop the κ from the state space in Problem (8). And substituting the above, we obtain the exact same problem as Problem (6). As long as the coupon cap does not effectively binds in the equilibrium path, the floating rate bond provides all the same incentive and spanning features as one-period bonds.

As we discussed before, the finite upper bound allows us to rule out the case where $\kappa = \infty$, and the government cannot issue any finite level debt.

So to summarize: with the EG timing, a floating rate bond is effectively equivalent to a one-period fixed-rate bond (as long as the coupon cap is not binding). However, there is a dimension in which the floating rate bond is an improvement: it makes the government more robust to self-fulfilling rollovers crisis. We proceed to discuss this next.

4 Rollover Risk

To introduce rollover risk, we alter the timing within a period. The government first auctions debt, and then decides to repay maturing debt. This timing makes the repayment decision contingent on the outcome of the auction. See panel (b) of Figure 1.

We begin with the fixed-rate coupon environment. Working backward through the period, suppose the government has issued $b' - (1-\lambda)b$ bonds at price q during the auction. At the time of settlement, the government's value of repayment is:

$$V^R(s,b,b',q) = u(y(s) - (\kappa+\lambda)b + q \times (b' - (1-\lambda)b)) + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) V(s',b'),$$

where we have re-purposed the notation to fit the current environment. We can let s index the price as well, so that $q = q(s,b,b')$, and drop q as an argument of the repayment value.

The default payoff is the same as in the EG benchmark.[10] The government defaults if $V^R(s,b,b') < V^D(s)$. The government's problem at the time of auction is:

$$V(s,b) = \max\left\{\max_{b'} V^R(s,b,b'), V^D(s)\right\}$$

Note that there is perfect foresight within a period, and hence the government knows what the payoffs to repayment and default are. Let $\mathcal{B}(s,b)$ denote the debt issuance policy, where $\mathcal{B}(s,b) = \arg\max_{b'} V^R(s,b,b')$, and $\mathcal{D}(s,b) = 1$ if $\max_{b'} V^R(s,b,b') < V^D(s)$ and zero otherwise.

To see the indeterminacy in this environment, fix the continuation equilibrium. Specifically, given (s,b), hold the function $\sum_{s' \in \mathbb{S}} \pi(s'|s)V(s',b')$ constant in the government's problem, as well as future policies. Let $\overline{q}(s,b')$ be the break-even price *conditional* on repayment in the current period. That is,

$$\overline{q}(s,b') \equiv R^{-1} \sum_{s' \in \mathbb{S}} \pi(s'|s)(1 - \mathcal{D}(s',b')) \left[\kappa + \lambda + (1-\lambda) q(s',b',\mathcal{B}(s',b'))\right].$$

Note that this is identical to (3); the only difference is that the policy functions may differ in an environment with rollover risk. This is the "good" equilibrium.

To see the "crisis" equilibrium, suppose that $q(s,b,b') = 0$ for all $b' \geq 0$. In this case for $b' \geq 0$,

$$V^R(s,b,b') \leq V^R(s,b,0) = u(y(s) - (\kappa+\lambda)b) + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) V(s',(1-\lambda)b).$$

[10] For simplicity, we assume that if the government auctions debt at a positive price and then defaults, the auction proceeds are lost to both parties. On the equilibrium path, this never occurs.

The government must pay the entire amount of maturing debt plus coupon out of current endowment. It then carries over non-maturing debt into next period. This is a failed auction. If

$$u(y(s) - (\kappa + \lambda)b) + \beta \sum_{s' \in \mathbb{S}} \pi(s'|s) V(s', (1 - \lambda)b) < V^D(s),$$

then a zero price is consistent with the lenders' break-even condition.[11] The lenders see that the government will default at settlement, and refuse to pay a positive price at auction. Such a scenario is possible if $(\kappa + \lambda)b$ is large relative to y.

For pairs of b and y such that both $q = 0$ and $q = \overline{q}$ are possibilities, following Cole and Kehoe (2000) we let a sunspot coordinate beliefs. That is, we let the state s contain a random variable that takes a value one for a crisis and zero otherwise.

In the case of short-term debt, $\lambda = 1$, the debt burden is particularly painful after a rollover crisis. This is the logic behind why short-term debt makes a government particularly vulnerable to a rollover crisis. Conversely, if $\lambda = 0$, for a given face value the repayment burden is light, and a crisis is possible only for very large b.

This sets up the canonical maturity dilemma. On the one hand, short-term debt provides correct incentives. On the other, it exposes the country to rollover risk, and, perhaps, offers less spanning of income risk. A floating rate coupon bond provides the same incentives as one-period debt, but defers the maturity payments, mitigating rollover risk. Indeed, if we ignore spanning (as in our simple model without income risk), then the floating rate perpetuities offers the best of both worlds – correct incentives but limited rollover risk.[12]

The only draw back is that a long-term bond may provide better hedging of income and other potential risks, but this is a quantitative question. In the next section, we therefore turn to a quantitative model that incorporates floating-rate debt and non-insurable income risk.

5 A Quantitative Model

In this section, we introduce income risk as well as rollover risk in a quantitative model. We explore five alternatives: a one-period bond EG model (henceforth EG-ST); a one-period bond model with rollover risk (CK-ST); the same two environments but with long-term fixed-rate bonds (EG-LT and CK-LT); and finally a long-maturity floating-rate bond (FR) *with rollover risk*. As we shall see, the long-maturity floating-rate bond significantly eliminates the risk of a rollover crisis,

[11] We assume the government cannot repurchase long-term bonds at zero price. See Aguiar and Amador (2013) for how this can be supported in equilibrium.

[12] A floating-rate equilibrium is constructed in the presence of rollover risk along the same lines as in the benchmark EG model. That is, we price a one-period bond, which now must compensate lenders for rollover risk as well as "fundamental" default risk, and set the coupon to compensate lenders for that risk.

so we do not need to present the floating-rate bond under the Eaton-Gersovitz timing (in addition to the Cole-Kehoe timing).

The benchmark parameterization is the same as Chatterjee and Eyigungor (2012) (henceforth, CE12).[13] The model is quarterly. The underlying process for log income follows:

$$\begin{aligned} \ln y_t &= x_t + z_t \\ x_t &= \rho x_{t-1} + \varepsilon_t \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) \\ z &\sim TruncatedN(0, \sigma_z^2). \end{aligned}$$

Following CE12, we set $\rho = 0.95$, $\sigma_\varepsilon = 0.027$, $\sigma_z = 0.01$. The persistent process x is approximated by Tauchen's method with a span of 3 standard deviations of the long-run distribution. The *iid* shock z is a truncated Normal with support $[-2\sigma_z, +2\sigma_z]$, and is included for computational reasons, as discussed by CE12.

In default, the endowment is reduced by a quadratic factor. Specifically,

$$\begin{aligned} \ln y_t^D &= x_t^D + z_t \\ x_t^D &= x_t - \max\{0, -0.189 x_t + 0.246 x_t^2\}. \end{aligned}$$

In the first period of default, we set $z = \underline{z}$, its minimum value.

The government's preferences are comprised by a constant relative risk aversion felicity with a risk-aversion parameter 2, and a discount factor $\beta = 0.95$. The risk-free interest rate is $R = 1.01$. The benchmark maturity is $\lambda = 0.05$, or an expected maturity of 20 quarters. For the one-period bond models, we set $\lambda = 1.0$. We set $\kappa = 0.01$ for all models with fixed-rate bonds.

Finally, in the environments with rollover risk, we set the probability of a sunspot to 10% quarterly, although the frequency of crises will be lower in equilibrium. A rollover crisis occurs only if the sunspot is realized and debt is large enough. We assume the probability of a crisis is *iid* over time. See Bocola and Dovis (2019) for a quantitative model in which the probability of a crisis follows a persistent process.

In Table 1, we report ergodic moments for the five models, plus an additional floating rate model in which $\overline{\kappa}$ is set at 0.015, fifty basis points higher than the risk free net interest rate of 0.01. A few things stand out. One is that with short-term debt, the presence of rollover risk looms large. Comparing EG-ST with CK-ST, debt is much lower in the latter and one hundred percent of the defaults are due to self-fulfilling runs. With long-term debt (EG-LT and CK-LT), rollover

[13]The code and additional computational information is available at https://github.com/manuelamador/floatingrate.

Table 1: Moments of the Ergodic Distribution

	FR	EG-ST	CK-ST	EG-LT	CK-LT	FR low $\overline{\kappa}$
$\mathbb{E}\left[b'/y\right]$	0.77	0.82	0.38	0.94	0.94	0.87
$\mathbb{E}\left[q \times b'/y\right]$	0.77	0.82	0.37	0.72	0.72	0.78
Default Rate†	0.002	0.003	0.002	0.067	0.067	0.033
$\mathbb{E}[r - r^{\star}]^{\dagger}$	0.003	0.003	0.002	0.080	0.080	0.038
StDev$(r - r^{\star})^{\dagger}$	0.004	0.004	0.003	0.044	0.044	0.029
$\mathbb{E}\kappa$	0.011	0.010	0.010	0.010	0.010	0.011
StDev(κ)	0.001	0	0	0	0	0.002
Max κ	0.023	0.010	0.010	0.010	0.010	0.015
Fraction of Defaults Due to Runs	0.775	0	1.00	0	0.003	0.003

† =Annualized

Note: This table reports key moments from the ergodic distribution of each model. All moments are conditional on being in good credit standing for the prior 20 quarters. The first row is the average level of debt issued as a fraction of the endowment. The second row is the average market value of debt issuance, again normalized by the level endowment. The third row is the annualized frequency of default. The fourth and fifth rows are the mean and standard deviation of implied spreads, respectively. Spreads are reported in annualized rates. The sixth through eighth rows are the mean, standard deviation, and maximum of the coupon, respectively. The final row is the fraction of defaults that occur due to a self-fulfilling rollover crisis.

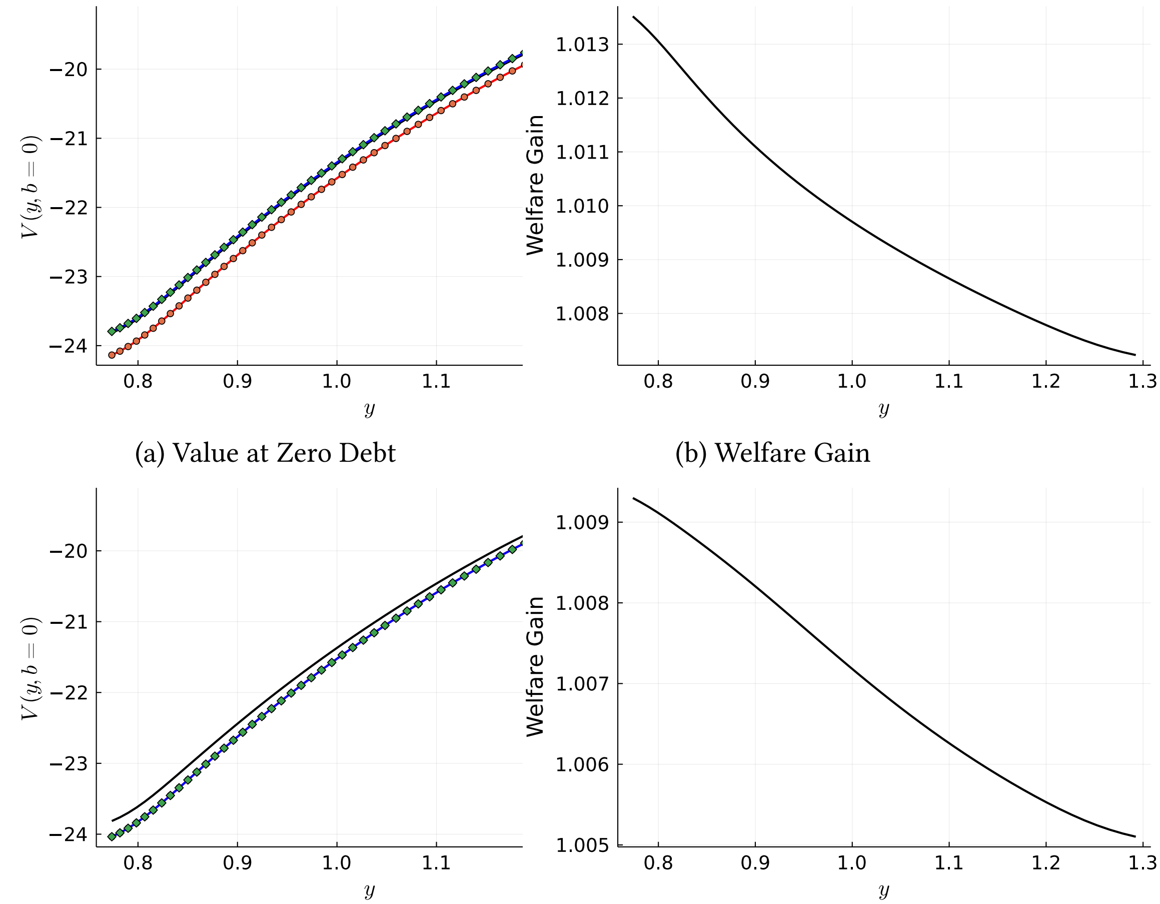
risk is essentially non-existent, but default is more frequent.

The floating rate model generates few defaults, with the floating rate coupon addressing dilution and the long-maturity essentially eliminating the adverse effects of rollover risk. The fraction of defaults due to runs decreases from 100% in CK-ST to 77.5% in the FR specification. But more importantly, the average level sustained in equilibrium is higher than in the CK-ST specification where the rollover risk played an important role. Interestingly, the volatility of the coupon is low, and the maximum coupon is only 0.02. However, in the the last column, we report the floating rate model with $\overline{\kappa} = 0.015$ and the outcome is quite different. The hard cap binds, and this opens the door to dilution risk, and the default rate increases.

Our focus is on the payoffs to the government under alternative arrangements. To evaluate this, we present the value at zero debt for alternative endowments: $V(\cdot, 0)$. In Figure 2 panel (a), we plot the value function for the two one-period models (EG-ST and EG-CK) as well as the floating-rate model. The horizontal axis traces out alternative initial endowment states.

The EG-ST and FR values are indistinguishable, while the Cole-Kehoe short-term bond model

Figure 2: Government Welfare



(a) Value at Zero Debt (b) Welfare Gain

(c) Value at Zero Debt (d) Welfare Gain

Note: Panel (a) depicts the equilibrium value function at $b = 0$ as a function of current endowment. The solid black line represents the floating-rate bond model, the blue line with diamond markers represents the short-term EG model, and the red line with circle markers represents the short-term CK model. Note that the black and blue lines are identical. Panel (b) represents the consumption-equivalent welfare gain for the government between the floating-rate model and the short-term CK model. Panel (c) repeats panel (a) but with the long-term versions of EG and CK. In this case, the EG and CK models are identical. Panel (d) repeats panel (b) comparing the floating-rate model with the long-term CK model.

has a distinctly lower value. The fact that rollover risk lowers welfare is intuitive, particularly with short maturity bonds. As anticipated by the analytical models, the floating-rate model preserves the good features of the one-period model while eliminating the negative effects of rollover risk.

In panel (b), we plot the consumption equivalent welfare gain between the CK-ST model and the FR model. For low endowment states, welfare increases by slightly more than 1% percent, while for high endowment states the gain is an order of magnitude less. Recall that welfare is evaluated at zero debt, and hence the likelihood of default (whether fundamental or self-fulfilling) lies well in the future.

In panels (c) and (d) of Figure 2 we repeat the same exercises for the long-term bond models. In panel (c), the EG-LT and CK-LT models generate the same value for the government. The reason is that the long-term bonds eliminate the vulnerability to rollover risk. However, the FR model dominates in welfare. This is because the long-term fixed-rate models suffer from dilution risk, something not present with a floating-rate coupon. Panel (d) presents the consumption equivalent welfare gain between FR and CK-LT (=EG-LT). We see that at low endowment states, the welfare gain is roughly one percent.

Another approach to evaluating the efficiency of alternative debt instruments is to trace out the frontier between lenders payoffs and the government's value at different levels of debt. Specifically, consider a state (y_{-1}, y, b). The government's value is $V(y_{-1}, y, b)$, where y_{-1} is an irrelevant state in the fixed-rate environments. The lenders' market value at the start of the period is:

$$MV(y_{-1}, y, b) \equiv (1 - \mathcal{D}(y_{-1}, y, b)) \times b \times [\kappa + \lambda + (1 - \lambda) q(y, \mathcal{B}(y_{-1}, y, b))] \,.$$

The value is zero if the government defaults ($\mathcal{D}(\cdot) = 1$). Otherwise, lenders receive the coupon and principal $(\kappa + \lambda)b$, and the market value of non-maturing debt is $q(y, b')b$, where b' is the equilibrium debt issuance policy.

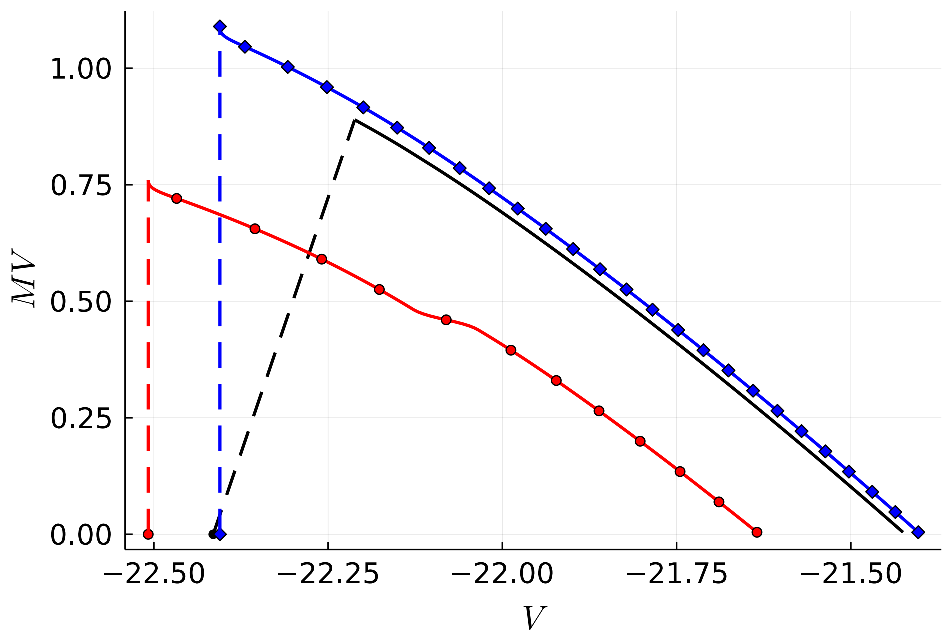
Figure 3 traces out the frontier between MV on the vertical axis and V on the horizontal axis as we vary b and hold y and y_{-1} at the mean value. Panel (a) contains the short-term fixed rate models, and panel (b) the long-maturity environments, with both panels containing the floating-rate case as well.

For each frontier, the point furthest to the left on the horizontal axis is the default value for the government. This point represents all b such that the government defaults and lenders receive zero. Note that the default value varies across environments due to the probability of re-entry. Hence, a lower re-entry value lowers the default value.

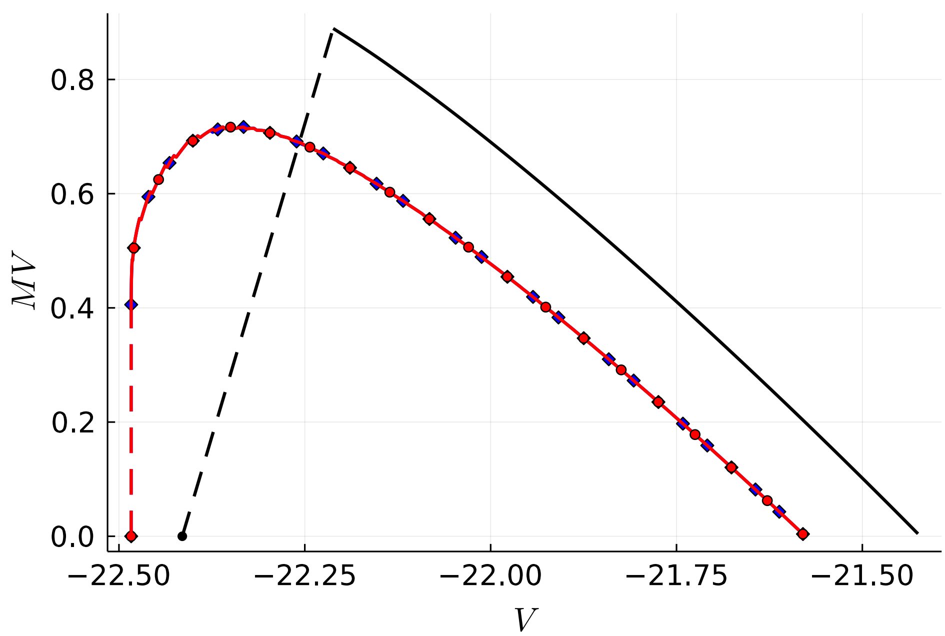
The remaining points represent positive values for the lenders. In panel (a), we see that the one-period EG model (EG-ST) slightly dominates the floating-rate model. The floating-rate frontier depends on the coupon, which in the figure depicted is evaluated at the mean endowment (that is, we assume y_{-1} equals the unconditional mean). The EG-ST frontier extends further to the left, indicating that repayment continues at higher levels of face value than in the floating-rate case. Again, this reflects that the floating-rate government must pay a larger coupon at these debt levels. But recall that in the EG models, there is no rollover risk. Once the possibility of rollover risk is introduced, the one-period fixed rate coupon bond does significantly worse: this is the CK-ST bond model, which is clearly dominated by both. Although the CK-ST model is prone to rollover risk, which depresses the frontier, the low default value (due to the low re-entry value) of CK-ST enables the government to sustain lower repayment values without defaulting, extending the frontier to the left.

Panel (b) repeats the frontier for the long-term bond model. Recall that in this case, the EG-LT model and the CK-LT model are equivalent, as maturity is such that there is no significant

Figure 3: Pareto Frontiers



(a) Short-Term Debt



(b) Long-Term Debt

Note: This depicts the frontier between lenders' value (vertical axis) and government value (horizontal axis) as b varies, evaluating y and lagged y at the mean. Panel (a) depicts the one-period bond models as well as the floating rate model. Panel (b) compares the long-term bond models with the floating-rate model. In each panel, the black line is the floating-rate model; the blue line with diamond markers is the EG model; and the red line with circle markers is the CK model. The dashed lines connect to the default value for the respective models.

rollover risk. However, there is the risk of debt dilution. For this reason, the floating-rate frontier dominates the other two. Note that the upward portions of the frontier for the fixed-rate bonds are on the "wrong" side of the debt Laffer curve. That is, debt forgiveness would increase both lender and government values. This reflects that legacy bondholders are being diluted. Such debt forgiveness is ruled out a priori because it cannot be implemented via voluntary market transactions due to the hold-out problem. Hatchondo, Martinez and Padilla (2014) provide an analysis of negotiated restructurings to alleviate this inefficiency.

6 Conclusion

In this paper we presented analytical and quantitative arguments in favor of long-term bonds with floating-rate coupons. We showed that such bonds combine the incentive properties of one-period bonds with the protection from rollover risk of a long-term bond. In the presence of rollover and dilution risk, such bonds provide government welfare that dominates both short-term and long-term bonds.

As noted in the introduction, while the analysis includes standard features in the literature, it omits some real world complications. Perhaps primary among these omissions are shocks to the global required rate-of-return.

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