Online Appendix to:
"GROWTH IN THE SHADOW OF EXPROPRIATION."

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This document contains the online appendices for "Growth in the Shadow of Expropriation." Section I contains the proofs of all propositions. Section II extends the benchmark model to incorporate exogenous productivity growth. Section III considers the case in which the capitalists are also political insiders and therefore enter the incumbent's utility function. Section IV considers the near sufficiency of the ratio $(1-\delta)/\theta$ to characterize the speed of convergence in the concave utility case.

I Proofs

This appendix presents proofs of statements made in the body of the paper. We begin with Proposition 1, postponing proof of lemma 1 until after the proof of lemma 2. The proof of proposition 3 is contained in the final subsection of the appendix, in which we discuss the dynamics with concave utility more generally.

For convenience, we restate the problem (P):

$$V(b_0) = \max_{\{c_t,k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{AP}$$

subject to:

$$b_0 \le \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - (r+d)k_t - c_t \right), \tag{A12}$$

$$\underline{W}(k_t) \le \sum_{s=t}^{\infty} \beta^{s-t}\delta^{s-t}\theta u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t}(1-\delta^{s-t})u(c_s),\ \forall t \tag{A13}$$

$$\underline{k} \le k_t,\ \forall t \tag{A14}$$

Let μ_0 be the multiplier on the budget constraint (A12), $\lambda_t(R^{-t}\mu_0/\theta)$ be the multiplier on the sequence of constraints on participation (A13) and $\phi_t R^{-t}$ be the multiplier on (A14).

For c_t, k_t to be an optimal allocation, there exist non-negative multipliers such that: (i) the following first order conditions hold:

$$\frac{1}{u'(c_t)} = \frac{(\beta R)^t}{\mu_0} + \sum_{s=0}^{t} (\beta R)^s \left(\delta^s(\theta-1)+1\right) \frac{\lambda_{t-s}}{\theta} \tag{A15}$$

$$\frac{\lambda_t}{\theta} \underline{W}'(k_t) = f'(k_t) - (r+d) + \phi_t, \ \forall t \geq 0; \tag{A16}$$

(ii) the constraints (A12)-(A14) hold; and (iii) the associated complementary slackness conditions hold. Given the convexity of the problem (Assumption 2), these conditions are also sufficient.

Proof of Proposition 1

To prove the proposition, suppose that k_t does not converge to k^*. Define $T_\epsilon = \{t | k_t < k^* - \epsilon\}$. It follows that for some $\epsilon > 0$, T_ϵ has infinite members. Then from (A15):

$$\frac{1}{u'(c_t)} = \frac{1}{\mu_0} + \sum_{s=0}^{t} \left(\delta^{t-s}(\theta-1)+1\right) \frac{\lambda_s}{\theta} \geq \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \leq t} \frac{\lambda_s}{\theta} \geq \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \leq t} C_\epsilon$$

where $C_\epsilon \equiv (f'(k^\star - \epsilon) - (r+d))/(\underline{W}'(k^\star - \epsilon)/\theta) > 0$, and the inequalities reflect $\lambda_s, \phi_s \geq 0$ for all s and $\lambda_s \geq C_\epsilon$ for $s \in T_\epsilon$. It follows then that $1/u'(c_t)$ diverges to infinity, and thus $u(c_t)$ converges to its maximum. But this implies that the participation constraints will stop binding at some finite t_0, which leads to λ_s that are zero for all $s > t_0$, a contradiction.

Proof of Proposition 2

From (17) evaluated at t, we have:

$$1 = \frac{\beta^t R^t}{\mu_0} + \sum_{s=0}^{t} \beta^s R^s \frac{\lambda_{t-s}}{\theta} + \sum_{s=0}^{t} \beta^s R^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta} \ \forall t \geq 0. \tag{A17}$$

Evaluated at $t = 0$, we have $\lambda_0 = 1 - 1/\mu_0$. At $t = 1$, we have $\lambda_1 = 1 - \beta R + \beta R(1 - \delta)(1 - 1/\theta)\lambda_0$. For $t > 1$, we let L represent the lag operator and write (A17) as

$$\begin{aligned}
1 &= \frac{\beta^t R^t}{\mu_0} + \frac{1}{\theta}\sum_{s=0}^{t} \beta^s R^s \left(1 + \delta^s(\theta - 1)\right) L^s \lambda_t \\
&= \frac{\beta^t R^t}{\mu_0} + \frac{1}{\theta}\left(\frac{1}{1 - \beta R L} + \frac{\theta - 1}{1 - \beta R \delta L}\right)\lambda_t \\
&= \frac{\beta^t R^t}{\mu_0} + \frac{1}{\theta}\left(\frac{1 - \beta R \delta L + (\theta - 1)(1 - \beta R L)}{(1 - \beta R L)(1 - \beta R \delta L)}\right)\lambda_t,
\end{aligned}$$

where the step from the first to the second line uses the fact that $\lambda_t = 0$ for $t < 0$. Multiplying through and rearranging yields equation (18) from the text:

$$\lambda_{t+1} = (1 - \beta R)(1 - \beta R \delta) + \beta R\left(1 - \frac{1 - \delta}{\theta}\right)\lambda_t \, \forall t \geq 1.$$

The steady state value can be computed in the usual way. Given that the slope of (18) is positive and less than one, convergence and monotonicity follow.

Proof of Corollary 1

Proposition 2 characterized the dynamics of λ_t. One can then use the first order condition for capital to derive the associated dynamics for k_t. For any given value of λ_t, define $K(\lambda_t)$ to be the solution to:

$$\lambda_t = \frac{f'(K(\lambda_t)) - (r + d)}{\underline{W}'(K(\lambda_t))/\theta} = \frac{f'(K(\lambda_t)) - (r + d)}{\bar{c}'(K(\lambda_t))}.$$

The convexity assumption (Assumption 2) guarantees that the above has a unique solution, and that $K(\lambda_t)$ is strictly decreasing in λ_t. Now, let $\overline{\lambda}$ be such that $K(\overline{\lambda}) = \underline{k}$. Then the optimal path for k_t will be:

$$k_t = \begin{cases} K(\lambda_t) & ; \text{for } \lambda_t < \overline{\lambda} \\ \underline{k} & ; \text{otherwise} \end{cases}$$

Given that λ_t is monotone, this implies that the path for k_t will also be monotone. Define $\overline{\theta}$ to be the value such that:

$$\overline{\lambda} = \frac{f'(\underline{k}) - (r+d)}{\overline{c}'(\underline{k})} = \frac{\overline{\theta}(1-\delta\beta R)(1-\beta R)}{\overline{\theta}(1-\beta R) + \beta R(1-\delta)}.$$

Hence, the long run level of capital will be:

$$k_\infty = \begin{cases} K(\lambda_\infty) & ;\text{for } \theta < \overline{\theta} \\ \underline{k} & ;\text{otherwise} \end{cases}$$

This proves the first part of Corollary 1. For the second part, note that higher debt implies a (weakly) higher multiplier μ_0, and a higher $\lambda_0 = 1 - 1/\mu_0$. Given that λ_1 and λ_t are monotonic in previous values, it follows that the entire path of λ_t increases with μ_0 and debt. That is, a higher level of debt leads to a lower level of capital at each point in time.

Proof of Lemma 2

Using the definitions, we have

$$\begin{aligned} V_t &= u_t + \beta V_{t+1} \\ W_t &= \theta u_t + \beta\delta W_{t+1} + \beta(1-\delta)V_{t+1}. \end{aligned}$$

Eliminating u_t from the above and re-arranging:

$$\begin{aligned} \theta\left(V_t - \beta\left(1 - \frac{1-\delta}{\theta}\right)V_{t+1}\right) &= W_t - \beta\delta W_{t+1} \\ \theta\left(1 - \beta\left(1 - \frac{1-\delta}{\theta}\right)F\right)V_t &= (1-\beta\delta F)\,W_t, \end{aligned}$$

where F is the forward operator. Solving for V_t and eliminating explosive solutions:

$$\begin{aligned}
\theta V_t &= \left(\frac{1-\beta\delta F}{1-\beta\left(1-\frac{1-\delta}{\theta}\right)F} \right) W_t \\
&= W_t + \beta(1-\delta)\left(1-\frac{1}{\theta}\right)\sum_{i=0}^{\infty}\beta^i\left(1-\frac{1-\delta}{\theta}\right)^i W_{t+1+i}.
\end{aligned}$$

Dividing through by θ yields the expression in the lemma.

Proof of Lemma 1

Define $\underline{W}(k)$ to be the incumbent's value function if it deviates given capital k. We can write this as

$$\underline{W}(k) = \theta u(\bar{c}(k)) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V}, \tag{I.1}$$

where $\underline{W}$ is the continuation value under the punishment if the incumbent retains power next period, and $\underline{V}$ is the continuation value if it loses power. We normalize $t = 0$ to be the time of the deviation, so we have $\underline{W} = W_1$ and $\underline{V} = V_1$. From Lemma 2, we have:

$$\theta\underline{V} = \theta V_1 = W_1 + \beta(1-\delta)\left(1-\frac{1}{\theta}\right)\sum_{i=0}^{\infty}\beta^i\left(1-\frac{1-\delta}{\theta}\right)^i W_{1+i}.$$

As the punishment must be self-enforcing, we have $W_t \geq \theta u(\bar{c}(k_t)) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V}$, at each t. Note that a second deviation is punished in the same way as the first. The fact that $\underline{W}(k)$ is the worst possible punishment implies that this maximizes the set of possible self-enforcing allocations, from which we are selecting the one with minimum utility. Substituting in the participation constraint in the above expression yields:

$$\begin{aligned}
\theta\underline{V} &\geq \theta u(\bar{c}(k_1)) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V} \\
&\quad + \beta(1-\delta)\left(1-\frac{1}{\theta}\right)\sum_{i=0}^{\infty}\beta^i\left(1-\frac{1-\delta}{\theta}\right)^i\left(\theta u(\bar{c}(k_{1+i})) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V}\right) \\
&\geq \left(\frac{1-\beta\delta}{1-\beta\left(1-\frac{1-\delta}{\theta}\right)} \right)\left(\theta u(\bar{c}(\underline{k})) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V}\right),
\end{aligned}$$

where the last inequality uses the fact that $k_t \geq \underline{k}$, for all t. Rearranging, we have

$$\underline{V} \geq \frac{(1-\beta\delta)\left(u(\bar{c}(\underline{k})) + \beta\delta\underline{W}\right)}{\theta(1-\beta) + \beta^2\delta(1-\delta)}. \tag{I.2}$$

Recall that $W_1 = \underline{W}$. Participation at $t = 1$ requires $W_1 \geq \theta u(\bar{c}(k_1)) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V}$, or using the fact that $k_1 \geq \underline{k}$:

$$\underline{W} \geq \theta u(\bar{c}(\underline{k})) + \beta\delta\underline{W} + \beta(1-\delta)\underline{V}.$$

Substituting (I.2) in for $\underline{V}$ and rearranging yields:

$$\underline{W} \geq \left(\frac{\theta - 1}{1-\beta\delta} + \frac{1}{1-\beta}\right) u(\bar{c}(\underline{k})).$$

Substituting back into (I.2), we have

$$\underline{V} \geq \frac{u(\bar{c}(\underline{k}))}{1-\beta}.$$

The left hand sides of these last two inequalities are the government's and private agent's utility, respectively, from the Nash equilibrium repeated ad infinitum. As repeated Nash is a self enforcing equilibrium and bounds from below the punishment payoff, it is the self-enforcing equilibrium that yields the lowest utility for the deviating government.

Proof of Proposition 4

The proof of this proposition follows directly from Lemma 2, the fact that k_t is monotone, and that $\underline{W}(k)$ is an increasing function of k.

Proof of Corollary 2

Suppose that k_t is increasing. Let $B_t = \sum_{s=t}^{\infty} R^{s-t}(f(k_s) - (r+d)k_s - c_s)$ denote the stock of debt outstanding in period t. Suppose, to generate a contradiction, that $B_{T+1} > B_T$ for some $T \geq 1$. Let $\{c_t, k_t\}$ denote the equilibrium allocation. Now consider the alternative allocation: $\tilde{c}_t = c_t$ and $\tilde{k}_t = k_t$ for $t < T$, and $\tilde{c}_t = c_{t+1}$ and $\tilde{k}_t = k_{t+1}$ for $t \geq T$. That is, starting with period T, we move up the allocation one period. As $\tilde{V}_0 - V_0 = \beta^T(\tilde{V}_T - V_T) = \beta^T(V_{T+1} - V_T) > 0$, the objective function has increased and where the last inequality follows from the monotonicity of V_t. Similarly, $\tilde{B}_0 - B_0 = R^{-T}(\tilde{B}_T - B_T) = R^{-T}(B_{T+1} - B_T) > 0$, the budget constraint is relaxed, where the last inequality follows from the premise $B_{T+1} > B_T$. For $t \geq T$, we have $\tilde{W}_t = W_{t+1} \geq \underline{W}(k_{t+1}) = \underline{W}(\tilde{k}_t)$, so participation holds for period T and after. For $t < T$, note that $W_t = \sum_{s=t}^{T-1} \beta^{s-t} \left[\delta^{s-t}(\theta - 1) + 1\right] u_s + \beta^T \delta^T W_T + \beta^T (1 - \delta^T t) V_{T+1}$. As $\tilde{W}_T > W_T$ and $\tilde{V}_T > V_T$, we have $\tilde{W}_t > W_t$ for all $t < T$. As $\tilde{k}_t = k_t$ for $t < T$, our new allocation satisfies the participation constraints of the governments along the path. Therefore, we have found a feasible allocation that is a strict improvement, a contradiction of optimality. A similar argument works for a decreasing path of k_t.

Proof of Proposition 5

By construction, the budget constraint implications of both policies are the same. Moreover, the deviation utility is unchanged for debt relief, while strictly higher for unconditional aid. Therefore, debt relief can be viewed as a relaxed version of the problem with unconditional aid, and so weakly dominates. If the participation constraint binds in the solution with debt relief, this allocation is unattainable with unconditional aid. Given the convexity of the problem, welfare will be strictly higher with debt relief in this case. The second part of the proposition follows from a simple change of variable in the original problem (P). Let $\{k_t, c_t\}_{t=0}^{\infty}$ denote the efficient allocation without aid, and $\{\tilde{k}_t, \tilde{c}_t\}_{t=0}^{\infty}$ denote the efficient allocation given a sequence of unconditional aid payments $\{y_t\}$. Note that the resource constraint in the presence of aid is $b_0 \leq \sum_t R^{-t} \left(f(\tilde{k}_t) - (r + d)\tilde{k}_t + y_t - \tilde{c}_t\right)$. Define $\hat{c}_t = \tilde{c}_t - y_t$, so that the resource constraint can now be written $b_0 \leq$
$\sum_t R^{-t} \left(f(\tilde{k}_t) - (r + d)\tilde{k}_t - \hat{c}_t\right)$, which is observationally equivalent to the non-aid problem. As the participation constraint is linear in $\tilde{c}_t$ and unconditional aid, we can subtract the discounted stream of y_t from both sides replace $\tilde{c}_t - y_t$ with $\hat{c}_t$, thereby eliminating y_t from the participation constraints. The objective function is also linear in $\tilde{c}_t$, so we can replace $\sum \beta^t \tilde{c}_t$ with $\sum \beta^t \hat{c}_t$ without changing the solution to the problem. With this change of variable, the problem with aid can be stated in terms of $\hat{c}_t$, without the presence of y_t. Therefore, the solution $\{\hat{c}_t, \tilde{k}_t\}_{t=0}^{\infty}$ will coincide with the non-aid allocation $\{c_t, k_t\}$. That is $\tilde{k}_t = k_t$ and $\hat{c}_t = c_t$ for all t. From the definition of $\hat{c}_t$, we therefore have $\tilde{c}_t = c_t + y_t$, as stated in the proposition.

Analysis of Nonlinear Dynamics and Proof of Proposition 3

We now characterize dynamics when utility is concave. For the case of $\delta = 0$, we can depict the dynamics in a two dimensional phase diagram. We then discuss a linearized system for the case of general $\delta < 1$. For concave utility, we impose the Inada conditions: $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

The non-linear system for general δ is:

$$
\begin{aligned}
W_t &= \theta u_t + \beta\delta W_{t+1} + \beta(1-\delta)V_{t+1} \\
V_t &= u_t + \beta V_{t+1} \\
c'(u_t) &= \frac{(\beta R)^t}{\mu_0} + \sum_{s=0}^{t} (\beta R)^s \frac{\lambda_{t-s}}{\theta} + \sum_{s=0}^{t} (\beta R)^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta} \\
\lambda_t &= \frac{f'(k_t) - (r+d)}{\underline{W}'(k_t)/\theta} = H(k_t),
\end{aligned}
$$

where $c(u)$ is the inverse utility function. For convenience, we assume that the constraint $k_t \geq \underline{k}$ is not binding, a point we discuss below. A sequence $\{u_t, k_t, V_t, W_t, \lambda_t\}$ plus a multiplier μ_0 that satisfies this system, the constraints (A12)–(A13) with the right hand side of (A13) replaced by W_t, complementary slackness, plus the boundary conditions $\lim_{s\to\infty} \beta^s V_{t+s} = 0$ and $\lim_{s\to\infty} \beta^s W_{t+s} = 0$, will be an optimal allocation. To see this, note that the latter two equations are the same as (A15) and (A16). The solutions to the first two difference equations that satisfy the boundary conditions yield the correct value function for incumbent utility, so constraint (A13) is equivalent to $\underline{W}(k_t) \leq W_t$.

It will be convenient to introduce the following notation:

$$
\begin{aligned}
\Lambda_t &\equiv \frac{(\beta R)^t}{\mu_0} + \beta R \sum_{s=0}^{t-1} (\beta R)^s \frac{\lambda_{t-1-s}}{\theta} \\
\Phi_t &\equiv \beta R \delta \sum_{s=0}^{t-1} (\beta R)^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta}.
\end{aligned}
$$

With this, we can write the first order condition for consumption as

$$
c'(u_t) = \Lambda_t + \Phi_t + \lambda_t.
$$

We have $\Lambda_0 = 1/\mu_0$ and$\Phi_0 = 0$, with

$$\Lambda_{t+1} = \beta R \left(\Lambda_t + \frac{\lambda_t}{\theta} \right) \tag{I.3}$$

$$\Phi_{t+1} = \beta R \delta \left(\Phi_t + (\theta - 1) \frac{\lambda_t}{\theta} \right). \tag{I.4}$$

Letting a prime denote next period's value, we can write the dynamic system the characterizes the equilibrium allocation as the set of first order difference equations:

$$\begin{aligned} -W' &= \frac{\theta - 1 + \delta}{\beta\delta} u + \frac{1-\delta}{\beta\delta} V - \frac{1}{\beta\delta} W \\ V' &= \frac{1}{\beta} V - \frac{1}{\beta} u \\ \Lambda' &= \beta R \left(\Lambda + \frac{\lambda}{\theta} \right) \\ \Phi' &= \beta R \delta \left(\Phi + (\theta - 1) \frac{\lambda}{\theta} \right); \end{aligned} \tag{D}$$

the first order conditions::

$$\begin{aligned} c'(u) &= \Lambda + \Phi + \lambda \\ \lambda &= H(k); \end{aligned}$$

the complementary slackness condition for $\lambda \geq 0$:[1]

$$\lambda(W - \underline{W}(k)) = 0;$$

and the boundary conditions

$$\begin{aligned} \lim_{s \to \infty} \beta^s V_{t+s} &= 0 \\ \lim_{s \to \infty} \beta^s W_{t+s} &= 0. \end{aligned}$$

For convenience, we invert the first order condition $c'(u) = \Lambda + \Phi + \lambda$ and introduce the

[1]We omit the slackness condition on the budget constraint, as this constraint will always hold with equality at an optimal allocation.

function $U : \mathbb{R}_+ \to [\underline{u}, \bar{u}]$:

$$u = U(\Lambda + \Phi + \lambda) = c'^{-1}(\Lambda + \Phi + \lambda). \tag{I.5}$$

We can state the following, which is part (i) of Proposition 3:

Lemma 1. *When $\beta R < 1$, there is a unique steady state for the system (D) for which $k_\infty < k^\star$. When $\beta R = 1$, there is a continuum of possible steady states which all have $k_\infty = k^\star$.*

Proof. Equations (D) imply:

$$\begin{aligned}
\lambda_\infty &= \frac{(1-\beta R)\theta}{\beta R}\Lambda_\infty \\
\Phi_\infty &= \frac{\beta R\delta(\theta-1)}{(1-\beta R\delta)\theta}\lambda_\infty \\
W_\infty &= \left(\frac{\theta-1}{1-\beta\delta} + \frac{1}{1-\beta}\right)u_\infty \\
V_\infty &= \frac{u_\infty}{1-\beta}.
\end{aligned}$$

Case (i) $\beta R < 1$: Suppose, to generate a contradiction, that $\lambda_\infty = 0$, so $W_\infty \geq \underline{W}(k^\star)$. From (AI.3)-(AI.4), we have $\Lambda, \Phi \to 0$. The first order condition for u implies that $c'(u_\infty) = 1/u'(c_\infty) = 0$, or that $c_\infty = 0$, which contradicts $W_\infty \geq \underline{W}(k^\star)$. This establishes that $\lambda_\infty > 0$. From the slackness condition, we then have $W_\infty = \underline{W}(k_\infty)$. Using the other identities to substitute, we can write this as:

$$U\left(\left(\frac{\beta R}{(1-\beta R)\theta} + \frac{\beta R\delta(\theta-1)}{(1-\beta R\delta)\theta} + 1\right)H(k_\infty)\right) = \\ \left(\frac{\theta-1}{1-\beta\delta} + \frac{1}{1-\beta}\right)^{-1}\underline{W}(k_\infty).$$

The left hand side is strictly decreasing in k_∞ and the right hand side is strictly increasing in k_∞. When $\underline{k} = 0$, the right hand side is zero at $k_\infty = 0$, while the left hand side is strictly positive at zero as $H(0) > 0$, so a unique steady state k exists for $\underline{k}$ sufficiently small. If $\underline{k}$ is such that the left hand side is less than the right hand side at $k_\infty = \underline{k}$, then $k_\infty = \underline{k}$ is the steady state. The remaining variables can be uniquely derived from k_∞.

Case (ii) $\beta R = 1$: From the above steady state relationships, we have $\lambda = 0$, so $k_\infty = k^\star$. Note that any Λ_∞ such that $u_\infty = U(\Lambda_\infty)$ is large enough to sustain $k^\star$ can be a steady state. In particular, if μ_0 is such that $k_0 = k^\star$, $\Lambda_t = \Lambda_0 = 1/\mu_0$ for all t and the system stays there indefinitely. That is, let $\bar{\mu}_0$ be such that $\left(\frac{\theta-1}{1-\beta\delta}+\frac{1}{1-\beta}\right)U\left(\frac{1}{\bar{\mu}_0}\right)=\underline{W}(k^\star)$. Then any $\mu_0 \leq \bar{\mu}_0$ is a steady state. $\square$

We now characterize the dynamics away from the steady state for the case of $\delta = 0$. From (AI.4), we have $\Phi_t = \Phi_0 = 0$ when $\delta = 0$. This allows us to reduce the dimensionality of the system. In particular, we will reduce the system to two variables: Λ and V. The variable Λ_t is the discounted sum of multipliers through $t-1$ plus the multiplier on the initial budget constraint. This term reflects the extent that consumption in period t relaxes the participation constraints for prior incumbents. In this manner, it represents promises made to prior incumbents and will serve as our state variable. The variable V_t is the present discounted value of private agent utility going forward from t.

From the definitions of W and V, we have $W_t = (\theta-1)u_t + V_t$ when $\delta = 0$. We can use this plus the complementary slackness condition to replace λ with a function $L(\Lambda, V)$. To this end, note that $W = (\theta-1)U(\Lambda+\lambda)+V$. Define $L(\Lambda, V) = 0$ if $(\theta-1)U(\Lambda)+V > \underline{W}(k^\star)$. When $W \leq \underline{W}(k^\star)$, we have $W = \underline{W}(k)$. In this case, we can define $L(\Lambda, V)$ as the λ that solves:

$$(\theta-1)U(\Lambda+\lambda)+V = \underline{W}(H^{-1}(\lambda)),$$

where we have inverted $\lambda = H(k)$ to map k into λ, which is possible as $H(k)$ is strictly decreasing by Assumption 2. To see that there is a unique solution to this equation, the left hand side is strictly increasing in λ while the right hand side is strictly decreasing. As we are considering the case when $(\theta-1)U + V \leq \underline{W}(k^\star)$, by assumption the left hand side is less than or equal to the right hand side at $\lambda = 0$. Note that L is continuous in both arguments. Moreover, straightforward manipulations show that L is non-increasing in both arguments, and strictly decreasing when $(\theta-1)U(\Lambda)+V \leq \underline{W}(k^\star)$, but that $\Lambda + L(\Lambda, V)$ is strictly increasing in Λ.

With this function in hand, our dynamic system can be written:

$$\begin{aligned} V' &= -\frac{U(\Lambda+L(\Lambda,V))}{\beta}+\frac{V}{\beta} \\ \Lambda' &= \beta R\left(\Lambda+L(\Lambda,V)\right). \end{aligned} \tag{D'}$$

For both equations, the right hand side is strictly increasing in Λ and V, so V' and Λ' are uniquely defined given (Λ, V). We depict the dynamics using a phase diagram in figure A.I. Panel (a) is the case $\beta R = 1$ and panel (b) treats $\beta R < 1$. The gray shaded area area corresponds to points such that $(\theta-1)U(\Lambda)+V \geq \underline{W}(k^\star)$. This area has a downward sloping border in the $V \times \Lambda$ plane as L is strictly increasing in both arguments when $(\theta - 1)U(\Lambda) + V = \underline{W}(k^\star)$.

The two bold lines correspond to points for which $V' = V$ and $\Lambda' = \Lambda$, respectively. From the first equation of D', we see that $V' = V$ if $V = U(\Lambda + L(\Lambda, V))/(1-\beta)$. In the region where $L(\Lambda, V) = 0$, this equation is satisfied along a upward sloping locus in the in the $V \times \Lambda$ plane, as U is a strictly increasing function. Outside this region, use can appeal to the fact that $\Lambda + L(\Lambda, V)$ is increasing in Λ to show that the locus is upward sloping in the unshaded region as well. From (D'), we see that for a given V, as we increase Λ to the right of this line, then $V' > V$, and vice versa for Λ to the left of this line. These dynamics are represented by the vertical arrows in the phase diagram.

In panel (a), $\Lambda' = \Lambda$ when $\lambda = 0$. Therefore, $\Lambda' = \Lambda$ at all points in the gray shaded region. In panel (b), when $\beta R < 1$, $\Lambda' = \Lambda$ along the downward sloping line. This locus is defined by $L(\Lambda, V) = (1 - \beta R)\Lambda$. This coincides with the gray region at $\Lambda = 0$, and is strictly below it for $\Lambda > 0$. In this region, L is decreasing in both arguments, so the locus is downward sloping in the $V \times \Lambda$ plane. As we increase V for a given Λ starting from a point on this locus, $\Lambda' < \Lambda$ outside the gray region and constant otherwise. The reverse is true below the locus. These dynamics are represented by the horizontal arrows in the unshaded region of the phase diagram.

The steady state is represented by the intersection of the two loci, which exists by lemma 1. The dynamics imply saddle path stability. As $\lim_{s\to\infty} \beta^s V_{t+s} = 0$ is a condition of optimality, the equilibrium allocation follows the saddle path. In panel (a), when $\beta R = 1$, there exists a continuum of steady states to the right of Λ_∞ corresponding to cases in which the system begins with low enough debt that we immediately have $\lambda_0 = 0$ and there are no further dynamics. In this case $k_t = k^\star$ and c_t is constant for all t. When we begin with enough debt that $\Lambda_0 < \Lambda_\infty$, the system converges along a saddle path to the Λ_∞. During this transition, the investment wedge ($\lambda = L(\Lambda, V)$) monotonically declines to zero. This is the case depicted by Λ_0 in the figure.

When $\beta R < 1$ (panel (b)), there exists a unique steady state at which capital is distorted away from $k^\star$, and for any initial condition the dynamics are monotonic towards this steady state. If b_0 is such that $\Lambda_0 < \Lambda_\infty$ (the case depicted by Λ_0 in the figure), V and Λ increase over time, and so λ declines over time and k increases; while if initial debt is sufficiently low ($\Lambda_0 > \Lambda_\infty$), then k (weakly) decreases over time.

Note that in both panels, convergence is monotonoic toward the steady state, which implies that capital also converges monotonically. We therefore can appeal to proposition

4 and corollary 2 for the transition dynamics of private agents' utility and external debt. Moreover, as $\Lambda_0 = 1/\mu_0$, the greater is μ_0 the lower the initial Λ_0 and V. This implies initial capital is decreasing in μ_0, and strictly decreasing if we begin with enough debt that $\lambda_0 = L(\Lambda_0, V_0) > 0$. As μ_0 is the multiplier on the resource constraint, this implies that initial capital is weakly decreasing in initial debt. We have now established part (ii) of Proposition 3. We turn next to part (iii), the speed of convergence.

Speed of Convergence

We explore the speed of convergence by studying the first order dynamics in the neighborhood of the steady state. To do so, we linearize the dynamic system and solve for the speed of convergence along the saddle path. Note that in the case of $\beta R = 1$, this means we are using the dynamics from "below" (the unshaded region in Figure A.I). We consider general $\delta \in [-1/N, 1]$ and linearize the system (D).[2] Letting $\hat{x} = x - x_\infty$, we have the four equation linearized system:

$$
\begin{aligned}
\hat{k}' &= \frac{1}{\beta\delta}\left[-\frac{\theta-1+\delta}{\underline{W}'(k_\infty)}\frac{H'(k_\infty)}{c''(u_\infty)} + 1\right]\hat{k} - \frac{\theta-1+\delta}{\beta\delta\underline{W}'(k_\infty)}\frac{1}{c''(u_\infty)}\hat{\Lambda} - \frac{\theta-1+\delta}{\beta\delta\underline{W}'(k_\infty)}\frac{1}{c''(u_\infty)}\hat{\Phi} - \frac{1-\delta}{\beta\delta\underline{W}'(k_\infty)}\hat{V} \\
\hat{V}' &= \frac{1}{\beta}\hat{V} - \frac{1}{\beta}\frac{H'(k_\infty)}{c''(u_\infty)}\hat{k} - \frac{1}{\beta}\frac{1}{c''(u_\infty)}\hat{\Lambda} - \frac{1}{\beta}\frac{1}{c''(u_\infty)}\hat{\Phi} \\
\hat{\Phi}' &= \beta R\delta\frac{\theta-1}{\theta}H'(k_\infty)\hat{k} + \beta R\delta\hat{\Phi} \\
\hat{\Lambda}' &= \beta R\frac{1}{\theta}H'(k_\infty)\hat{k} + \beta R\hat{\Lambda}
\end{aligned}
$$

Let $\kappa = \frac{\underline{W}'(k_\infty)c''(u_\infty)}{H'(k_\infty)}$, which captures the nonlinearity of utility if $c'' > 0$. Note that $\kappa \leq 0$ given that $H'(k) \leq 0$. We renormalize Λ and Φ to be $\Lambda/H'(k^\infty)$ and $\Phi/H'(k^{ss})$, and write the linear system in matrix form:

$$
\begin{bmatrix} \hat{k}' \\ \hat{V}' \\ \hat{\Phi}' \\ \hat{\Lambda}' \end{bmatrix} = \begin{bmatrix} -\frac{1}{\beta\delta}\left[(\theta-1+\delta)\frac{1}{\kappa} - 1\right] & -\frac{1-\delta}{\beta\delta\underline{W}'(k^{ss})} & -\frac{\theta-1+\delta}{\beta\delta\kappa} & -\frac{\theta-1+\delta}{\beta\delta\kappa} \\ -\frac{\underline{W}'(k^{ss})}{\beta\kappa} & \frac{1}{\beta} & -\frac{\underline{W}'(k^{ss})}{\beta\kappa} & -\frac{\underline{W}'(k^{ss})}{\beta\kappa} \\ \beta R\delta\frac{\theta-1}{\theta} & 0 & \beta R\delta & 0 \\ \beta R\frac{1}{\theta} & 0 & 0 & \beta R \end{bmatrix} \times \begin{bmatrix} \hat{k} \\ \hat{V} \\ \hat{\Phi} \\ \hat{\Lambda} \end{bmatrix}
$$

[2] By considering dynamics around the steady state, we are assuming that the convergence results derived for $\delta = 0$ extend to arbitrary δ.

The characteristic equation is:

$$Ch(x) \equiv x(x\theta - \beta R(\theta - 1 + \delta))(-\theta + x\beta(\theta - 1 + \delta)) \\ + (x - \beta R)(x\beta - 1)(x - \beta R\delta)(x\beta\delta - 1)\theta\kappa = 0$$

The linear case as a limiting case

Note that when $\kappa \to 0$ (so that the utility becomes linear), the roots of the characteristic equation are:

$$x \in \left\{0, \frac{\theta}{\beta(\theta - 1 + \delta)}, \beta R\left(1 - \frac{1 - \delta}{\theta}\right)\right\}$$

where the last one is the one we found in the linear case and corresponds to the highest eigenvalue less than one.

Proof of Proposition 3 Part (iii)

To prove part (iii) of Proposition 3, we need to show that for all $\kappa < 0$ there is always a root of the characteristic equation that is less than one but higher than $\beta R\left(1 - \frac{1-\delta}{\theta}\right)$. Towards this goal, note that for $\kappa < 0, \theta > 1$, and $\delta < 1$, we have: $Ch(0) < 0$; $Ch\left(\beta R\left(1 - \frac{1-\delta}{\theta}\right)\right) > 0$; $Ch(\beta R) < 0$; $Ch(1) < 0$; $Ch\left(\frac{\theta}{\beta(\theta-1+\delta)}\right) > 0$; and $\lim_{x\to\infty} Ch(x) = -\infty$. Therefore, by continuity of the polynomial, we have two roots less than one, one between 0 and $\beta R\left(1 - \frac{1-\delta}{\theta}\right)$, and the other between $\beta R\left(1 - \frac{1-\delta}{\theta}\right)$ and βR. There are also two roots greater than one, the first between 1 and $\frac{\theta}{\beta(\theta-1+\delta)}$ and the other greater than $\frac{\theta}{\beta(\theta-1+\delta)}$. Note that the largest root less than one is between $\beta R\left(1 - \frac{1-\delta}{\theta}\right)$ and βR. Thus the system is saddle path stable in the neighborhood of the steady state, with the the speed of convergence of the system bounded above by $-\log\left(\beta R\left(1 - \frac{1-\delta}{\theta}\right)\right)$.

II Exogenous Growth

In this appendix we extend the model to include exogenous growth and show that the benchmark results are unaffected up to a re-normalization.

Suppose that $y_t = f(k_t, (1+g)^t l_t)$, where g is the rate of exogenous labor-augmenting technical progress. Constant returns to scale in production implies that $y_t = (1 +$

$g)^t f((1+g)^{-t}k_t, l_t)$ or $(1+g)^t f(\hat{k}_t, l_t)$, where $\hat{x}_t \equiv \frac{x_t}{(1+g)^t}$, for $x = k, c$. The firm's first order condition can be written:

$$\begin{aligned} f_k(k_t, (1+g)^t l_t) &= r + d \\ f_k(\hat{k}_t, l_t) &= r + d, \end{aligned}$$

as f_k is homogeneous of degree zero in k and l. We also have $\hat{\underline{k}}_t = (1+g)^{-t}\underline{k}_t$, so that $(1-\overline{\tau})f_k(\hat{\underline{k}}_t, l_t) = (1-\overline{\tau})f_k(\underline{k}_t, (1+g)^t l_t) = r + d$. The budget constraint can be re-written:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t}(1+g)^t \left[f(\hat{k}_t, l_t) - (r+d)\hat{k}_t - \hat{c}_t \right],$$

where we need $r > g$ to ensure finiteness of the budget set.

Let us assume that $u(c)$ is homogeneous of degree $1-\sigma$, then the objective function can be written:

$$\sum_{t=0}^{\infty} \beta^t u(c_t) = \sum_{t=0}^{\infty} \beta^t (1+g)^{(1-\sigma)t} u(\hat{c}_t),$$

where we need $\beta(1+g)^{1-\sigma} < 1$. Turning to the deviation utility:

$$\begin{aligned} \overline{c}(k_t) &= f(k_t, (1+g)^t l_t) - (1-\overline{\tau})f_k(k_t, (1+g)^t l_t)k_t \\ &= (1+g)^t \left[f(\hat{k}_t, l_t) - (1-\overline{\tau})f_k(\hat{k}_t, l_t)\hat{k}_t \right]. \end{aligned}$$

and we can define $\hat{\overline{c}}(\hat{k}_t) \equiv (1+g)^{-\mathrm{t}}\overline{c}(k_t) = f(\hat{k}_t, l_t) - (1-\overline{\tau})f_k(\hat{k}_t, l_t)\hat{k}_t$. So, the deviation utility is:

$$\begin{aligned} \underline{W}(k_t) &= \theta u(\overline{c}(k_t)) + \beta \left(\frac{\delta(\theta-1)}{1-\beta\delta} + \frac{1}{1-\beta} \right) u(\overline{c}(\underline{k})) \\ &= (1+g)^{(1-\sigma)t} \left[\theta u(\hat{\overline{c}}(\hat{k}_t)) + \beta \left(\frac{\delta(\theta-1)}{1-\beta\delta} + \frac{1}{1-\beta} \right) u(\hat{\overline{c}}(\hat{\underline{k}})) \right]. \end{aligned}$$

Define $\underline{\hat{W}}(\hat{k}_t) = (1+g)^{(\sigma-1)t}\underline{W}(k_t)$, we have

$$\underline{\hat{W}}(\hat{k}_t) = \theta u(\hat{\overline{c}}(\hat{k}_t)) + \beta \left(\frac{\delta(\theta-1)}{1-\beta\delta} + \frac{1}{1-\beta} \right) u(\hat{\overline{c}}(\hat{\underline{k}})).$$

The planning problem can be written:

$$\max \sum_{t=0}^{\infty} \beta^t (1+g)^{(1-\sigma)t} u(\hat{c}_t)$$

subject to:

$$\begin{aligned}
b_0 &\leq \sum_{t=0}^{\infty} R^{-t}(1+g)^t \left[f(\hat{k}_t, l_t) - (r+d)\hat{k}_t - \hat{c}_t \right] \\
\underline{\hat{W}}(\hat{k}_t) &\leq \theta u(\hat{c}_t) + \beta(1+g)^{(1-\sigma)} \sum_{s=t+1}^{\infty} \beta^{s-t-1} \left(\theta\delta^{s-t} + 1 - \delta^{s-t} \right) (1+g)^{(1-\sigma)(s-t-1)} u(\hat{c}_s) \\
\underline{\hat{k}}_t &\leq \hat{k}_t.
\end{aligned}$$

Now define $\hat{R} \equiv \frac{1+r}{1+g}$ and $\hat{\beta} \equiv \beta(1+g)^{(1-œ)}$. Then, the planner's problem above can be re-written:

$$\max \sum_{t=0}^{\infty} \hat{\beta}^t u(\hat{c}_t)$$

subject to:

$$\begin{aligned}
b_0 &\leq \sum_{t=0}^{\infty} \hat{R}^{-t} \left[f(\hat{k}_t, l_t) - (r+d)\hat{k}_t - \hat{c}_t \right] \\
\underline{\hat{W}}(\hat{k}_t) &\leq \theta u(\hat{c}_t) + \hat{\beta} \sum_{s=t+1}^{\infty} \hat{\beta}^{s-t-1} \left(\theta\delta^{s-t} + 1 - \delta^{s-t} \right) u(\hat{c}_s) \\
\underline{\hat{k}}_t &\leq \hat{k}_t.
\end{aligned}$$

Note that this problem is isomorphic to the original problem, (P) from the main text.

This discussion is important, not only to show that the results are robust to sustained technological improvements (a fact of the data), but also it highlights the following: a steady state in our model, once augmented with exogenous growth, will be a balanced growth path that features constant debt to output ratios and an output level that will be growing at the rate of g. In this environment, a long the transition to the steady state a country that grows at a slower rate than g will accumulate liabilities as fraction of its output, and the opposite will hold for a country that grows faster than g. If we take g, to a first approximation, to be equal to the growth rate of the U.S., then one should expect that countries that grew faster (slower) than the U.S. should have increased (decreased) external assets relative to GDP. This is exactly what Figure I in the main text shows.

III Capitalist Insiders

In this section of the appendix we extend the benchmark model to include domestic capitalists that enter the welfare functions of both the private agents setting initial policy and the subsequent governments. Recall that a key distinguishing feature of a capitalist in our environment is the ability to manage firms, a feature which prevented the government from converting savings into productive capital itself. Specifically, suppose that a subset of the domestic population has entrepreneurial ability which enables them to operate the production technology. We assume that all firms are managed by domestic entrepreneurs, but continue to assume the economy is open in that firm financing may originate abroad.

More concretely, consider an entrepreneur who manages a firm with capital stock k. This capital stock is financed through a combination of equity and debt financing, where the entrepreneur may own some of the equity. An entrepreneur hires workers and pays holders of debt and equity using after tax profits. We extend the limited commitment paradigm to encompass domestic entrepreneurs. That is, an entrepreneur can renege on the firm's contracts and divert resources to his or her own private gain. Let $\underline{U}^e(k)$ denote the lifetime utility of a manager who deviates given a firm's capital stock k. We provide a specific formulation of $\underline{U}^e(k)$ below; at this point, there is no need to put additional structure on the deviation utility of the entrepreneurs. Given the lack of commitment, firm financing must be self-enforcing. If c_t^e is the entrepreneur's consumption absent deviation, then the entrepreneur faces a financing constraint of the form $\underline{U}^e(k_t) \leq \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^e)$, for every t. This constraint is the individual firm's counterpart to the government's borrowing constraint, and corresponds to the constraint studied in Alburquerque and Hopenhayn (2004). Note that $\underline{U}^e(k)$ is the utility from deviation for the entrepreneur given the equilibrium actions of all other agents, including the government.

We study the private agents' planning problem.[3] Let the private agents' welfare function be given by $\gamma u(c^w) + (1-\gamma)u(c^e)$, where c^w and c^e are the per capita consumption of workers and entrepreneurs, respectively, and $\gamma \in (0,1]$ is the Pareto weight placed on workers. For ease of exposition, we assume the government places weight γ on workers

[3]The efficient allocation from the planning problem can be decentralized with appropriate taxes and transfers. We omit the details.

as well, but this could be relaxed. The planning problem can be written as:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\gamma u(c_t^w) + (1-\gamma)u(c_t^e)\right] \tag{P'}$$

subject to

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - c_t^w - c_t^e - k_{t+1} + (1-d)k_t\right) - (1+r)k_0$$

$$\underline{W}(k_t) \leq \theta \left[\gamma u(c_t^w) + (1-\gamma)u(c_t^e)\right] + \sum_{s=1}^{\infty} \beta^s \left(\theta\delta^s + 1 - \delta^s\right) \left[\left(\gamma u(c_{t+s}^w) + (1-\gamma)u(c_{t+s}^e)\right)\right], \ \forall t$$

$$\underline{U}^e(k_t) \leq \sum \beta^s u(c_{t+s}^e), \ \forall t.$$

The aggregate resource constraint states that the present value of output minus consumption and net investment must equal initial net foreign debt. This constraint is the same as (12), although written in a slightly different way. The second constraint is the government's participation constraint, which is modified to include both types of agents. We assume that the incumbent's preference for current consumption is uniform across agents. The final constraint is the entrepreneur's participation constraint ensuring that firm financing is self enforcing. Note that even though capitalists enter the welfare function of the government there is a temptation for the current incumbent to expropriate capital when $\theta > 1$.

Before solving the planning problem, we discuss how the government's deviation utility $\underline{W}(k)$ is affected by the presence of insider capitalists. We maintain our assumption that if the government deviates, the economy reverts to the Markov Perfect Equilibrium (MPE) under financial autarky. To set notation, let k denote the current capital stock inherited by the current incumbent, and k' the capital stock bequeathed to the next government. Let $V(k')$ denote the continuation value of the current incumbent if it leaves k' to the next government. That is, $V(k_t) = \sum_s \beta^s \left(\theta\delta^s + 1 - \delta^s\right) \left[\gamma u(c_{t+s}^w) + (1-\gamma)u(c_{t+s}^e)\right]$, where the sequence of consumptions are chosen by future incumbent governments given the inherited state variable k. Similarly, let $U^e(k')$ denote the continuation value of entrepreneurs conditional on k'. The current incumbent's problem is therefore

$$\underline{W}(k) = \max_{c^e, c^w, k'} \theta \left[\gamma u(c^w) + (1-\gamma)u(c^e)\right] + \beta V(k') \tag{III.6}$$

subject to

$$
\begin{aligned}
c^w + c^e + k' &\le f(k) + (1-d)k \\
u(c^e) + \beta U^e(k') &\ge \underline{U}^e(k).
\end{aligned}
$$

Note that we have set $\overline{\tau} = 1$, so the government has access to total output. We continue to use the notation $\underline{U}^e(k)$ to denote the entrepreneurs' deviation utility, although this is a slight abuse of notation – the value to an entrepreneur diverting with capital stock k will depend on the path of taxation, which in general will be different in the MPE. Other than this last constraint, the MPE is the closed economy neo-classical growth model with a quasi-hyperbolic decision maker discussed in section 5 of the main text.

Returning to the planning problem (P'), we let μ_0, $R^{-t}\frac{\lambda_t \mu_0}{\lambda \theta}$, and $R^{-t}\mu_0 \eta_t$ be the multipliers on the three constraints. The first order conditions are:

$$
1 = \gamma u'(c_t^w)\left(\frac{\beta^t R^t}{\mu_0} + \sum_{s=0}^{t} \beta^s R^s \frac{\lambda_{t-s}}{\gamma\theta} + \sum_{s=0}^{t} \beta^s R^s \delta^s \frac{(\theta-1)\lambda_{t-s}}{\gamma\theta}\right) \tag{III.7}
$$

$$
1 = (1-\gamma)u'(c_t^e)\left(\frac{\beta^t R^t}{\mu_0} + \sum_{s=0}^{t} \beta^s R^s \frac{\lambda_{t-s}}{\gamma\theta} + \sum_{s=0}^{t} \beta^s R^s \delta^s \frac{(\theta-1)\lambda_{t-s}}{\gamma\theta}\right. \tag{III.8}
$$

$$
\left. + \frac{1}{1-\gamma}\sum_{s=0}^{t} \beta^s R^s \lambda_{t-s}\right) \tag{III.9}
$$

$$
f'(k_t) = r + d + \frac{\lambda_t}{\gamma\theta}\underline{W}'(k_t) + \eta_t \underline{U}^{e'}(k_t). \tag{III.10}
$$

Before analyzing the problem in detail, a few points are worth mentioning. The benchmark case can be recovered by setting $\gamma = 1$ and relaxing the entrepreneurs borrowing constraint $\eta_t = 0$. Even if γ is less than one, the first order condition for workers remains essentially the same as before (compare (III.7) and (15)) – the only difference is a scaling factor. Moreover, conditions (III.7) and (III.9) can be combined to yield:

$$
\left(\frac{1-\gamma}{\gamma}\right)\frac{u'(c_t^e)}{u'(c_t^w)} + u'(c_t^e)\sum_{s=0}^{t} \beta^s R^s \eta_{t-s} = 1. \tag{III.11}
$$

This condition says that the plan allocates consumption to workers and entrepreneurs partially according to their Pareto weights, but entrepreneurs may be given additional resources when their borrowing constraint binds.

III.1 The Linear Case Revisited

We now reconsider our benchmark results with linear utility. The case of $\gamma \geq 1/2$ provides a straightforward extension of our basic model as there exists an interior optimum. If $\gamma > 1/2$, then the government strictly prefers workers to entrepreneurs as a group, and transferring resources from the entrepreneurs to the workers relaxes the government's constraint. Similarly, transferring resource from entrepreneurs to workers raises the planner's objective function. However, there is a limit on how many resources can be transferred, as the entrepreneurs always have the option to deviate. This ensures that entrepreneurial consumption is not driven to minus infinity in the linear case. We therefore assume $\gamma \geq 1/2$ in what follows. In the linear case, we also assume that $\underline{U}^e(k) = f(k) + (1-d)k$. That is, an entrepreneur that deviates simply consumes its output and un-depreciated capital. In this formulation, the entrepreneur's deviation utility is independent of government actions.

In the linear case, we can rewrite (III.7) as:

$$1 = \beta^t R^t \frac{\gamma}{\mu_0} + \sum_{s=0}^{t} \beta^s R^s \frac{\lambda_{t-s}}{\theta} + \sum_{s=0}^{t} \beta^s R^s \delta^s (\theta - 1) \frac{\lambda_{t-s}}{\theta}.$$

Note that this expression is the same as (17), except that $1/\mu_0$ has been replace by γ/μ_0. Recall that μ_0 only affects λ_0, but does not influence the dynamics in the linear case. Therefore, the only change in the path of λ_t is that the period 0 constraint is $\lambda = 1 - \gamma/\mu_0$ rather than $1 - 1/\mu_0$. Thus the dynamics of λ_t are the same as before, save for the initial term now has an explicit weight for the workers, γ.

Turning to (III.11), we have

$$\sum_{s=0}^{t} \beta^s R^s \eta_{t-s} = 1 - \frac{1-\gamma}{\gamma}.$$

This implies that $\eta_0 = 1 - \frac{1-\gamma}{\gamma}$, and $\eta_t = (1 - \beta R)\eta_0$ for $t > 0$. If $\gamma = 1/2$, then $\eta_t = 0$ for all t. This follows as workers and entrepreneurs receive equal weights and have linear utility, so the optimal plan will transfer resources from workers to entrepreneurs until the entrepreneur's constraint is slack. If $\beta R = 1$, then the entrepreneur's constraint binds only in the initial period for any $\gamma > 1/2$. With linear utility and patience, the entrepreneur is willing to delay consumption into the future (i.e., post a bond), relaxing the borrowing constraint. However, this does not imply that capital is first best – even if the borrowing constraint does not bind, the entrepreneur is subject to government taxation. In all cases, η_t is constant after the first period and does not depend on the political parameters θ and δ: the entrepreneur's lack of commitment does not generate dynamics beyond the first period. Therefore, θ and δ only influences the dynamics of the economy through λ, the multiplier on the government's participation constraint.

As in the benchmark case, the dynamics of λ_t pin down the dynamics of capital. Specifically, from the first order condition for capital we have $f'(k_t) - r - d = \frac{\lambda_t}{\gamma\theta}\underline{W}'(k_t) + \eta_t \underline{U}^{e'}(k_t)$. After manipulating the envelope and first order conditions from (III.6), we have $\underline{W}'(k) = \theta(1-\gamma)(f'(k) + 1 - d)$, where we have used the fact that $\underline{U}^{e'}(k) = f'(k) + 1 - d$ and that $\gamma \geq 1/2$ to guarantee an interior solution. Substituting into the first order condition for capital yields:

$$\lambda_t = \frac{\gamma}{1-\gamma}\left(\frac{f'(k_t) - r - d}{f'(k_t) + 1 - d}\right) - (1 - \beta R)\left(\frac{2\gamma - 1}{1-\gamma}\right) \tag{III.12}$$

for all $t \geq 1$. As in the benchmark model, λ_t is inversely related to k_t.

This appendix has shown that the results derived in Section 3.1 carry over directly to an environment in which domestic insiders manage firms.

IV Near sufficiency of $(1-\delta)/\theta$

In this section we numerically compute the speed of convergence of the linearized system for concave utility and show that the ratio $(1-\delta)/\theta$ is the major determinant.

The parameters are as follows (same as in the paper). A period is 5 years, and $u(c) = \log(c)$, $f(k) = k^{0.33}$, $\beta R = 1$, $R = 1.2$, $d = 0.2$ and $\bar{\tau} = 0.6$.

The table below shows the (5 year) speed of convergence of the saddle path in the linearized model for different values of δ and θ so that the ratio $(1-\delta)/\theta$ is constant.

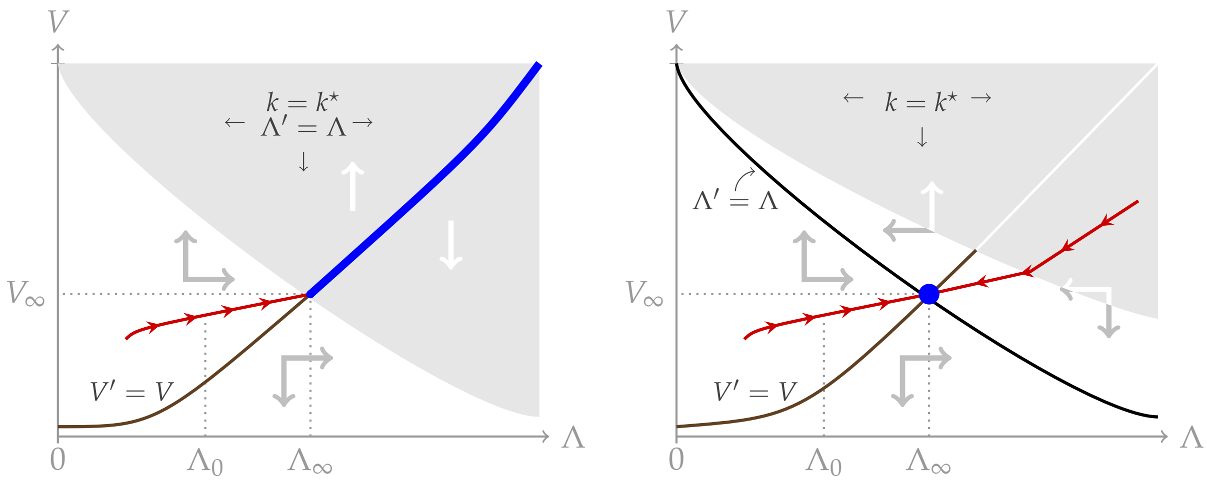
θ	δ	ratio	speed
3.00	0.00	3	0.27
2.85	0.05	3	0.28
2.70	0.10	3	0.28
2.55	0.15	3	0.29
2.40	0.20	3	0.29

θ	δ	ratio	speed
7.00	0.00	7	0.11
6.65	0.05	7	0.12
6.30	0.10	7	0.12
5.95	0.15	7	0.12
5.60	0.20	7	0.12

References

Alburquerque, Rui and Hugo Hopenhayn, "Optimal Lending Contracts and Firm Dynamics," *Review of Economic Studies*, 2004, *71* (2), 285–315.

Figures



(a) $\beta R = 1$

(b) $\beta R < 1$

Figure A.I: Phase diagram for $\delta = 0$

The shaded region represents points for which $k = k^{\star}$. The upward sloping bold line represents $V' = V$. In panel (a), the shaded region also represents points for which $\Lambda' = \Lambda$. In panel (b), $\Lambda' = \Lambda$ for points along the downward sloping bold line. The intersection of the $V' = V$ line with the shaded region represents steady states in panel (a). In panel (b), the intersection of the two lines defines the unique steady state. The (red) line marked with arrows represents the stable manifold (saddle path). The point Λ_0 depicts one possible initial value, which is determined by initial debt.